Quantum-interference magnetoconductivity in the variable-range-hopping regime

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The influence of interference effects on the low-field magnetoconductivity of disordered semiconductors in the variable-range-hopping regime is considered. The contribution to the (magnetic) field dependence of the phonon-assisted tunneling rate between two sites is calculated, which arises from interference between the amplitude for a direct transition and that for an indirect one involving a single third site. This process is analogous to that considered by Holstein for the hopping Hall effect. An expression for the low-field magnetoconductivity is derived in the effective-medium approximation and is shown to be positive and linear in the field. As in the multisite model of Nguyen *et al.*, the linear magnetoconductivity arises from processes with complete destructive interference in the absence of the field. The coefficient increases with decreasing temperature. For Mott hopping the temperature dependence can be approximated by a power law with an exponent close to 1.

I. INTRODUCTION

Hopping transport, i.e., carrier motion arising from phonon-assisted tunneling of electrons between localized states of disordered semiconductors is considered to be a process in which the electrons lose their phase memory after each step.[1-3] On the other hand, it has been shown by Holstein[4] that interference processes are possible in hopping transport if more than two sites are involved in a phonon-assisted tunneling event. This can lead to a nonvanishing Hall effect.[4] Recently, Nguyen, Spivak, and Shklovskii (NSS) (Refs. 5 and 6) pointed out that similar processes may influence also the magnetoconductivity in the hopping regime. They performed a numerical simulation of the interference process that occurs during a single hop. They demonstrated that under certain conditions the interference effect can lead to an anomalously negative magnetoresistance and to Aharonov-Bohm oscillations in multiply connected samples. By means of percolation theory, they have shown that the magnetoresistance is linear in the field and arises from those hops which have zero amplitude in the absence of the field as a consequence of destructive interference.

Experimental evidence for quantum interference hopping magnetoconductivity has been found in n-type channels of field-effect transistors,[7] in films of polycrystalline semiconductors,[8] and in impurity bands of doped crystalline semiconductors.[9] Also, earlier data of doped inversion layers[10] and recent data of amorphous Mo-Ge can be interpreted in this fashion.[11] Aharonov-Bohm oscillations which have been found recently in grid-structured amorphous $PbTeO_x$ films[12] can possibly be explained in terms of the above-mentioned mechanism.

NSS (Ref. 5) studied a model in which a large number of intermediate localized states participate in the interference process (multisite model). Similar models were also considered by other authors.[13,14]

Let us estimate how many intermediate centers one can expect in a typical variable-range hopping situation. Consider first a doped crystalline semiconductor. For having localized impurity states, according to Mott,[1] one has $n_s^{1/d} a_H < 0.25$, where $a_H = 1/\alpha$ is the effective Bohr radius, n_s is the number of donors per unit area (volume), and d is the dimensionality. The mean distance between donors $a = n_s^{-1/d}$ obeys, therefore, $\alpha a > 4$. In the variable-range hopping regime phonon-assisted tunneling occurs not between neighboring donors but rather over a length $r = (T_0/T)^{1/(d+1)}/2\alpha$. Typical experimental values for T_0/T are of the order of 10^3, which gives values for $2\alpha r$ of the order of 10. It is clear that r cannot easily be increased appreciably by lowering the temperature because of its weak temperature dependence. There will be only very few additional donors inside r. A similar reasoning holds for amorphous semiconductors: The hopping centers are several localization lengths apart and values of $k_B T_0$ are in the eV regime, so that also here a typical hopping length will exceed the mean distance between centers only by a factor of 2 or 3. This means that in situations of experimental relevance[7-11] only very few localized states are present within the distance of a typical electron jump.

In the following we shall consider a model with only *one* additional defect between the hopping sites. Applying the two-site effective-medium approximation (EMA) of Movaghar *et al.*,[15,16] it is confirmed that in the variable-range hopping regime one has to average the logarithm of the hopping rate over the energetical and spatial position of the additional defect as deduced by Shklovskii and co-workers[2,5,6] from percolation theory. This logarithmic averaging leads to an enormous sensitivity to configurations with destructive interference in the absence of the field and to a linear and positive magnetoconductivity.

II. THREE-SITE HOPPING PROBABILITIES

The following derivation of the three-site phonon-assisted tunneling rate is analogous to that of Holstein[4] for the hopping Hall effect. The difference is as follows: Holstein demonstrated that for obtaining a finite Hall effect a process with more than one phonon is needed; the second phonon must also be resonantly absorbed or emitted in order to obtain an indirect amplitude which is out of phase by $\pi/2$ compared to the direct one at zero field.

In what follows it is shown that the three-site hopping probability relevant to the interference magnetoconductivity can be obtained by performing a (renormalized) perturbation expansion of the wave function instead of going to higher order in the electron-phonon interaction.

