Theory of magnetotransport in two-dimensional electron systems with unidirectional periodic modulation

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A consistent theory of magnetotransport and collision broadening for a two-dimensional electron system with a periodic modulation in one direction is presented. The theory is based on the self-consistent Born approximation for the scattering by randomly distributed short-range impurities and explains recent experiments which revealed, in addition to the familiar Shubnikov–de Haas oscillations at stronger magnetic field, a new type of low-field oscillation, also periodic in B^{-1} but with a period depending on both the electron density and the period of the spatial modulation. It is shown that the antiphase oscillations observed for the resistivity components ρ_{xx} and ρ_{yy} have as a common origin the oscillating bandwidth of the modulation-broadened Landau bands, which reflects the commensurability of the period of the spatial modulation and the extent of the Landau wave functions. Recent magnetocapacitance experiments are also well understood within this theory.

I. INTRODUCTION

Aiming at electronic devices of smaller size and higher performance, current semiconductor research spends considerable effort on the investigation of electronic systems of reduced dimensionality. As an interesting intermediate case between one- and two-dimensional systems, samples containing a two-dimensional electron gas (2D EG) with a unidirectional periodic modulation on a submicrometer scale became recently accessible to experiment. Devices with a microstructured gate can be fabricated, which allows tuning of the density modulation by an applied gate voltage between the limits of a weakly modulated 2D EG on one side and an array of weakly coupled quasi-one-dimensional stripes on the other.[1] More recently, Weiss *et al.*[2] have used an ingenious holographic modulation technique, exploiting the persistent photoconductivity effect in $GaAs/Al_xGa_{1-x}As$ heterostructures at low temperatures, to produce (weakly) modulated 2D electron systems of high mobility with a well-known period a (typically of the order of 300 nm) much smaller than the electron mean free path d ($\sim 10\ \mu$m at $T=4.2$ K).[2] In these systems, a systematic investigation of the effect of the modulation on the dc-transport properties became possible, and interesting novel magnetoresistance oscillations were detected.[2]

A 2D EG (density $N_s=3.4\times10^{11}$ cm^{-2}) with periodic modulation in the x direction ($a=294$ nm) but homogeneous in the y direction, shows at low temperature ($T=4.2$ K) in a perpendicular magnetic field $\mathbf{B}$ (z direction), in addition to the usual Shubnikov–de Haas (SdH) oscillations at larger magnetic field ($B\geq 0.5$ T), pronounced low field oscillations ($B\leq 1.0$ T) of the resistivity component ρ_{xx}, and weaker, but also clearly resolved, oscillations of ρ_{yy} with a phase shift of 180° with respect to those of ρ_{xx}.[2] Similar to the SdH oscillations, which appear for $0.5\ \text{T}<B<1.0$ T as superimposed on the novel low field oscillations, these new oscillations are periodic in $1/B$, but with a larger period depending on both N_s and a. This period is obtained from the minima of ρ_{xx}, which can be characterized by the commensurability condition[2]

$$2R_c=(\lambda-\tfrac{1}{4})a,\quad \lambda=1,2,3,\ldots \tag{1.1}$$

between the cyclotron diameter at the Fermi level, $2R_c=2v_F/\omega_c=2l^2k_F$, and the period a of the modulation. Here $k_F=\sqrt{2\pi N_s}$ is the Fermi wave number, $l=\sqrt{\hbar c/eB}$ the magnetic length, and $\omega_c=\hbar/ml^2$ the cyclotron frequency with the effective mass $m=0.067m_0$ of GaAs.

The first theoretical explanations[3,4] of this effect addressed only the large amplitude oscillations of ρ_{xx}, which have also been observed in gated samples.[4] They noticed that a weak modulation potential $V(x)=V_0\cos(Kx)$ with $a=2\pi/K$ and $V_0\ll E_F=\hbar^2k_F^2/2m$, lifts the degeneracy of the Landau levels (LL's) and leads to modulation-broadened Landau bands with eigenstates which carry current in the y direction, a local Hall drift due to the crossed magnetic field in the z direction and the oscillating electric field in the x direction. In thermal equilibrium, these alternating local Hall currents add to a vanishing macroscopic net current. In the presence of an applied electric field, however, a Landau-band contribution to the conductivity σ_{yy} results which is absent in a homogeneous 2D EG, and which increases with increasing band width. The latter is an oscillatory function of the band index n due to the fact that the eigenstates effectively average the periodic potential over an interval of the order of the cyclotron diameter $2R_c=2l\sqrt{2n+1}$, the extent of the wave function in the x direction. Evaluating the conductivity with the *ad hoc* assumption

of a constant transport relaxation time τ, one can explain the novel oscillations of ρ_{xx},[3,4] but not those of ρ_{yy}.[3] The reason is that this assumption neglects quantum oscillations and, e.g., for the homogeneous 2D EG yields the classical Drude resistivities without any SdH oscillations. Since, on the other hand, the oscillatory local Hall drift can also be understood in classical terms, it may not be too surprising that this result, novel oscillations of ρ_{xx} with minima given by Eq. (1.1), and no oscillations of ρ_{yy} (and ρ_{xy}) can also be derived from a semiclassical calculation based on Boltzmann's equation in the constant-relaxation-time approximation.[5]

It is, however, important to note that the assumption of a constant relaxation time has no justification. For the homogeneous electron gas in a quantizing magnetic field it is well known[6,7] that a reasonable transport theory should calculate the collision broadening of Landau levels and the transport coefficients, which at low temperature are both governed by scattering of electrons from randomly distributed impurities, in a consistent manner. In the simplest of such "conserving" approximations, the self-consistent Born approximation (SCBA) for the scattering by short-range (δ function) potentials, the inverse transport time equals th Landau-level width $[\hbar/\tau=\Gamma(E)]$ and is proportional to the density of states (DOS),[6] i.e., exhibits magnetic quantum oscillations and is not constant at all.

The main purpose of the present work is to demonstrate that such a consistent quantum-mechanical treatment of collision broadening and transport coefficients of the modulated system explains in a natural way all the different types of oscillations seen in experiment, the novel antiphase oscillations of ρ_{xx} and ρ_{yy} at low magnetic fields, and also the superimposed SdH oscillations which occur at higher magnetic fields. In contrast to the quasiclassical (the constant τ) approach, no additional mechanism is needed to explain the novel oscillations of ρ_{yy}. On the contrary, all the novel oscillations have the same origin, the oscillatory dependence of the bandwidth of modulation-broadened Landau levels on the level index n. Narrow bands lead to large peak values of the DOS and thus to maxima of ρ_{yy} which, via the scattering rate, directly reflects the oscillations of the DOS. On the other hand, the local Hall drift and thus the Landau-band contribution to the conductivity σ_{yy}, which dominates in high-mobility samples the resistivity ρ_{xx}, becomes small for narrow bands, and ρ_{xx} exhibits minima where ρ_{yy} becomes maximum.

The generalization of the established theory of collision broadening and transport properties to modulated systems leads to some mathematical difficulties, since important selection rules, which hold for homogeneous systems, are no longer valid. We discuss these difficulties for the single-particle Green's function in Sec. III and we formulate and evaluate a tractable approximation scheme which overcomes these difficulties while still containing the essential physics of collision broadening and satisfying the necessary requirements of consistency and analyticity in the complex energy plane.

In Sec. II we define the model and in Sec. V we discuss our results. In addition to the novel oscillatory effects at lower magnetic fields, we also consider the fine structure of SdH peaks at larger B values. Actually, this question has already been discussed theoretically several years ago,[8] long before the low field oscillations were detected, but its experimental investigation is still preliminary.[9] In addition to the numerical results of Sec. V, we give in the Appendix some analytical results which hold in the limit of larger Landau quantum numbers, i.e, for low magnetic fields.

Some particular results of this work have been published in advance[10] or quoted in a different context,[11,12] but a systematic derivation based on the standard perturbational approach to transport theory is presented here for the first time.

