Phonon-assisted magneto-optical transitions in two-dimensional systems

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We have calculated the phonon-assisted dielectric response of p-type two-dimensional systems in the presence of a magnetic field. This permits us to calculate the magneto-optical absorption at the mixed cyclotron resonance in which absorption or emission of a boson accompanies a cyclotron transition. We use the Luttinger-Kohn Hamiltonian to represent the hole carriers and through a canonical transformation eliminate the carrier-boson interaction to first order. The expression for the phonon-assisted cyclotron-resonance absorption coefficient is obtained using linear-response theory. The calculation is valid in the weak-coupling limit when only a one-boson process needs to be considered. From the above formulation we have calculated the line shape for the usual cyclotron resonance and a phonon-assisted sideband in the Faraday configuration for left and right circularly polarized light. An additional peak in the absorption coefficient due to phonon-assisted transition is predicted in p-type GaAs/$Ga_{1-x}Al_xAs$ quantum wells.

I. INTRODUCTION

Molecular-beam-epitaxy techniques allow the manipulation of band discontinuities and quantum wells that occur at the interface between different layers of semiconductors. They form the basis for a broad class of two-dimensional (2D) electron and hole systems. These materials have recently become very important for studies on the basic physics of 2D electron and hole systems, as well as practical applications for new semiconductor devices. There is considerable interest in the study of optical properties[1–5] of such systems in the presence of a magnetic field, e.g., interband and intraband magneto-optical properties, magnetophonon effect, cyclotron resonance, magnetoexcitons, etc. The electronic properties of these systems are strongly modified by the application of an external magnetic field, and its effects are measured in various optical experiments. These experiments not only give information about novel conditions in a magnetic field, but in many cases also help towards a better understanding of the zero-field properties.

In the past few years considerable effort[6–9] has been devoted to the study of electron-phonon interaction on the optical properties of 2D systems in the presence of a magnetic field. One of us has reported[8,9] calculations on the frequency-dependent conductivity tensor for p- and n-type 2D systems interacting with acoustic phonons. Other investigations[4–7] on the magneto-optical properties of these systems are mainly concerned with effects like polaron effect, subband coupling, and nonparabolicity of the band. Some studies also reported anomalies in carrier effective mass and Landau-level broadening due to electron-phonon interaction. Another major magneto-optical effect relevant to any discussion of interaction process is that of the boson-assisted magneto-optical (BAMO) resonance transition. Xiaoguang *et al.*[10] have studied the polaron cyclotron resonance spectrum using a memory-function approach. They obtained the cyclotron frequency and the cyclotron mass of the single polaron from the position of certain peaks in the magneto-optical absorption spectrum. Recently, Cai *et al.*[11] discussed phonon-assisted electron tunneling through a semiconductor barrier. In this paper we turn our attention to the calculation of boson-assisted cyclotron resonance transition in p-type 2D systems. Here boson stands for phonon, plasmon, or coupled plasmon-phonon mode.

The boson-assisted magneto-optical transitions in three-dimensional (3D) semiconductors have been studied extensively.[12–15] Bass and Levinson, Enck *et al.*, and Johnson and Dickley were the first to demonstrate the existence of a phonon-assisted cyclotron resonance (PACR) transition.[12] McCombe *et al.*[13] observed the PACR in $Hg_{1-x}Cd_xTe$, while Nagasaka *et al.*[13] found evidence for a similar resonance in n-type CdS. More recently Goodwin and Seiler studied[14] in detail the PACR in InSb for left circularly polarized (LCP) and right circularly polarized (RCP) light. They found that in the Faraday geometry, the resonances were approximately equal in amplitude for both LCP and RCP light.

In this paper, we present our calculations on the dielectric response of p-type 2D systems exhibiting BAMO resonance. We have evaluated the frequency-dependent conductivity tensor using linear-response theory. The current operators are evaluated by making use of a canonical transformation which eliminates the carrier-boson interaction to first order in the coupling constant. Expressions for conductivity and the absorption coefficient are obtained in the Faraday configuration for LCP and RCP light. The theory predicts extra peaks in the magneto-optical spectrum due to BAMO transitions besides the usual cyclotron resonance. These are due to transitions between two Landau levels accompanied by emission and absorption of bosons. Numerical calculations have been performed for p-type GaAs/$Ga_{1-x}Al_xAs$

quantum well in the extreme quantum limit. The remaining part of the paper is organized as follows. In Sec. II we outline the theory for the calculation of conductivity tensor, and the absorption coefficient. Our results for p-type materials are presented in Sec. III, where we also provide commentary on the nature of our calculation,and comparison with other theoretical accounts.