Following Ref. 4 we consider a Hamiltonian of the form

$$H=H_e+H_{e\text{-ph}}+H_{\text{ph}} , \tag{1}$$

where $H_{e\text{-ph}}$ is the electron-phonon interaction, treated in deformation potential approximation[4] and H_{ph} is the phonon Hamiltonian. The electron Hamiltonian is assumed to be of the form

$$H_e=\sum_i |\varphi_i\rangle \varepsilon_i \langle\varphi_i| + \sum_{i\neq j} |\varphi_i\rangle V_{ij} \langle\varphi_j| , \tag{2}$$

where ε_i are randomly fluctuating local energies and V_{ij} are overlap integrals between local Wannier states $|\varphi_i\rangle$ with radius α^{-1}. The V_{ij} are assumed to be of the form

$$V_{ij}=V_0(r_{ij})\exp(-\alpha r_{ij}) , \tag{3}$$

where r_{ij} is the distance between the centers of gravity of those states.

$V_0(r)$ is supposed to be a weak function of the intersite separation. This model may describe two physical situations of experimental relevance.

(i) An impurity band in a crystalline semiconductor. The states $|\varphi_i\rangle$ in this case are effective hydrogen states calculated in effective mass approximation. The deviation of the ε_i from the donor (or acceptor) level is caused by compensation and by the overlap.

(ii) The region of localized states ("mobility gap") of an amorphous semiconductor. In this case the $|\varphi_i\rangle$ are defect states determined by the chemical nature and the amount of disorder in the material. The validity of (3) can be justified *a posteriori* from the ubiquity of Mott's $T^{1/4}$ law in amorphous semiconductors.

The eigenstates of (2) can be calculated by renormalized perturbation theory[17] to be of the form

$$|\psi_i\rangle=|\varphi_i\rangle+\sum_{k\neq i}|\varphi_k\rangle\left[\frac{V_{ki}}{E_i-E_k}+\sum_{n\neq k\neq i}\frac{V_{kn}V_{ni}}{(E_i-E_k)(E_i-E_n)}+\cdots\right] ; \tag{4}$$

where E_i are the renormalized local energies.[17] To include triangular interference processes we go to second order in the overlap integrals (for details concerning matrix elements of $H_{e\text{-ph}}$ see Ref. 4):

$$\langle\psi_j|H_{e\text{-ph}}|\psi_i\rangle=F_1 D_\lambda^{\pm}\left[\langle\varphi_j|e^{i\mathbf{q}_\lambda\cdot\mathbf{r}}|\varphi_i\rangle+\frac{1}{E_i-E_j}V_{ij}(e^{i\mathbf{q}_\lambda\cdot\mathbf{r}_j}-e^{i\mathbf{q}_\lambda\cdot\mathbf{r}_i})\right.$$
$$\left.+\frac{e^{i\mathbf{q}_\lambda\cdot\mathbf{r}_j}}{E_i-E_j}V_{jn}V_{ni}\frac{1}{E_i-E_n}+\frac{e^{i\mathbf{q}_\lambda\cdot\mathbf{r}_i}}{E_j-E_i}V_{jn}V_{ni}\frac{1}{E_j-E_n}+\frac{e^{\mathbf{q}_\lambda\cdot\mathbf{r}_n}}{E_i-E_n}V_{ni}V_{jn}\frac{1}{E_j-E_n}\right] , \tag{5}$$

where F_1 is the deformation potential. The quantity $D_\lambda^{\pm}$ is given by

$$D_\lambda^{\pm}=\pm i\mathbf{q}_\lambda\cdot\mathbf{e}_\lambda(\hbar/2MN\omega_\lambda)^{1/2}|1-e^{\pm\hbar\omega_\lambda/k_BT}|^{-1/2} . \tag{6}$$

$\hbar\omega_\lambda$ is the energy, $\hbar\mathbf{q}_\lambda$ the momentum, and $\mathbf{e}_\lambda$ the polarization of a phonon λ. M and N are the mass and number of the atoms, respectively and $+$ $(-)$ denotes absorption (emission) of a phonon.

In the presence of an applied magnetic field $\mathbf{B}$ the V_{ij} acquire a phase factor $\exp(i\alpha_{ji})$, where the angle α_{ji} is given by

$$\alpha_{ji}=\frac{e}{2\hbar}\mathbf{B}\cdot\mathbf{r}_j\times\mathbf{r}_i \tag{7}$$

so that we have $V_{ji}(B)=V_{ji}(B=0)\exp(i\alpha_{ji})\equiv V_{ji}\exp(i\alpha_{ji})$. The hopping probability is now obtained as

$$W_{ij}^{(n)}=\frac{2\pi}{\hbar}\sum_{\pm\lambda}|\langle\psi_j|H_{e\text{-ph}}|\psi_i\rangle|^2\delta(E_i-E_j\pm\hbar\omega_\lambda)$$
$$=4\frac{F_1^2\overline{|D_{ij}|^2}}{(E_i-E_j)^2}\left|V_{ij}+\left[\frac{1}{E_i-E_n}+\frac{1}{E_j-E_n}\right]V_{in}V_{nj}\exp(i\phi_{jni})\right|^2 . \tag{8}$$

Here the bar denotes averaging over the phonons, and $\phi_{jni}=\alpha_{jn}+\alpha_{ni}-\alpha_{ij}=(e/\hbar)\mathbf{B}\cdot\mathbf{A}_{jni}$, where $\mathbf{A}_{jni}$ is the vector area of the triangle $i-n-j$. $|D_{ij}|^2$ is the modulus squared of expression (6) with $\pm\hbar\omega_\lambda=E_j-E_i$, averaged over the possible polarization directions. In deriving (8) the first term of (5) has been dropped because it is much smaller than the others in the limit $\mathbf{q}_\lambda\cdot\mathbf{r}_{ij}\gg 1$ (see Ref. 4). The last term of Ref. 5 has been discarded because it cannot interfere with the direct process since it has no common phase factor. [At this stage one should note[6] that these approximations can become questionable if the second term inside the bars of (8) cancels the first (destructive interference). We shall come back to this point in Sec. IV B.]

