Magnetic field and temperature dependence of the self-trapping energy of a polaron in a polar-crystal slab in arbitrary magnetic field strength

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Taking into account the interaction of an electron with both bulk longitudinal-optical (BO) and surface longitudinal-optical (SO) phonons, we study the self-trapping energy of a magnetopolaron in a polar-crystal slab as a function of magnetic field and finite temperature, by using the generalized Larsen perturbation-theory method. The temperature dependence of the self-trapping energies $E_{\rm BO}$ and $E_{\rm SO}$ are found to be strongly dependent upon the strength of the magnetic field. The results also show that the self-trapping energy $E_{\rm SO}$, as well as the self-trapping energy $E_{\rm BO}$, plays an important role. However for thin slabs, $E_{\rm SO}$ plays the main role.

I. INTRODUCTION

Recently, with the development of techniques to fabricate heterostructures and superlattices, the properties of a polaron in a polar-crystal slab have aroused great interest, especially regarding the influence on the properties of the polaron by a magnetic field and by temperature.

In the past, most work on polarons in a polar-crystal slab was devoted to the calculation of the ground-state energy and the effective mass of polarons at zero temperature, and to the discussion of the dependence of the polaron's properties on the electron-phonon coupling strength.[1-5] Licari and Evrand derived a Hamiltonian for the electron-phonon interaction in a slab, which included the interaction of electron with bulk longitudinal-optical (BO) phonons and surface longitudinal-optical (SO) phonons.[1] Some studies[2-5] of the polaron in a polar-crystal slab have been carried out, but none of these took into account the electron–SO-phonon interaction. Thinking that when the thickness of the slab is comparable to the radius of the polaron, the electron–SO-phonon interaction is even stronger than the electron–BO-phonon interaction, some authors[6-11] have studied the electron–SO-phon interaction together with the electron–BO-phonon interaction. In recent years the properties of a polaron in a magnetic field have also attracted attention.[12,13] Larsen[13] developed a perturbational theory for two-dimensional (2D) magnetopolarons and Gu and his collaborators[8,9] applied Larsen's method to investigate the interface and slab magnetopolaron, simultaneously taking into account the interaction of electrons with both BO and SO phonons. However, the temperature dependence of the properties of a magnetopolaron was not reported in this earlier work.

In recent years there has been renewed interest in the temperature dependence of the properties of polarons. The different assumptions on the mechanism of the electron-phonon interaction and the different theoretical methods applied have led to ever significant different dependences of the polaron mass on temperature. In the early studies, Yokota[14] approximately calculated the energies of the polaron by using the Hartree method, and came to the conclusion that the polaron mass decreases with increasing temperature. This result was also obtained in Refs. 15–18 at sufficiently low lattice temperatures. However, by using the Gurari variational method, Fulton[19] obtained a contrary result. In Refs 12 and 20–23, the polaron mass was also found to be an increasing function of temperature at sufficiently low lattice temperatures. A compromise between these opposite temperature dependences of the polaron mass is obtained by extending Feynman's polaron theory[24] to finite temperature.[25] With Feynman's path-integral polaron theory, it was found[21,25,26] that with increasing temperature the polaron mass first increases at low temperatures, subsequently reaching a maximum value at a certain finite temperature, and for still higher temperatures it starts to decrease. Recently, Wu *et al.*[27] also found this behavior.

The electron–optical-phonon interaction in a polar-crystal slab plays an important role in determining the temperature dependence of the properties of a magnetopolaron. The purpose of this paper is to study how the electron–optical-phonon interactions affect the temperature dependence of the properties of a magnetopolaron in a polar-crystal slab. Taking into account the interaction of an electron with both BO and SO phonons, we have derived for the first time the self-trapping energy of a magnetopolaron in a polar-crystal slab as a function of the strength of the magnetic field and temperature in arbitrary magnetic field strength at finite temperature by using the generalized Larsen perturbational-theory method. Taking GaAs as an example, we make the calculations of the self-trapping energies $E_{\rm BO}$ and $E_{\rm SO}$, which arose from the electron–BO-phonon interaction and the electron–SO-phonon interaction, respectively, as a function of magnetic field strength B, temperature T, and the thickness of the slab, N, in arbitrary B at finite T. The results show a strong temperature dependence of $E_{\rm BO}$ and $E_{\rm SO}$ on the magnetic field strength B. As the temperature increases, $|E_{\rm BO}|$ and $|E_{\rm SO}|$ decrease in the weak magnetic field, while they increase in the strong magnetic field. The results also show that the electron–SO-phonon interaction plays an important role, especially when the slab is thin. The present paper is organized as follows: in Sec. II we write out the effective Hamiltonian and self-trapping energy for the system, Sec. III contains our numerical results and our conclusion is presented in Sec. IV.

