

## Kinetics of radiative recombination in quantum wells

B. K. Ridley

*Department of Physics, University of Essex, Colchester CO4 3SQ, United Kingdom*

(Received 30 October 1989)

A theory of radiative-recombination kinetics which treats free carriers, excitons, and photon recycling in a quantum-well system is presented. An expression for the temporal decay of excess carriers which encompasses large- and small-signal regimes is derived. When excitons are present the decay can be approximated by two exponentials in general, and in the large-signal regime the photoluminescence time constant is half as long as that associated with photoconductivity. Explicit expressions for the recombination coefficients are given and their magnitudes discussed for nondegenerate and degenerate populations in GaAs. Excitons are shown to enhance the temperature dependence. A simple model of exciton screening is used to illustrate the dependence of radiative time constants on background carrier density, which deviates significantly from the conventional free-carrier dependence. The magnitudes of radiative time constants in real systems depend, in addition to material characteristics, upon the details of exciton screening, the overlap of the electron and hole wave functions in the quantum well, and the probability of photon reabsorption, all of which are specimen specific. It is pointed out that the transition from a degenerate to a nondegenerate population may be misinterpreted in terms of Auger processes.

### I. INTRODUCTION

It is well known that the quantum confinement of electrons and holes in a quantum well increases the importance of excitons in absorption and photoluminescence experiments. The effect of exciton formation on the radiative recombination of excess carriers has, however, not been analyzed. One would expect that in situations in which excitons are not screened out, electrons and holes would rapidly form excitons and a dynamic equilibrium would be set up between these and free carriers. In general, there will be two paths along which radiative recombination takes place, and the proportion of excitonic to free-carrier recombination will increase as the temperature is lowered. Exciton involvement is thus expected to add to the temperature dependence of the radiative lifetime. Moreover, the more carriers are bound together as excitons the less they can contribute to photoconductivity (PC), and so a consideration of the role of excitons is therefore important to the interpretation of PC experiments. Also, at high excitations it is vital to understand the form of the photoluminescence (PL) or PC transient decay curves to be expected for purely radiative processes so that confident assessment of Auger effects can be made. In extracting magnitudes for the radiative-recombination coefficients it is necessary to be aware of the effect of photon reabsorption.

It is the purpose of this paper to describe a theory of recombination kinetics that takes into account the effect of exciton formation together with the effect of photon recycling in a simple and transparent manner, and describes the transients to be expected in small-signal and large-signal experiments of PL and PC with nondegenerate or degenerate populations.

In Sec. II the basic equations of recombination kinetics are introduced. In Sec. III transient solutions are sought

when Auger and impurity processes are neglected. The radiative-recombination coefficients for free carriers and for excitons are discussed in Sec. IV and the dependence of time constants on temperature and on carrier density is illustrated for GaAs quantum wells with the help of a rough model of exciton screening. The main results are discussed and summarized in Sec. V.

### II. RECOMBINATION KINETICS

The rate equation for the electron density can be written in the form

$$\frac{dn}{dt} = \left[ \frac{dn}{dt} \right]_r - \nabla \cdot \mathbf{j}_n, \quad (1)$$

where the first term on the right is the recombination rate and the second is the divergence of the electron current density  $\mathbf{j}_n$  which is given by

$$\mathbf{j}_n = n \mathbf{v}_n - D_n \nabla n, \quad (2)$$

where  $\mathbf{v}_n$  is the drift velocity and  $D_n$  is the diffusion coefficient. Similar equations can be written for the densities of holes, excitons, and photons, viz.,

$$\frac{dp}{dt} = \left[ \frac{dp}{dt} \right]_r - \nabla \cdot \mathbf{j}_p, \quad \mathbf{j}_p = p \mathbf{v}_p - D_p \nabla p; \quad (3)$$

$$\frac{dn_x}{dt} = \left[ \frac{dn_x}{dt} \right]_r - \nabla \cdot \mathbf{j}_x, \quad \mathbf{j}_x = -D_x \nabla n_x; \quad (4)$$

$$\frac{dn_v}{dt} = \left[ \frac{dn_v}{dt} \right]_r - \nabla \cdot \mathbf{j}_v, \quad \mathbf{j}_v = n_v \mathbf{v}_v - D_v \nabla n_v. \quad (5)$$

In general this set of equations should be supplemented by equations describing the flow of energy and of the

phonon population, particularly when the out-of-equilibrium populations are generated optically by above-band-gap photons, but we intend to ignore this aspect and assume that energy relaxation is more rapid than any of the rates that we intend to consider. This assumption will usually be invalid at low temperatures and low carrier concentrations, where energy relaxation is via the emission of acoustic phonons, which is relatively slow, but otherwise it ought to be reasonably good. Equations (1)–(5) also carry the implicit assumption that each population can be treated as a coherent identity, which implies that equilibrating processes within each population are more rapid than individual particle recombination and trapping rates. This will not be true of the photon population, since photons do not interact, but we will be concerned usually with one or two narrow frequency ranges, so we do not have to deal explicitly with a wide photon distribution.