Expression (8) can be rewritten as

$$W_{ij}^{(n)}=\frac{\nu_0(E_{ij}^2)}{|1-e^{E_{ji}/k_BT}|}V_{ij}^2 g_{ij}^{(n)}\equiv W_{ij}^{(0)}g_{ij}^{(n)} \tag{9}$$

with

$$g_{ij}^{(n)}=\left|1+\left[\frac{1}{E_i-E_n}+\frac{1}{E_j-E_n}\right]V_{in}V_{nj}V_{ij}^{-1}\exp(i\phi_{jni})\right|^2 . \tag{10}$$

$W_{ij}^{(0)}$ is the usual Miller-Abrahams one-phonon hopping probability per unit time. (The prefactor ν_0 is further considered as a constant.) $g_{ij}^{(n)}$ is the factor which arises as a consequence of the interference process. $g_{ij}^{(n)}$ can be larger (constructive interference) or smaller than 1 (destructive interference) depending on the sign of the prefactor of the second term inside the bars. We shall see below that the case of destructive interference is most important for the low-field magnetoconductivity.

III. CALCULATION OF THE CONDUCTIVITY

For calculating the field-dependent hopping conductivity we utilize the effective-medium approximation (EMA) developed by Movaghar *et al.*[15,16] for hopping transport in disordered semiconductors. In the dc and low-temperature–density limit this method has been shown to yield the same results as the percolation method.[2,3] It can, however, be also applied to the ac case and to the whole density and temperature range.

Since the transition probabilities we are dealing with describe an effective two-site process (from the point of view of the hopping mechanism) leading from site i to j we can use the two-site EMA. We have only to make sure that in averaging over the additional sites n we exclude the energy range around the Fermi level, E_F which is used by the sites i and j in the hopping process. This range can be estimated to be $E_0=2\alpha r k_B T$ above and below E_F.

Studying hopping near the Fermi level we can use the symmetrized version of the theory,[16] i.e., we put

$$W_{ij}^{(0)}=W_{ji}^{(0)}=\nu_0 e^{-2\alpha r_{ij}}e^{-E_{ij}/k_BT} , \tag{11}$$

where $E_{ij}=|E_i-E_j|$ is the effective barrier between i and j and ν_0 a prefactor of the order of a phonon frequency. The conductivity is given as[15,16]

$$\sigma=n_c\left[1-\frac{n_c}{n_s}\right]\frac{e^2}{k_BT}\frac{1}{2d}R^2\nu_0\sigma_1 . \tag{12}$$

Here n_s is the number of sites per volume and n_c is the number of carriers per volume. R^2 is given by

$$R^2=\left\langle\left\langle\frac{r_{ij}^2}{1/\sigma_1+\nu_0/W_{ij}^{(n)}}\right\rangle\right\rangle\Big/\left\langle\left\langle\frac{1}{1/\sigma_1+\nu_0/W_{ij}^{(n)}}\right\rangle\right\rangle \tag{13}$$

and σ_1 is the dimensionless conductivity which obeys the self-consistent EMA equation

$$\sigma_1=a_p n_s\left\langle\left\langle\frac{1}{1/\sigma_1+\nu_0/W_{ij}^{(n)}}\right\rangle\right\rangle . \tag{14}$$

$a_p=\exp(-1)$ is a density renormalization which compensates double counting. $\langle\langle\cdots\rangle\rangle$ denotes an average with respect to the sites i, j, and n. Defining $\langle\cdots\rangle$ to be an average over the conditional probability for finding a third site n if i and j are fixed we have

$$\langle\langle\cdots\rangle\rangle\equiv\langle\int N(E_{ij})dE_{ij}d^d\mathbf{r}_{ij}\cdots\rangle , \tag{15}$$

where $N(E)$ is the density of states. In the following we restrict ourselves to the cases $d=2$ and $d=3$. Defining now the hopping length r and the interference exponent η by

$$r=-\ln(\sigma_1)/2\alpha , \tag{16}$$

$$\eta_{ij}^{(n)}=-\ln(g_{ij}^{(n)}) . \tag{17}$$

We can rewrite Eq. (14) in the form

$$1=a_p 2\pi(d-1)\left\langle\int_0^\infty dE_{ij}N(E_{ij})\int_0^\infty dr_{ij}r_{ij}^{d-1}\left[1+\exp\left[\frac{E_{ij}}{k_BT}+2\alpha r_{ij}-2\alpha r+\eta_{ij}^{(n)}\right]\right]^{-1}\right\rangle . \tag{18}$$