II. THEORY

We consider the quantum-mechanical problem of carrier motion in the xy plane in 2D systems. A magnetic field is applied parallel to the growth direction of the superlattice, which is the z direction. The Hamiltonian of the interacting carrier-boson system is ($\hbar=1$)

$$H=H_0+\sum_{\mathbf{Q}}\omega_{\mathbf{Q}}b^{\dagger}_{\mathbf{Q}}b_{\mathbf{Q}} -i\sum_{\mathbf{Q}}\sum_{\substack{nn'\\pp'}}\gamma(\mathbf{Q})D^{p'p}_{n'n}(\mathbf{Q})c^{\dagger}_{n'p'}c_{np}(b_{\mathbf{Q}}-b^{\dagger}_{\mathbf{Q}}) , \quad (1)$$

in which the first term represents the Luttinger-Kohn Hamiltonian for the holes[8,16,17]

$$H_0=\begin{bmatrix} P+Q & S & R & 0 \\ S^{\dagger} & P-Q & 0 & R \\ R^{\dagger} & 0 & P-Q & -S \\ 0 & R^{\dagger} & -S^{\dagger} & P+Q \end{bmatrix} , \quad (2)$$

where

$$P\pm Q=E-\left[\frac{\gamma_1\pm\gamma_2}{2ml^2}\right](c^{\dagger}_{np}c_{np}+c_{np}c^{\dagger}_{np})\left[\frac{\gamma_1\mp 2\gamma_2}{2m}\right]k_z^2+V(z)+J_z\kappa\mu_B B ,$$

$$R=\frac{\sqrt{3}}{4ml^2}[\gamma(c_{np})^2-\mu(c^{\dagger}_{np})^2],\quad S=\frac{\sqrt{6}\gamma_3}{ml}k_z c_{np} , \quad (3)$$

$$\gamma=\gamma_2+\gamma_3,\quad \mu=\gamma_3-\gamma_2 ,$$

and γ_1, γ_2, and γ_3 are material parameters determining the effective masses. The first two terms in Eq. (1) represent the noninteracting Hamiltonian for the hole carrier and boson system. The third term is the Hamiltonian for holes coupled to boson modes. Landau and subband quantum numbers are denoted by n and p, respectively. E_{np} is the energy of the nth Landau level in the pth subband. $c^{\dagger}_{np}$ and c_{np} are the Landau creation and annihilation operators, $b^{\dagger}_{\mathbf{Q}}$ and $b_{\mathbf{Q}}$ are boson creation and annihilation operators, respectively. $\mathbf{Q}=(Q_\perp,Q_z)$ where $Q_\perp$ and Q_z are components of the boson wave vector along and perpendicular to the 2D surface, respectively. In the above Hamiltonian, also included is the interaction of spin magnetic moment with the external magnetic field through $J_z\kappa\mu_B B$, where κ is the Luttinger constant and μ_B is the Bohr magneton, and $V(z)$ is a self-consistent effective potential. Landau and subband quantum numbers are denoted by n and p, respectively. The Landau wave function for the holes in single-quantum structures is given by[8,9,17]

$$|np\rangle=\begin{bmatrix} f^1_{np}|n-2\rangle \\ f^2_{np}|n-1\rangle \\ f^3_{np}|n\rangle \\ f^4_{np}|n+1\rangle \end{bmatrix} , \quad (4)$$

where the function f^i_{np} depend on the hole motion in the z direction, and $|n\rangle$ is the usual harmonic oscillator function for cyclotron motion in the xy plane. The matrix elements appearing in the carrier-boson coupling Hamiltonian are given by

$$D^{p'p}_{n'n}(\mathbf{Q})=F^{11}_{p'p}(Q_z)J_{n'-2,n-2}(Q_\perp) +F^{22}_{p'p}(Q_z)J_{n'-1,n-1}(Q_\perp)+F^{33}_{p'p}(Q_z)J_{n',n}(Q_\perp) +F^{44}_{p'p}(Q_z)J_{n'+1,n+1}(Q_\perp) , \quad (5)$$

where

$$J_{n'n}(Q_\perp)=(-1)^{n'-n}\sqrt{n'!/n!}\,x^{(n'-n)/2}L_n^{n'-n}(x)e^{-x/2} ;$$

$$x=\frac{Q_\perp^2 l^2}{2} , \quad (6)$$

$$F^{\alpha\beta}_{p'p}(Q_z)=\langle f^{\alpha}_{n'p'}|\exp(iQ_z z)|f^{\beta}_{np}\rangle , \quad (7)$$

l is the Landau radius, and $L_n^{n'}(x)$ are the associated Laguerre polynomials.