II. MODEL AND ENERGY SPECTRUM

The system we have in mind is a 2D EG forming a plane inversion layer in GaAs near its interface with $Al_zGa_{1-x}As$. Charge neutrality is guaranteed by a layer of ionized donors in the $Al_xGa_{1-x}As$ behind an undoped spacer layer. In the holographically structured samples,[2,3] the distribution of ionized donors is periodically modulated in one direction (the x direction) parallel to the interface. For the theoretical description of the system, two aspects of the impurity distribution are important, which can be characterized by different length scales. On a "microscopic" scale ($\sim$10 nm) the impurities, i.e., mainly ionized donors behind the spacer layer, seem to be distributed randomly and lead to collision broadening effects, i.e., imaginary parts of the self-energy. On a larger scale ($\sim$300 nm), the charged-donor density is periodic and leads, via screening effects, to a periodic effective potential seen by the electrons, and thus to a modification of the single-particle energy spectrum, i.e., a real part of the self-energy. In principle, it should be possible to treat both aspects on the same footing, taking into account a suitable correlation of the impurity distribution. In practice, however, such a theory has not yet been worked out, and we will treat the different aspects differently. We will simulate the long-range modulation by a simple (sinusoidal) periodic potential, just as if it were produced by a grating gate, and we will take into account its effect on the single-particle energy spectrum in principle exactly. The short-range fluctuations on the other hand, which give rise to collision broadening, will be simulated by randomly distributed scatters and treated perturbatively.

We are mainly interested in the limit of weak modulation and small magnetic fields. Then, assuming strong confinement of the 2D EG in the z direction perpendicular to the interface, we can describe the system by a strictly 2D model, i.e., we assume that the (occupied) wave functions are of the form $\Psi(\mathbf{r})=\psi(x,y)\chi_0(z)$, and that $\chi_0(z)$ and the corresponding electric subband energy are independent of modulation and magnetic field. Of course, this assumption is not strictly true, e.g., it is insufficient for a theory of magnetocapacitance,[13] but is a reasonable approximation for the investigation of transport properties in the inversion layer,[7] and allows us to average the Hamiltonian over the z direction, leading to

effective 2D interaction potentials.[7,12]

As a further simplification we assume that the effective modulation potential seen by the electrons is independent of the magnetic field. This is a poor approximation for strong magnetic fields at which the SdH oscillations are well resolved, since then the screening of the external electrostatic potential by the 2D EG becomes strongly dependent on the filling of Landau levels.[14] For smaller magnetic fields, however, where the oscillations of the Landau DOS are smeared out by thermal and by collision broadening effects, this approximation is reasonable.

Using the Landau gauge $\mathbf{A}=(0,xB,0)$ for the vector potential, we thus describe the electron-impurity system by the 2D Hamiltonian

$$H=H_0+V_I\ , \tag{2.1}$$

with

$$H_0=\frac{1}{2m}\left[-\hbar^2\frac{d^2}{dx^2}+\left[\frac{\hbar}{i}\frac{d}{dy}+\frac{e}{c}Bx\right]^2\right]+V_0\cos(Kx) \tag{2.2}$$

and with

$$V_I=\sum_j u(\mathbf{r}-\mathbf{R}_j;Z_j)\ , \tag{2.3}$$

a random array of in-plane scattering potentials due to impurities at distance Z_j from the 2D EG. The eigenfunctions of H_0 are of the form $\psi_{kn}(x,y)=L_y^{-1/2}\exp(iky)\varphi_{nx_0}(x)$, with L_y a normalization length. Note that, owing to the symmetry of the problem, the center coordinate $x_0=l^2k$ remains a good quantum number, as in the unmodulated case. The $\varphi_{nx_0}(x)$ are eigenfunctions of the one-dimensional Hamiltonian

$$H_{x_0}=-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+\tfrac{1}{2}m\omega_c^2(x-x_0)^2+V_0\cos(Kx)\ , \tag{2.4}$$

with eigenvalues $\varepsilon_n(x_0)=\varepsilon_n(x_0+a)=\varepsilon_n(-x_0)$. Using the well-known[7] oscillator wave functions $\varphi^{(0)}_{nx_0}(x)=\langle x|nx_0\rangle$ of the unmodulated system ($V_0\equiv 0$) as a basis set, one calculates (for $n'\le n$ and with $X=\frac{1}{2}K^2l^2$) the matrix elements of the modulation potential to be

$$V_{n'n}(x_0)=\langle n',x_0|V_0\cos(Kx)|n,x_0\rangle=V_0\left[\frac{n'!}{n!}\right]^{1/2}\exp(-\tfrac{1}{2}X)X^{(n-n')/2}L_{n'}^{n-n'}(X)\mathrm{Re}(e^{iKx_0}i^{n-n'})\ , \tag{2.5}$$

where $L_{n'}^{n-n'}(X)$ is a Laguerre polynomial.[15] The solid lines of Fig. 1 depict, for a realistic set of parameters, the energy spectrum of H_0 obtained from numerical diagonalization of H_0 using Eq. (2.5) with a sufficiently large set of basis functions. The thick broken lines show, for comparison, the result of the first-order perturbation calculation with respect to V_0, given by the diagonal elements of Eq. (2.5),[8]

$$\varepsilon_n(x_0)\approx E_n(x_0)=E_n+U_n\cos(Kx_0)\ , \tag{2.6}$$

with $E_n=\hbar\omega_c(n+\frac{1}{2})$, the unperturbed Landau energies, and $U_n=V_0\exp(-\frac{1}{2}X)L_n(X)$. It is seen that the first-order approximation becomes very good for large quantum numbers n, i.e., high energies. We have checked that the energy value, beyond which the first-order approximation becomes excellent, increases with decreasing magnetic field B. A realistic value[2,3] of the Fermi energy is $E_F\approx 11$ meV. Thus, near E_F, and for the parameter values of Fig. 1, the first-order approximation is excellent for $B>0.1$ T, but it breaks down for $B\to 0$.[12]

The most important effect of the novel magnetoresistance oscillations is that the modulation potential lifts the degeneracy of the Landau levels and leads to Landau bands of finite width. The bandwidth ($\approx 2U_n$) depends on the band index n in an oscillatory manner. Formally, this is due to the properties of the Laguerre polynomials. Physically it reflects the fact that with increasing n, the spatial extent of the wave function increases

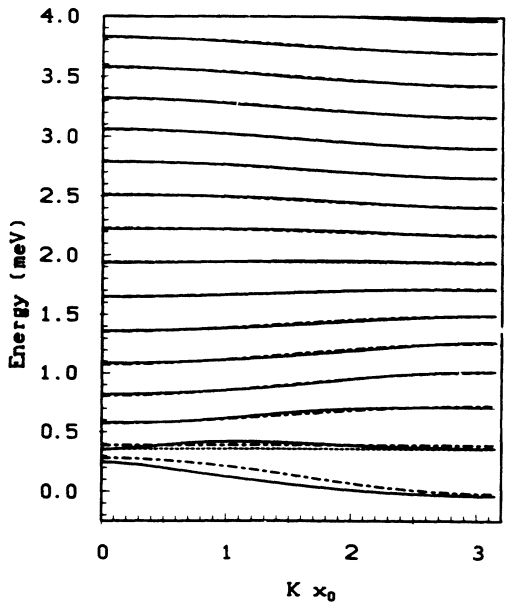

FIG. 1. Calculated energy spectrum vs center coordinate x_0 for $B=0.15$ T and periodic potential $V_0\cos(Kx_0)$ with $V_0=0.25$ meV and $a=2\pi/K=294$ nm. Solid lines, exact; dashed-dotted lines, first-order approximation of Eq. (2.6). The flat-band energies (thin dotted lines) $E_\lambda=\frac{1}{8}(a/l)^2\hbar\omega_c(\lambda-\frac{1}{4})^2$ calculated from Eq. (2.8) are $E_1=0.36$ meV, $E_2=1.96$ meV, $E_3=4.83$ meV, etc.