In (18) only energies of the order $2\alpha r\,k_BT$ are relevant so that we can put $N(E_{ij})\approx N_F$ outside the integral. [At very low temperatures $N(E)$ becomes zero at $E=E_F$ as a result of correlation effects ("Coulomb gap"[2]) and varies as $(E-E_F)^2$. This case is easily included within the present formalism. For reasons of clarity we restrict ourselves here to the "classical" Mott variable-range hopping regime where $N_F\neq 0$.] Furthermore, we set in (10) $1/(E_i-E_n)+1/(E_j-E_n)=2/(E_F-E_n)$. Performing now a partial integration we obtain

$$1=4\alpha a_p k_BT\pi\frac{d-1}{d}N_F \times\left\langle\int_0^\infty dr_{ij}r_{ij}^d\frac{1+(1/2\alpha)d\eta_{ij}^{(n)}/dr_{ij}}{1+\exp(2\alpha r_{ij}+\eta_{ij}^{(n)}-2\alpha r)}\right\rangle . \tag{19}$$

We expand now the Fermi function $\{1+\exp[\eta_{ij}^{(n)}+2\alpha(r_{ij}-r)]\}^{-1}$ with respect to η (which is legitimate if typical values of η are small compared to $2\alpha r$, see the Appendix of Ref. 6) and neglect the term proportional to $\langle\eta\dot{\eta}\rangle$. The term proportional to η is multiplied by a function which becomes proportional to $\delta(r_{ij}-r)$ in the limit of large $2\alpha r$. We therefore can approximately rewrite (19) as

$$1=4\alpha a_p k_BT\pi\frac{d-1}{d}N_F \times\int_0^\infty dr_{ij}r_{ij}^d\frac{1}{1+\exp[2\alpha(r_{ij}-r)+\langle\eta\rangle]} \tag{20}$$

with

$$\langle\eta\rangle=\langle\eta_{ij}^{(n)}\rangle|_{r_{ij}=r} .$$

We replace now the Fermi function in (20) by a step function. (We shall see later in the discussion of the field dependence of the conductivity at very small fields that this procedure must be considered in more detail.) This yields

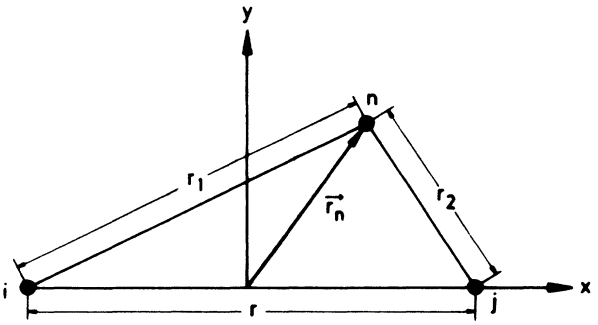

FIG. 1. Interference triangle together with the coordinate system fixed to the triangle which is used for evaluating the average over the relative position of site n with respect to the hopping sites i and j.

$$1=(T/T_0)(2\alpha r-\langle\eta\rangle)^{d+1} \tag{21}$$

with

$$T_0=\frac{(2\alpha)^d d(d+1)}{(d-1)a_p N_F 2\pi} . \tag{22}$$

Defining now r_0 to be the hopping length in the absence of interference effects, we obtain Mott's law

$$2\alpha r_0=(T_0/T)^{1/(d+1)} \tag{23}$$

and, to first order in $\langle\eta\rangle/2\alpha r_0$

$$r=r_0+\langle\eta\rangle/2\alpha . \tag{24}$$

The conductivity becomes

$$\sigma=n_c\left[1-\frac{n_c}{n_s}\right]\frac{e^2}{k_B T}\frac{1}{2d}R^2\nu_0 \times\exp\left[-\left[\frac{T_0}{T}\right]^{1/(d+1)}-\langle\eta\rangle\right] \tag{25}$$

with

$$R^2=r_0^2 d(d+1)/(d+2)(d+3)a_p . \tag{26}$$

If we neglect all other sources of a field dependence except that coming from the interference mechanism the magnetoconductivity for small $\langle\eta\rangle(B)-\langle\eta\rangle(0)\equiv\Delta\langle\eta\rangle_B$ becomes

$$\sigma_B=\frac{\sigma(B)-\sigma(0)}{\sigma(0)}=-\Delta\langle\eta\rangle_B . \tag{27}$$

All these results confirm those obtained by NSS from percolation theory.

IV. RESULTS AND DISCUSSION

A. Linear magnetoconductivity

Let us now specify the averaging procedure over the third site. $\langle\eta\rangle$ is to be calculated as follows:

$$\langle\eta\rangle=-\int_{|E|>E_0}dE\,N(E)\int_\Omega d^d\mathbf{r}_n\ln\left|1-\frac{W(r,\mathbf{r}_n)}{E}\exp[i\phi(r,\mathbf{r}_n)]\right|^2 . \tag{28}$$