The coupling constant $\gamma(\mathbf{Q})$ can be obtained from the bare interaction

$$v(\mathbf{Q})=\frac{4\pi e^2}{Q^2\epsilon_b(\omega)} , \quad (8)$$

where ϵ_b is the dielectric function for the bosons. For instance, substituting ϵ_b by phonon $\epsilon(\omega)$ yields the well-known Fröhlich result for optical phonons. Similarly one can obtain an expression for $\gamma(\mathbf{Q})$ for plasmons and coupled phonon-plasmon modes by utilizing the appropriate dielectric function in ϵ_b.

The interaction Hamiltonian for carriers and electromagnetic radiation can be written as

$$H_{\text{int}}=\frac{i}{\omega}e^{-i\omega t}\mathbf{E}_0\cdot\mathbf{j}(\mathbf{q}) , \quad (9)$$

where $\mathbf{j}(\mathbf{q})=\frac{1}{2}[\mathbf{j}\exp(i\mathbf{q}\cdot\mathbf{r})+\exp(i\mathbf{q}\cdot\mathbf{r})\mathbf{j}]$. $\mathbf{E}_0$ is the electric field vector of the electromagnetic wave, $\mathbf{q}=(q_\perp,q_z)$,

$q_\perp$ and q_z being the photon wave vectors parallel and perpendicular to the 2D plane, respectively. We use a circular polarization representation whose basis vectors are $\hat{e}_\pm=(\hat{e}_x \pm i\hat{e}_y)/\sqrt{2}$ and $\hat{e}_x$. Here the unit vectors $\hat{e}_x$ and $\hat{e}_y$ are taken in the xy plane. The components of the current operator relative to this basis become $j_-=el(\partial H/\partial c_{np})$, and $j_+=el(\partial H/\partial c_{np}^\dagger)$.

The system we are investigating, responding to the electromagnetic radiation, is that of interacting carriers and bosons. We are chiefly interested in the low-temperature regime, where carrier-level broadening will be due primarily to impurity scattering and acoustical-photon scattering. Hence only the one-boson interaction processes to the lowest order will be important. We remove the carrier-boson interaction term from the Hamiltonian H to first order [in the coupling term $\gamma(\mathbf{Q})$] by a canonical transformation $\tilde{H}=\exp(iS)H\exp(-iS)$. The generator of the canonical transformation S is given by

$$S=\sum_{\substack{n'n\\p'p}}\sum_{\mathbf{Q}}\gamma(\mathbf{Q})D_{n'n}^{p'p}c_{n'p'}^\dagger c_{np} \times\left[\frac{b_\mathbf{Q}}{E_{n'p'}-E_{np}-\omega_\mathbf{Q}}-\frac{b_{-\mathbf{Q}}^\dagger}{E_{n'p'}-E_{np}+\omega_\mathbf{Q}}\right] . \tag{10}$$

One can now use the transformed system to calculate the current operator appearing in the conductivity tensor. The transformed current operator, to first order in carrier-boson interaction, is

$$\tilde{j}_\pm(\mathbf{q})=j_\pm^0(\mathbf{q})+i[S,j_\pm^0(\mathbf{q})]=j_\pm^0(\mathbf{q})+j_\pm^1(\mathbf{q}) . \tag{11}$$