($\sim 2R_n = 2l\sqrt{2n+1}$), and the latter effectively senses an average of the periodic modulation potential over an interval of width $2R_n$. Indeed, if one takes the average of the modulation potential over the classical cyclotron orbit with radius R_n and center coordinate x_0, one obtains the large-n limit of Eq. (2.6),[4]

$$U_n = V_0 J_0(KR_n) \approx V_0 \pi^{-1/2}(nX)^{-1/4}\cos(2\sqrt{nX} - \pi/4) . \tag{2.7}$$

The energy values at which one expects flat Landau bands can be estimated from the asymptotic formula for zeros of the Laguerre polynomials,[15] $L_n(X)=0$ if $X = X_\lambda^{(n)}$,

$$X_\lambda^{(n)} \approx \tfrac{1}{4}[\pi(\lambda - \tfrac{1}{4})]^2/(n+\tfrac{1}{2}), \quad \lambda = 1,2,3,\ldots \tag{2.8}$$

or directly from Eq. (2.7). Expressing this in terms of the cyclotron radius R_n, one obtains Eq. (1.1) as the condition for flat bands (with R_n instead of R_c). The corresponding flat band energies $E_n = \hbar\omega_c(n+\frac{1}{2})$ with n calculated from Eq. (2.8) (i.e., n not necessarily an integer) are indicated as horizontal dotted lines in Fig. 1.

A fact related closely to the finite width of Landau bands is that the eigenstates of H_0 carry current in the y direction. The expectation value of the velocity operator v_y in such an eigenstate $|n,x_0\rangle$ is

$$(n,x_0|v_y|n,x_0) = -\frac{1}{m\omega_c}\frac{d\varepsilon_n}{dx_0} = \frac{1}{\hbar}\frac{d\varepsilon_n}{dk} , \tag{2.9}$$

and vanishes only for flat bands. The physical meaning of this result is, of course, a local Hall current, or, in classical terms, a guiding-center drift of the cyclotron motion in the presence of the periodic electric modulation field. There is no corresponding current in the x direction, and $(n,x_0|v_x|n,x_0)\equiv 0$ as in the unmodulated system.

In the following calculations, we will replace the exact eigensolution of Eq. (2.4) by the first-order approximation, i.e., we use Eq. (2.6) for the energy spectrum and

$$|n,x_0) = |n,x_0\rangle + \sum_{m\neq n}\frac{V_{nm}(x_0)}{E_n - E_m}|m,x_0\rangle \tag{2.10}$$

for the eigenstates. This yields for the velocity matrix elements needed below to calculate conductivities

$$\frac{\sqrt{2}}{l\omega_c}\left[n,x_0\left|\begin{Bmatrix}-iv_x\\ v_y\end{Bmatrix}\right|n',x_0\right] = \sqrt{n+1}\,\delta_{n',n+1} \mp \sqrt{n}\,\delta_{n',n-1} + (1-\delta_{n',n+1})\frac{\sqrt{n'}V_{n,n'-1} - \sqrt{n+1}V^*_{n+1,n'}}{(n-n'+1)\hbar\omega_c}$$
$$\mp (1-\delta_{n',n-1})\frac{\sqrt{n'+1}V_{n,n'+1} - \sqrt{n}\,V^*_{n-1,n'}}{(n-n'-1)\hbar\omega_c} , \tag{2.11}$$

with V_{nm} given by Eq. (2.5). The diagonal matrix elements of v_y are consistent with Eqs. (2.6) and (2.9), whereas $(n,x_0|v_x|n,x_0)=0$ holds in this approximation, too. We want to emphasize that, in principle, there is no necessity to use the first-order approximation with respect to V_0 for the energy eigenvalues and velocity matrix elements. We use this approximation, since it is quantitatively very good for the parameter values of our interest, and since it allows us to calculate, in the absence of random impurity scattering, all relevant quantities analytically.

III. COLLISION BROADENING

We now consider the effect of random impurity scattering on the Green's function $\hat{G}^{\pm}(E) = (E \pm i0^+ - H)^{-1}$. For simplicity we assume only one type of impurity potential, i.e., take the same value of Z_j in Eq. (2.3) for all impurities, and we assume that the positions $\mathbf{R}_j$ of the impurities, with area density n_I, are uncorrelated. The average over impurity positions is taken term by term in the perturbation expansion of $\hat{G}^-$ with respect to V_I, as usual.[6,7,16] The average Green's function $G^- = \langle \hat{G}^- \rangle_{\text{imp}}$ satisfies Dyson's equation and can be expressed in terms of a self-energy operator, $G^-(E) = [E - H_0 - \Sigma^-(E)]^{-1}$.

In the self-consistent Born approximation (SCBA)[6,7] (i.e., approximation No. 4 of Ref. 16) the self-energy is proportional to G^-, and a nonlinear closed equation for G^- and Σ^- is obtained, which is visualized diagrammatically in Fig. 2(a).

For the unmodulated system ($V_0 \equiv 0$) it can be shown that both the self-energy operator Σ^- and the Green's function G^- are diagonal in the Landau representation $|n,x_0\rangle$, which diagonalizes H_0, and that they are independent of x_0, provided the impurity potential $u(\mathbf{r})$ has rotational symmetry.[6] Furthermore, for the short-range potential $u(\mathbf{r}) = u_0\delta^{(2)}(\mathbf{r})$, $\Sigma^-(E)$ is in the SCBA also independent of n, i.e., a multiple of the unit operator.

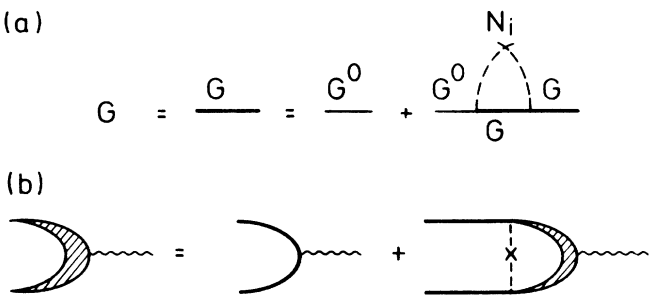

FIG. 2. Diagrammatic representation of the self-consistent Born approximation. (a) Green's function with self-energy according to Eq. (3.1); (b) current vertex F from Eq. (4.4).

Thus, for $V_0 \equiv 0$, $\Sigma^-(E)$ commutes with H_0, one can write Dyson's equation in the form $G_n^-(E)=[E-E_n-\Sigma_n^-(E)]^{-1}$, and the center coordinate x_0 only determines the degeneracy.

For the modulated system, $V_0 \neq 0$, Σ^- does not commute with H_0, and there is no representation in which both Σ^- and G^- are simultaneously diagonal. The only obvious symmetry is the translational invariance of the average Green's function in the y direction. Thus, $G^-(E)$ must be diagonal in x_0. In the eigenbasis of H_0 we thus can write the SCBA for the self-energy as

$$\Sigma_{nm}^{x_0}(E)=\sum_{n',m'}\int_0^a dx_0' \frac{1}{a}\Gamma^2_{nm,n'm'}(x_0,x_0')G_{n'm'}^{x_0'}(E) , \quad (3.1)$$

with

$$\Gamma^2_{nm,n'm'}(x_0,x_0')=n_I\int d^2R\,(n,x_0|u(\mathbf{r}-\mathbf{R})|n',x_0') \times(m',x_0'|u(\mathbf{r}-\mathbf{R})|m,x_0) . \quad (3.2)$$

For an iterative solution of Eq. (3.1), one has to invert the matrix $(G^{-1})_{nm}^{x_0}=[E-E_n(x_0)]\delta_{nm}-\Sigma_{nm}^{x_0}$ in each iteration step. Even the assumption of short-range (δ-function) scattering potentials does not simplify the complicated matrix structure noticeably.