The variations with time that arise out of nonuniformity require detailed analysis of the particular structure and of the excitation process. We intend to avoid this by assuming that uniform conditions in the vicinity of a quantum well establish themselves very quickly. At most carrier densities of interest, deviations from electrical neutrality caused by the differential motion of excited electrons and holes will be rapidly countered to give ambipolar diffusion. In quantum-well systems the distances involved (e.g., of a cladding layer) are typically of order 1000 Å, implying diffusion times of 100 ps or less for diffusion coefficients 1 cm<sup>2</sup>s<sup>-1</sup> or more. Capture into quantum wells is expected to be very rapid ( $\approx 1$  ps). Diffusion between wells will be relatively slow, often slow enough for each well to be considered in isolation, with recombination rates largely unaffected by the difference of diffusion currents in and out. As regards photons emitted by radiative recombination in the wells, the high speed of diffusion and high internal reflectivity of the semiconductor will ensure a large degree of uniformity of photon density. Net losses via absorption in the contacts, barriers, buffer layer or substrate, or via transmission out of the sample can be simply described by a time constant  $\tau_{vs}$ . We therefore discard all the divergences in Eqs. (1), (3), (4), and (5).

The recombination-trapping rate for electrons within a well can be written

$$\left(\frac{dn}{dt}\right)_r = \left(\frac{dn}{dt}\right)_{\text{Auger}} + \left(\frac{dn}{dt}\right)_{\text{radiative}} + \left(\frac{dn}{dt}\right)_{\text{exciton}} + \left(\frac{dn}{dt}\right)_{\text{impurity}} \quad (6)$$

and similar equations can be written for holes and excitons. We can write the Auger rate in nondegenerate material as follows:

$$\left(\frac{dn}{dt}\right)_{\text{Auger}} = -A_r n^2 p - A_p p^2 n + (e_n n + e_p p) N_J, \quad (7)$$

where  $A_n, A_p$  are the Auger recombination coefficients,  $e_r, e_p$  are the impact-ionization coefficients, and  $N_J$  is the

joint effective density of states. For all situations at low fields, impact ionization can be neglected. Auger coefficients are typically of order 10<sup>-30</sup> cm<sup>6</sup>s<sup>-1</sup> in medium-gap semiconductors like GaAs,<sup>1,2</sup> and so for concentrations of carriers much less than about 10<sup>19</sup> cm<sup>-3</sup> Auger recombination rates will be much smaller than those for radiative recombination and can be ignored.

The radiative-recombination rate can be written

$$\left(\frac{dn}{dt}\right)_{\text{radiative}} = -Bnp + e_v N_{cv} n_v, \quad (8)$$

where  $B$  is the free-carrier radiative-recombination coefficient, typically of order 10<sup>-10</sup> cm<sup>3</sup>s<sup>-1</sup>,  $e_v$  is the photoionization coefficient and  $N_{cv}$  is the reduced effective density of states. We assume that nonradiative band-to-band transitions occur at negligible states. The exciton rate is

$$\left(\frac{dn}{dt}\right)_{\text{exciton}} = -c_x np + e_x N_J n_x, \quad (9)$$

where  $c_x$  is the coefficient describing the rate of capture of an electron by a hole to form an exciton, and  $e_x$  is the corresponding dissociation rate. If we assume that the thermodynamic population density of excitons is given by

$$n_{x0} = N_J e^{-(E_g - E_x)/k_B T}, \quad (10)$$

where  $E_g$  is the band-gap energy and  $E_x$  is the exciton binding energy, then detailed balance provides the relationship

$$\phi_x = \frac{c_x}{e_x} = e^{E_x/k_B T}. \quad (11)$$

The quantity  $\phi_x$ , which may be referred to as the exciton trapping factor, is related to the fraction of the carrier population that are bound as excitons.