The first term $j_\pm^0(\mathbf{q})$ is the current operator in the absence of carrier-boson coupling Hamiltonian and the second term denoted by $j_\pm^1(\mathbf{q})$ is written as

$$j_\pm^1(\mathbf{q})=\sum_{\substack{nn'\\pp'}}\sum_{\mathbf{Q}}\{[F_{n'p',np}^\pm(\omega_\mathbf{Q},\mathbf{q})c_{n'p'}^\dagger c_{np}b_\mathbf{Q}] +[F_{n'p',np}^\pm(-\omega_\mathbf{Q},\mathbf{q})c_{n'p'}^\dagger c_{np}b_\mathbf{Q}]\} , \tag{12}$$

where

$$F_{n'p',np}^\pm(\omega_\mathbf{Q},\mathbf{q})=i\gamma(\mathbf{Q})\sum_{n''p''}\frac{D_{n'n''}^{p'p''}(\mathbf{Q})\langle n''p''|j_\pm^0(\mathbf{q})|np\rangle}{E_{n'p'}-E_{n''p''}-\omega_\mathbf{Q}} . \tag{13}$$

The expression for $F_{n'p',np}^\pm(-\omega_\mathbf{Q},\mathbf{q})$ may be obtained from Eq. (13) by replacing $\omega_\mathbf{Q}$ by $-\omega_\mathbf{Q}$. The matrix elements of $\langle n''p''|j_\pm^0(\mathbf{q})|np\rangle$ are provided in Ref. 8.

We calculate the conductivity tensor $\sigma_{-+}(\omega)$ in the circular polarization representation within linear-response theory, using the Kubo formula

$$\sigma_\mp(\omega)=\frac{1}{2\omega}\int_0^\infty dt\, e^{i(\omega+i\eta)t}\langle[\tilde{j}_-(\mathbf{q},t),\tilde{j}_+(\mathbf{q},0)]\rangle . \tag{14}$$

To obtain the conductivity tensor from Eq. (14) we use the Landau wave functions in the evaluation of the trace. Replacing $\tilde{j}_\pm(\mathbf{q})$ with the zeroth- and first-order contributions [Eq. (11)] results in four commutators:

$$\langle[\tilde{j}_-(\mathbf{q},t),\tilde{j}_+(\mathbf{q},0)]\rangle=\langle[j_-^0,j_+^0]\rangle+\langle[j_-^0,j_+^1]\rangle +\langle[j_-^1,j_+^0]\rangle+\langle[j_-^1,j_+^1]\rangle , \tag{15}$$

of which the first term gives rise to $\sigma_{-+}^0(\omega)$ and the last one to $\sigma_{-+}^1(\omega)$. The other terms involving j^0 and j^1 in Eq. (15) give zero contribution to the conductivity tensor, since the boson operators $b_\mathbf{Q}$ and $b_\mathbf{Q}^\dagger$ have zero trace. Explicitly, the conductivity tensor due to boson-assisted transitions between Landau states becomes

$$\sigma_\mp^1(\omega)=\frac{i}{2\omega}\sum_{\substack{nn'\\pp'}}\sum_{\mathbf{Q}}\left[\frac{R_{np,n'p'}^{\mp}N_{np,n'p'}^{Q}}{E_{n'p'}-E_{np}-\omega_\mathbf{Q}+\omega+i\Gamma_{n'p'}}+\frac{R_{np,n'p'}^{\pm}N_{np,n'p'}^{Q}}{E_{n'p'}-E_{np}+\omega_\mathbf{Q}+\omega+i\Gamma_{n'p'}}\right] , \tag{16}$$

where

$$R_{np,n'p'}^{rs}=F_{np,n'p'}^{r}(\omega_\mathbf{Q},\mathbf{q})F_{n'p',np}^{s}(-\omega_\mathbf{Q},\mathbf{q}) , \tag{17}$$

$$N_{np,n'p'}^{Q}=N_\mathbf{Q}(f_{np}-f_{n'p'})+f_{np}(1-f_{n'p'}) . \tag{18}$$

Here Γ_α is the imaginary part of the self-energy function and the real part of self-energy has been included in E_α for brevity. Also, we have abbreviated $f_{np}=f(E_{np})$, and $N_\mathbf{Q}=N_0(\omega_\mathbf{Q})$, the Fermi and Bose distribution functions evaluated at carrier and boson energies.