II. EFFECTIVE HAMILTONIAN AND SELF-TRAPPING ENERGY

Now we consider a slab of polar crystal, whose thickness is $2d$. The static uniform magnetic field $\mathbf{B}$ is alone the z direction (see Fig. 1). We employ the symmetric gauge vector potential $\mathbf{A}=B(-y/2,x/2,0)$ for the magnetic field. Under the isotropic effective-mass approximation, the Hamiltonian of the system, which includes the interaction of the electron with both BO and SO phonons in the slab, can be written as[8]

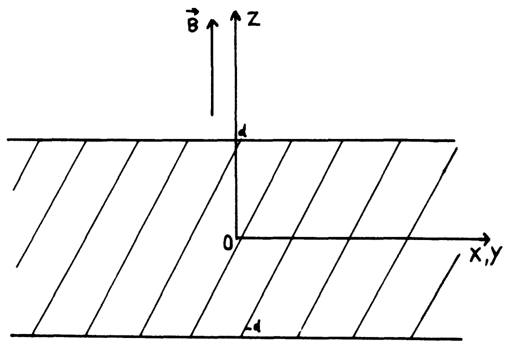

FIG. 1. Geometry of a polar-crystal slab.

$$H=H_{\parallel}+H_{\perp}, \quad H_{\parallel}=H_0=H_0+H_I , \tag{1a}$$

$$H_0=H_{xy}+H_{\rm ph}, \quad H_L=H_{e\text{-BO}}+H_{e\text{-SO}} , \tag{1b}$$

where

$$H_{xy}=\frac{1}{2m_b}\left[p_\chi-\frac{\beta^2}{4}y\right]^2+\frac{1}{2m_b}\left[p_y+\frac{\beta^2}{4}x\right]^2 \tag{1c}$$

is the contribution from the electron moving in the x-y plane,

$$H_{\rm ph}=\sum_{k,m,p}\hbar\omega_{\rm BO}a^\dagger_{k,m,p}a_{k,m,p}+\sum_{q,p}\hbar\omega_{s,p}b^\dagger_{s,p}b_{s,p} \tag{1d}$$

is the contribution from BO and SO phonons,

$$H_{\perp}=p_z^2/2m_b+V_{\rm im}(z) \tag{1e}$$

is the z direction contribution, where

$$V_{\rm im}(z)=\frac{e^2}{2\epsilon_\infty d}\left[\frac{\epsilon_\infty-1}{\epsilon_\infty+1}\right]\left[\frac{z^2}{z^2-d^2}\right] \tag{1f}$$

is the surface image-potential energy,[8] and $H_{e\text{-BO}}$ (Ref. 1) is the electron–BO-phonon interaction given by

$$H_{e\text{-BO}}=\sum_k\left[F^*\exp(-i\mathbf{k}\cdot\boldsymbol{\rho})\left[\sum_{m=1,2,5,\ldots}\frac{\cos(m\pi z/2d)}{[k^2+(m\pi/2d)^2]^{1/2}}a^\dagger_{k,m,+}+\sum_{m=2,4,6,\ldots}\frac{\sin(m\pi z/2d)}{[k^2+(m\pi/2d)^2]^{1/2}}a^\dagger_{k,m,-}\right]+\text{H.c.}\right] . \tag{1g}$$

$H_{e\text{-SO}}$ (Ref. 1) is the electron–SO-phonon interaction given by

$$H_{e\text{-SO}}=\sum_q\left[\frac{\sinh(2qd)}{q}\right]^{1/2}\exp(-qd)\{C^*\exp(-i\mathbf{q}\cdot\boldsymbol{\rho})[G_+(q,z)b^\dagger_{q,+}+G_-(q,z)b^\dagger_{q,-}]+\text{H.c.}\} \tag{1h}$$

and

$$G_+(q,z)=\frac{\cosh(qz)/\cosh(qd)}{(\epsilon_\infty+1)-(\epsilon_\infty-1)\exp(-2qd)}\left[\frac{(\epsilon_\infty+1)-(\epsilon_\infty-1)\exp(-2qd)}{(\epsilon_0+1)-(\epsilon_0-1)\exp(-2qd)}\right]^{1/4} , \tag{1i}$$

$$G_-(q,z)=\frac{\sinh(qz)/\sinh(qd)}{(\epsilon_\infty+1)+(\epsilon_\infty-1)\exp(-2qd)}\left[\frac{(\epsilon_{in}+1)+(\epsilon_\infty-1)\exp(-2qd)}{(\epsilon_0+1)+(\epsilon_0-1)\exp(-2qd)}\right]^{1/4} . \tag{1j}$$

In the above expressions, m_b is the band mass of the electron we have assumed that it is isotropic for simplicity; **k** and **q** are the plane wave vectors of BO and SO phonons, respectively, in the x-y plane. The subscript p stands for the parity. For even parity, p is positive, and for odd parity, p is negative. m is the quantum number of BO-phonon wave vectors in the z direction. When p is positive (or negative), m is an odd (or even) positive integer. $a^{\dagger}_{k,m,p}$ ($a_{k,m,p}$) and $b^{\dagger}_{q,p}$ ($b_{q,p}$) are the creation (annihilation) operators of BO and SO phonons, respectively. $\omega_{\rm BO}$ and $\omega_{s,p}$ are the frequencies of BO and SO phonons, respectively.