Here $E=E_n-E_F$ and $E_0=2\alpha r k_B T$. The vector $\mathbf{r}_n$ is the vector which points from the midpoint between sites i and j to site n in the coordinate system defined in Fig. 1. The energy W is given by

$$W(r,\mathbf{r}_n)=2V(r_1)V(r_2)V(r)^{-1} . \tag{29}$$

Here $V(r_{ij})=V_{ij}$ and r_1 and r_2 are the legs of the interference triangle defined by

$$r_{1,2}=[(r/2\pm x)^2+y^2]^{1/2} . \tag{30}$$

$\phi(r,\mathbf{r}_n)$ is $e/\hbar$ times the flux through the triangle

$$\phi(r,\mathbf{r}_n)=\frac{e}{\hbar}B_z\tfrac{1}{2}ry=\phi(r,y) . \tag{31}$$

The normalization volume Ω must fulfill $\Omega n_s=1$. Since it must have the length r, one obtains the conditions $y<1/rn_s$ for $d=2$ and $y<(rn_s)^{-1/2}$ for $d=3$. Let us compare the region where the integrand is different from zero with Ω. Similar to the findings of NSS, the region where the function $W(r,r_1,r_2)$ differs appreciably from zero is an elliptic (or ellipsoidal in 3D) region with length r and diameter $y_0\equiv(r/\alpha)^{1/2}$ ("cigar-shaped region"[5]). The present model is devised for the case where the average number of additional sites inside this region is smaller than 1. This number can be estimated to be $c=n_s r^{3/2}/\alpha^{1/2}$ ($d=2$) and $c=n_s r^2/\alpha$ ($d=3$). The corresponding ratios of the diameter of the cigar-shaped region to that of the normalization volume are proportional to c ($d=2$) and $c^{1/2}$ ($d=3$). Therefore, in evaluating the integral over r_n we will not care about the restriction $|r_n|<\Omega$ and push the integration boundary to infinity.

For obtaining the low-field behavior of $\langle\eta\rangle$ we replace $\exp(i\phi)$ by $1+i\phi$ and perform a Taylor expansion of $\langle\eta\rangle$ with respect to ϕ. With the help of the identity

$$\frac{1}{x+iy}=P\left[\frac{1}{x}\right]-i\pi\,\mathrm{sgn}(y)\delta(x) , \tag{32}$$

which is valid in the limit $|y|\to 0$, we obtain

$$\Delta\langle\eta\rangle_B=-2\pi\int_{|E|>E_0}dE\,N(E)\int_{-\infty}^{+\infty}d^d\mathbf{r}_n[|\phi(r,\mathbf{r}_n)W(r,\mathbf{r}_n)|\delta(E-W(r,\mathbf{r}_n))]+O(\phi^2) . \tag{33}$$

This means that $\langle\eta\rangle_B$ contains a term which is proportional to the modulus of the applied field.

Combining (27) with (33) we obtain—in agreement with NSS—a positive linear magnetoconductivity. Within the present model it is explicitly given as

$$\sigma_B = 2\pi \int_{|E|>E_0} dE\, N(E) \int_{-\infty}^{+\infty} d^d \mathbf{r}_n [|\phi(r,\mathbf{r}_n) W(r,\mathbf{r}_n)| \delta(E - W(r,\mathbf{r}_n))] \ . \tag{34}$$

It is clear from (34) that the linear magnetoconductivity comes from those intermediate sites for which $g_{ij}^{(n)}=0$ in the absence of the field. In other words, the linear magnetoconductivity is produced by those triangular configurations which have complete destructive interference in the absence of the applied field. Also, this feature agrees with the results of the NSS multisite theory. Within this single-site model, the intermediate state must be energetically situated below the Fermi level if $V(r)<0$ and above it if $V(r)>0$. In the case of n-doped crystalline semiconductors, $V<0$ so that the interference magnetoconductivity probes the states *below* the Fermi level.

In order to be able to evaluate the integral explicitly we keep only the exponential dependence of $V(r_1)$ and $V(r_2)$, i.e., we set

$$V_0(r_1)=V_0(r_2)=V_0(r/2) \ . \tag{35}$$

In $d=2$ this leads to (see the Appendix)

$$\sigma_B = \frac{\pi}{2}\frac{e}{\hbar}|B_z|\frac{r^3}{\alpha}\int_{E_{\min}}^{E_{\max}} dE\, N(E) P_2\left[\frac{w_E}{\alpha r}\right] , \tag{36}$$

with

$$P_2(x)=\tfrac{2}{3}+2x+x^2 \tag{37}$$

and

$$w_E=-\ln|E/E_1| \ . \tag{38}$$

In (36) B_z in the component of the field perpendicular to the two-dimensional plane. The integration boundaries $E_{\min}$ and $E_{\max}$ are given by

$$E_{\min}=\begin{cases} -E_1, & V(r)<0 \\ E_0, & V(r)>0 \end{cases} \tag{39a}$$

$$E_{\max}=\begin{cases} -E_0, & V(r)<0 \\ E_1, & V(r)>0 \end{cases} \tag{39b}$$

with

$$E_1=|2V_0(r/2)^2/V_0(r)| \ . \tag{40}$$

The analogous formulas for $d=3$ are

$$\sigma_B = \frac{\pi}{2}\frac{e}{\hbar}|B|\frac{r^3}{\alpha}\left[\frac{r}{\alpha}\right]^{1/2}\int_{E_{\min}}^{E_{\max}} dE\, N(E) w_E^{1/2} P_3\left[\frac{w_E}{\alpha r}\right] , \tag{41}$$

with

$$P_3(x)=\left[1+\frac{x}{2}\right]^{1/2}(\tfrac{3}{4}+2x+x^2) \ . \tag{42}$$

In comparing (36) with (41) one notes the important point that in $d=3$ the effect does not depend on the direction of **B**, whereas in $d=2$ it does. This is a consequence of the orbital nature of the interference process and can be utilized to experimentally identify quantum interference effects in hopping transport.[5-9]

