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Fluctuations in an impure unconventional superconductor

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We study the paraconductivity in an impure unconventional superconductor. We find that the Azlamazov and Larkin term in three dimensions consists of two contributions, one involving a sum of s-wave-like terms, and an extra piece involving differences of different coherence lengths. In two dimensions the result is found to be universal, being proportional to the number of components of the order parameter. The Maki-Thompson terms are found to be less divergent and hence negligible due to pair-breaking effects.

The oxide superconductor discovered two years ago[1] created a stir in the interest in superconductivity including some speculations of unconventional pairing states. The short coherence lengths in these systems also allow the study of fluctuation effects.[2-8] In Ref. 2 the fluctuation contribution to the specific heat is measured. In the O(N) model ($N=2n$), comparing this fluctuation contribution above and below T_c would give one the number of components n of the order parameter directly.[9] Unfortunately, this is not true if one uses a proper Ginzburg-Landau theory,[10] thus one can only put reasonable limits on n. Measurements on paraconductivity[3-7] and diamagnetism[8] have only been analyzed using the theory appropriate to an s-wave superconductor,[11-15] and the possible implication of non-s-wave pairing has not been investigated. In particular, one would like to obtain the physical parameters, in particular n, of these new superconductors.

In this paper we would like to correct this unsatisfactory state of affairs. In particular, we investigate the effects of *impurities* on paraconductivity in an unconventional superconductor. Since impurities are known to be pair breaking for non-s-wave pairing,[16] one expects *qualitatively* different effects of the impurities than in an s-wave superconductor, as born out in the present investigation. For definiteness we first consider a spherically symmetric d-wave superconductor, thus ignoring crystal effects. As we shall explain below, most of our results (with proper reinterpretation) have rather general validity. We assume a pairing interaction of

$$H=-g\sum_{\mathbf{k},\mathbf{k}',\mathbf{q}}P_2(\hat{\mathbf{k}}\cdot\hat{\mathbf{k}}')a^\dagger_{\mathbf{k}+\mathbf{q}/2\uparrow}a^\dagger_{-\mathbf{k}+\mathbf{q}/2\downarrow}a_{-\mathbf{k}'+\mathbf{q}/2\downarrow}a_{\mathbf{k}'+\mathbf{q}/2\uparrow}\,, \tag{1}$$

where P_2 is the Legendre polynomial. The pair propagator $K_{\mathbf{k}\mathbf{k}'}(\mathbf{q},i\Omega_\nu)$ is as shown in Fig. 1. Assuming s-wave impurity scatters, one can easily solve for the two-electron propagator $D_{\mathbf{k}\mathbf{k}'}(\epsilon_n;q,i\Omega_\nu)$ displayed as (α) in Fig. 1 in terms of the two-electron propagator without the impurity ladder, $D^0_{\mathbf{k}}$ shown as (β) in Fig. 1. Since the bare interaction (1) is independent of the magnitude of the momenta $\mathbf{k}$ and $\mathbf{k}'$, one needs only the quantity

$$Q_{\hat{\mathbf{k}}\hat{\mathbf{k}}'}(\epsilon_n;\mathbf{q},i\Omega_\nu)\equiv\sum_{|k|,|k'|}D_{\mathbf{k}\mathbf{k}'}(\epsilon_n;\mathbf{q},i\Omega_\nu)\,, \tag{2}$$

and similarly for $Q^0_{\hat{\mathbf{k}}}$, which is given by

$$Q^0_{\hat{\mathbf{k}}}(\epsilon_n;\mathbf{q},i\Omega_\nu)=2\pi iN(0)\frac{\text{sgn}(\epsilon_n)H(\epsilon_n(\epsilon_n+\Omega_\nu))}{(2i\epsilon_n+i\Omega_\nu)-v_F\hat{\mathbf{k}}\cdot\mathbf{q}+i/\tau\,\text{sgn}\epsilon_n}\,, \tag{3}$$

