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Ballistic transport in a novel one-dimensional superlattice

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We point out that fabrication and experimental study of a one-dimensional semiconductor superlattice (1DSL) structured in a ballistic constriction is now feasible and present a theory of its transport properties. We predict sharp switching between quantized conductance plateaus as the Fermi level moves through the 1DSL miniband gaps and strong resonant conductance oscillations. Surprisingly, these effects are predicted to be observable even for 1DSL's with very few periods and to be nearly independent of the geometry of the constriction openings.

Remarkable advances in microfabrication have occurred in the last few years,[1] making it possible to confine the electrons of a two-dimensional electron gas (2D EG) in a semiconductor heterostructure to regions with a lateral extent of 100 nm or less, resulting in narrow quantum wires, constrictions, and quantum dots. The Fermi wavelength of the confined electrons is close to the dimensions of these nanostructures, so the quantization of the electron energy levels is very important. Also, the new confinement techniques used, the small size of the structures, and the very high mobility of the parent 2D EG largely eliminate defect scattering, making the electron transport ballistic or quasiballistic at low temperatures. Notable examples of the novel physical phenomena recently discovered in these ballistic nanostructures are the quantized conductance of short ballistic constrictions reported by van Wees *et al.*[2] and Wharam *et al.*,[3] the disappearance of the Hall voltage across ultranarrow conductors reported by Roukes *et al.*,[4] the nonlocal coherent bend-resistance effects reported by Timp *et al.*,[5] and conductance oscillations indicative of charge density waves in quantum wires reported by Scott-Thomas *et al.*[6] The purpose of this article is to point out that the experimental study of an important new class of semiconductor nanostructures, the one-dimensional ballistic superlattices (1DSL's), is now within reach and to present detailed theoretical predictions of their transport properties. The physics of three-dimensional semiconductor superlattices is a topic of great current interest,[7] and tunneling in two dimensions has recently yielded very interesting results.[8] The still lower dimensionality and very high degree of quantum coherence in the 1DSL's are unique features which we believe will make the study of these new systems very exciting.[9]

Figure 1(a) shows schematically a possible realization of the proposed 1DSL. The shaded areas represent a negatively biased metal gate placed on top of a GaAs-$Al_xGa_{1-x}As$ heterostructure containing a 2D EG, in an arrangement similar to the split-gate devices used to observe the quantized conductance of ballistic constrictions.[2,3] As in those devices, the negative bias depletes the 2D EG under the gate, leaving only a narrow ballistic channel joining the 2D EG regions at the top and bottom of the figure. In the present case, however, the gap in the split gate is bridged so that an electron in the constriction experiences a periodically modulated potential [Fig. 1(b)]. For strong gate biases, the constriction becomes a string of quantum dots (the unshaded rectangles). Experiments on conduction through a constriction containing a single quantum dot of this kind have recently been reported by Smith *et al.*[10] At present, quantized conductances characteristic of ballistic transport are observed in constrictions up to ≈ 1 μm long, and a spacing of at least ≈ 0.2 μm between the bridges is needed to achieve a strong modulation of the electron

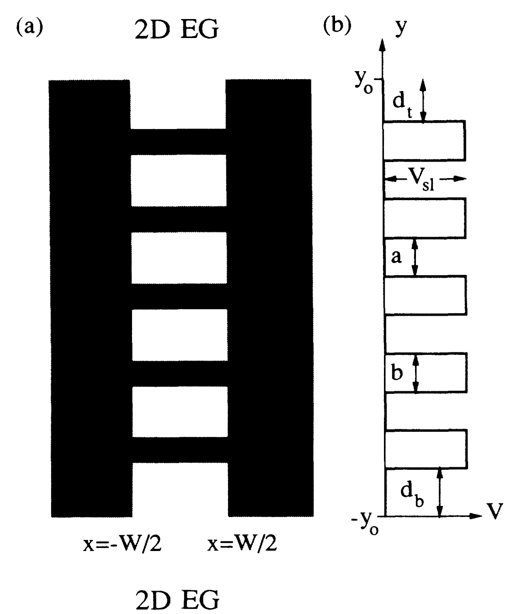

FIG. 1. Constriction extends along y axis, width W. Shaded areas represent negatively biased gate.

density along the constriction, implying that 1DSL's with four to five periods can be readily produced today. Surprisingly, our calculations reveal that for realistic model parameters the superlattice minibands and gaps should have striking and readily observable effects on the conductance even of 1DSL's having so few periods. We predict strong resonant effects, as well as a novel switching behavior of the conductance between quantized plateaus, due to the turning on and off of conduction in the various 1D subbands of the constriction, as the Fermi level is swept through the superlattice minibands. These phenomena should be fairly robust to finite temperatures, and almost independent of the constriction orifice tapering.