We did not attempt to evaluate the complicated quantum-number dependence of the self-energy. Instead we made the ansatz of a quantum-number-independent self-energy, so that the average Green's function can be written in H_0 representation as

$$G_{nx_0}^-(E)=\frac{1}{E-E_n(x_0)-\Sigma^-(E)} , \quad (3.3)$$

with an effective c-number self-energy determined by the simple self-consistency equation

$$\Sigma^-(E)=\Gamma_0^2\sum_n\int_0^a dx_0\frac{1}{a}G_{nx_0}^-(E) , \quad (3.4)$$

where a high-energy cutoff is needed to make $\mathrm{Re}\Sigma^-$ well defined.[6,17] We restrict the n sum to $n \leq 2E_F/\hbar\omega_c$. Then, both $G_{nx_0}^-(E)$ and $\Sigma^-(E)$ are analytical functions with non-negative imaginary parts in the complex half-plane $\mathrm{Im}E<0$, $\Sigma^-(E)$ bounded for $|E|\to\infty$, and the sum rule

$$\int_{-\infty}^{\infty} dE\; A_{nx_0}(E)=1 \quad (3.5)$$

is satisfied for the spectral function $A_{nx_0}(E)=\pi^{-1}\mathrm{Im}G_{nx_0}^-(E)$, i.e., our ansatz conserves the number of states, as it should. We think that this ansatz contains the most important features of collision broadening, since for the nonmodulated limit, $V_0 \to 0$, and with

$$\Gamma_0^2=\frac{1}{2\pi}\hbar\omega_c\frac{\hbar}{\tau} \quad (3.6)$$

it reduces to the SCBA for δ potentials, with τ the corresponding lifetime for zero magnetic field.[7] Using Eq. (2.6), one evaluates Eq. (3.4) as

$$\Sigma^-(E)=\sum_n\frac{\Gamma_0^2}{\{[E-E_n-\Sigma^-(E)]^2-U_n^2\}^{1/2}} , \quad (3.7)$$

and obtains for the DOS (including spin degeneracy)

$$D(E)=\frac{2}{2\pi l^2}\int_0^a dx_0\frac{1}{a}\sum_n A_{nx_0}(E)=\mathrm{Im}\left[\frac{\Sigma^-(E)}{\pi^2 l^2\Gamma_0^2}\right] . \quad (3.8)$$

In the limit of vanishing collision broadening, $\Gamma_0^2 \to 0$, Eq. (3.8) reduces to

$$D(E)|_{\Gamma_0\to 0}=\frac{1}{\pi^2 l^2}\sum_n\frac{\Theta(U_n-|E-E_n|)}{[U_n^2-(E-E_n)^2]^{1/2}} , \quad (3.9)$$

with 1D Van Hove singularities at the low- and high-energy edges of the modulation-broadened Landau bands.

Thus, our basic approximation, Eqs. (3.3) and (3.4), contains both the effects of collision broadening and of modulation broadening of Landau levels, with reasonable limits if one of these mechanisms is turned off. In the following we will see that it uniquely determines a calculation scheme for the transport coefficients and that it provides, without any further approximations or assumptions, a good qualitative understanding of all the magnetoresistance oscillations mentioned in the Introduction.

We can, however, not expect this approximation to yield quantitatively correct results. As mentioned, it reduces in the absence of the periodic modulation to the SCBA for short-range (δ-function) impurity potentials, in which the effect of impurities is described by a single lifetime parameter τ, which determines the broadening of the Landau levels [see Eq. (3.6)], and at the same time has the meaning of a transport relaxation time. Systematic measurements on modulation-doped GaAs /$Al_xGa_{1-x}As$ heterostructures of the same type as those showing the novel magnetoresistance oscillations have shown, however, that the Landau-level broadening Γ is much larger than one would estimate, using Eq. (3.6), from the transport relaxation time τ_r at zero magnetic field, $\Gamma \gg \hbar/\tau_r$.[18] This is in agreement with other experimental investigations[19,20] and indicates that long-range Coulomb potentials due to ionized donors behind the spacer in the $Al_xGa_{1-x}As$ provide the dominant scattering mechanism in these high-mobility systems.[19–21]

Nevertheless, in order to keep the calculations mathematically simple and numerically tractable, we consider in the following only the approximation defined by Eqs. (3.3) and (3.4), which also implies that we neglect the x_0 dependence of the self-energy. From this approximation we cannot expect quantitative agreement with the experiments, but can expect a qualitative understanding of the underlying physics.

Figure 3 shows results for the DOS obtained from numerical solution of Eq. (3.7). For sufficiently small collision broadening (small Γ_0), the DOS peaks due to individual Landau bands do not overlap. This oscillatory dependence of the bandwidth of the modulation-broadened Landau levels (LL) is seen directly from the width of the peaks, but the corresponding modulation of the peak height, which is a consequence of the fact that

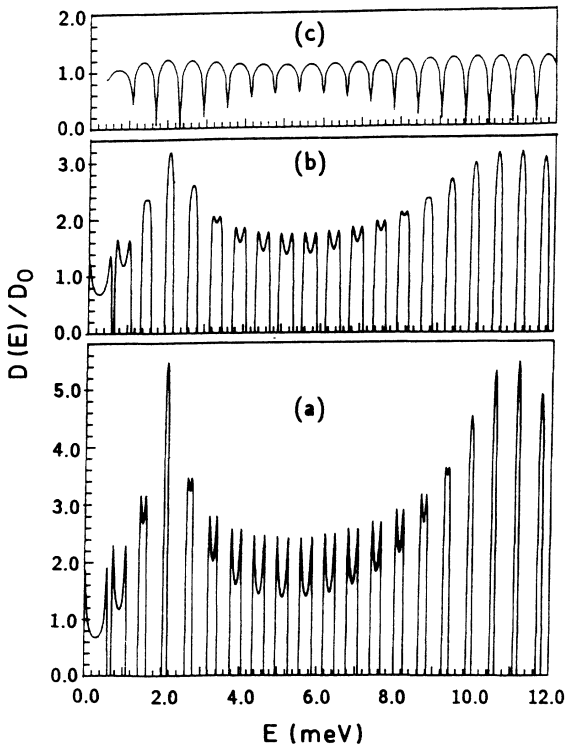

FIG. 3. Density of states in units of the zero magnetic field DOS $D_0=m/(\pi\hbar^2)$ for $B=0.35$ T, $V_0=0.35$ meV, and (a) $\Gamma_0=0.0590\sqrt{B[\mathrm{T}]}$ meV, (b) $\Gamma_0=0.1025\sqrt{B[\mathrm{T}]}$ meV, and (c) $\Gamma_0=0.3074\sqrt{B[\mathrm{T}]}$ meV.

each LL contains the same number of states, is much more impressive. Maximum peak height is obtained near flat band energies. If the modulation-induced bandwidth becomes larger than the collision broadening, a double-peak structure is resolved, the remainder of the 1D Van Hove singularities. The sharp edges of the individual DOS peaks, which are well known from the SCBA for the unmodulated 2D EG,[7] are an artifact of our approximation, which neglects coherent multicenter scattering completely.

If the collision broadening is so large that the DOS peaks start to overlap (see top of Fig. 3), in addition to the peak heights also the minimum values of the DOS between peaks show an oscillatory modulation owing from the bandwidth oscillations. Recently we have used these results to calculate the magnetocapacitance of holographically modulated samples, and we found nice agreement with the experimental results, which clearly exhibit the expected modulation of maximum as well as minimum values of the magnetocapacitance oscillations.[11] An analytic expression, describing this modulation in the large-n limit, is derived in the Appendix.