Let us now rewrite expressions (36) and (41) as follows: If we denote the region where $W(r,r_1,r_2)$ differs appreciably from zero ("cigar-shaped region",[5,6,8] by $\Psi_d \equiv {y_0}^{d-1} r$ and the area of the maximum interference triangle which can be imbedded into this region by $S=\frac{1}{2}ry_0$ we can cast Eqs. (36) and (41) into the form

$$\sigma_B = \pi\frac{e}{\hbar}|B_d| S\Psi_d n_s \nu_d = 2\pi\phi(r,y_0) c\nu_d \ , \tag{43}$$

where $c=n_s\Psi_d$ is the above-defined average number of additional sites, and we have introduced the dimensionless quantities.

$$\nu_2=\frac{1}{n_s}\int_{E_{\min}}^{E_{\max}} dE\, N(E) P_2\left[\frac{w_E}{\alpha r}\right] \tag{44a}$$

and

$$\nu_3=\frac{1}{n_s}\int_{E_{\min}}^{E_{\max}} dE\, N(E) w_E^{1/2} P_3\left[\frac{w_E}{\alpha r}\right] . \tag{44b}$$

In (43) $B_2=B_z$ and $B_3=B$.

If we assume that the temperature dependence of r is essentially the same as that of r_0 as given by Eq. (22) we obtain for the explicit temperature dependence of σ_B:

$$S\Psi_d=\tfrac{1}{2}r^2\left[\frac{r}{\alpha}\right]^{d/2}=\frac{1}{2}\left[\frac{T_0}{2T}\right]^{(2+d/2)/(1+d)}\alpha^{-(d+2)} \ , \tag{45}$$

which gives a T^{-1} law in $d=2$ and a $T^{-7/8}$ law in $d=3$. The energy integrals weakly increase with temperature so that we can expect a T^{-s} behavior where s is a little bit smaller than the exponent given in Eq. (45).

B. Linear versus quadratic field dependence

It has been argued by Sivan *et al.*[13] that the logarithmic averaging procedure, which leads to the linear magnetoconductivity, is not valid in the low-field limit and that one has, instead, a B^2 dependence. Shklovskii and Spivak[6] admit that at exponentially small fields this is true, but in the experimentally relevant region the logarithmic averaging procedure retains its validity. We can study this problem with the help of our EMA expression (19). Considering only the first term in the numerator of (19) we can evaluate the integral over r_{ij} exactly to obtain[18]

$$(2\alpha)^{d+1}\int_0^\infty dr_{ij}r_{ij}^d\left[1+\frac{\sigma_1}{g_{ij}^{(n)}}\exp(2\alpha r_{ij})\right]^{-1}$$
$$=\begin{cases}-d!H_{d+1}\left[\dfrac{g_{ij}^{(n)}}{\sigma_1}\right], & \dfrac{g_{ij}^{(n)}}{\sigma_1}<1\\[2ex](-1)^{d+1}d!H_{d+1}\left[\dfrac{\sigma_1}{g_{ij}^{(n)}}\right]+\dfrac{1}{d+1}\ln^{d+1}\left[\dfrac{g_{ij}^{(n)}}{\sigma_1}\right]-2d!\displaystyle\sum_{\nu=1}^{[d+1]}\frac{1}{(d+1-2\nu)!}\ln^{d+1-2\nu}\left[\frac{g_{ij}^{(n)}}{\sigma_1}\right]H_{2d+2}(1), & \dfrac{g_{ij}^{(n)}}{\sigma_1}>1\end{cases}$$

(46)

where $H_n(x)=\sum_{\nu=1}^{\infty}(-x)^\nu/\nu^n$ and $[x]$ is the largest natural number smaller than x. Clearly the logarithmic terms disappear if $g_{ij}^{(n)}/\sigma_1<1$. In the presence of an applied field $g_{ij}^{(n)}$ has a lower bound of $\phi(r,r_n)^2$. We expect, therefore, a quadratic field dependence of the conductivity if the field is so small that $\phi^2(r,\mathbf{r}_n)<\exp(-2\alpha r)$ holds for typical values $|\mathbf{r}_n|=y_0$. If $\phi^2(r,y_0)$ is larger, the logarithms appear in the self-consistent EMA expression. For $\phi^2\gg\exp(-2\alpha r)$ the term $[1/(d+1)]\ln^{d+1}(g_{ij}^{(n)}/\sigma_1)$ becomes dominant and we recover (21) and (24). These findings agree with those of Shklovskii and Spivak[6] derived from percolation theory. These authors, however, mention another important point in this context: In the limit of very small $g_{ij}^{(n)}$ the terms neglected in deriving (8) may be important. Their estimate leads to a crossover field which is still exponentially small but larger than that given by $\phi^2(r,y_0)=\exp(-2\alpha r)$.