The impurity rate is of the form (in the absence of Auger processes)

$$\left(\frac{dn}{dt}\right)_{\text{impurity}} = \sum_i [-c_{ni}(N_i - n_i)n + e_{ni}N_c n_i], \quad (12)$$

where  $c_{ni}, e_{ni}$  are the capture and generation coefficients (related by detailed balance),  $N_i$  is the trap density,  $n_i$  is the density of electrons in the trap,  $N_c$  is the effective density of states in the conduction band. For deep traps, the capture coefficient is typically 10<sup>-10</sup> cm<sup>3</sup>s<sup>-1</sup> or less. In good quality material with background impurity concentrations of 10<sup>16</sup> cm<sup>-3</sup> and less, the rates associated with deep center trapping can be neglected. In doped material, shallow donors or acceptors can present large cross sections, especially at low temperatures, and even contribute to the overall radiative recombination rate if the doping is direct and not indirect as in modulation doping. Clearly, impurities can affect the recombination kinetics substantially, particularly at low temperatures. Here, since our prime concern is with intrinsic radiative processes, we will assume that the role of impurities can be ignored. This assumption allows us to equate the elec-

tron and hole rates.

Turning now to the exciton population rate, we can write this as follows:

$$\left[ \frac{dn_x}{dt} \right]_r = c_x np - e_x N_J n_x - B_x N_J n_x + e_{vx} N_J n_{vx}, \quad (13)$$

where  $B_x N_J$  is the radiative annihilation rate and  $e_{vx} N_J$  is the creation rate. (The joint effective density of states has been included here solely for dimensional conformity of notation.) Strictly speaking, we should distinguish the photons that are associated with free-carrier processes ( $n_v$ ) from those associated with excitons ( $n_{vx}$ ). Thus the photon rates are

$$\left[ \frac{dn_v}{dt} \right]_r = Bnp - e_v N_{cv} n_v - \frac{n_v}{\tau_{vs}} = Bnp - \frac{n_v}{\tau_v}, \quad (14)$$

$$\left[ \frac{dn_{vx}}{dt} \right]_r = B_x N_J n_x - e_{vx} N_J n_{vx} - \frac{n_{vx}}{\tau_{vsx}} = B_x N_J n_x - \frac{n_{vx}}{\tau_{vx}}, \quad (15)$$

where  $\tau_{vs}, \tau_{vsx}$  are the lifetimes of a photon associated with escape from the active region or absorption in the active region other than in the quantum wells and  $\tau_v, \tau_{vx}$  are the lifetimes including reabsorption in the wells. In many experimental situations the active region consists of a multiple-quantum-well (MQW) structure. We will assume, for simplicity, that once a photon disappears from the active region it never returns. This will certainly be a valid assumption for GaAs MQW systems in which the buffer layer or substrate absorbs the photons emitted. The validity is more problematical for  $\text{Ga}_{1-x}\text{In}_x\text{As}$  MQW and similar systems that contain buffer layers and substrates that do not absorb.

### III. TRANSIENT SOLUTION

We assume that electron-hole pairs are generated at  $t=0$  and instantaneously thermalize to the lattice temperature. The decay of the excess population under the conditions described in Sec. II is determined by the following equations (repeated here for convenience),

$$\frac{dn}{dt} = \frac{dp}{dt} = -Bnp + e_v N_{cv} n_v - c_x np + e_x N_J n_x, \quad (16)$$

$$\frac{dn_x}{dt} = c_x np - e_x N_J n_x - B_x N_J n_x + e_{vx} N_J n_{vx}, \quad (17)$$

$$\frac{dn_v}{dt} = Bnp - \frac{n_v}{\tau_v}, \quad (18)$$

$$\frac{dn_{vx}}{dt} = B_x N_J n_x - \frac{n_{vx}}{\tau_{vx}}. \quad (19)$$

This is a set of nonlinear, first-order, coupled differential equations, which we go about solving by exploiting order-of-magnitude differences in the rates.

Exciton formation and dissociation are extremely fast processes at room temperature. Studies of linewidth of excitonic resonances<sup>3,4</sup> and direct measurements of disso-

ciation times<sup>5</sup> give magnitudes in GaAs of 300 to 400 fs. We would expect cross sections for capture of an electron by a hole into the ground state of the exciton to be in the range  $10^{-12}$  to  $10^{-14}$  cm<sup>2</sup> corresponding to capture coefficients in the range  $10^{-5}$  to  $10^{-7}$  cm<sup>3</sup>s<sup>-1</sup>. These excitonic rates are much larger than rates associated with radiative recombination. We will assume, therefore, that the free carrier and exciton populations rapidly establish a dynamic equilibrium that persists throughout radiative recombination.