IV. CONDUCTIVITIES

A. General formalism

We use conventional linear response theory to calculate the current density j resulting as a response to an applied electric field E. We consider only the spatial averages of both j and E, i.e., we take the $q\rightarrow 0$ limit of the corresponding spatial Fourier transforms. Then the response of our electron-impurity system to an adiabatically switched-on electric field with frequency ω is given by a Kubo-type formula for the conductivity tensor,[6,22]

$$\sigma_{\mu\nu}(\omega)=\frac{1}{i\omega}[\chi_{\mu\nu}(\omega)-\chi_{\mu\nu}(0)] , \tag{4.1}$$

where ω is understood to have a positive infinitesimal imaginary part ($\omega\rightarrow\omega+i0^+$), and, in our single-electron approximation, the susceptibility $\chi_{\mu\nu}$ can be written as a trace over single-particle states,

$$\chi_{\mu\nu}(\omega)=-\frac{2e^2}{L_xL_y}\int_{-\infty}^{\infty} dE\, f(E)\mathrm{tr}\{\delta(E-H)[v_\mu\hat{G}(E+\hbar\omega)v_\nu+v_\nu\hat{G}(E-\hbar\omega)v_\mu]\} . \tag{4.2}$$

The matrix elements of the velocity operators v_x and v_y are given in Eq. (2.11) in the first-order approximation. One confirms that the general sum rule $\chi_{\mu\nu}(0)=\delta_{\mu\nu}e^2N_s/m$ also holds in the present modulated case. Since we are here only interested in the static conductivity, we will take the limit $\omega\rightarrow 0$.

In the first attempt[3] to understand the novel magnetoresistance oscillations in the framework of Kubo's formulas, Eqs. (4.1) and (4.2) were evaluated in a crude damping approximation. The impurity potentials were neglected in the Hamiltonian $H\rightarrow H_0$, and scattering effects were simulated by the substitution $\omega=i/\tau$, where $\tau=\hbar/\gamma$ has the meaning of a transport relaxation time. With this approximation, the novel oscillations of ρ_{xx} could be explained, but not those of ρ_{yy}. The reason for this failure was supposed[3] to be the neglect of quantum oscillations of the scattering rate γ. Indeed, this crude damping approximation yields for the homogeneous unmodulated 2D EG, for which it is easy to evaluate Eq. (4.2) explicitly, just the classical Drude result

$$\sigma_{xx}^D=\sigma_{yy}^D=\frac{\sigma_0}{1+(\omega_c\tau)^2},\quad \sigma_0=\frac{e^2N_s\tau}{m} ,$$
$$\sigma_{yx}^D=\omega_c\tau\sigma_{xx}^D . \tag{4.3}$$

In the present paper we avoid such *ad hoc* assumptions and evaluate the impurity average of eq. (4.2) in a systematic approximation consistent with the treatment of collision broadening in Sec. III. Since $\delta(E-H)=(2\pi i)^{-1}[\hat{G}^-(E)-\hat{G}^+(E)]$, impurity averages of the type

$$F(E,E';v_\nu)=\langle \hat{G}(E)v_\mu\hat{G}(E')\rangle_{\rm imp} \tag{4.4}$$

have to be calculated, where E and E' include positive or negative infinitesimal imaginary parts. Expanding the $\hat{G}$'s in powers of V_I and taking the average term by term leads to a linear integral equation for F, the Bethe-Salpeter equation.[6]

B. Present consistent approximation

The Bethe-Salpeter equation in the SCBA, shown in Fig. 2(b), is trivially solved in our present approximation, Eqs. (3.3) and (3.4), which means that the kernel (3.2) is replaced by $\delta_{nm}\delta_{n'm'}\Gamma_0^2$, where Γ_0^2 is a constant, independent of all the quantum numbers. Then, one can prove that current vertex corrections vanish and Eq. (4.4) reduces to

$$F(E,E';v_\mu)=G(E)v_\mu G(E') . \tag{4.5}$$

Inserting this into Eqs. (4.2) and (4.1), we obtain, in the limit $\omega\to 0$, for the diagonal components of the conductivity tensor after integrating by parts,

$$\sigma_{\mu\mu}=\int_{-\infty}^{\infty} dE\left[-\frac{df(E)}{dE}\right]\sigma_{\mu\mu}(E) , \tag{4.6}$$

where $f(E)=\{\exp[(E-\mu)/k_BT]+1\}^{-1}$ is the Fermi function and

$$\sigma_{\mu\mu}(E)=\frac{\hbar e^2}{l^2}\int_0^a dx_0\frac{1}{a}\sum_{n,n'}|(n,x_0|v_\mu|n',x_0)|^2A_{n,x_0}(E)A_{n'x_0}(E) . \tag{4.7}$$

For the Hall conductivity we obtain

$$\sigma_{yx}=-\frac{\hbar e^2}{il^2}\int_{-\infty}^{\infty}dE\,f(E)\int_0^a dx_0\frac{1}{a}\sum_{n,n'}(nx_0|v_y|n'x_0)(n'x_0|v_x|nx_0)A_{nx_0}(E)\frac{2}{\pi}\mathrm{Re}\left[\frac{dG^-_{n'x_0}}{dE}\right] . \tag{4.8}$$

In the numerical calculations we used the identity

$$dG^-_{nx_0}/dE = -[G^-_{nx_0}(E)]^2\Big/\left[1-\Gamma_0^2\sum_n\int_0^a G^-_{nx_0}(E)^2dx/a\right] .$$

The transport coefficients were calculated from Eqs. (4.6)–(4.8), using the solution of Eqs. (3.3) and (3.7). For given density N_s of the 2D EG and for given temperature T, the chemical potential μ was calculated from the equation

$$\int_{-\infty}^{\infty}dE\,f(E)D(E)=N_s . \tag{4.9}$$

This typical results for the conductivities are depicted in Fig. 4. Finally, we inverted the conductivity tensor to obtain the resistivities

$$\rho_{xx}=\frac{\sigma_{yy}}{\mathcal{D}},\quad \rho_{yy}=\frac{\sigma_{xx}}{\mathcal{D}},\quad \rho_{xy}=\frac{\sigma_{yx}}{\mathcal{D}} , \tag{4.10}$$

with $\mathcal{D}=\sigma_{xx}\sigma_{yy}+\sigma_{xy}^2$. The results, for two different temperatures, are shown in Figs. 5 and 6. For comparison, we have also included the thermodynamic DOS

$$D_T(\mu)=\frac{\partial N_s}{\partial\mu}=\int dE\left[-\frac{df}{dE}\right]D(E) , \tag{4.11}$$

which determines the magnetocapacitance of the sys-

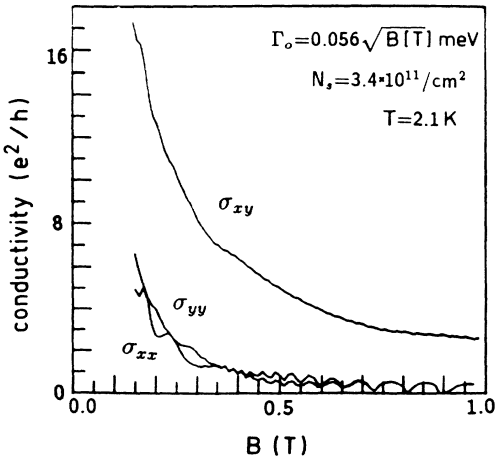

FIG. 4. Calculated conductivity vs magnetic field, where $V_0=0.25$ meV and $a=294$ nm; the values of σ_{xy} have been reduced by a factor of 5.

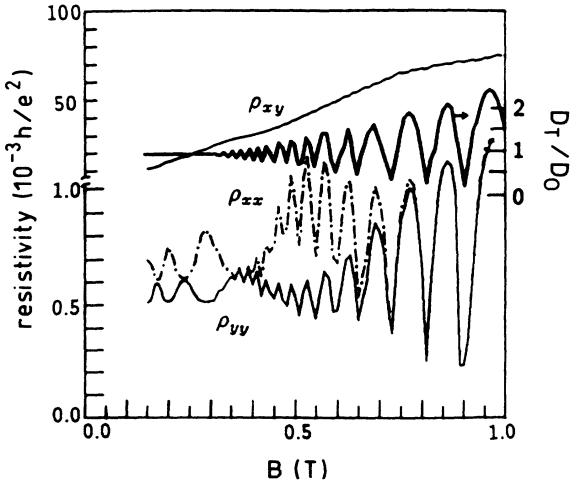

FIG. 5. Calculated resistivity in units of $10^{-3}h/e^2=25.8\ \Omega$. The thick solid line represents the thermodynamic DOS in units of $D_0=m/\pi\hbar^2$ (scale on right-hand side). Parameters as in Fig. 4.