The relation between the frequency of bulk transverse-optical phonons, $\omega_{\rm TO}$, and one of the BO phonons, $\omega_{\rm BO}$, is determined by the Lyddane-Sachs-Teller (LST) relation

$$\omega_{\rm BO}^2/\omega_{\rm TO}^2=\epsilon_0/\epsilon_\infty \tag{2a}$$

and the frequency of BO and SO phonons satisfies the equality

$$\omega_{s,\mp}^2=\left[\frac{(\epsilon_0+1)\pm(\epsilon_0-1)\exp(-2qd)}{(\epsilon_\infty+1)\pm(\epsilon_\infty-1)\exp(-2qd)}\right]\omega_{\rm BO}^2\ , \tag{2b}$$

where ϵ_∞ and ϵ_0 are the optical and static dielectric constants of the polar crystal, respectively, and

$$\beta^2=2eB/c\ , \tag{3}$$

$$F^*=\left[\frac{4\pi e^2}{V}\hbar\omega_{\rm BO}\left[\frac{1}{\epsilon_\infty}-\frac{1}{\epsilon_0}\right]\right]^{1/2}, \tag{4a}$$

$$C^*=-\left[\frac{2\pi e^2}{A}\hbar\omega_{\rm TO}(\epsilon_0-\epsilon_\infty)\right]^{1/2}, \tag{4b}$$

where V is the volume of the crystal 1 and A its surface area.

We regard $H_\parallel$ as the "transverse" Hamiltonian and $H_\perp$ as the "normal" Hamiltonian. For a thin slab, the motion of the electron along the z axis is almost in a certain state. Therefore, we can use an approximation which is analogous to the adiabatic approximation[28] to find the effective Hamiltonian of the system. Let us first seek for the energy of the "transverse" motion that depends on the parameter z. Then regard it as an "adiabatic potential" and add it to the normal $H_\perp$ to find the effective Hamiltonian of the magnetopolaron.

In the following, we shall use Larsen's[13] method to calculate $H_\parallel$. We introduce two harmonic-oscillator operators

$$A=\frac{1}{\sqrt{\hbar}\beta}\left[\left[p_x-\frac{\beta^2}{4}y\right]-i\left[p_y+\frac{\beta^2}{4}x\right]\right], \tag{5a}$$

$$D=A^\dagger-i\frac{\beta}{2\sqrt{\hbar}}(x+iy)\ , \tag{5b}$$

which satisfy the commutation relations of bosons,

$$[A,A^\dagger]=[D,D^\dagger]=1 \text{ and } [A,D]=[A,D^\dagger]=0\ . \tag{5c}$$

In the first step, we treat $H_\parallel=H_0+H_I$. We shall take $H_0=H_{xy}+H_{\rm ph}$ as the unperturbed Hamiltonian and H_I as the perturbational one.

At finite temperature, we choose $|\{N_{k,m,p}\},\{N_{q,p}\}\rangle$ for the wave function of the phonon state, in which $N_{k,m,p}$ and $N_{q,p}$ represent the number of BO and SO phonons, respectively.

As shown in Ref. 29, the phonon frequency will decrease with the increasing temperature, but if the temperature is restricted to the range lower than the room temperature ($T<300$ K), the relative change of the frequency ($|\Delta\omega|/\omega$) is only 1%. Then we can take the phonon frequency approximately as constant. In addition, we also approximately omit the electron-phon interaction energy because of its very small values compared with $\hbar\omega_{\rm BO}$ (or $\hbar\omega_{\rm TO}$) except for the strong-coupling case. So, with the consideration of taking the phonon frequencies as the constant values and in the absence of the electron-phonon interactions at finite temperature, we assume that the eigenvalues of $a^\dagger a$ and $b^\dagger b$ in the phonon state $|\{N_{k,m,p}\},\{N_{q,p}\}\rangle$ are approximately equal to the thermal equilibrium values,[30] i.e.,

$$\bar{N}_k=\langle a^\dagger_{k,m,p}a_{k,m,p}\rangle=[\exp(\hbar\omega_{\rm BO}/k_BT)-1]^{-1}\ , \tag{6a}$$

$$\bar{N}_{q,p}=\langle b^\dagger_{q,p}b_{q,p}\rangle=[\exp(\hbar\omega_{s,p}/k_BT)-1]^{-1}\ , \tag{6b}$$

where k_B is the Boltzman constant.

Having introduced the operators A and D, we can represent the Landau levels as products of two independent one-dimensional (1D), harmonic-oscillator states, which are

$$|n\rangle_A=(n!)^{1/2}(A^\dagger)^n|0\rangle_A$$

and (7)

$$|M\rangle_D(=M!)^{1/2}(D^\dagger)^M|0\rangle_D\ ,$$

where n is the Landau quantum number, and M is the angular momentum projection quantum number relative to the z axis. In the occupation-number picture, we have

$$A^\dagger|n\rangle_A=(n+1)^{1/2}|n+1\rangle_A,\quad A|n\rangle_A=n^{1/2}|n-1\rangle_A\ , \tag{8a}$$

$$D^\dagger|M\rangle_D=(M+1)^{1/2}|M+1\rangle_D,\qquad D|M\rangle_D=M^{1/2}|M-1\rangle_D\ . \tag{8b}$$

At finite temperature, the unperturbed-state wave function is

$$|n,M,N_{k,+},N_{k,-},N_{q,+},N_{q,-}\rangle=|n\rangle_A\,|M\rangle_D\,|N_{k,+}\rangle\,|N_{k,-}\rangle\,|N_{q,+}\rangle\,|N_{q,-}\rangle\ , \tag{9}$$

and the unperturbed-state expected energy value is

$$E_n(n+\tfrac{1}{2})\hbar\beta^2/2m_b+\sum_{k,m,p}\bar{N}_k\hbar\omega_{\rm BO}+\sum_{q,p}\hbar\omega_{s,p}\ . \tag{10}$$

In the following, we only consider the $n=0$ Landau ground state for the sake of calculational ease; the case in the first excited state will be shown in the next paper studying the cyclotron resonance of a magnetopolaron in a polar-crystal slab. After a tedious but direct calculation (see Appendix), we can obtain the effective Hamiltonian as

$$H_{\rm eff}(z)=p_z^2/2m_b+V_{\rm im}(z)+E_0+\Delta E_0^{(2)}(z)\ , \tag{11}$$

where $\Delta E_0^{(2)}(z)$ is given in the Appendix.