Thus, solving Eq. (17) by neglecting all but the formation and dissociation rates we obtain the relation between free carrier and exciton populations, viz.,

$$n_x = \frac{c_x np}{e_x N_J} = \phi_x \frac{np}{N_J}, \quad (20)$$

where  $\phi_x$  is defined in Eq. (11). We can use the relationship in all radiative rates. To the next order of approximation Eq. (17) gives

$$c_x np - e_x N_J n_x = \frac{d}{dt} \left[ \frac{\phi_x np}{N_J} \right] + B_x \phi_x np - e_{vx} N_J n_{vx}, \quad (21)$$

which can be used in Eq. (16) to eliminate the exciton formation and dissociation rates. We then obtain

$$\frac{d}{dt} \left[ n + \phi_x \frac{np}{N_J} \right] = -(B + B_x \phi_x) np + e_v N_{cv} n_v + e_{vx} N_J n_{vx}, \quad (22)$$

$$\frac{dn_v}{dt} = Bnp - \frac{n_v}{\tau_v}, \quad (23)$$

$$\frac{dn_{vx}}{dt} = B_x \phi_x np - \frac{n_{vx}}{\tau_{vx}}. \quad (24)$$

We now exploit the rapidity of the photon processes, which arise as a consequence of the photon's high velocity. Even with an absorption-edge constant of order  $10$  cm<sup>-1</sup> the rate of absorption is of order  $10^{11}$  s<sup>-1</sup>. The transit rate across a 1- $\mu$ m-thick active layer is of order  $10^{14}$  s<sup>-1</sup>, but because of high internal reflection only a small fraction succeeds in escaping. Nevertheless, the rate of escape remains high ( $\leq 10^{12}$  s<sup>-1</sup>). Thus the rate associated with photon processes will be typically of order  $10^{12}$  s<sup>-1</sup> or more. In most cases this rate is large compared with the emission rates, which means that the photon population stays in equilibrium with the free carriers, and we can write

$$n_v \approx Bnp\tau_v, \quad n_{vx} \approx B_x \phi_x np\tau_{vx} \quad (25)$$

or, to a better approximation

$$n_v = Bnp\tau_v - \tau_v \frac{d}{dt} (Bnp\tau_v), \quad (26)$$

$$n_{vx} = B_x \phi_x np\tau_{vx} - \tau_{vx} \frac{d}{dt} (B_x \phi_x np\tau_{vx}). \quad (27)$$

Substitution in Eq. (22) gives us

$$\frac{d}{dt} \left[ n + \phi \frac{np}{N_J} \right] = -[B(1-\phi_v) + B_x \phi_x (1-\phi_{vx})] np, \quad (28)$$

where

$$\begin{aligned} \phi &= \phi_x + BN_J \tau_v \phi_v + B_x \phi_x N_J \tau_{vx} \phi_{vx}, \\ \phi_v &= e_v N_{cv} \tau_v, \quad \phi_{vx} = e_{vx} N_J \tau_{vx}. \end{aligned} \quad (29)$$

Equation (28) now embodies exciton formation and photon reabsorption. Photon "recycling" has been considered by several authors<sup>6-8</sup> who point out that radiative lifetimes can be significantly enhanced by this process. Our approach is quite different from other treatments but leads to the same conclusion. The factors  $\phi_v, \phi_{vx}$  are the probabilities that a photon is reabsorbed in the well and when equal to unity the decay disappears entirely, according to Eq. (28). The enhancement of radiative lifetime in GaAs/Al<sub>x</sub>Ga<sub>1-x</sub>As double heterostructures was observed by Ettenberg and Kressel,<sup>9</sup> the effect becoming noticeable when the active layer was 1 μm thick or more, a result explained in a calculation by Asbeck.<sup>8</sup> In general, the effects of photon reabsorption cannot be ignored.

The solution of Eq. (28) for the case of *p*-type material such that

$$p = p_0 + n, \quad (30)$$

where  $p_0$  is the equilibrium density of holes, is

$$\frac{n}{(n+p_0)^{1-r}} = \frac{n(0)}{[n(0)+p_0]^{1-r}} \exp(-t/\tau), \quad (31)$$

where  $n(0)$  is the electron concentration at  $t=0$ . The factor  $r$  is given by

$$r = \frac{2\phi p_0 / N_J}{1 + \phi p_0 / N_J}. \quad (32)$$

It thus varies between 0 and 2 depending on the size of the quantity  $\phi p_0 / N_J$  relative to unity. The time constant is given by

$$\frac{1}{\tau} = \frac{(B^* + B_x^* \phi_x) p_0}{1 + \phi p_0 / N_J}, \quad (33)$$

where  $B^* = B(1-\phi_v)$  and  $B_x^* = B_x(1-\phi_{vx})$  are the radiative-recombination coefficients modified by photon reabsorption. (The corresponding solution for *n*-type material can readily be obtained.) In deriving Eq. (31) we have assumed that the excitonic binding energy and the other factors contributing to  $\phi$  do not depend upon carrier density, and therefore time. Note that this assumption will not be valid in the large-signal regime if screening of the exciton is important.