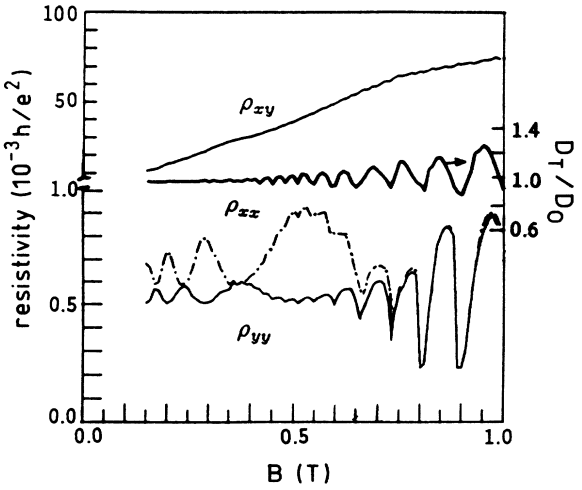

FIG. 6. Same as Fig. 5, but for the higher temperature $T=4.2$ K.

tem.[11]

In addition to the SdH oscillations, which are resolved in Fig. 5 for $B>0.3$ T, and which are in phase for ρ_{xx}, ρ_{yy}, and D_T, the novel oscillations are clearly seen in the resistivities with minima of ρ_{xx} and maxima of ρ_{yy} at $B=0.88$, 0.38 , 0.24 , and 0.18 T, corresponding to $\lambda=1$, 2, 3, and 4 in Eq. (1.1). The extrema for $\lambda=1$ are obscured by the strong SdH oscillations. Before we discuss the results in detail, we want to recall briefly the corresponding results for an unmodulated system, which are helpful for the understanding of the situation of our present interest.

C. Unmodulated case

If we omit the modulation, $V_0\equiv 0$, the quantities in Eqs. (4.7)–(4.8) are independent of x_0, and the matrix elements of v_x and v_y, given by the first two terms of Eq. (2.11), are off diagonal in the Landau eigenbasis. In this homogeneous isotropic case, the sum in Eq. (4.7) can be rearranged to yield, for both $\mu=x$ and $\mu=y$,[6]

$$\sigma^H_{\mu\mu}(E)=\frac{e^2}{m}\frac{\hbar\Gamma}{(\hbar\omega_c)^2+\Gamma^2}\tilde{n}(E)\ , \tag{4.12}$$

with $\Gamma\equiv\Gamma(E)=2\,\mathrm{Im}\Sigma^-(E)$, and

$$\tilde{n}(E)=(E-\Delta)D(E)+\frac{1}{2\pi}\Gamma D_0\ , \tag{4.13}$$

where $\Delta=\mathrm{Re}\Sigma^-(E)$ and $D_0=m/\pi\hbar^2$ is the DOS for zero magnetic field. With $\tau=\hbar/\Gamma(E)$, Eq. (4.12) resembles the Drude result, Eq. (4.3), if $\tilde{n}(E)$ is replaced by the density N_s. Indeed, if for small magnetic field the SdH oscillations are smeared out by collision broadening effects, $\tilde{n}(E)$ approaches, for $E\gg|\Sigma^-(E)|$, the integrated DOS, and , with $\tilde{n}(E_F)=N_s$, Eq. (4.12) reduces to the zero-temperature Drude result. Furthermore, Eq. (4.8) can be shown to reduce to [6,7]

$$\sigma^H_{yx}=\frac{e^2}{m\omega_c}N_s-\int_{-\infty}^{\infty}dE\left[-\frac{df}{dE}\right]\frac{\Gamma(E)}{\hbar\omega_c}\sigma^H_{xx}(E)\ , \tag{4.14}$$

which completes the analogy and relation to the classical Drude result.

For stronger magnetic fields, $\hbar\omega_c\gg\Gamma$, the SdH oscillations are resolved and Eq. (4.12) reduces to the well-known result $\sigma^H_{xx}(E)\propto\Gamma(E)D(E)$, i.e., within the SCBA the low-temperature conductivity $\sigma^H_{xx}\propto\Gamma_0^2D(E_F)^2$ is nonzero only due to elastic scattering of electrons by impurities from occupied states into empty states at the Fermi level.

V. DISCUSSION OF RESULTS

The novel magnetoresistance oscillations caused by a periodic modulation of the 2D EG in one direction are most easily understood for high-mobility systems, where collision broadening effects are small, $\Gamma_0\ll\hbar\omega_c$, and in the limit of weak modulation $\Gamma_0\ll V_0\ll\hbar\omega_c$.

Owing to Eq. (2.9), a qualitatively new, n-diagonal contribution to σ_{yy} arises,

$$\Delta\sigma_{yy}(E)=\frac{e^2\hbar}{l^2}\int_0^a\frac{dx_0}{a}\sum_n|(nx_0|v_y|nx_0)|^2[A_{nx_0}(E)]^2\ , \tag{5.1}$$

which is absent in the unmodulated case. The $A^2_{nx_0}$ factor behaves like the square of the DOS, so that the contribution $\Delta\sigma_{yy}$ becomes increasingly important with decreasing collision broadening ($\Gamma_0\to 0$), and leads to SdH oscillations which are in phase with those of the DOS. As discussed in connection with Fig. 3, the periodic modulation of the 2D EG leads to a modulation of the peak height of the DOS oscillations with maxima near flat-band energies. The diagonal matrix elements of v_y, on the other hand, vanish for flat bands, so that $\Delta\sigma_{yy}(E)$ exhibits SdH-type oscillations modulated by a prefactor approaching zero where the amplitude of the DOS oscillations becomes maximum. Thus, the amplitude oscillations of $\Delta\sigma_{yy}(E)$ and those of the DOS have a phase shift of 180°.

Since the spectral function $A_{nx_0}(E)$ is a broadened δ function, with maximum value $2/\pi\Gamma$ if the energy dependence of Γ is neglected, one may to a crude approximation replace $[A_{nx_0}(E)]^2$ by $(\pi\gamma)^{-1}\delta(E-\varepsilon_n(x_0))$ with a constant γ. Inserting this into Eqs. (5.1) and (4.6) yields, for $\Delta\sigma_{yy}$, Eq. (8) of Ref. 3. If one further assumes the thermal broadening ($\sim k_BT$) to be larger than $\hbar\omega_c$ but less than the distance of adjacent flat-band energies, one obtains in the semiclassical large-n limit, using Eq. (2.7) the approximate result[4,12]

$$\Delta\sigma_{yy}\approx\frac{e^2}{2\pi\hbar}\frac{V_0^2}{\gamma\hbar\omega_c}\frac{4}{ak_F}\cos^2\left[2\pi\frac{R_c}{a}-\frac{\pi}{4}\right]\ , \tag{5.2}$$

with zeros given by Eq. (1.1).

The off-diagonal ($n'\neq n$) contributions to Eqs. (4.7) and (4.8) are, for $V_0\ll\hbar\omega_c$, still dominated by the first two (V_0-independent) terms of Eq. (2.11). For a qualita-

tive understanding of these contributions, it is sufficient to neglect their explicit x_0 dependence and to take into account the effect of the periodic modulation only via the self-energy. In this approximation we can use Eq. (4.12) for σ_{xx} and the regular, off-diagonal contribution to σ_{yy}, and Eq. (4.14) for σ_{yx}. Since, for $\Gamma \ll \hbar\omega_c$, $\sigma_{xx}^H(E) \propto D(E)^2$, the SdH oscillations are in phase with the DOS oscillations and moreover, the modulation-induced oscillations of the peak heights are also in phase.