$V_{\rm im}(z)+\Delta E_0^{(2)}(z)$ in the $|z|<d$ range is approximately a square-well potential. Thus the z-direction wave function of the electron can be approximately described by the wave function of a particle moving freely in an infinite one-dimensional square well with a $2d$ width, i.e.,

$$\Phi_L(z)=(1/d)^{1/2}\sin[L\pi(z+d)/2d],\quad |z|\le d\ . \tag{12}$$

The self-trapping energy of a magnetopolaron in a polar-crystal slab can be found as

$$E_{\rm BO}=\langle\Phi_L(z)|[\Delta E_{k,+}^{(2)}(z)+\Delta E_{k,-}^{(2)}(z)]|\Phi_L(z)\rangle$$

$$=-\frac{e^2}{4d}\left[\frac{1}{\epsilon_\infty}-\frac{1}{\epsilon_0}\right]\sum_i\frac{1}{i!}\left[\left[\frac{\bar N_k+1}{i\lambda_B^2+1}+\frac{\bar N_k}{i\lambda_B^2-1}\right]\left[\sum_{m=1}^{N/2}I_B(i,m,d)+I_B(i,L,d)\right]\right]\ , \tag{13a}$$

$$E_{\rm SO}=\langle\Phi_L(z)|[\Delta E_{q,+}^{(2)}(z)+\Delta E_{q,-}^{(2)}(z)]|\Phi_L(z)\rangle$$

$$=-\frac{e^2}{2d}(\epsilon_0-\epsilon_\infty)\sum_i\frac{1}{i!}\left\{\left[\left[\frac{\bar N_{q,+}+1}{i\lambda_{s,+}^2+1}+\frac{\bar N_{q,t}}{i\lambda_{s,+}^2-1}\right]I_{s,+}(i,d)\right]+\left[\left[\frac{\bar N_{q,-}+1}{i\lambda_{s,-}^2+1}+\frac{\bar N_{q,-}}{i\lambda_{s,-}^2-1}\right]I_{s,-}(i,d)\right]\right\}\ , \tag{13b}$$

where

$$I_B(i,m,d)=\int_0^\infty dy\left[\frac{\hbar y}{\beta^2}\right]^2\exp\left[-\frac{\hbar y}{\beta^2}\right]\frac{1}{y+(m/2d)^2}\ , \tag{13c}$$

$$I_{s,+}(i,d)=\int_0^\infty dy\frac{\exp(-y)}{y}\exp\left[-\frac{\hbar y^2}{4d^2\beta^2}\right]\left[\frac{\hbar y^2}{4d^2\beta^2}\right]^i\frac{\tanh(y/2)[y+\sinh(y)/(1+y^2/L^2\pi^2)]}{[(\epsilon_\infty+1)+(\epsilon_\infty-1)\exp(-y)][(\epsilon_0+1)+(\epsilon_0-1)\exp(-y)]}\ , \tag{13d}$$

$$I_{s,-}(i,d)=\int_0^\infty dy\frac{\exp(-y)}{y}\exp\left[-\frac{\hbar y^2}{4d^2\beta^2}\right]\left[\frac{\hbar y^2}{4d^2\beta^2}\right]^i\frac{\coth(y/2)[\sinh(h)/(1+y^2/L^2\pi^2)-y]}{[(\epsilon_\infty+1)-(\epsilon_\infty-1)\exp(-y)][(\epsilon_0+1)-(\epsilon_0-1)\exp(-y)]}\ . \tag{13e}$$

N in Eqs. (13) satisfies the equality $Na=2d$, where a is the lattice constant of the crystal.

$E_{\rm BO}$ and $E_{\rm SO}$ are the self-trapping energies of a magnetopolaron in a polar-crystal slab arising from electron–BO-phonon interactions and electron–SO-phonon interaction, respectively. It is easy to show that the unlimited series in the above equation are all convergent.

III. NUMERICAL RESULTS

GaAs is taken as an example to evaluate numerically the self-trapping energies of a magnetopolaron in a polar-crystal slab. The characteristic parameters in our numerical computation are taken from Ref. 31: $\epsilon_0=12.83$, $\epsilon_\infty=10.9$, $\hbar\omega_{\rm BO}=36.7$ meV, $a=5.654$ Å, and the electron-phonon coupling constant $\alpha=0.067$.