Figure 1 shows semilogarithmic plots of Eq. (31) for large-signal excitation. Provided the mobilities of the excess carriers remains constant, these curves are depictions of the photoconductive (PC) decay. There are several points of interest.

(1) The curve with  $r=0$  is the classical free-carrier

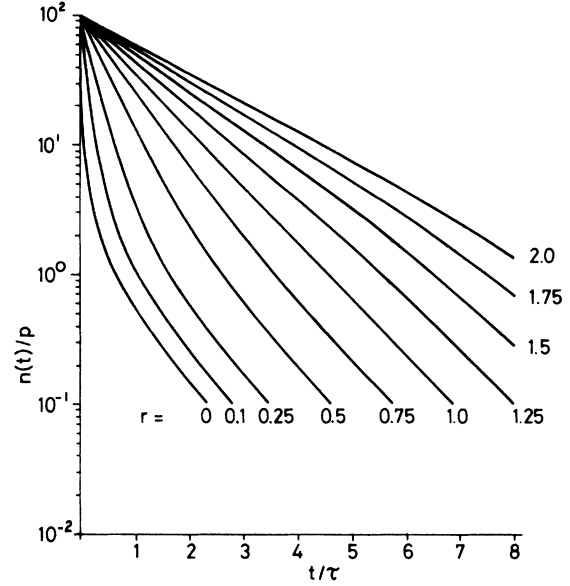


FIG. 1. Semilogarithmic plot of electron density vs time for various values of  $r$ .

radiative-recombination behavior. There is no excitonic recombination or photon recycling.

(2) In the absence of photon recycling the condition  $r=2$  represents purely excitonic recombination.

(3) The condition  $r=1$  leads to an exponential decay whatever the excitation level.

(4) For all  $r > 0$  the decay can be resolved approximately into two exponentials

$$n \approx n(0) e^{-t/r\tau}, \quad n \gg p_0 \quad (34)$$

$$n \approx \frac{n(0)}{\left[1 + \frac{n(0)}{p_0}\right]^{1-r}} e^{-t/\tau}, \quad n \ll p_0. \quad (35)$$

The photoluminescence (PL) intensity is proportional to the rate of loss of photon density, which varies with time according to Eqs. (25), (34), and (35), e.g.,

$$\frac{n_v}{\tau_{vs}} = B(1-\phi_v)[n(0)]^2 e^{-2t/r\tau}, \quad n \gg p_0 \quad (36)$$

$$\frac{n_v}{\tau_{vs}} = \frac{B(1-\phi_v)p_0 n(0)}{\left[1 + \frac{n(0)}{p_0}\right]^{1-r}} e^{-t/\tau}, \quad n \ll p_0. \quad (37)$$

Note that in the large signal regime the PC time constant is twice the PL time constant. It is interesting to observe that an exponential decay component can arise in the highly nonlinear large-signal regime, as this is clearly a consequence of exciton formation.

#### IV. RADIATIVE-RECOMBINATION COEFFICIENTS

The free-carrier radiative-recombination rate is given by standard theory<sup>10</sup> as follows:

$$\frac{dn}{dt} = -W_0 \int_{E_{\max}}^0 \hbar\omega f_e f_h N(E_e) dE_e, \quad (38)$$

where

$$W_0 = \frac{2e^2 \eta p_{cv}^2 G}{3\pi \epsilon_0 \hbar^2 c^3 m^2}, \quad (39)$$

