Self-affine fractal dimensions of film surfaces

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We study the fractal property of films formed by random deposition with the use of computer simulation. The films are shaped by packing particles of unit squares on segments divided into unit lengths (cells). In the course of packing, clusters with various forms appear, and they become large in their areas, surface, and bottom lengths. Their shapes are not self-similar but self-affine, and the forms cannot be represented by a unique value of fractal dimension. Their fractal dimensions are defined in many ways, and are different according to the definitions of the fractal dimension. In this paper, by applying three kinds of fractal dimension, we obtain the self-affine fractal dimensions of film surfaces formed by random deposition on substrates of segment.

It has been reported that irregular patterns such as percolation clusters, diffusion-limited aggregations (DLA), and surfaces of deposited film have fractal properties. Percolation clusters and DLA's grow isotropically. The thickness and length of deposited films grow, but the speed of the growth of length is greater than that of thickness, and therefore deposited films grow anisotropically. The shapes such as percolation cluster and DLA which grow isotropically are self-similar, while film surfaces which grow anisotropically are self-affine.[1] Their fractal dimensions can be defined in many equations, and the values of fractal dimension are different by the methods of definition.

We have studied the fractal property of film surfaces, using computer simulation.[2,3] The model of film surfaces is formed by depositing rigid square particles with an edge of unit length on a segment divided into unit lengths (cells). The deposition is carried out without any correlation among the deposited square particles. In Fig. 1 we present an example of the shaped patterns, and there are 11 clusters numbered from 1 to 11 on the segment divided into 100 unit cells. The representative sizes of each cluster are the area A, surface length L, and bottom length B, and the sizes grow as deposition proceeds. The smallest and largest cluster of Fig. 1 are, respectively, number 8 ($A=3$, $L=7$, and $B=1$) and number 5 ($A=83$, $L=70$, and $B=26$).

We can define the fractal dimension by various methods;[4] we apply the following three equations as the definitions of the fractal dimension. The first equation is

$$L=c_{\parallel}B^{D_{\parallel}}\ , \tag{1}$$

where $c_{\parallel}$ is a proportionality constant, and $D_{\parallel}$ is the fractal dimension for the relation between one-dimensional sizes L and B which is an independent variable of the parallel direction to segment substrates.

The second equation is

$$L=c_{\perp 1}A^{D_{\perp 1}/2}\ , \tag{2}$$

where $D_{\perp 1}$ is a fractal dimension for the relation between L and A which is a two-dimensional independent variable. Equations (1) and (2) were applied in our previous papers.[2,3] However, in this paper, by introducing the concept of the self-affinity, we present (1) and (2) with new notations $D_{\parallel}$ and $D_{\perp}$.

The third equation is an equation newly applied here and is written as

$$B=c_{\perp 2}A^{D_{\perp 2}/2}\ , \tag{3}$$

where $D_{\perp 2}$ is another fractal dimension for the relation between a one-dimensional quantity B and a two-dimensional quantity A, and

$$D_{\perp 2}=D_{\perp 1}/D_{\parallel}\ . \tag{4}$$

In Fig. 2, we plot L against B on a log-log scale. Figure 2 shows that a fractal property exists for the long clusters. Applying the least-squares fit method, we fit a straight line to the data points for $B \geq 1000$ in Fig. 2, and present $c_{\parallel}$, $D_{\parallel}$, and the correlation coefficient γ:

$$c_{\parallel}=2.6\ , \tag{5}$$

$$D_{\parallel}=1.05\ , \tag{6}$$

$$\gamma=0.999\ . \tag{7}$$

In Fig. 3 we plot L against A on a log-log scale. Figure 3 also shows that another fractal dimension exists for large clusters. By applying the least-squares fit method,

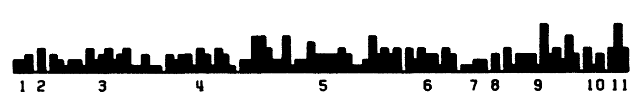

FIG. 1. An example of patterns formed on a segment divided into 100 unit cells. There are 11 clusters numbered from 1 to 11. The area, surface, and bottom length of clusters are denoted, by A, L, and B, respectively. For example, A, L, and B of number 1 are equal to 7, 9, and 3.