The results presented in Figs. 2–6 show that the temperature dependence of the self-trapping energies of a magnetopolaron in a polar-crystal slab depend tremendously on the strength of the magnetic field B. In the present case $\hbar\omega_{\rm BO}=36.7$ meV, and $\hbar\omega_{s,+}$ and $\hbar\omega_{s,-}$ are both approaches to $\hbar\omega_{\rm BO}$; that is, the difference among them is small, so we can approximately define that the weak magnetic field, i.e., $\omega_c<\omega_{\rm BC},\omega_{s,p}$, is in the case in $B<20$ T ($\hbar\omega_c=35.2$ meV), and the strong magnetic field, i.e., $\omega_c>\omega_{BO},\omega_{s,p}$, is in the case $B>21.5$ T ($\hbar\omega_c=37.8$ meV) and the resonant range is in the case in $20<B<21.5$ T. Figure 2 describes the self-trapping energy $E_{\rm BO}$ as a function of temperature T for a different slab thickness N in a strong magnetic field $B=22$ T ($\hbar\omega_c=38.8$ meV), and Fig. 3 describes $E_{\rm BO}$ as a function of N at different temperature T in weak magnetic field

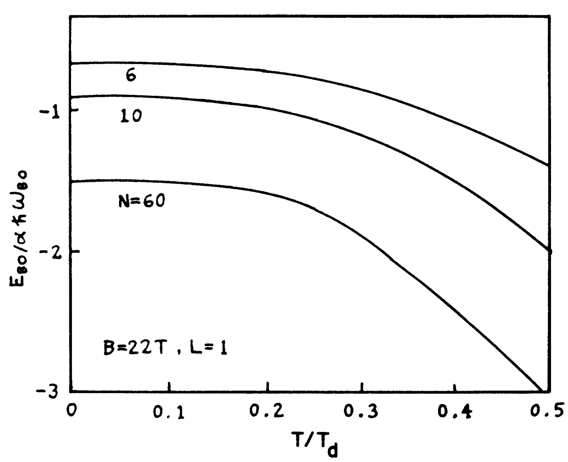

FIG. 2. $E_{\rm BO}$ vs T for the different thickness of the slab N in a strong magnetic field, $B=22$ T, where $T_d=426$ K.

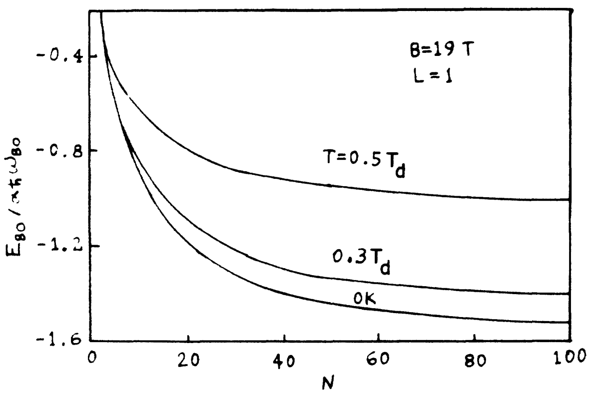

FIG. 3. E_{BO} vs N for different temperature T in the weak magnetic field $B=19$ T, where $Na=2d$, $T_D=426$ K.

$B=19$ T ($\hbar\omega_c=33.5$ meV). At the same time, Fig. 4 describes the self-trapping energy E_{SO} as a function of T for different N in a weak magnetic field $B=19$ T, and Fig. 5 describes E_{SO} as a function of N at different T in strong magnetic field $B=22$ T. We can see that the abstract of the self-trapping energies $|E_{BO}|$ and $|E_{SO}|$ both decrease with an increase of the temperatures in the weak magnetic field; that is to say the self-trapping of the magnetopolaron will be weakened with increasing temperature in the weak magnetic field. On the other hand, we obtain the contrary results in the strong magnetic field; that is, the self-trapping of a magnetopolaron will be strengthened with the increasing temperature in the strong magnetic field. From Figs. 3 and 5, we can see that E_{BO} and E_{SO} are both dependent obviously on slab thickness N and their changing with the changing of temperature is also dependent on N. The thicker the polar-crystal slab, more obvious changing of the self-trapping energy E_{BO} with the changing temperature; the thinner the polar-crystal slab, the more obvious the changing of the self-trapping energy E_{SO} with the changing temperature. The most important thing is that when the slab

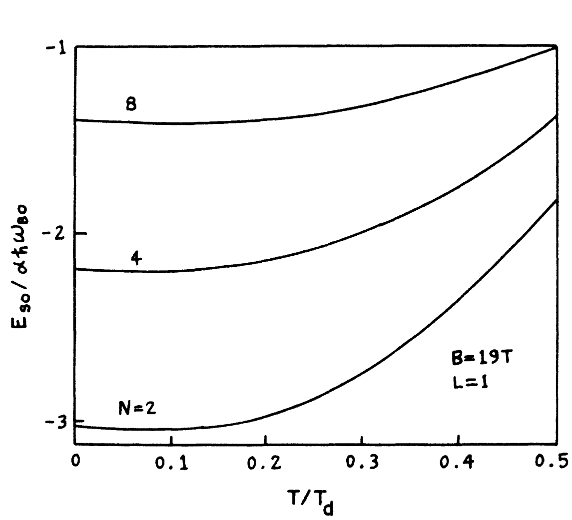

FIG. 4. E_{SO} vs T for the different thickness of the slab N in a weak magnetic field, $B=19$ T, where $T_D=426$ K.