Since at liquid-He temperatures ($T \sim 2$–4 K) the SdH oscillations are not resolved for $B < 0.4$ T, it is important to understand the temperature dependence of the novel, modulation-induced oscillations. To this end, we compare in Fig. 7, for several values of the temperature and as functions of the chemical potential μ, the thermodynamic DOS D_T, defined in Eq. (4.11), and the quantity

$$D_T^2(\mu) = \int dE \left[-\frac{df}{dE} \right] D(E)^2 , \tag{5.3}$$

which is proportional to σ_{xx}. Since the area under the individual $D(E)$ peaks is the same for all peaks, (i.e., since all Landau levels contain the same number of states), the oscillations of D_T are completely washed out if the thermal average extends over only a few neighboring Landau bands ($k_B T \approx \hbar\omega_c$). For such T values, $D_T^2(\mu)$ shows, however, still pronounced oscillations with maxima at flat-band energies, since the area under the individual $D(E)^2$ peaks strongly depends on the peak height, with maxima at flat bands and minima at broad bands. To smear out these modulation-induced oscillations, much higher temperatures are required, with $k_B T$ of the order of the distance ΔE_λ between adjacent flat-band energies $E_\lambda = \frac{1}{8}(a/l)^2 \hbar\omega_c (\lambda - \frac{1}{4})^2$. A similar comparison[12] shows that the modulation-induced oscillations of $\Delta\sigma_{yy}$ also survive to these higher temperatures. In our present high-mobility approximation, the off-diagonal contribution to σ_{yy} agrees with σ_{xx}; the total σ_{yy} is, however, dominated by $\Delta\sigma_{yy}$, and has minima of the modulation-induced oscillations where σ_{xx} has maxima.

According to Eq. (4.14), we also expect oscillatory contributions to the Hall conductivity σ_{yx}, but these are by a factor of $\Gamma/\hbar\omega_c$ smaller than σ_{xx} and are much smaller than the leading first term on the right-hand side of Eq. (4.14). Thus, we find that the resistivity components ρ_{xx} and ρ_{yy}, according to Eq. (4.10), reflect the antiphase oscillations of σ_{yy} and σ_{xx}, respectively, whereas the Hall resistance ρ_{xy} is essentially given by its classical value $m\omega_c/e^2 N_s$, corrected only by small-amplitude oscillations with maxima when condition (1.1) is satisfied. If the mobility is not extremely high, the oscillatory part of ρ_{xy} may be affected by σ_{xx} and σ_{yy}, and more complicated oscillation patterns result. Regardless, these small oscillations are hardly seen in experiment.[2,3]

In summary, all the oscillations shown by the numerical results of Figs. 5 and 6 are well understood by these qualitative considerations. We notice that, contrary to Ref. 3, where SdH-type oscillations were obtained only for the additional contribution $\Delta\sigma_{yy}$, the amplitude of the SdH oscillations calculated in the present work does not become small near $B = 0.88$ T, where $\Delta\sigma_{yy}$ vanishes. This is now in agreement with the experimental finding.[2,3]

From the discussion of the temperature dependence (Fig. 7), we also understand that, for a given temperature, the novel modulation-induced oscillations of ρ_{xx} and ρ_{yy} are well resolved down to very low values of the magnetic field, for which the SdH oscillations are no

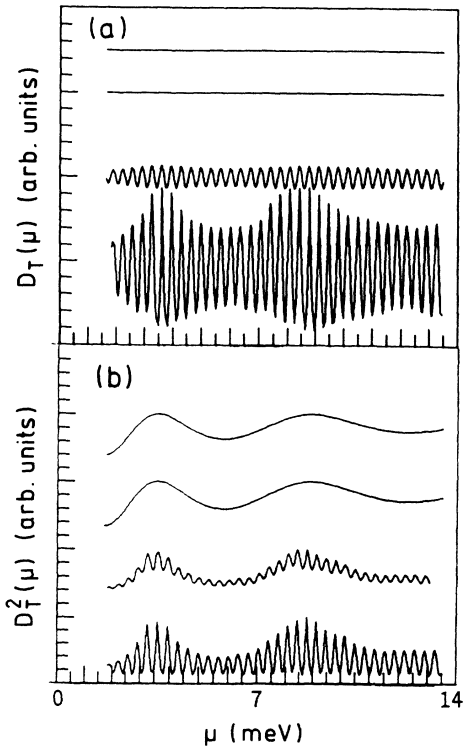

FIG. 7. Thermal average of (a) DOS $D(E)$ and (b) $[D(E)]^2$ vs chemical potential μ for temperature $T = 0.5$, 1, 2, and 4 K (from bottom to top). The curves are normalized and shifted so that (a) the average value is 1, and (b) the maximum value is 1.

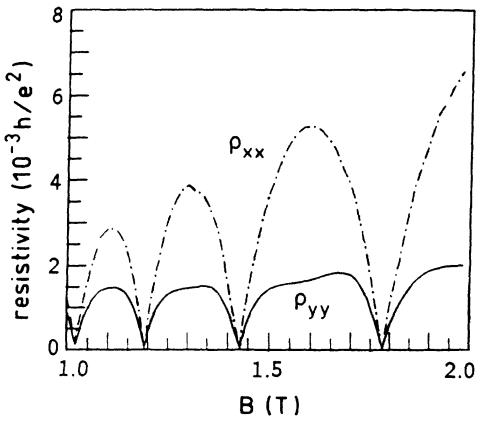

FIG. 8. Calculated resistivity vs magnetic field at higher fields and larger modulation amplitude; parameters as in Fig. 4, but $V_0 = 0.5$ meV. Note the weak double-peak structure of ρ_{yy}, the remnant of broadened Van Hove singularities.

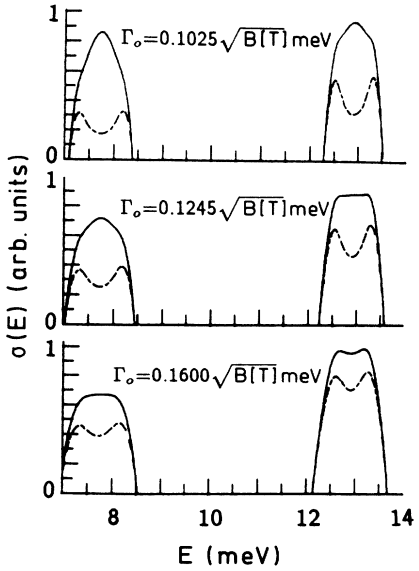

FIG. 9. Calculated conductivity as a function of energy, where $V_0=0.4$ meV, $a=294$ nm, and $B=3.0$ T. The solid line is for σ_{yy} and the broken line is for σ_{xx}.

longer resolved and for which the thermodynamic DOS appears to be constant (see Figs. 5 and 6). This explains the experimental fact[9,11] that, in contrast to the magnetoresistivities, the magnetocapacitance shows no effect of the periodic modulation at the low magnetic fields, where the SdH-type oscillations are not resolved. The only effect of the modulation potential seen in the magnetocapacitance is an amplitude modulation of the SdH-type oscillations, which is clearly seen for sufficiently strong modulation potential. For the small V_0 value chosen in Figs. 5 and 6, this amplitude modulation of D_T is hard to see, although it is present.

The small-amplitude, short-period wiggles seen most clearly in the ρ_{xy} curves are partly due to numerical inaccuracies in the calculation of the chemical potential and partly due to the magnetic field-dependent cut-off procedure used for the calculation of the selfenergy. These could easily be removed at the expense of larger computer time.

For larger magnetic field ($B>1$ T), $\Delta\sigma_{yy}$ increases and ρ_{xx} and ρ_{yy} differ again, as shown in Fig. 8. Systematic experimental investigations are not yet available for this regime. The fine structure of the conductivities within a single SdH peak, has, however, been discussed previously within a different approach[8] omitting collision broadening effects. Within our approach, we find different line shapes of $\sigma_{yy}(E)$ possible, depending on the values of magnetic field and mobility, which determines the relative importance of the diagonal contribution $\Delta\sigma_{yy}$, whereas $\sigma_{xx}(E)$ essentially reflects the DOS. Some possible line shapes are shown in Fig. 9. This may be related to strong changes in the line shape of ρ_{xx} found in experiments in which the electron density and the mobility, and probably also the strength of the modulation potential, have been changed by a gate voltage.[9]

VI. CONCLUDING REMARKS

We have presented a self-consistent theory of collision broadening and magnetotransport for a 2D EG in a unidirectional periodic potential, which gives a satisfactory explanation of all the novel oscillatory effects recently observed in such systems.[2–4,11] The theory contains only two parameters which cannot directly be determined from the experiment, the amplitude V_0 of the periodic potential and an effective strength Γ_0 of the impurity scattering. We choose Γ_0 so that the average resistance at small magnetic fields agrees with experiment and V_0 so that the amplitude of the novel oscillations of ρ_{xx} compares favorably with experiment. This determines, within narrow limits, a V_0 value, which has been checked[3,12] to be reasonable concerning the experimental procedure of microstructuring.[2,3,11] The collision broadening of the Landau bands and the amplitude of all other oscillations are then fixed.