where $N(0)$ is the density of states for one spin, τ the relaxation time for scattering with impurities, and H is the step function. One finds

$$Q_{\hat{\mathbf{k}}\hat{\mathbf{k}}'}=Q^0_{\hat{\mathbf{k}}}4\pi\delta_{\hat{\mathbf{k}},\hat{\mathbf{k}}'}+Q^0_{\hat{\mathbf{k}}}\frac{(2\epsilon_n+\Omega_\nu+\tau^{-1}\text{sgn}\epsilon_n)H(\epsilon_n(\epsilon_n+\Omega_\nu))}{[2\pi N(0)\tau]\{2\epsilon_n+\Omega_\nu+[v_F^2q^2\,\text{sgn}\epsilon_n/(2\epsilon_n+\Omega_\nu+\tau^{-1}\text{sgn}\epsilon_n)^2]\}}Q^0_{\hat{\mathbf{k}}'}\,. \tag{4}$$

Notice that the second term is independent of the relative direction of $\hat{\mathbf{k}}$ and $\hat{\mathbf{k}}'$.

Substituting this into the series in Fig. 1(a), due to the angular dependence of the interaction, one sees that the second term in (4) only contributes terms of at least $O(q^4)$ and hence is negligible. This reflects the pair-breaking effects of the impurities, and has far reaching consequences, as we shall see below.

Since the bare interaction only acts on the $l=2$ subspace of the momenta, so does the pair propagator. More-

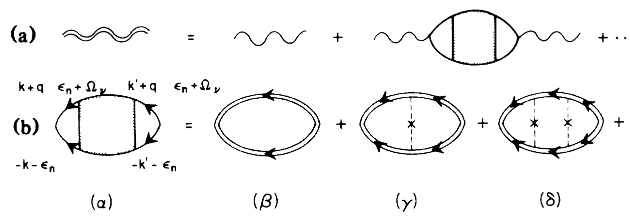

FIG. 1. (a) The pair propagator. A single wavy line is the bare interaction. (b) The two-electron propagator; double solid line represents electron Green's function dressed by impurities.

over, $K_{\mathbf{kk}'}$ is independent of $|\mathbf{k}|$ and $|\mathbf{k}'|$. It is convenient to write

$$K_{\mathbf{kk}'}(\mathbf{q},i\Omega_\nu)=4\pi\sum_{mm'}Y_2^m(\hat{\mathbf{k}})K_{mm'}(\mathbf{q},i\Omega_\nu)Y_2^{m'*}(\hat{\mathbf{k}}')\ , \quad (5)$$

and similarly for $Q_{\hat{\mathbf{k}}\hat{\mathbf{k}}'}(\epsilon_n;\mathbf{q},i\Omega_\nu)$. Defining

$$\bar{Q}_{mm'}(\mathbf{q},i\Omega_\nu)=T\sum_{\epsilon_n}Q_{mm'}(\epsilon_n;\mathbf{q},i\Omega_\nu)\ ,$$

and similarly for $\bar{Q}^0_{mm'}$, the pair propagator is obtained as (hereafter all matrices are in m space unless the momenta are explicitly indicated)

$$K(\mathbf{q},i\Omega_\nu)=[5/g-\bar{Q}(\mathbf{q},i\Omega_\nu)]^{-1}\ . \quad (6)$$

Because of the discussion below (4), we can approximate $\bar{Q}$ by $\bar{Q}^0$ in this equation, the value of which, for small $\mathbf{q}$ Ω_ν, is given by

$$\bar{Q}^0(\mathbf{q},i\Omega_\nu)=U(\hat{\mathbf{q}})\tilde{Q}(|q|,i\Omega_\nu)U^+(\hat{\mathbf{q}})\ , \quad (7)$$

where $U(\hat{\mathbf{q}})$ is the rotation matrix from $\hat{\mathbf{z}}$ to $\hat{\mathbf{q}}$ in the $l=2$ representation, $\tilde{Q}$ is a diagonal matrix depending only on the magnitude of $\mathbf{q}$:

$$\tilde{Q}_{mm'}(|q|,i\Omega_\nu)=N(0)\left[4\pi T\sum_{n\geq 0}^{\omega_D}\frac{1}{(2\epsilon_n+\tau^{-1})}-\tilde{\eta}_m q^2-r\frac{\pi|\Omega_\nu|}{8T}\right]\delta_{mm'}\ . \quad (8)$$