$f_e, f_h$  are the electron and hole occupation probabilities of states connected by the  $\mathbf{k}$ -conservation rule,  $\hbar\omega$  is the photon energy, and  $E_e$  is the kinetic energy of the electron in the conduction band. The basic rate,  $W_0$ , written here in mks units, incorporates the assumption that there is no restriction on the polarization of the emitted light. The quantities  $e, \epsilon_0, \hbar, c$ , and  $m$  are all fundamental constants,  $\eta$  is the refractive index. The factor of 3 arises out of an average of the momentum matrix element  $p_{cv}$  over all directions of polarizations. The momentum matrix element connects conduction-band and valence-band states and its value is usually assumed to be that at the band edge and independent of  $\mathbf{k}$ . For example, in GaAs  $p_{cv}^2/2m = 5.4$  eV.<sup>11</sup> However, it has become conventional to incorporate spin selection into the momentum matrix element and so we take  $p_{cv}^2/2m = 2.7$  eV but this means doubling the hole occupation probability of a given spin and  $\mathbf{k}$  state, hence the factor of 2 in Eq. (39). This value for the matrix element compares with 2.88 eV used by Casey and Stern<sup>12</sup> and 2.60 eV used by Dumke.<sup>10</sup> The factor  $G$  is the overlap integral of the electron and hole envelope functions in the quantum well.<sup>13</sup>

For nondegenerate distributions Eq. (38) becomes

$$\frac{dn}{dt} = -Bnp, \quad (40)$$

with

$$B = \frac{W_0 E_g N_{cv}}{N_c N_v}. \quad (41)$$

In bulk GaAs  $G=1$ ,  $E_g=1.42$  eV at 300 K,  $N_{cv}/N_c = (0.059/0.067)^{3/2}$ ,  $N_v = 9.5 \times 10^{18}$  cm<sup>-3</sup> and if we include a factor of 2 to account for both light and heavy holes, we obtain  $B = 2.6 \times 10^{-10}$  cm<sup>3</sup> s<sup>-1</sup>. In a 55 Å quantum well with only heavy holes involved, assuming a density-of-states mass of 0.14  $m$  for holes<sup>14</sup> a reduced mass of 0.045  $m$  and, assuming  $G=1$ , we obtain  $B = 3.6 \times 10^{-10}$  cm<sup>3</sup> s<sup>-1</sup>. This increase over the bulk value is due to the smaller hole mass in the plane of the quantum wells. However, the overlap factor in a 55 Å well will be less than unity, making  $B$  comparable with the bulk. The value for  $B$  may be compared with the value of  $1.7 \times 10^{-10}$  cm<sup>3</sup> s<sup>-1</sup> obtained by Matsusue and Sakaki<sup>15</sup> for 90 Å wells. In degenerate systems  $f_e = f_h = 1$  up to the Fermi energy. In two dimensions (2D) the Fermi wave vectors for electrons and holes will be equal when  $n = p$ . Under these conditions we obtain the minimum free-carrier lifetimes  $\tau_0$  given by (for  $E_f \ll E_g$ )

$$\tau_0 \approx (W_0 E_g)^{-1}, \quad (42)$$

which for bulk GaAs is 0.68 ns at 300 K. (This may be compared with 0.35 ns obtained by Dumke<sup>11</sup> for bulk material when both hole bands are available, hence halving the time.) The decay under these conditions is exponential. Equation (42) will also be the electron lifetime when the holes are degenerate but the electrons are not. When the electrons are degenerate but the holes are not Eq. (42) will be the hole lifetime.

The exciton recombination rate is

$$dn_x/dt = -\frac{1}{2}W_0(E_g - E'_x)F_x n_x \approx -\frac{1}{2}W_0 E_g F_x n_x. \quad (43)$$

The factor of  $\frac{1}{2}$  arises because only excitons with zero spin can undergo radiative annihilation.  $E'_x$  is the binding energy when the exciton is in the triplet state, which is the state involved in a radiative transition (since the photon carries off a quantum of angular momentum). We assume  $(E_x - E'_x)/k_B T \ll 1$  and in what follows we take  $E'_x \ll E_g$ . The factor  $F_x$  is determined by the exciton linewidth  $\Delta$  in the vicinity of zero kinetic energy and is given by<sup>16</sup>

$$F_x = \frac{8\mu}{M} \frac{E_x}{\Delta} (1 - e^{-\Delta/k_B T}) \approx \frac{8\mu}{M} \frac{E_x}{k_B T}, \quad \Delta \rightarrow 0 \quad (44)$$

where  $\mu$  is the reduced mass and  $M$  is the total mass. Since  $\Delta$  is of order 0.5 meV it is usually permissible to make the approximation in Eq. (44).