In order to study quantitatively the conductance of such structures, we perform exact numerical integrations of the system Hamiltonian. Our model consists of a finite narrow constriction defined by $|y| < y_0$ and $|x| < W/2$, and connecting two semi-infinite 2D electron-gas regions which act as electron reservoirs (Fig. 1). The Hamiltonian of the system is written in the effective-mass approximation with a confining potential $U(x,y)$ defining the constriction ($U=0$ in the 2D semi-infinite regions). Self-consistent calculations[11] of typical electrostatic confining potentials in these systems yield a rather broad quarticlike function, very similar to a square-well confinement in the case of wide constrictions. We will assume the latter for simplicity, an approximation which is widely used in the literature.[12] In addition, we introduce a y-periodic part in U as in a square Kronig-Penney arrangement with well (barrier) dimension a (b) and superlattice potential height $V_{\rm sl}$. Our conclusions do not depend on the detailed choice of $U(x,y)$.[13]

The conductance of the system is evaluated by considering an electron impinging on the constriction with a given wave vector $\mathbf{k}_\lambda = (K,k)$ such that its wave function in the injecting 2D region (bottom in Fig. 1) is given by a superposition of plane waves with complex coefficients, $\psi_\lambda^B(\mathbf{r}) = e^{iky}e^{iKx} + \sum_{K'} a^B_{\lambda K'} e^{-ik'y}e^{iK'x}$, where $k' = (2mE_\lambda/\hbar^2 - K'^2)^{1/2}$. The basis set also includes evanescent waves in the y direction (k' imaginary) to account for the loss of translational symmetry introduced by the constriction walls. Similarly, the wave function in the top 2D region is given by a sum of outgoing terms with coefficients $\{a^T_{\lambda K}\}$. In the constriction, we use well-known infinite square-well eigenfunctions $\{\chi_n\}$ as the basis set for the transverse x direction. Taking into account the periodicity of the superlattice modulation, the wave function in the jth well of the channel is given by

$$\psi_\lambda^C(\mathbf{r}) = \sum_n \{c^+_{\lambda nj} e^{iq_n(y-jl)} + c^-_{\lambda nj} e^{-iq_n(y-jl)}\}\chi_n(x),$$

where $l = a+b$ is the period of the superlattice, $q_n = [2m(E_\lambda - E_n)/\hbar^2]^{1/2}$, and $E_n = \hbar^2\pi^2 n^2/2mW^2$. From the usual wave-function-matching conditions, a set of coupled equations linking $\{a^B\}$, $\{a^T\}$, and the superlattice coefficients at the ends of the constriction is obtained. The connection between the two ends of the superlattice is achieved by a transfer-matrix technique which can also be used in other superlattice geometries (such as a periodic modulation of the constriction width). The resulting set of equations can be solved in terms of only the constriction coefficients, in a procedure similar to that reported previously.[12] The infinite set of equations is cast into a matrix form and inverted numerically in a truncated mode scheme which yields very good convergence ($< 0.1\%$) with relatively small matrices. Details will be presented elsewhere.

The total probability current through the constriction for each incoming λ mode is obtained as

$$< \psi_\lambda^C | \hat{j}_y | \psi_\lambda^C > = \frac{\hbar}{m}\left(\sum_n^R q_n(|c^+_{\lambda nj}|^2 - |c^-_{\lambda nj}|^2) - 2\sum_n^I |q_n| \,\mathrm{Im}(c^+_{\lambda nj} c^{-*}_{\lambda nj})\right),$$

where the superscripts R and I in the summations indicate that q_n is real or imaginary, respectively. The conductance of the system in linear response (and at $T=0$) is then given by the contributions from all incoming λ modes at the Fermi energy,

$$G = 2\int_{-K_F}^{K_F} \frac{me^2}{h^2 k} < \psi_\lambda^C | \hat{j}_y | \psi_\lambda^C > dK,$$

with $K_F = (2mE_F/\hbar^2)^{1/2}$, and $k = (K_F^2 - K^2)^{1/2}$.