Here ω_D is the cutoff frequency,

$$\tilde{\eta}_m=\tilde{\eta}_d[1-\tfrac{2}{7}(m^2-2)]\ , \quad (9)$$

$$\tilde{\eta}_d=\frac{1}{6}\left[\frac{V_F}{\pi T}\right]^2\frac{7\zeta(3)}{8}\chi_d\left[\frac{1}{4\pi\tau T}\right]\ , \quad (10)$$

$$\chi_d(x)=\sum_{n=0}^{\infty}(n+\tfrac{1}{2}+x)^{-3}\Big/\sum_{n=0}^{\infty}(n+\tfrac{1}{2})^{-3}\ , \quad (11)$$

$$r=\psi'\left[\frac{1}{2}+\frac{1}{4\pi\tau T}\right]\Big/\psi'(\tfrac{1}{2})\ , \quad (12)$$

where ψ is the digamma function.

Determining the divergence of K at $\mathbf{q}=0$, $\Omega_\nu=0$ is of course just the Cooper problem with d-wave pairing in the presence of impurities. This condition determines the transition temperature T_c:

$$1=N(0)\frac{g}{5}\left[T\sum_{|\epsilon_n|<\omega_D}\frac{2\pi}{2|\epsilon_n|+\tau^{-1}}\right]_{T=T_c}\ . \quad (13)$$

Notice the presence of the τ^{-1} in the denominator, representing the suppression of T_c by impurities. Eliminating the pairing interaction in favor of T_c, one gets

$$K=U\tilde{K}U^+\ , \quad (14a)$$

where

$$\tilde{K}^{-1}_{mm'}=N(0)\left[t+\tilde{\eta}_m q^2+r\frac{\pi|\Omega_\nu|}{8T}\right]\delta_{mm'}\ , \quad (14b)$$

$t\equiv(T-T_c)/T_c$ is the reduced temperature. These results should be compared with that of Aslamazov and Larkin,[11] and Maki.[12] Here there are three distinct values of coherence lengths ($\tilde{\eta}_m^{1/2}$) (also notice the difference of χ_d and the corresponding quantity of the s wave). Moreover, there is an extra factor of r in front of the frequency term. This, as well as the τ^{-1} in (13), represents pair-breaking effects. In s-wave pairing where the impurities are not pair breaking, the second term of (4), representing "in-scattering" processes, contributes an extra term in (13) so that the total involves $|2\epsilon_n|$ instead of $|2\epsilon_n+\tau^{-1}|$ in the denominator. In our case the absence of these processes results in the suppression of T_c as well as the factor r in (14). The electrons, once they are scattered, end up on the "wrong" momentum space and do not contribute to the pairing (on the average, up to q^2).

One can easily obtain the fluctuation contribution to the specific heat in the manner of Ref. 11. In three dimensions (3D) we find

$$C_d'(3\text{D})=\left[\sum_m\left[\frac{\tilde{\eta}_d}{\tilde{\eta}_m}\right]^{3/2}\right]\left[\frac{\eta_s}{\tilde{\eta}_d}\right]^{3/2}C_s'(3\text{D})\ , \quad (15)$$

where $C_s'(3\text{D})$ is the corresponding expression for an s-wave superconductor. Our result is just what one would obtain by independently adding the contribution from each m component. Since our "mass term" is independent of m, our result agrees with that of Ref. 10. The numerical value of the sum over m is 16.65.