We may now express the reciprocal time constant appearing in Eq. (33) as follows:

$$\frac{1}{\tau} = \frac{1}{\tau_0} \frac{G\mu}{M} \left[ \frac{M}{(m_e m_{hh})^{1/2}} (1 - \phi_v) + \frac{4E_x}{k_B T} e^{E_x/k_B T} (1 - \phi_{vx}) \right] \frac{p_0}{N_J + \phi p_0}. \quad (45)$$

The main temperature dependence is contained in  $N_J$  ( $\sim T$ ) and  $\phi$  in addition to what is explicit. Well-width dependence is contained in  $\tau_0, G, E_x$ , and  $N_J$ . In the absence of excitonic and recycling effects ( $E_x = 0, \phi = 0$ ) the temperature dependence is determined by that of the density of states, and this dependence has been observed.<sup>17</sup> When excitons exist, the free carriers freeze out as the temperature drops and the rate of decay in the small-signal regime ultimately becomes independent of carrier density. This leads to a quite different temperature dependency as Fig. 2 shows. For simplicity we have assumed that the free carrier and excitonic photon recycling factors are the same, and we have incorporated the common factor into a new parameter, viz.,

$$G^* = G(1 - \phi_v). \quad (46)$$

At a given temperature the dependence on carrier density is complicated by the screening effect on the exciton binding energy. Roughly, we would expect excitonic effects to disappear at densities  $n_x$  such that  $n_x a_x^2 \geq 1$ , where  $a_x$  is the exciton radius and many-body calculations broadly confirm this expectation.<sup>14,18</sup> Here we illustrate the effect using a very rough model with  $\phi \approx \phi_x$ .

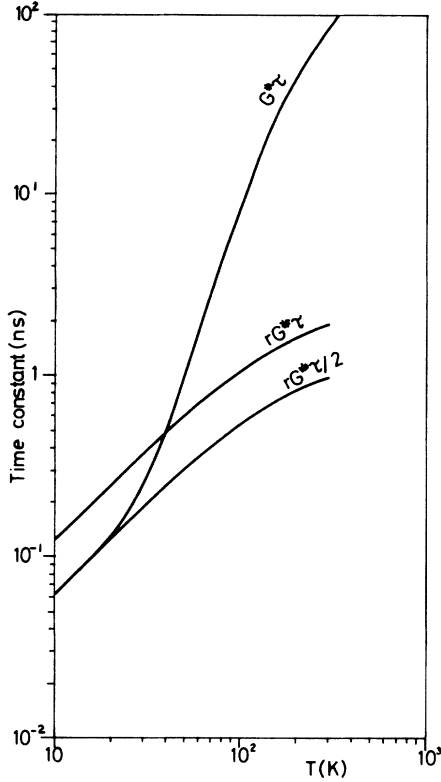


FIG. 2. Temperature dependence on the radiative time constant. For a dark hole density of  $1 \times 10^{10} \text{ cm}^{-2}$  excitons are assumed to be unscreened and exist at all temperatures with a binding energy of 10 meV and a linewidth  $\Delta \ll k_B T$ .  $G^* \tau \propto$  small-signal lifetime,  $rG^* \tau \propto$  large-signal PC time constant,  $rG^* \tau/2 \propto$  large-signal PL time constant, and  $G^* = G(1 - \phi_v)$ , where  $G$  is the overlap integral and  $\phi_v$  is the photon recycling factor applicable to GaAs/Al<sub>x</sub>Ga<sub>1-x</sub>As 55 Å wells.

We suppose that the excitonic binding energy at room temperature is reduced by a Debye-like factor, viz.,

$$E_x = E_{x0} e^{-q_s a_x}, \quad (47)$$

where  $q_s$  is the 2D screening factor for nondegenerate carriers, namely,

$$q_s = \frac{e^2 n_s}{2\epsilon k_B T}, \quad (48)$$

where  $\epsilon$  is the static permittivity. We then take  $a_x^2 = \hbar^2 / 2\mu E_x$  and solve for  $E_x$ . The result for GaAs at room temperature is shown in Fig. 3 and is roughly in accord with the result of more elaborate calculations. Applying this to the case of a GaAs quantum well at 300 K leads to Fig. 4. The onset of excitonic effects towards low carrier densities shows up distinctly as a change of slope. Note that our crude model only influences the details of this change of slope, but not its ontology.

## V. DISCUSSION AND SUMMARY

The foregoing analysis of the kinetics of radiative recombination illustrates the importance of excitonic

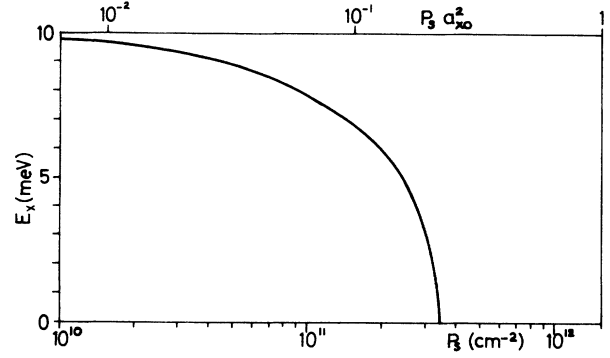


FIG. 3. Model dependence of exciton binding energy on carrier density in GaAs quantum wells at 300 K.

recombination and photon recycling in determining the shape of the transient decay, its speed, its temperature dependence, and its dependence on background carrier density. It has also highlighted the importance of exciton screening, the overlap of the electron and hole wave functions in the well, and the base rate afforded by a degenerate population.