In what follows we use normalized quantities. The length and energy scales are given by the constriction width W and the first transverse-motion level E_1 ($= \hbar^2\pi^2/2mW^2$). Figure 2 shows the conductance versus K_F for a system with five barriers, $a=b=0.5$, $V_{\rm sl}=1$, and equally offset at both ends ($d_t = d_b = 0.25$) within a constriction of total length $2y_0 = 5$. In this plot, the

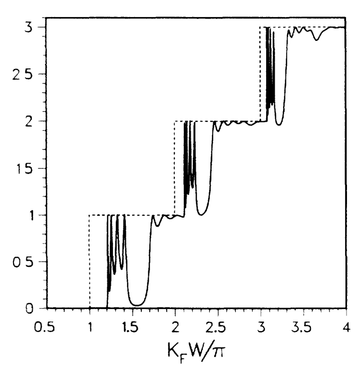

FIG. 2. Conductance for 1DSL with five periods vs Fermi wave number (solid line). Dashed line, conductance steps in ideal unmodulated constriction. Notice strong resonances and subband onsets moved upwards by superlattice effects.

steps near integer values of $K_F W/\pi$ arise from the successive occupation of transverse subbands as E_F increases. The dashed staircaselike curve represents the conductance of an ideal constriction without any superlattice modulation and adiabatically matched to the 2D electron reservoirs.[14] In the case of a superlattice with a small number of periods, G exhibits strong transmission resonances which drastically affect the quantized plateaus. This structure can be understood qualitatively in a tight-binding scheme as arising from the quasibound states in the wells of the superlattice, for each of the transverse subbands. Because of interwell tunneling, these quasibound states overlap and produce weakly split groups of transmission resonances (with one less peak per group than the number of barriers in the system). Each group of resonances evolves into a continuous *miniband* in the limit of a many-period superlattice.

Considering that the results in Fig. 2 are for a *very short* (five period) superlattice, the extremely strong signatures produced by the first miniband gap in *each* transverse subband are quite remarkable. As the Fermi level is swept through the gap ($K_F W/\pi \simeq 1.5$, 2.3, and 3.2), where the current carried by that subband is depressed almost to zero, the conductance drops rapidly by a quantum. This severe modulation of the conductance can be seen as a generalization of the strong coherent multiple-scattering effects explored recently, both theoretically and experimentally, in a constriction with obstacles.[10,15,16]

The structure in the conductance is also modulated by the *longitudinal* resonances arising from the multiple reflections at the ends of the constriction. This modulation is more important for smaller values of $V_{\rm sl}$, as expected, and evolves in that limiting case into the pattern of longitudinal resonances described previously.[12] For comparison, let us now consider a multichannel version of the Imry-Büttiker formula[17] generalized to the case of a superlattice, $G_{\rm IB} = (2e^2/h)\sum_n T_n$, where T_n is the transmission coefficient for subband n. $G_{\rm IB}$ assumes that multiple subbands in the system conduct in parallel without interfering, and describes the behavior of the conductance in a constriction if one were to ignore the details of the injection process which occurs at the openings and mixes the different subbands.[15,17] This expression then applies approximately to constrictions with adiabatically tapered orifices. Figure 3 shows the region of the third plateau for the system of Fig. 2, both for G and for $G_{\rm IB}$. The overall trend is for $G_{\rm IB}$ to follow G fairly closely, except that $G_{\rm IB}$ has slightly shifted and broader resonances. The differences become progressively more important with increasing E_F within a given plateau since the superlattice potential has decreasing influence on the overall transmission matrix. This difference has a cumulative effect in higher plateaus. However, the two curves are generally similar, strongly suggesting that the complex structure in G should be quite robust to geometrical variations in the constriction openings. This is an important point since it is well known that orifice tapering normally suppresses the sharp longitudinal resonance features in the conductance of unmodulated constrictions.[12]