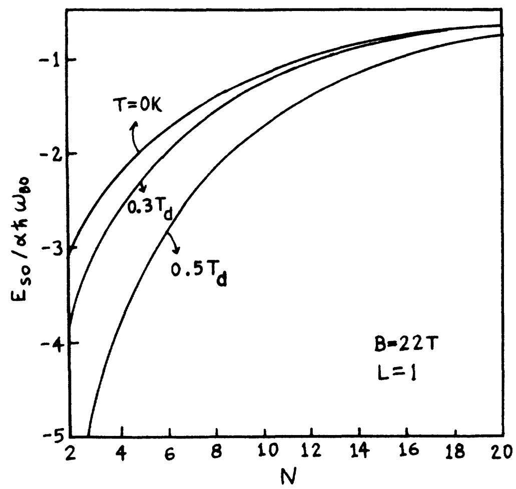

FIG. 5. E_{SO} vs N for different temperature T in a strong magnetic field, $B=22$ T, where $Na=2d$, $T_D=426$ K.

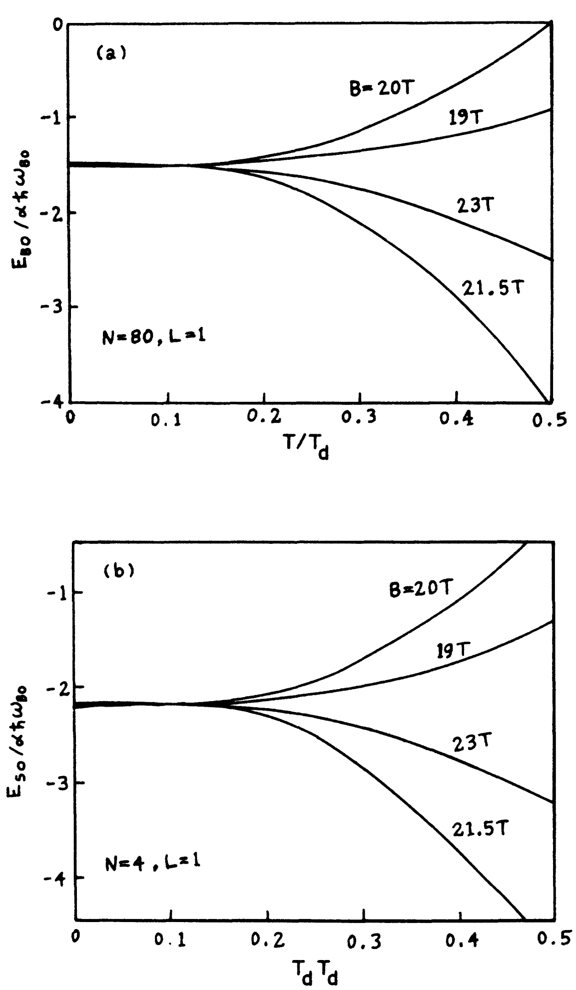

FIG. 6. (a) E_{BO} vs T in different magnetic field strengths B for slab thickness $N=80$, where $Na=2d$, $T_D=426$ K. (b) E_{SO} vs T in different magnetic field strengths B for slab thickness $N=4$, where $N_a=2d$, $T_D=426$ K.

thickness is small, especially when the slab thickness approaches the radius of the polaron, the value of E_{SO} and its changing with temperature are both noticeable. On the other hand, the value of E_{BO} and its changing with temperature are both insignificant when the slab thickness is very small.

Figure 6(a) describes the E_{BO} as a function of temperature in different magnetic field strengths B for the slab thickness $N=80$. Figure 6(b) describes the E_{SO} as a function of temperature in different magnetic field strengths B for the slab thickness $N=4$. We can see from Fig. 6 that in the weak-magnetic-field range, i.e., $B<20$ T, the abstraction of the self-trapping energy $|E_{BO}|$ and $|E_{SO}|$ are both decreasing functions of the temperature; however, in the strong-magnetic-field range, i.e., $B>21.5$ T, $|E_{BO}|$ and $|E_{SO}|$ are both increasing functions of temperature. We can also see that the closer B is to the resonant range, the more obvious the effect of the temperature on the self-trapping energies. The case when B is in the resonant range, i.e., $20<B<21.5$ T, will be discussed in a planned study of the cyclotron resonance of a magnetopolaron in a polar-crystal slab or quantum well.

IV. CONCLUSION

Taking into account the interaction of an electron with both BO phonons and SO phonons in a polar-crystal slab, we use a generalized Larsen perturbational-theory method to study the magnetic field and temperature dependence of the self-trapping energy of a magnetopolaron. The results show that the changing of the self-trapping energies of a magnetopolaron in a polar-crystal slab with temperature will be different with different magnetic fields. As the temperature increases, the self-trapping of a slab magnetopolaron will be weakened in the weak-magnetic-field range, but it will be strengthened in the strong-magnetic-field range. When B is close to the resonant range, the self-trapping energies of a slab magnetopolaron become highly sensitive to temperature. Having neglected the effect of the absorbing virtual-phonon process in the process of electron-phonon interaction, Fulton[19] obtained that the mass of a polaron increased with temperature at intermediate temperatures. But in the present paper we show numerically that the absorbing virtual-phonon process must be considered, especially for the magnetopolaron. The absorption process will weaken the self-trapping of the magnetopolaron in the weak magnetic field, but it will strengthen the self-trapping of a magnetopolaron in the strong magnetic field. The emission process will always strengthen the self-trapping of a magnetopolaron.