C. Discussion of experimental data

The clearest evidence of the existence of the quantum interference mechanism is obtained from experiments in which a dimensional crossover occurs, i.e., a crossover from an isotropic to an anisotropic magnetoconductivity. Such experiments have been performed with n channels in field-effect transistors[7] and with thin films of polycrystalline semiconductors.[8] In the former samples the crossover occurs by reducing the thickness of the channel with the help of the gate voltage; in the latter the thickness of the films was varied. In the transistor experiments[7] a huge effect of $\sigma_B^{\max}\sim 30-50\,\%$ is observed. The field dependence is in all cases linear, followed by a crossover to a negative magnetoconductivity caused by shrinking. In the polycrystalline indium oxide films[8] the effect is very small, $\sigma_B^{\max}\sim 1\%$, but the evidence for the dimensional crossover is clear. In this experiment the B dependence can be described better by a B^2 law than by a linear law. The reason for this is obvious. Considering expression (43) for the magnetoconductivity and assuming $c\nu_d$ to be of the order to 1 we can estimate the phase $\phi(r,y_0)$ to be in the range of about 1%. Taking their reported values of $2\alpha r$, which are in the range 5–10, we have $\phi^2\approx\exp(-2\alpha r)$, which means that one is in the crossover region between a quadratic and linear B dependence. In both experiments a real quantitative analysis cannot be done because not all of the parameters which characterize the sample are known. The situation is much better in the recent experiments of Qiu-Yi Ye *et al.*[9] These authors reported on extremely anisotropic negative magnetoresistance data. The sample relevant to the present discussion consists of δ-layer superlattices with 20 successive Si-donor sheets spaced by 1000 Å of nominally undoped single-crystalline GaAs. The width of the sheets does not exceed that of a monatomic layer and the average distance between donors is 380 Å. The Bohr radius is $a_B=1/\alpha=100$ Å. Below 5 K a

$$\sigma\propto\exp[-(T_0/T)^{1/3}]$$

law is observed with $T_0=340$ K. This gives a value of $2\alpha r=5.5$ at $T=2$ K, which shows that the transport process is just at the onset of variable-range hopping. The observed magnetoconductivity at low fields is indeed positive and linear in the field, followed by a high-field negative magnetoconductivity due to the shrinking effect.

Since the sample is experimentally so well characterized we can try a quantitative comparison of the theory with the experiment. Let us consider first the quantity ν_2 in (43) as defined by (44a). $E_{\min}=-E_1\approx V_0(r)$ should be of the order of the binding energy so that $E_{\min}$ lies below the impurity band. The degree of compensation is reported to be very low ($K\sim 2-5\,\%$) so that $E_0\approx E_F$ lies near the top of the band. Retaining only the leading term in $P_2(x)$ in expression (37) yields essentially $\nu_2\approx\frac{2}{3}$. With the preceding values for r, a_B, and a we obtain a number of additional sites inside the cigar-shaped region $c=0.32$. For $d\phi/dB_z$ we obtain at this temperature the value 0.37 T^{-1}. Inserting these numbers into (43) yields $d\sigma_B/dB_z=0.5\ T^{-1}$. This compares well with the observed value of 0.66 T^{-1}. Also the observed temperature dependence of $d\sigma_B/dB_z\propto T^{-0.79}$ is in agreement with the T^{-s} law ($s\lesssim 1$) estimated from expression (45).

Another interesting feature of their data is mentioned by Qiu-Yi Ye *et al.*: They measure a finite low-field negative magnetoresistance in the $B_\parallel$ configuration, i.e., with the field parallel to the δ planes. This could be explained either by a finite width of the layer or by a finite background bulk impurity concentration. In the former case, the anisotropy $\sigma_B^\perp/\sigma_B^\parallel$ should increase with decreasing temperature (as is, indeed, observed in the In-O films of Ref. 8); in the latter it should decrease. The observed temperature dependence of $\sigma_B^\parallel\propto T^{-1.69}$ leads to a decrease of the anisotropy with decreasing T and therefore confirms that there are background bulk impurities.

V. CONCLUDING REMARKS

In the above we have worked out a theory of quantum interference hopping magnetotransport considering the interference between a direct transition between two sites and an indirect one via a single third site during a hopping event. The results obtained from this model in the effective-medium approximation are qualitatively the same as that obtained by Nguyen *et al.*[5] from a multisite model using percolation theory: For magnetic fields larger than those given by $|\phi(r,y_0)|=\exp(-\alpha r)$, a linear and positive magnetoconductivity is obtained which arises from configurations with complete destructive interference in the absence of the applied magnetic field. Thanks to the simplicity of the present model and the symmetrized two-site EMA we have obtained formulas for the two- and three-dimensional magnetoconductivity which allow for a quantitative comparison with experiments on well-defined systems such as δ-doping structures. It should, however, be realized that neither in the NSS treatment nor in the present one have many-body effects been addressed. This point is important because in the case $V_0(r)<0$ the intermediate donor levels which lead to the interference effect are occupied by electrons. Shklovskii and Spivak[6] have discussed this point at some length. Both by means of a Hubbard model calculation as well as using real Coulomb potentials they demonstrate that the many-body resonant integral which involves a third occupied site has the same sign and roughly the same magnitude as that involving an empty one. It has also been demonstrated some time ago[19] that the electron-electron interaction can induce spin-dependent hopping events which also leads to anomalies in the hopping magnetoconductivity. Another point which cannot be discussed within the present model is the simultaneous treatment of the orbital low-field interference effect and the high-field shrinking effect. This has been achieved by NSS with the help of a numerical calculation. The result is that the more impurities are inside the cigar-shaped region the "later" (i.e., at higher magnetic fields) the shrinking effect becomes dominant.