Now one can find the expression for the absorption coefficient due to phonon-assisted absorption. The absorption coefficient $\alpha_-^1(\omega)$ is obtained from the real part of the conductivity tensor as $\alpha_-^1(\omega)=(E_0^2/2)\mathrm{Re}[\sigma_\mp^1(\omega)]$.[8] The formula for the absorption coefficient is considerably simplified (computationally speaking) if we choose electromagnetic wave vector $\mathbf{q}$ parallel to the magnetic-field direction (i.e., Faraday configuration). The absorption coefficient in the Faraday configuration reads

$$\alpha_-^1(\omega)=\frac{|\mathbf{E}_0|^2}{4\omega}\sum_{\mathbf{Q}}\sum_{\substack{nn'\\pp'}}\frac{|\gamma(\mathbf{Q})|^2\Gamma_{n'p'}N^{Q}_{np,n'p'}}{(E_{np}-E_{n'p'}+\omega_{\mathbf{Q}})^2[(E_{np}-E_{n'p'}\pm\omega_{\mathbf{Q}}+\omega)^2+\Gamma^2_{n'p'}]}$$
$$\times[D^{p'p}_{n',n-1}(\mathbf{Q})I^{1-}+D^{p'p}_{n',n-3}(\mathbf{Q})I^{3-}][D^{p'p}_{n',n+1}(\mathbf{Q})I^{1+}+D^{p'p}_{n',n+3}(\mathbf{Q})I^{3+}]\ , \tag{19}$$

where

$$\begin{aligned}I^{1-}&=\alpha_1[F^{11}_{p'p}(q_z)\sqrt{n-1}+F^{44}_{p'p}(q_z)\sqrt{n+2}]+\alpha_2[F^{22}_{p'p}(q_z)\sqrt{n}+F^{33}_{p'p}(q_z)\sqrt{n+1}]\\&\quad+\alpha_3k_z[F^{12}_{p'p}(q_z)-F^{34}_{p'p}(q_z)]+\alpha_4[F^{13}_{p'p}(q_z)\sqrt{n}+F^{24}_{p'p}(q_z)\sqrt{n+1}]\ ,\\I^{3-}&=\alpha_5[F^{13}_{p'p}(q_z)\sqrt{n+1}+F^{24}_{p'p}(q_z)\sqrt{n+2}]\ ,\end{aligned} \tag{20}$$

in which

$$\alpha_1=(\gamma_1+\gamma_2)\frac{e}{ml},\quad \alpha_2=(\gamma_1-\gamma_2)\frac{e}{ml},\quad \alpha_3=\sqrt{6}\gamma_3\frac{e}{ml},\quad \alpha_4=\sqrt{3}\gamma\frac{e}{ml},\quad \alpha_5=-\sqrt{3}\mu\frac{e}{ml}\ . \tag{21}$$

The expressions for I^{1+} and I^{3+} are complex conjugates of I^{1-} and I^{3-}, and they follow from the matrix elements of the current operator $j^0_\pm$ in the Faraday configuration. Their detailed derivation is given by Singh and Tang.[8]

Note from the expression for $\alpha_-^1(\omega)$ that the absorption peaks will appear at $\omega=E_{n'p'}-E_{np}\pm\omega_{\mathbf{Q}}$. The positive term corresponds to a transition from the state $|n'p'\rangle$ to state $|np\rangle$ accompanied by the emission of a boson. The negative term corresponds to the absorption of a boson. When a transition takes place within one subband ($p\rightarrow p$) and different Landau levels (i.e., say $n\rightarrow n+1$) in the presence of a boson, we call it boson-assisted cyclotron resonance. When $p\neq p'$ and one has a $\Delta n=\pm1$ transition we call it combined boson-assisted cyclotron resonance. As indicated by $I^{3\pm}$ terms, there are transitions when $\Delta n=\pm3$; they arise because of the mixing of $J_z=\pm\frac{3}{2}$ and $\pm\frac{1}{2}$ states in the Luttinger-Kohn Hamiltonian.[8]

Finally, for completeness, we give the absorption coefficient in the absence of carrier-boson interaction:

$$\alpha_-^0(\omega)=\frac{|\mathbf{E}_0|^2}{4\omega}\sum_{\substack{nn'\\pp'}}(\delta_{n',n-1}I^{1-}+\delta_{n',n-3}I^{3-})(\delta_{n',n+1}I^{1+}+\delta_{n',n+3}I^{3+})$$
$$\times\left[\frac{f(E_{n'p'})\Gamma_{np}}{(E_{n'p'}-E_{np}+\omega)^2+\Gamma^2_{np}}+\frac{f(E_{np})\Gamma_{n'p'}}{(E_{np}-E_{n'p'}+\omega)^2+\Gamma^2_{n'p'}}\right]\ , \tag{22}$$

where $I^{1\pm}$ and $I^{3\pm}$ have the same meaning as in Eq. (19). This completes our derivation of the conductivity tensor and absorption coefficient for p-type materials. We have given results for the LCP light only, but similar expressions can be obtained for RCP light through the transformation $\sigma_\pm(\omega)=\sigma_\pm(-\omega)$.