In a polar-crystal slab, the electron–SO-phonon interaction plays an important role in the self-trapping energy of a magnetopolaron, especially when the slab is thinner. From the numerical results we can see that when the thickness of the slab is comparable to the radius of polaron, the electron–SO-phonon interaction is stronger than the electron–BO-phonon interaction and the changing of the self-trapping energy E_{SO} with temperature is also more obvious than that of the self-trapping energy E_{BO}. The thinner the slab, the more obvious the phenomenon stated above. On the other hand, when the slab is very thick, the electron–SO-phonon interaction is relatively small compared to the electron–BO-phonon interaction, and the changing of E_{SO} with temperature is also slower than that of E_{BO}. So we can neglect the effect of the electron–SO-phonon interaction for a very thick slab, but we must consider it for the thin slab. For example, in the quasi-two dimensional (Q2D) system, we must consider the electron–SO-phonon interaction.

The method used in this paper can be available for arbitrary magnetic field strength B, and the range of the temperature is approximately restricted to $T<0.5T_D$, where $T_D=426$ K is the Debye temperature. The method can be conveniently generalized to study the quantum-well problem.

APPENDIX

Having introduced the operator A and D in Eq. (5), H_{xy} and H_I can be rewritten as

$$H_{xy}=(\hbar\beta^2/2m_b)(A^\dagger A+\tfrac{1}{2}) , \tag{A1}$$

$$H_I=\sum_k \left[F^* L_k M_k \left[\sum_{m=1,3,5,\ldots} \frac{\cos(m\pi z/2d)}{[k^2+(m\pi/2d)^2]^{1/2}} a^\dagger_{k,m,+} + \sum_{m=2,4,6,\ldots} \frac{\sin(m\pi z/2d)}{[k^2+(m\pi/2d)^2]^{1/2}} a^\dagger_{k,m,-} \right] + \text{H.c.} \right] + \sum_q \left[\frac{\sinh(2qd)}{q} \right]^{1/2} \exp(-qd)\{ C^* L_q M_q [G_+(q,z) b^\dagger_{q,+} + G_-(q,z) b^\dagger_{q,-}] + \text{H.c.}\} , \tag{A2}$$

where

$$L_k=\exp[(\hbar^{1/2}/\beta)(k_x+ik_y)A-(\hbar^{1/2}/\beta)(k_x-ik_y)A^\dagger] , \tag{A3}$$

$$M_k=\exp[(\hbar^{1/2}/\beta)(k_x-ik_y)D-(\hbar^{1/2}/\beta)(k_x+ik_y)D^\dagger] , \tag{A4}$$

$$L_q=\exp[(\hbar^{1/2}/\beta)(q_x+iq_y)A-(\hbar^{1/2}/\beta)(q_x-iq_y)A^\dagger] , \tag{A5}$$

$$M_q=\exp[(\hbar^{1/2}/\beta)(q_x-iq_y)D-(\hbar^{1/2}/\beta)(q_x+iq_y)D^\dagger] . \tag{A6}$$

In order to calculate the correction in second order conveniently, H_I can be divided into the following four terms:

$$H_I = H_{k,+} + H_{k,-} + H_{q,+} + H_{q,-} , \tag{A7}$$

where

$$H_{k,+} = \sum_k \left[F^* L_k M_k \left[\sum_{m=1,3,\ldots} \frac{\cos(m\pi z/2d)}{[k^2+(m\pi/2d)^2]^{1/2}} a^\dagger_{k,m,+} \right] + \text{H.c.} \right] , \tag{A8}$$

$$H_{k,-} = \sum_k \left[F^* L_k M_k \left[\sum_{m=2,4,\ldots} \frac{\sin(m\pi z/2d)}{[k^2+(m\pi/2d)^2]^{1/2}} a^\dagger_{k,m,-} \right] + \text{H.c.} \right] , \tag{A9}$$

$$H_{q,+} = \sum_q \left[\frac{\sinh(2qd)}{q} \right]^{1/2} \exp(-qd)[C^* L_q M_q G_+(q,z) b^\dagger_{q,+} + \text{H.c.}] , \tag{A10}$$

$$H_{q,-} = \sum_q \left[\frac{\sinh(2qd)}{q} \right]^{1/2} \exp(-qd)[C^* L_q M_q G_-(q,z) b^\dagger_{q,-} + \text{H.c.}] . \tag{A11}$$

Because H_I can be written as the four parts stated above, the crossed terms, which are from the states with different wave vectors or different parities, will disappear in the perturbation energy to second order. Then the correction to second order is

$$\Delta E_n^{(2)} = \Delta E^{(2)}_{n,k,+} + \Delta E^{(2)}_{n,k,-} + \Delta E^{(2)}_{n,q,+} + \Delta E^{(2)}_{n,q,-} , \tag{A12}$$