Quantitatively, the agreement between our theory and the experiment is not perfect. The amplitude of the novel oscillations of ρ_{yy} at low magnetic fields, the lowest B value at which the SdH oscillations are first resolved, and the amplitudes of the SdH-type oscillations of both the resistivities and the magnetocapacitance indicate that in the experiment the linewidth of the Landau levels is larger than we calculate from a fit of Γ_0 to the zero-field mobility. This finding is consistent with the situation familiar from nonmodulated samples.[18–20] There, too, the collision broadening is much larger than estimated from the zero magnetic field mobility within the single-parameter SCBA for point scatterers, indicating the importance of long-range Coulomb scatterers in high-mobility samples.[19–21]

The present theory also has problems in explaining in detail the line shape of the magnetocapacitance oscillations, which come out too sharp near the minima, as a consequence of the sharp band edges obtained in the SCBA, owing to the neglect of coherent multi-center scattering.[7] We also expect problems with predicting the detailed line shape of SdH peaks in strong magnetic fields, since localized states, which eventually lead to the quantized Hall effect and are completely neglected in our approximation, may become increasingly important.

Nevertheless, we think that the nice qualitative agreement of our results with the experimental findings in the low magnetic field region leaves no doubt that our theory contains the basic mechanism for the novel magnetoresistance oscillations: the modification of the Landau energy spectrum by the periodic potential, which leads to the beating effects superimposed on the DOS oscillations and, as a direct consequence, to the novel oscillations of ρ_{yy}, and also to a band conduction responsible for the novel oscillations of ρ_{xx} with a phase shift of 180° relative to those of ρ_{yy}.

Note added in proof. Recently, P. Vasilopoulos and F. M. Peeters [Phys. Rev. Lett. **63**, 2120 (1989)] calculated resistivities for the same physical situation, but with an approach neglecting collision broadening. They obtained for ρ_{yy} small-amplitude oscillations (more than 2 orders of magnitude smaller than those of ρ_{xx}) on top of a

nonoscillating background (similar to our result for the DOS), but not the weakly-temperature-dependent result shown in our Figs. 5 and 6.

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APPENDIX

Here we derive approximate analytical formulas for the self-energy $\Sigma^-(E)$ and the DOS $D(E)$ which explicitly demonstrate the beating effect owing to the spatial modulation on the SdH-type quantum oscillations, and which hold in the large-n limit for small magnetic field at finite energies, $E \gg \hbar\omega_c$.

The main contribution to the sum in Eq. (3.4) comes from the energy band $E_n(x_0)$ [cf Eq. (2.6)] closest to E. We assume $n_E = E/\hbar\omega_c \gg 1$ and use the asymptotic form (2.7) to approximate in Eq. (3.4) the energy spectrum

$$E_n(x_0) \approx \hbar\omega_c(n+\tfrac{1}{2}) + U(E)\cos(Kx_0) , \tag{A1}$$

with

$$U(E) = V_0 \pi^{-1/2} \left[\tfrac{1}{2} K^2 l^2 \frac{E}{\hbar\omega_c} \right]^{-1/4} \times \cos\left[Kl \left[\frac{2E}{\hbar\omega_c} \right]^{1/2} - \frac{4}{\pi} \right] . \tag{A2}$$

For the imaginary part of Eq. (3.4) we can now, to a good approximation, extend the n sum from $-\infty$ to ∞ and evaluate it by the method of residues. We do the same for the real part and, thereby, introduce effectively a cutoff, since the symmetric sum of n and $-n$ terms converges. We thus obtain

$$\Sigma^-(E) \approx \int_0^a \frac{dx_0}{a} \sum_{n=-\infty}^{\infty} \frac{\Gamma_0^2}{E - \Sigma^-(E) - U(E)\cos(Kx_0) - \hbar\omega_c(n+\frac{1}{2})}$$
$$= \frac{\pi\Gamma_0^2}{\hbar\omega_c} \int_0^a dx_0 \frac{1}{a} \cot\left[\frac{\pi}{\hbar\omega_c} [E - \Sigma^-(E) - U(E)\cos(Kx_0)] - \frac{\pi}{2} \right] , \tag{A3}$$

or, separating real and imaginary parts,

$$\Delta(E) + \frac{i}{2}\Gamma(E) = \frac{\pi\Gamma_0^2}{\hbar\omega_c} \int_0^a dx_0 \frac{1}{a} \frac{-\sin u + i \sinh v}{\cosh v + \cos u} , \tag{A4}$$

where $v = \pi\Gamma(E)/\hbar\omega_c$, and

$$u = 2\pi[E - \Delta(E) - U(E)\cos(Kx_0)]/\hbar\omega_c . \tag{A5}$$

In the case of large collision broadening, $\pi\Gamma \gg \hbar\omega_c$, we can expand with respect to the small quantity $\exp(-v)$, and solve Eq. (A4) by iteration. Up to first order in exponentially small terms, one obtains

$$\tfrac{1}{2}\Gamma(E) = \frac{\Gamma_0^2 \pi}{\hbar\omega_c} \left\{ 1 - 2\exp\left[-2 \left[\frac{\pi\Gamma_0}{\hbar\omega_c} \right]^2 \right] \int_0^a dx_0 \frac{1}{a} \cos\left[\frac{2\pi}{\hbar\omega_c} [E - U(E)\cos(Kx_0)] \right] \right\} , \tag{A6}$$

$$\Delta(E) = \frac{\Gamma_0^2 \pi}{\hbar\omega_c} 2\exp\left[-2 \left[\frac{\pi\Gamma_0}{\hbar\omega_c} \right]^2 \right] \int_0^a dx_0 \frac{1}{a} \sin\left[\frac{2\pi}{\hbar\omega_c} [E - U(E)\cos(Kx_0)] \right] , \tag{A7}$$

and, expanding up to second order in the modulation strength V_0, one gets from Eq. (3.8) for the DOS

$$D(E) = \frac{m}{\pi\hbar^2} \left[1 + 2\exp\left[-2 \left[\frac{\pi\Gamma_0}{\hbar\omega_c} \right]^2 \right] \cos\left[\frac{2\pi E}{\hbar\omega_c} - \pi \right] \left\{ 1 - \frac{a}{l} \left[\frac{V_0}{\hbar\omega_c} \right]^2 \left[\frac{\hbar\omega_c}{2E} \right]^{1/2} \cos^2\left[2\pi \frac{l}{a} \left[\frac{2E}{\hbar\omega_c} \right]^{1/2} - \frac{\pi}{4} \right] \right\} \right] . \tag{A8}$$

For $V_0 = 0$, this is exponentially damped de Haas–van Alphen (dHvA) oscillations around the zero-B DOS, with maxima at the Landau energies $E_n = \hbar\omega_c(n+\frac{1}{2})$. The modulation potential leads to a beating effect, an oscillation, which becomes weaker with increasing energy as $(a/l)[(\hbar\omega_c)/(2E)]^{1/2} = [\frac{1}{2}(m\omega_c^2 a^2)/E]^{1/2}$. The dHvA amplitude is in general reduced and has the value of the unmodulated system only at the flat-band energies $E_\lambda = \frac{1}{8} m\omega_c^2 a^2 (\lambda - \frac{1}{4})^2$ corresponding to Eq. (2.8).

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