III. RESULTS AND DISCUSSION

Having developed a theory for the linear response of a collection of hole carriers and boson modes in single-quantum-well structures, we now apply it to a specific case. We consider the GaAs/$Ga_{1-x}Al_x$GaAs heterojunction, in which the boson modes are polar optical phonons, with coupling

$$\gamma_{\mathrm{OP}}(\mathbf{Q})=2\pi e^2\omega_L\left[\frac{1}{\epsilon_\infty}-\frac{1}{\epsilon_0}\right]|\mathbf{Q}|\ . \tag{23}$$

The optical-phonon frequency is denoted by ω_L and ϵ_∞ and ϵ_0 are the dielectric constants of the material. We calculate the conductivity tensor in the extreme quantum limit where subband indices are $p=p'=1$ and the Landau level is $n=0$. If we assume that the Fermi level lies between Landau levels $n=0$ and $n=1$, then $f(E_{np})=1$ for $n=0$, $p=1$, and $f(E_{np})=0$ for $n=1$, $p=1$, at low temperatures. Furthermore, we shall neglect the $\Delta n=\pm3$ transition, and only consider the $\Delta n=\pm1$ case.

In the following, we present our results on the absorption coefficient of p-type GaAs/$Ga_{1-x}Al_xAs$ heterojunctions. There are two types of holes, heavy and light, present in this material, distinguished by their effective masses. In our calculation, we use the theoretical effective masses given by Broido and Sham,[16] but neglect their dependence on the applied magnetic field. We display in Fig. 1, the absorption coefficient $\alpha_-(\omega)$ (LCP light in Faraday configuration) for heavy holes (effective mass $m_h=0.4m_e$). The first peak at $\omega/\omega_c=1$ is the usual cyclotron resonance peak, the second one occurring at $\omega/\omega_c=1+\omega_L/\omega_c\approx13.6$ is due to the phonon-assisted cyclotron resonance. The results presented here at $T=2$ K and $B=10$ T. The optical-phonon frequency ω_L is taken to be 36.2 meV, and for the Landau-level broadening (the imaginary part of the self-energy) Γ_{01} we have assumed a constant value of 0.5 meV. Note that neither the position of the peaks nor their relative intensities are affected by the choice of Γ_{01}, only the width depends on Γ_{01} at a given magnetic-field strength B. From Fig. 1 we observe that the relative intensity of the second peak due to the BAMO resonance is about a fifth of the cyclotron resonance. It should be observable without difficulty in a

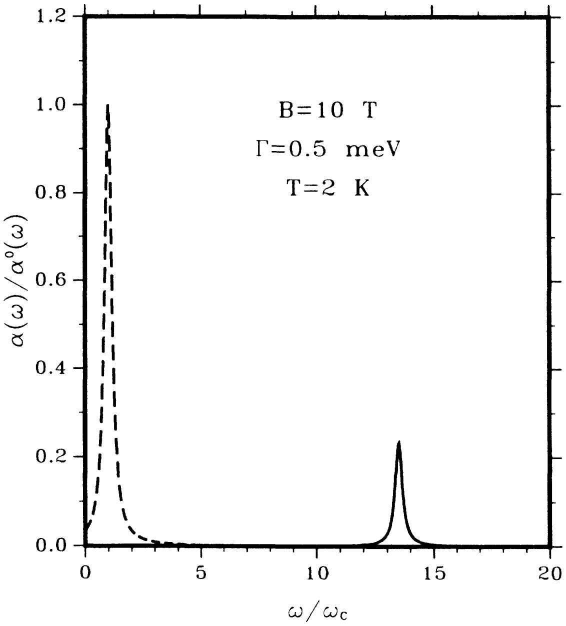

FIG. 1. The absorption coefficient $\alpha_-(\omega)$ divided by $\alpha_-^0(\omega)$ vs ω/ω_c for heavy holes in the GaAs/Ga$_{1-x}$Al$_x$As superlattice at $T=2$ K and $B=10$ T. The first peak (dashed line) originates from $\alpha_-^0(\omega)$, the second peak (solid line) is the phonon-assisted resonance coming from $\alpha_-^1(\omega)$.