We shall first evaluate the Aslamazov and Larkin[11] (AL) contribution as shown in Fig. 2. As in Ref. 11, the two electron loops are just given by the $\mathbf{q}$ derivative of $D_{\mathbf{kk}'}$ of Fig. 1. The current-current response function (at zero wave vector) reads

$$R_{ij}^{(\text{AL})}(i\omega_0)=4\int\frac{d^dq}{(2\pi)^d}T\sum_{\Omega_\nu}\text{Tr}\left[\left[\frac{\partial}{\partial\mathbf{q}_i}\bar{Q}\right]K(\mathbf{q},i\Omega_\nu)\left[\frac{\partial}{\partial\mathbf{q}_j}\bar{Q}\right]K(\mathbf{q},i\Omega_\nu+i\omega_0)\right]\ , \quad (16)$$

which can be evaluated using the unitary transformations (7) and (14). For three dimensions one can average over all $i=j$ components, and use the spherical coordinates

$$\nabla_{\mathbf{q}}=\hat{\mathbf{q}}\frac{\partial}{\partial q}+\frac{\hat{\boldsymbol{\theta}}}{q}\frac{\partial}{\partial\theta}+\frac{\hat{\boldsymbol{\phi}}}{q\sin\theta}\frac{\partial}{\partial\phi}\ .$$

The q derivatives act only on $\tilde{Q}$ and $\tilde{K}$ and give the contribution

$$R_l^{(\text{AL})}(i\omega_0)=\tfrac{4}{3}\int\frac{d^3q}{(2\pi)^3}\sum_m T\sum_{\Omega_\nu}\frac{\partial\tilde{Q}_m}{\partial q}\tilde{K}_m(\mathbf{q},i\Omega_\nu)\frac{\partial\tilde{Q}_m}{\partial q}\tilde{K}_m(\mathbf{q},i\Omega_\nu+i\omega_0)\ , \quad (17)$$

and this is a sum of contributions from each component. However, the $\hat{\mathbf{q}}$ dependences of U give rise to an extra contribution. Since if all coherence lengths were equal, then U is obsolete and (17) would be the only contribution, this extra contribution will involve differences of the various $\tilde{\eta}_m$'s (raised to the appropriate powers). After the analytic continuation and momentum integrals, one obtains the (AL) paraconductivity at zero frequency (in units of $e^2/\hbar$)

$$\sigma'^{(\mathrm{AL})}(3\mathrm{D}) = \frac{r}{32\eta_d^{1/2}t^{1/2}}\left(\sum_m \left(\frac{\eta_d}{\tilde{\eta}_m}\right)^{1/2} + \frac{16}{3}\left\{\left[\left(\frac{\eta_d}{\tilde{\eta}_1}\right)^{1/2} + \left(\frac{\eta_d}{\tilde{\eta}_2}\right)^{1/2} - 2\left(\frac{2\eta_d}{\tilde{\eta}_1+\tilde{\eta}_2}\right)^{1/2}\right] + \tfrac{3}{2}\,(\tilde{\eta}_2 \leftrightarrow \tilde{\eta}_0)\right\}\right), \quad (18)$$

exactly as the structure expected above. The (positive definite) extra contribution arises from the form of the supercurrent in the superfluid, and is absent in the specific heat [Eq. (15)]. The numerical value in the large bold parentheses is 7.00.

In 2D the normal to the film, taken as $\hat{\mathbf{z}}$, is a unique direction. The response function needed is the average of R_{xx} and R_{yy}. We find the interesting fact that the derivatives on $U(\hat{\mathbf{q}})$ give a term which *vanishes* for $\hat{\mathbf{q}}$ in the x-y plane. Hence in the 2D regime (where $q_z = 0$ and the $\mathbf{q}$ integral reads $d^{-1}\int[dq_x dq_y/(2\pi)^2]$), we obtain the simple result of a sum over the number of components [cf. (17)], and since each component contributes an amount which is independent of $\tilde{\eta}_m$, we obtain the universal result

$$\sigma'^{(\mathrm{AL})}(2\mathrm{D}) = r(1/16dt)n\,. \quad (19)$$

Extra terms analogous to those in (18), which in principle can exist and would involve terms like $\ln\{[(\tilde{\eta}_1+\tilde{\eta}_2)/2]^2/\tilde{\eta}_1\tilde{\eta}_2\}$, does not arise.