where

$$\begin{aligned}
\Delta E^{(2)}_{n,k,+} &= \sum_{m',M',N'_{k,+}} \frac{|\langle n,M,N_{k,+},N_{k,-},N_{q,+},N_{q,-}|H_{k,+}|n',M',N'_{k,+},N_{k,-},N_{q,+}N_{q,-}\rangle|^2}{E_n - E_{n'}} \\
&= \sum_k \left[F^2 \left[\sum_{m=1,3,5,\ldots} \frac{\cos^2(m\pi z/2d)}{k^2+(m\pi/2d)^2} \right] \left[\sum_{n'} \frac{(\bar{N}_k+1)\, {}_A\langle n|L_k|n'\rangle_A\, {}_A\langle n'|L_k^\dagger|n\rangle_A}{(n-n')\hbar\beta^2/2m_b - \hbar\omega_{\text{BO}}} \right.\right. \\
&\qquad \left.\left. + \sum_{n'} \frac{\bar{N}_k\, {}_A\langle n|L_k|n'\rangle_A\, {}_A\langle n'|L_k^\dagger|n\rangle_A}{(n-n')\hbar\beta^2/2m_b + \hbar\omega_{\text{BO}}} \right] \right] .
\end{aligned} \tag{A13}$$

$\Delta E^{(2)}_{n,k,-}$, $\Delta E^{(2)}_{n,q,+}$, and $\Delta E^{(2)}_{n,q,-}$ can be expressed just the same as (A13), and for the sake of clarity, we do not write them out.

In the above expression, the first term corresponds to the emission of a virtual phonon during the electron-phonon interaction, and the second term corresponds to the absorption of a virtual phonon in the process of the electron-phonon interaction.[19]

For the $n=0$ Landau ground state the index function of operators A and D in $L_k^\dagger$ (L_k) and $L_q^\dagger$ (L_q) can be expanded into unlimited series, and after a tedious but direct calculation we can obtain the following results:

$$\Delta E^{(2)}_{k,+} = -\frac{e^2}{2d} \left[\frac{1}{\epsilon_\infty} - \frac{1}{\epsilon_0} \right] \sum_i \left[\frac{1}{i!} \left[\frac{\bar{N}_k+1}{i\lambda_B^2+1} + \frac{\bar{N}_k}{i\lambda_B^2-1} \right] I_{k,+}(i,z) \right] , \tag{A14}$$

$$\Delta E^{(2)}_{k,-} = -\frac{e^2}{2d} \left[\frac{1}{\epsilon_\infty} - \frac{1}{\epsilon_0} \right] \sum_i \left[\frac{1}{i!} \left[\frac{\bar{N}_k+1}{i\lambda_B^2+1} + \frac{\bar{N}_k}{i\lambda_B^2-1} \right] I_{k,-}(i,z) \right] , \tag{A15}$$

$$\Delta E^{(2)}_{q,+} = -2e^2(\epsilon_0 - \epsilon_\infty) \sum_l \left[\frac{1}{i!} \left[\frac{\bar{N}_{q,+}+1}{i\lambda_{s,+}^2+1} + \frac{\bar{N}_{g,+}}{i\lambda_{s,+}^2-1} \right] I_{q,+}(i,z) \right] , \tag{A16}$$

$$\Delta E^{(2)}_{q,-} = -2e^2(\epsilon_0 - \epsilon_\infty) \sum_i \left[\frac{1}{i!} \left[\frac{\bar{N}_{q,-}+1}{i\lambda_{s,-}^2+1} + \frac{\bar{N}_{q,-}}{i\lambda_{s,-}^2-1} \right] I_{q,-}(i,z) \right] , \tag{A17}$$

where

$$I_{k,+}(i,z) = \int_0^\infty dy \left[\left[\frac{\hbar y}{\beta^2} \right]^i \exp\left[-\frac{\hbar y}{\beta^2} \right] \sum_{m=1,3,\ldots} \frac{\cos^2(m\pi z/2d)}{y+(m\pi/2d)^2} \right] , \tag{A18}$$

$$I_{k,-}(i,z) = \int_0^\infty dy \left[\left[\frac{\hbar y}{\beta^2} \right]^i \exp\left[-\frac{\hbar y}{\beta^2} \right] \sum_{m=2,4,\ldots} \frac{\sin^2(m\pi z/2d)}{y+(m\pi/2d)^2} \right] , \tag{A19}$$

$$I_{q,+}(i,z)=\int_0^\infty dy\left[\left(\frac{\hbar y}{\beta^2}\right)^i \exp\left(2yd-\frac{\hbar y}{\beta^2}\right)\frac{\tanh(yd)\cosh^2(yz)}{[(\epsilon_\infty+1)-(\epsilon_\infty-1)\exp(-2qd)][(\epsilon_0+1)-(\epsilon_0-1)\exp(-2qd)]}\right], \quad \text{(A20)}$$

$$I_{q,-}(i,z)=\int_0^\infty dy\left[\left(\frac{\hbar y}{\beta^2}\right)^i \exp\left(-2yd-\frac{\hbar y}{\beta^2}\right)\frac{\coth(yd)\sinh^2(yz)}{[(\epsilon_\infty+1)+(\epsilon_\infty-1)\exp(-2qd)][(\epsilon_0+1)+(\epsilon_0-1)\exp(-2qd)]}\right], \quad \text{(A21)}$$

where

$$\lambda_B^2=\omega_c/\omega_{BO},\quad \lambda_{s,+}^2=\omega_c/\omega_{s,+},\quad \lambda_{s,-}^2=\omega_c/\omega_{s,-}\ , \quad \text{(A22)}$$

and

$$\omega_c=eB/m_b c\ . \quad \text{(A23)}$$

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