From the experimentalist's point of view the simplest system is one which is degenerate with its single time constant (provided Auger processes do not assume importance) at all excitation levels. Another advantage is the reduction of photon reabsorption due to the Moss-Burstein shift. However, it will not always be possible to

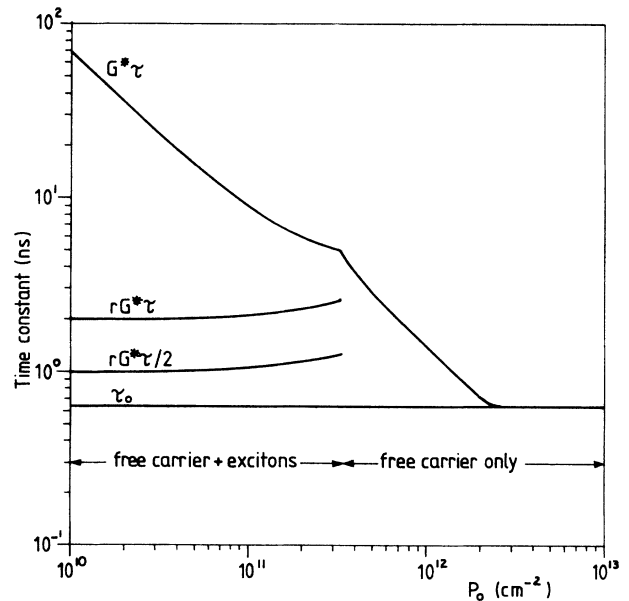


FIG. 4. Dependence of radiative time constant on dark hole density in *p*-type GaAs quantum wells ( $L=55 \text{ \AA}$ ) at 300 K.  $G^* \tau \propto$  small-signal lifetime,  $rG^* \tau \propto$  large-signal PC time constant,  $rG^* \tau/2 \propto$  large-signal PL time constant,  $G^* = G(1 - \phi_v)$  (where  $G$  is the overlap integral and  $\phi_v$  is the photon recycling factor), and  $\tau_0$  denotes lifetime in degenerate material.

avoid multiple subband effects. At lower background densities, but high enough to preclude excitons, analysis is most easily carried out in the small-signal regime. This remains true in principle at even lower densities in the exciton regime, but increasingly difficult in practice for two reasons. One is the obvious one of maintaining signal strength while keeping to the small-signal regime. The other is that radiative time constants become very long, allowing impurity processes to dominate. Working in the large-signal regime avoids these defects but it runs into the problem of exciton screening if the excitation is too large, making the exciton binding energy effectively a function of time.

Our approach has made a number of simplifications that will not be justified in many experimental situations. We have assumed throughout that only the energetically lowest electron and hole subbands are involved. Involvement of the light hole band would enhance the recombination rate for electrons, and this would add another component to the temperature dependence. In wide wells several subbands will be involved. We have made no attempt to bridge the nondegenerate and/or degenerate divide, but this would be important to do if one wished to

interpret high-excitation experiments designed to measure Auger recombination rates. Note that at a carrier concentration of order  $10^{19} \text{ cm}^{-3}$ , typically used in such experiments, the degenerate radiative recombination rate in bulk material ( $\approx 3 \times 10^9 \text{ s}^{-1}$ ) could be mistakenly interpreted in terms of an Auger coefficient of  $3 \times 10^{-29} \text{ cm}^6 \text{ s}^{-1}$ . We have also assumed that thermalization is always complete, which will not be so at low temperatures (typically below 30 K), where acoustic phonon energy relaxation is slow. Since the carrier temperature appears in the exciton trapping factor, slow cooling will have a profound effect on exciton-mediated recombination.

In a following paper we will compare theory and experiment in the case of a GaAs/ $\text{Al}_x\text{Ga}_{1-x}\text{As}$  MQW system.<sup>19</sup>

#### ACKNOWLEDGMENTS

The author would like to express his appreciation of the many contributions made by Dr. P. J. Bishop to this project. He would also like to thank the U.S. Office of Naval Research for partial support of this research.

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