The form of Eqs. (16), as well as the argument below it, and (17) is rather general, therefore the *form* of (18) should still be valid when one takes the crystal symmetry into account (also notice that the dimension of conductivity is $e^2/\hbar$ divided by a length). In 2D the coherence length is replaced by d and hence one obtains a universal term (19) corresponding to the contribution (17). At this point, however, we do not have an argument whether the logarithmic terms discussed below (19) can arise or not in the general case.

To complete our discussion we discuss the Maki[12]-Thompson[13] (MT) contributions, as shown in Fig. 3. The evaluation of these diagrams runs parallel to the s-wave case. There the contribution to the paraconductivity arises from an integral of the form

$$\int \frac{d^dq}{(2\pi)^d}\frac{1}{Dq^2}\frac{1}{(t+\eta q^2)}\,, \quad (20)$$

where the diffusion pole $(Dq^2)^{-1}$ (D denotes the diffusion constant) arises from the impurity ladder correction (i.e., the in-scattering terms) to the pairing interaction (Fig. 4). In our case it is more convenient to consider the diagrams with and without the impurity ladder separately. For the first type the $(Dq^2)^{-1}$ term is simply absent. For the second type one sees that one always has an integral of the form

$$\int \frac{d\Omega_{\hat{\mathbf{k}}_1}}{4\pi} Q^0_{\hat{\mathbf{k}}_1} P_2(\hat{\mathbf{k}}_1\cdot\hat{\mathbf{k}}')$$

[cf. Eq. (4)] which provides a factor of τDq^2, canceling that of the diffusion pole. Thus a calculation parallel to the s-wave case only gives rise to integrals of the form (20) with Dq^2 replaced by τ^{-1}. Hence in 3D there is no contribution to the paraconductivity, and in 2D the contribution is $\sim\ln t$ and negligible compared with the t^{-1} AL terms. (Moreover, in 2D there is no necessity of any regularization procedures as in the case of the s wave.[13,17]) Thus basically the pair breaking by impurities (the absence of the in-scattering contribution as $q \to 0$) makes the MT term insignificant compared with the AL term. This conclusion is completely general, and should be true as long as the relevant-pairing interaction averages to zero over the Fermi surface.

We have also performed the calculation for a p-wave superconductor. As expected the above qualitative conclusions still hold, and Eq. (19) holds *quantitatively*.

In the high-T_c superconductors it is generally believed that the CuO_2 planes are sufficiently decoupled that one should use the Lawrence and Doniach[14] (LD) theory of layered superconductors. Our calculation can easily be extended to this case. Rather similar to the s-wave case[14] we find that our AL result is simply Eq. (19) with d replaced by the distance between the layers and reduced by the interlayer coupling factor (notice, however, that $r = 1$ for an s-wave superconductor):

$$\sigma'^{(\mathrm{AL})}(\mathrm{layer}) = r\frac{1}{16dt}n\left[1+\left(\frac{2\xi_c(0)}{dt^{1/2}}\right)^2\right]^{-1/2}. \quad (21)$$

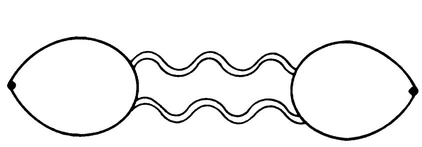

FIG. 2. The Aslamazov-Larkin term. Noncrossing impurity lines are implicit.