Figure 4 shows results for a stronger superlattice modulation to illustrate the effects that larger minigaps can have on the conductance of the system. In Fig. 4(a) $V_{\rm sl} = 3$, keeping all other parameters the same as those of Fig. 2. Here, the second miniband gap for the first transverse subband is well developed and it produces a strong depression in G for $K_F W/\pi \simeq 2.5$, where it overlaps the first minigap of the second subband. The drastic changes in the conductance curve appearing at still higher energies occur whenever the Fermi energy falls in miniband

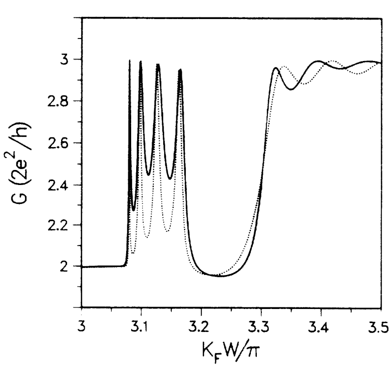

FIG. 3. Conductance $G_{\rm IB}$ (solid line) and G (dotted) for third plateau in system of Fig. 2. Sharper superlattice features in G are due to longitudinal resonance effects.

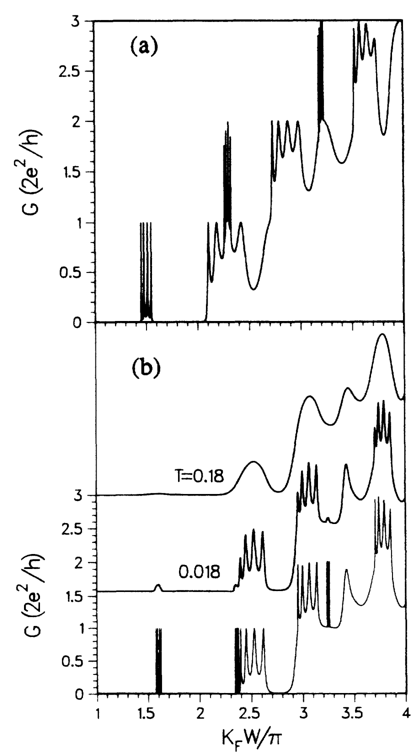

FIG. 4. Conductance for five-period superlattices with (a) $V_{\rm sl} = 3$, and (b) $V_{\rm sl} = 5$. Temperature effects are shown to smooth out only the sharpest features. $T = 0.018 \simeq 120$ mK in typical system (see text).

gaps for each of the different transmitting channels. It is clear that by controlling the values of the different parameters it is possible to achieve a rather complicated conductance showing on and off "switchings" of the different quantization values. Figure 4(b) shows results for $V_{sl} = 5$, where we see the first miniband of the *second* subband appear at $K_F \simeq 2.3$, *before* the second miniband of the first subband. The larger minigaps cause G to nearly vanish at $K_F \simeq 2.8$.

Figure 4(b) also shows the effect of finite temperatures on the conductance curves. This is calculated by thermally averaging the zero-temperature results with the appropriate Fermi factors, $G(\mu,T) = \int G(E,0)(-\partial f/\partial E)\,dE$, where $f((E-\mu)/kT)$ is the Fermi function. Results are given for two different temperatures: $kT = 0.018$ and $kT = 0.18$ (in units of E_1), which correspond to 0.12 and 1.2 K, respectively, if one assumes W=100 nm and m=0.067 (that of GaAs). We see that for 120 mK (a typical temperature used in these experiments to quench possible universal conductance fluctuations[18]), the main resonance features are still clearly developed, although their strength is somewhat diminished, especially for the first miniband in each channel. When the temperature is of the order of the separation between resonances, these are smoothed out and only the larger gaps remain. Since the particular resonance features are dependent on structural parameters, it should be possible to find a relatively large range of temperatures where the switching phenomenon exhibited at $T = 0$ is still present, especially if the potential modulation is strong.[13]

A large number of remarkable physical phenomena have been observed in the study of three-dimensional superlattices. We anticipate that the increased carrier coherence obtained by operating in the ballistic regime, and the reduced dimensionality of the structures proposed here, should make them particularly attractive as a topic of exciting future research. The novel features of parallel conducting subbands should also result in interesting new physics. Nonlinear effects arising from the application of strong fields should, in particular, be of great interest.

Note added. We are pleased to note that since this article was submitted for publication, the first experimental study of a one-dimensional semiconductor superlattice has been reported by Kouwenhoven *et al.*[19]

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