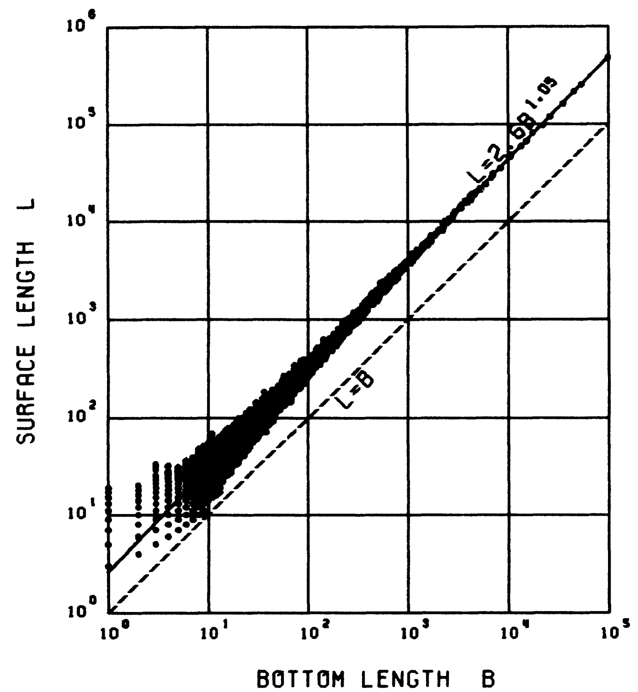

FIG. 2. Scatter plot of the surface length L vs bottom length B.

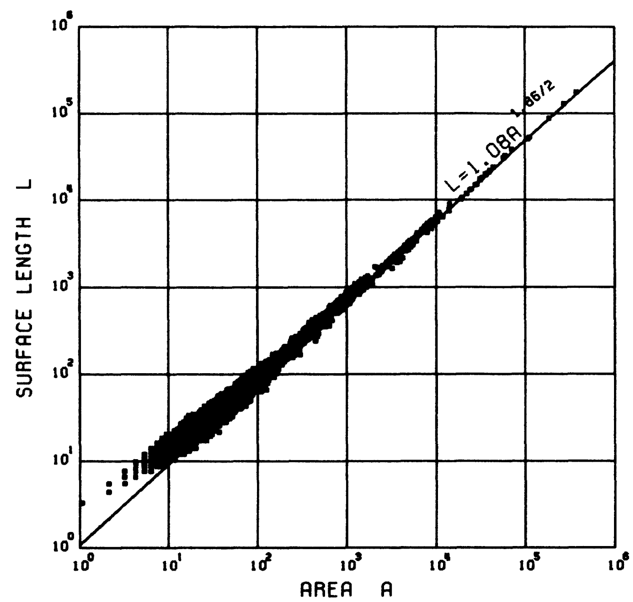

FIG. 3. Scatter plot of the surface length L vs area A. The unit of L is the multiplier of lattice constant, and the unit of A is the multiplier of the area of unit particle.

we fit a straight line to the data points for $A \geq 1000$ in Fig. 3, and present $c_{\perp 1}$, $D_{\perp 1}$, and γ:

$$c_{\perp 1} = 1.1 \ , \tag{8}$$

$$D_{\perp 1} = 1.86 \ , \tag{9}$$

$$\gamma = 0.999 \ . \tag{10}$$

From Eqs. (4), (6), and (9), we obtain the third fractal dimension

$$D_{\perp 2} = 1.77 \ . \tag{11}$$

In this paper we define three equations (1), (2), and (3) for the fractal dimension of deposited film surfaces, using the representative sizes (surface length L, bottom length B, and area A) of the clusters.

The values of the self-affinity fractal dimension of the large clusters are given by (6), (9), and (11), and the decreasing order of the values is

$$D_{\perp 1} > D_{\perp 2} > D_{\parallel} \ . \tag{12}$$

Can we analytically obtain these values of the fractal dimension? Can $D_{\parallel}$ be expressed by a function of only $D_{\perp 1}$ or $D_{\perp 2}$? Can we shape two different groups of film surface with the same $D_{\parallel}$ ($D_{\perp}$) and different $D_{\perp}$ ($D_{\parallel}$)? These are study themes of the future.

In this paper we treat only a lattice filling problem as opposed to a continuum filling problem which is a more common operation. Also the fractal property of deposited films formed by a continuum filling is a theme of the future.

[1] R. F. Voss, in *The Science of Fractal Images*, edited by H.-O. Peitgen and D. Saupe (Springer-Verlag, New York, 1988), p. 21.
[2] M. Nakamura, Phys. Rev. B **40**, 2549 (1989).
[3] M. Nakamura, Phys. Rev. B **40**, 3358 (1989).
[4] B. B. Mandelbrot, *The Fractal Geometry of Nature* (Freeman, San Francisco, 1983).