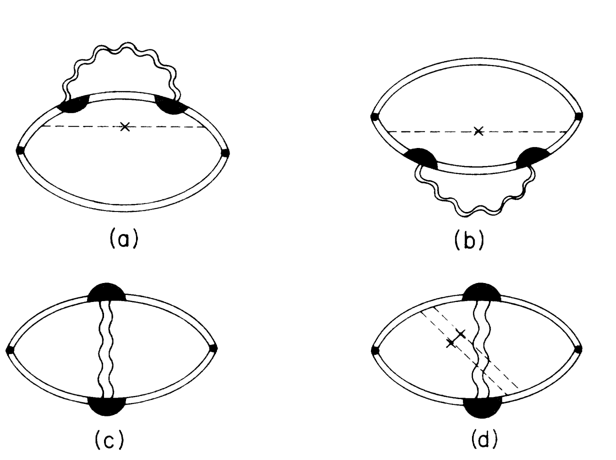

FIG. 3. The Maki-Thompson terms. Shaded areas indicate vertex correction by impurities. In (a) and (b) both the diagrams with and without the explicit impurity line are included.

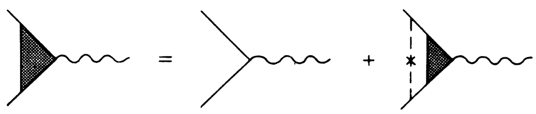

FIG. 4. The vertex correction to the pairing interaction by impurities.

The corresponding MT contribution, investigated recently by Hikami and Larkin,[15] is, of course, again negligible in our case.

In conclusion we find that, for a non-*s*-wave superconductor, the paraconductivity in 3D consists of a term involving the sum of inverse coherence lengths and an extra term involving their differences, which vanishes when all the coherence lengths are equal. In 2D the result is universal. Further, the magnitude of the AL contribution is reduced by the factor r in Eq. (12). The Maki-Thompson terms can be ignored. The measurement of this paraconductivity, especially in 2D, should be very useful in understanding the oxide superconductors. Experiments have been done on the Y 1:2:3 (Refs. 3–6) and the Bi 2:2:1:2 (Ref. 7) compounds, where the layered superconductor calculation should apply. In Refs. 3 and 4 it is found that the paraconductivity can be fitted to the *s*-wave AL contribution [Eq. (21) with $r=1$], provided that an arbitrary reduction factor of $\sim 3-7$ is included. Moreover, there is no evidence of any MT contribution. In view of Eq. (21) it may be that the order parameter is a single component ($n=1$), albeit an *unconventional* one. (This sort of order parameter, however, is only allowed with crystal-symmetry considerations). Hagen *et al.*,[5] however, find that their result cannot be fitted to (the AL contribution of) the LD theory. Friedmann *et al.*,[6] analyzing their data in a different way, find that various theories can be fitted to their data and hence the situation is not very conclusive. For the Bi 2:2:1:2 compound an ambiguity arises as to whether each individual CuO_2 layer or double layers form the quasi-2D systems in the LD theory. In Ref. 7 it is found that the paraconductivity is *twice* the AL value one would expect if each double layer forms a quasi-2D system. Under that assumption it is then tempting to conclude that $n \geq 2$. More careful experiment and data analysis is needed to arrive at more concrete conclusions.

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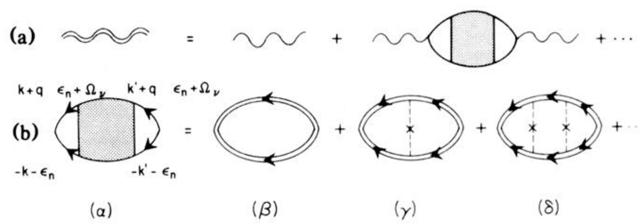

FIG. 1. (a) The pair propagator. A single wavy line is the bare interaction. (b) The two-electron propagator; double solid line represents electron Green's function dressed by impurities.

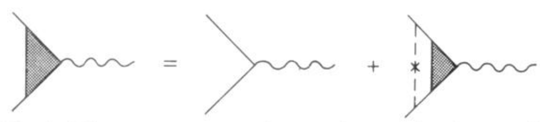

FIG. 4. The vertex correction to the pairing interaction by impurities.