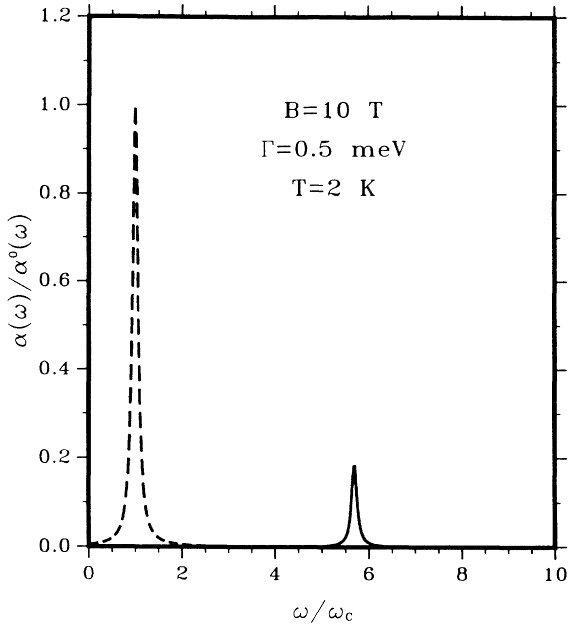

FIG. 2. The absorption coefficient $\alpha_-(\omega)$ divided by $\alpha_-^0(\omega)$ vs ω/ω_c for light holes in the GaAs/Ga$_{1-x}$Al$_x$As superlattice, at $T=2$ K and $B=10$ T. The first peak (dashed line) originates from $\alpha_-^0(\omega)$, the second peak (solid line) is the phonon-assisted resonance coming from $\alpha_-^1(\omega)$.

low-temperature, high-magnetic-field ($B\simeq 10$ T) experiment. According to Eq. (19) there should be another peak at $\omega/\omega_c=1-\omega_L/\omega_c$ due to phonon-absorption process. The intensity of this peak is very small because of the thermal factor $N_0(\omega_L)$; there are very few phonons to contribute to an absorption process at low temperatures. The temperature at which phonon absorption becomes appreciable is $k_BT\simeq\omega_L$. Therefore our model predicts only a phonon emission peak to be observed in a low-temperature experiment. In Fig. 2, we show the same effect for light holes with effective mass $m_l=0.15m_e$. In addition to the usual cyclotron resonance, we observe a second peak at $\omega/\omega_c=1+\omega_L/\omega_c\simeq 5.7$ due to phonon emission. Note that the phonon-assisted peaks occur at different positions in Figs. 1 and 2, since the cyclotron frequency ω_c is different in both cases owing to the difference in the effective masses of heavy and light holes. Xiaoguang *et al.*[10] have studied the cyclotron resonance spectrum of a 2D polaron within the memory-function formalism, obtaining cyclotron resonance frequency and mass in the weak electron-phonon coupling limit. They have found that the absorption spectrum shows phonon-assisted peaks around $\omega=\omega_L+n\omega_c$, but have not reported the intensity of such peaks relative to the main cyclotron resonance, rendering a direct comparison with our results difficult.

The use of a constant value for Γ_{np} can easily be relaxed to make the whole calculation parameter free. We have noted earlier that the Landau-level broadening at low temperatures will be due primarily to impurity scattering and acoustic-phonon effects. Writing out the hole self-energy function due to hole–acoustic-phonon interaction in the self-consistent Born approximation as

$$\Sigma_{np}(\omega)=\sum_{\substack{n'p'\\ \mathbf{Q}}}\gamma(\mathbf{Q})|D_{nn'}^{pp'}(\mathbf{Q})|^2 \times\left[\frac{N_0(\omega_{\mathbf{Q}})}{\omega+\omega_{\mathbf{Q}}-E_{n'p'}+\Sigma_{n'p'}(\omega)}+\frac{1+N_0(\omega_{\mathbf{Q}})}{\omega-\omega_{\mathbf{Q}}-E_{n'p'}+\Sigma_{n'p'}(\omega)}\right], \quad (24)$$

where $\gamma(\mathbf{Q})$ now describes the hole–acoustic-phonon interaction, and taking the imaginary parts of both sides, one obtains a self-consistent equation for the Landau-level broadening function Γ_{np}, which may be simplified in special cases. Similarly, one can express the broadening due to disorder when the impurity potential is specified.[15] In the elastic-scattering approximation and from dimensional analysis, one deduces[5] that $\Gamma_{np}\propto(B/m^*)^{1/2}$, where m^* is the effective mass of the holes. This shows that the Landau-level broadening should increase with the applied magnetic field, and for a given B, it should be greater for the light holes than the heavy holes. Therefore, if such effects were included in our calculation of the absorption coefficient, we would obtain a broader phonon-assisted peak for the light holes than the corresponding peak for the heavy holes.