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APPENDIX: CALCULATION OF THE INTEGRAL APPEARING IN EXPRESSION (34)

We only treat the case $V(r)<0$. The calculation for $V(r)>0$ is completely analogous.

1. $d=2$

For obtaining the expressions (36) and (37) from (34) we have to evaluate the integral

$$I_2(E)=2\pi\int_{-\infty}^{+\infty}dx\int_{-\infty}^{+\infty}dy\left|\frac{e}{\hbar}B_z\tfrac{1}{2}ry\right|E_1e^{-w}\times\delta(E+E_1e^{-w})\,, \tag{A1}$$

where B_z is the field component rectangular to the plane, $E_1=|2V_0(r/2)^2/V_0(r)|$, and $w=\alpha(r_1+r_2-r)$. r_1 and r_2 are the legs of the triangle in Fig. 1 defined by $r_{1,2}=[(r/2\pm x)^2+y^2]^{1/2}$. Changing to the variables $v=(r_1-r_2)/r$ and w we obtain

$$I_2(E)=2\pi\frac{e}{\hbar}|B_z|\frac{r}{\alpha}\int_0^\infty dw\,e^{-w}\int_0^1 dv\,E_1r_1r_2\delta(E+E_1e^{-w})\,, \tag{A2}$$

where we have $r_{1,2}=\frac{1}{2}(r+w/\alpha\pm vr)$. Introducing another variable $u=E_1\exp(-w)$ we obtain

$$\begin{aligned}I_2(E)&=2\pi\frac{e}{\hbar}|B_z|\frac{r}{\alpha}\int_0^{E_1}du\int_0^1 dv\frac{1}{4}\left[\left[r+\frac{w}{\alpha}\right]^2-v^2r^2\right]\delta(E+u)\\&=2\pi\frac{e}{\hbar}|B_z|\frac{r}{\alpha}\int_0^1 dv\frac{1}{4}\left[\left[\frac{w_E}{\alpha}\right]^2+2r\frac{w_E}{\alpha}+r^2(1-v^2)\right]\Theta(E+E_1)\\&=\frac{\pi}{2}\frac{e}{\hbar}|B_z|\frac{r^3}{\alpha}\left[\frac{2}{3}+\frac{2w_E}{\alpha r}+\left[\frac{w_E}{\alpha r}\right]^2\right]\Theta(E+E_1)\,.\end{aligned} \tag{A3}$$

2. $d=3$

For evaluating the spatial average in three dimensions we consider the variable y in Fig. 1 as the radial coordinate in a cylindrical coordinate system with symmetry axis along the x direction. If α is the corresponding angular variable, Θ the angle between **B** and the normal vector belonging to the plane spanned by the triangle, and β the angle between **B** and the x axis we have $\cos\Theta=\sin\alpha\sin\beta$. Since the x axis, which is fixed to the triangle, can be any direction we have to average over $|\sin\beta|$, which gives a factor of $2/\pi$. The integral over $|\sin\alpha|$ gives a factor of 4, so that we are left with

$$\begin{aligned}I_3(E)&=8\frac{e}{\hbar}|B|r\int_0^\infty y^2dy\int_{-\infty}^{+\infty}dx\,E_1e^{-w}\delta(E+E_1e^{-w})\\&=8\frac{e}{\hbar}|B|(r/\alpha)\int_0^\infty dw\,E_1e^{-w}\times\int_0^1 dv\,yr_1r_2\delta(E+E_1e^{-w})\,,\end{aligned} \tag{A4}$$

where y in the v-w system is given by $y=\frac{1}{2}[(2r+w/\alpha)(w/\alpha)(1-v^2)]^{1/2}$. Consequently,

$$I_3(E)=\frac{e}{\hbar}|B|\frac{r}{\alpha}\left[\left[2r+\frac{w_E}{\alpha}\right]\frac{w_E}{\alpha}\right]^{1/2}\int_0^1 dv(1-v^2)^{1/2} \times\left[\left[\frac{w_E}{\alpha}\right]^2+2r\frac{w_E}{\alpha}+r^2(1-v^2)\right]\Theta(E+E_1)\ . \quad \text{(A5)}$$

Therefore,

$$I_3(E)=\frac{\pi}{2}\frac{e}{\hbar}|B|\frac{r^3}{\alpha}\left[\frac{r}{\alpha}\right]^{1/2}\left[w_E\left[1+\frac{w_E}{2\alpha r}\right]\right]^{1/2} \times\left[\frac{w_E}{\alpha r}\left[2+\frac{w_E}{\alpha r}\right]+\frac{3}{4}\right]\Theta(E+E_1)\ , \quad \text{(A6)}$$

which leads to expression (41) combined with (42).

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