Enhancement in nonlinear effects in percolating nonlinear resistor networks

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The nonlinear response is studied in a percolating network of superconductor and normal conductor with nonlinear *I-V* characteristic below the percolation threshold of the superconductor. The crossover current density $J_{\text{L-NL}}$, defined as the current density at which the linear and nonlinear responses of the network become comparable, is found to have a power-law dependence $J_{\text{L-NL}} \sim (p_c - p)^H$ as the percolation threshold is approached from below. With use of a model of the percolating network *below* p_c analogous to the nodes-links-blobs picture, H is found to be $H = \nu_d(d-1)-1$, where ν_d is the correlation-length exponent and d is the dimensionality of the lattice.

I. INTRODUCTION

The subject of percolation phenomena has attracted much attention over the past decade.[1,2] Studies in the geometric aspects of percolating systems, for example, has led to the use of the idea of fractal objects in physical systems. Transport properties in percolating systems are usually studied within models of random resistor networks.[2] In a network which consists of randomly occupied normal conducting and insulating bonds (N/I systems), the network conducts only above the percolation threshold where there exists a connected path of conducting bonds. Similarly, in a network with superconducting and normal conducting or insulating bonds (S/N or S/I systems), the network becomes superconducting above the percolation threshold of the superconducting bonds. For linear *I-V* response for the conducting bonds, one can define critical exponent $t(s)$ describing the divergence of the network resistance (conductance) in a N/I (S/N) system in the vicinity of the threshold. It is these richnesses in the interplay between geometric and transport properties that have led to a deeper understanding of the physics in macroscopic disordered systems.

In this Brief Report, we consider the geometrical effects of a percolating system on the nonlinear transport properties in a superconductor–normal-conductor nonlinear resistor network. Nonlinear composite systems have recently attracted much interest.[3] Stroud and Hui studied the finite-frequency nonlinear dielectric response of a mixture of nonlinear dielectric in a linear host in the dilute limit of nonlinear constituent, and demonstrated the relation between the nonlinear-random-network problem and the noise problem in linear random network.[4] Zeng *et al.*[5] proposed a general effective-medium-type approximation for calculating the effective nonlinear susceptibility of a mixture for all concentrations. Recently, Blumenfeld and Bergman[6] pointed out that the results in Ref. 4 can be used to derive a characteristic value of the current at which the nonlinear and nonlinear responses in a N/I mixture become comparable. It is the purpose of this paper to study the effects of percolation on the effective nonlinear response of a S/N network below the percolation threshold of the superconductor.

Below the percolation threshold p_c, the conductivity of the whole network is still finite. The *I-V* response of the network, however, is nonlinear due to the nonlinearity of the individual nonlinear bonds. We define a crossover current density $J_{\text{L-NL}}$ as the current density at which the linear response and the nonlinear response of the network become comparable. Using a picture of the percolating network below p_c analogous to the nodes-links-blobs picture often used above p_c, a power-law dependence of $J_{\text{L-NL}}$ is derived. It is found that, near the percolation threshold, $J_{\text{L-NL}} \sim (p_c - p)^H$, where $H = \nu_d(d-1)-1$, where p is the fraction of superconducting bonds, ν_d is the correlation-length exponent in d dimension, and d is the dimensionality. This result shows that the nonlinear response of the network becomes more pronounced as the threshold is approached due to the restricted geometry of the path through which current flows in the vicinity of the threshold.

II. DERIVATION

We consider a d-dimensional hypercubic lattice with fraction p of superconducting bonds and fraction $1-p$ of normal conducting bonds. Each normal bond is assumed to have identical *I-V* response of the form

$$v = ri + bi^{\alpha} \quad (\alpha > 1)\,, \tag{1}$$

where v and i are the voltage across the bond and the current in the bond, respectively. The second term in Eq. (1) represents the nonlinear response of the nonlinear normal bond. Response of the form Eq. (1) has been studied previously in a N/I mixture and also in the finite-frequency regime,[4,7] although most studies in nonlinear percolating systems assumed nonlinear response of the form *without* the linear term in Eq. (1).[8] The form of Eq. (1) is of interest because most materials have linear response in the limit of small current and become nonlinear in the presence of large current. For materials with inversion symmetry, the lowest order nonlinearity refers to $\alpha=3$.

We first consider the case of a full lattice of nonlinear bonds. Let L be the linear dimension of the lattice and a be the lattice constant. If a current I is injected into the lattice, the voltage V across the network is given by

$$V=rI/(L/a)^{d-1}+b[I/(L/a)^{d-1}]^{\alpha}, \quad (2)$$

as $I/(L/a)^{d-1}$ is the current in each path connecting the lattice from one side to another in a full lattice. The injected current density is given by $J=I/L^{d-1}$. Let $J^0_{\text{L-NL}}$ be the current density at which the linear response and the nonlinear response in Eq. (2) become equal in magnitude, then for a full lattice