We have also investigated the dependence of the ab-

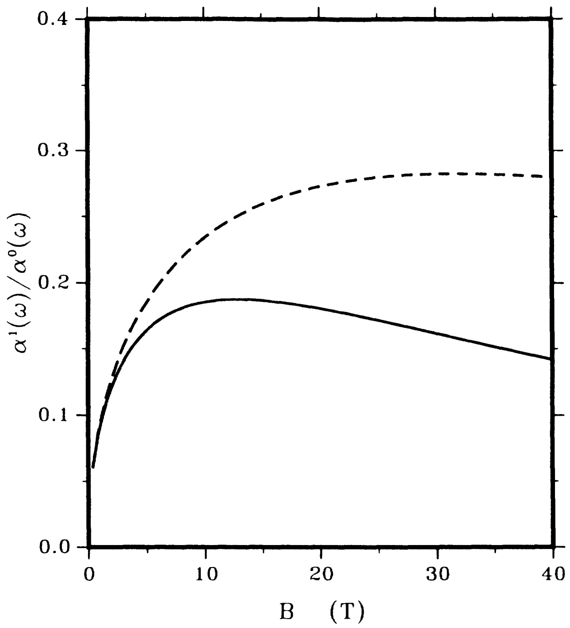

FIG. 3. The ratio of the peak values of the absorption coefficients $\alpha_-^1(\omega_L+\omega_c)$ and $\alpha_-^0(\omega_c)$ vs the magnetic field B for heavy (solid line) and light (dashed line) holes.

sorption coefficient on the magnetic field. Figure 3 shows the ratio of the peak values of $\alpha_-^1(\omega)$ and $\alpha_-^0(\omega)$ as a function of B for heavy and light holes. The peak values calculated at $\omega=\omega_L+\omega_c$ and $\omega=\omega_c$ for $\alpha_-^1(\omega)$ and $\alpha_-^0(\omega)$, respectively. We observe that the peak ratio of the phonon-assisted transition to the cyclotron resonance reaches a maximum at $B\simeq 13$ T for the light holes. The heavy holes also show a similar behavior around $B\simeq 30$ T, although it is barely visible on the scale of Fig. 3. This suggests an optimum value of the applied field to be used in the experiments to observe the BAMO transitions. In principle, the Landau-level broadening Γ_{np}, and effective masses of heavy and light holes should depend on B. We have not included these effects into our calculation. Omission of such factors may be responsible for the observed behavior of $\alpha_-(\omega)$ in Fig. 3. In any case, it is interesting to note that the intensity of phonon-assisted cyclotron resonance transition for a heavy hole is greater than that of a light hole.

In conclusion, we have evaluated the conductivity tensor which determines all the magneto-optical properties of a material due to the boson-assisted transition between Landau levels. In magneto-optical experiments at finite temperature, in addition to carrier-photon interaction, bosons in the material also interact with the carrier-photon system to create absorption and emission resonance. For example, a carrier in Landau level n will absorb a photon, while simultaneously emitting or absorbing an optical phonon of energy ω_L, thus making a transition to some higher Landau level n'. The same may happen for plasmons or more generally still for hybrid modes involving plasmon–optical-phonon interaction. Our theory allows, in principle, a general treatment of the boson-assisted cyclotron resonance in 2D systems. The introduction of new resonance will also modify the spectrum of elementary excitations in the material, introducing new continuum of such excitations. Experimentally, the boson-assisted transition can be recognized by the fact that when magnetic fields tends to zero, their energies converge to ω_Q. The authors are not aware of any experimental results on BAMO transitions in 2D systems, therefore a direct comparison of our results with experiment is not possible. It would be interesting to extend the present formalism to multiple-quantum-well structures, where the calculation of energy levels will be considerably more complex.

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