$$J^0_{\text{L-NL}}=(r/b)^{1/(\alpha-1)}a^{-(d-1)}. \quad (3)$$

As more and more normal bonds are removed and replaced by superconducting bonds, the percolation threshold p_c of the superconductor is approached. Below p_c, the conductance of the network is still finite. The I-V characteristic of the network is nonlinear due to the assumed nonlinearity in each of the individual normal bonds. To consider the percolation effect on the nonlinear response, we invoke a picture[9] for the percolating S/N network similar to that of the "nodes-links-blobs" picture in N/I networks.[10] For $p\leq p_c$, the percolating material (superconducting clusters in this case) can be approximated by an array of nodes separated by the correlation length ξ. Each node is the center of a superconducting cluster of linear size of order ξ. Adjacent clusters are separated by a thin layer of normal bonds. At some places, there is only one normal bond separating the superconducting clusters. These bonds are the "singly disconnected bonds" (SDB's). The number of SDB's diverges as $(p_c-p)^{-1}$ as the percolation threshold is approached from below.[10,11] This picture of the percolating network below p_c has been used successfully to study the problem of fluctuations in resistance,[11] i.e., the noise problem, in percolating networks. Also, the present author and co-workers used this model to study the nonuniversal breakdown behavior in superconducting and dielectric composites.[12]

Using this model of percolating systems below p_c, the nonlinearity of the network can be studied. Let I be the current injected into the lattice, the current passes through each path from one side to another in the array of superconducting clusters and thin layers of normal bonds is approximately given by $I/(L/\xi)^{d-1}$, as there are $(L/\xi)^{d-1}$ paths assuming a regular array. Let L_1 be the number of SDB's in the thin layer separating adjacent superconducting clusters. As mentioned above, $L_1\sim(p_c-p)^{-1}$. The current in each SDB is thus given by $\tilde{i}=I/(L/\xi)^{d-1}L_1$, if we neglect the effects of multiply connected regions, i.e., places where adjacent clusters are separated by more than one bond.[13] The voltage $\tilde{v}$ across the layer of normal bonds separating two adjacent clusters is given by

$$\tilde{v}=r\tilde{i}+b\tilde{i}^{\alpha}. \quad (4)$$

The voltage V across the system is then given by

$$V=(L/\xi)\tilde{v} \quad (5)$$

due to the assumed geometry of the percolating network below p_c.

Substituting the form of $\tilde{i}$ and Eq. (4), Eq. (5) can be rewritten as

$$V=r\frac{L}{\xi}\frac{\xi^{d-1}}{L_1}J+b\frac{L}{\xi}\left[\frac{\xi^{d-1}}{L_1}\right]^{\alpha}J^{\alpha}, \quad (6)$$

where $J=I/L^{d-1}$ is the injected current density. We define the crossover current density $J_{\text{L-NL}}$ as the current density at which the linear and nonlinear terms in Eq. (6) become equal in magnitude. Hence,

$$J_{\text{L-NL}}=\left[\frac{r}{b}\right]^{1/(\alpha-1)}L_1\xi^{-(d-1)}. \quad (7)$$

The correlation length ξ diverges as $\xi\sim a(p_c-p)^{-\nu_d}$, where ν_d is the exponent characterizing the divergence in a d-dimensional system. Substituting this result and $L_1\sim(p_c-p)^{-1}$ into Eq. (7), we obtain

$$J_{\text{L-NL}}=J^0_{\text{L-NL}}(p_c-p)^{H}, \quad (8)$$

with the exponent H given by

$$H=\nu_d(d-1)-1. \quad (9)$$

The crossover current density $J_{\text{L-NL}}$ thus decreases to zero following a power law as the percolation threshold is approached from below and H is the exponent characterizing the power-law behavior. Note that with the present model for the percolating network, H does not depend on α, i.e., the crossover current density behaviors the same way independent of the detail of the nonlinear behavior of individual normal bonds.

III. DISCUSSION

Near the percolation threshold, $J_{\text{L-NL}}$ vanishes with a power law with exponent $H=\nu_d(d-1)-1$. To show that this is the case, we use the standard values for ν_d[14,15] which give $H=\frac{1}{3}$ and 0.76 for $d=2$ and 3, respectively.[16] The exponent H increases as dimensionality increases and takes on the value $H=\frac{3}{2}$ for $d=6$, the upper critical dimension of percolation. Hence, $H>0$ for all d.

The result Eqs. (8) and (9) implies that due to the restricted geometry near the threshold, the nonlinear behavior is enhanced relative to the case of a full lattice of nonlinear bonds. This result will be of potential practical use because such percolating S/N composite has a high conductance and yet is highly nonlinear. It is well known that the system of N/S composites can be related to a N/I composite if the dielectric constants are considered instead of the conductivities, as the imaginary part of the dielectric function of a normal component diverges at the low-frequency limit and thus plays the role of superconductor in a S/N composite.[17] Thus, with slight modification, the result here can be used in $N/1$ composites in the low-frequency regime. It will be interesting to see if other methods such as the effective- medium approximation,[5] real-space renormalization analysis,[18] and computer stimulations give similar result as the simple scaling theory presented here.

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