Hall effect near the metal-insulator transition

D. W. Koon* and T. G. Castner
Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627-0011
(Received 5 October 1989)

Hall-coefficient and dc-conductivity measurements have been made, with use of the van der Pauw geometry, on uncompensated Si:As samples on both sides of the metal-insulator transition ($7.77\times10^{18}<N<32.8\times10^{18}$ cm^{-3}, $8.55\times10^{18}<n_c<8.60\times10^{18}$ cm^{-3}) in the temperature range 300 to 0.5 K. Much of the data was taken in temperature sweeps between 4.2 and 0.5 K at magnetic fields between 0.5 and 15 T. The insulating samples exhibit variable-range-hopping (VRH) behavior for $R_H(N,H,T)$ that is similar to the VRH behavior of $\sigma(N,H,T)$ and is Mott VRH in the temperature range of these experiments. The ratio of the Hall VRH characteristic temperature T_{0H} and the Mott characteristic temperature T_0 as $H\rightarrow0$ and $N\rightarrow n_{c-}$ is in good agreement with the theoretical prediction of Gruenewald *et al.* that $(T_{0H}/T_0)^{1/4}=\frac{5}{8}$. The metallic results indicate $R_H(n,H,T)\simeq R_H(n,H)[1+m_H(n,H)T^{1/2}]$ at sufficiently low temperature, analogous to earlier results for $\sigma(n,H,T)$ and suggest a coefficient of the $T^{1/2}$ term for σ_{xy} of order $m_{xy}\sim1.5m_{xx}$. The values of $R_H^{-1}(n, H\rightarrow0, T\rightarrow0\text{ K})$ do not show the apparent critical behavior observed for Ge:Sb, Kr:Bi, and *a*-Si:Pt and are essentially in agreement with the weak-localization theoretical predictions of Fukuyama and of Shapiro and Abrahams. It is speculated that the differing "critical behavior" of these metal-insulator systems results from a spin-orbit contribution (extraordinary contribution) to the Hall coefficient.

I. INTRODUCTION

In the last decade there have been substantial advances in our understanding of electrical transport near the metal-insulator transition (MIT). The theoretical prediction[1] of the scaling of $\sigma(n, T\rightarrow0\text{ K})$ as $n\rightarrow n_{c+}$, where n is the carrier density ($n\approx N$ for the weakly compensated case, $N_A \ll N_D$; N_A is the acceptor density, N_D the donor density, and our convention will be n for $N_D>n_c$ and N for $N_D<n_c$) and n_c is the critical density for the onset of metallic behavior, has been confirmed in many doped semiconductors[2–9] and in granular metal systems.[10–14] Many of these systems[8–14] have yielded $\mu\approx1.0$ [$\mu=d\ln\sigma/d\ln\epsilon$ where $\epsilon=(n-n_c)/n_c$], in agreement with the weak-localization theoretical prediction,[1] but others, particularly Si:P,[2] Si:As,[3,4] Si:P+As,[5] Si:Sb,[6] and Ge:As (Ref. 7) produce results with μ close to $\frac{1}{2}$. On the insulating side of the MIT, $\sigma(N<n_c, T\rightarrow0\text{ K})\rightarrow0$, and, at a finite but sufficiently low temperature $\sigma(n<n_c, T)$ exhibits variable-long-range hopping (VRH) behavior of either the Mott[15] or the Efros-Shklovskii type.[16] The characteristic temperature for this VRH conduction, T_0, approaches 0 K as $N\rightarrow n_{c-}$, resulting from the scaling of the localization length. There have also been many magnetoresistance studies of these MIT systems on both sides of n_c. On the other hand, the Hall coefficient has been less systematically studied in the vicinity of the MIT, particularly at very low temperatures. Weak-localization predictions by Fukuyama[17] and by Shapiro and Abrahams[18] lead to no scaling of the Hall coefficient R_H, while, on the other hand, theories,[19,20] incorporating *e-e* interactions lead to different predictions that might be interpreted as implying scaling of R_H as $n\rightarrow n_{c+}$. Furthermore, calculations[21] based on classical percolation theory have suggested that R_H should exhibit scaling behavior with a characteristic exponent. In the last few years there have been a number of Hall-effect studies on different MIT systems. Results by Field and Rosenbaum[22] for Ge:Sb yielded $R_H^{-1}\propto(n/n_c-1)^{0.7}$ for n to within 8% of n_c. Rohde and Micklitz[23] have reported scaling of R_H^{-1} with virtually the same exponent for amorphous $Kr_{1-x}Bi_x$, while Löbl *et al.*[24] have found scaling behavior of R_H^{-1} for a Si:Pt, but with a somewhat larger exponent. Stankiewicz *et al.*[25] have also reported scaling of R_H^{-1} for the magnetic semiconductor EuTe.[26] Nevertheless, Tousson and Ovadyahu[27] find no evidence for scaling of R_H^{-1} for In_2O_{3-x} films. Our own measurements of R_H for metallic *n*-type Si:As samples were motivated by the differing results near n_c and a preliminary account[28] of our results for Si:As does not indicate scaling of R_H^{-1} as $n\rightarrow n_{c+}$.

The primary and original motivation of the present study was to investigate the behavior of $R_H(N,T,H)$ on the insulating side of the MIT in the critical regime characterized by the VRH conduction.[29] Following the pioneering calculation of Holstein,[30] it was recognized that it took at least three sites and interference between different paths to produce a Hall effect in the hopping regime. Later theoretical work[31,32] extended Holstein's results to consider percolation paths between clusters and to obtain explicit expressions for $R_H(N,T)$. In 1981 Gruenewald *et al.*,[33] employing percolation theory, were able to calculate both $\sigma(N,T)$ and the Hall mobility $\mu_H(N,T)$ and demonstrate in the VRH regime that these quantities are characterized by different characteristic temperatures.

Early Hall data[34,35] in the hopping regime came before Mott's prediction[15] of VRH conduction and the data were thereore not analyzed in terms of Mott's temperature dependence, even though some of the data were in the VRH regime. Fritzsche,[34] in doped Ge, observed a sharp slope reversal in R_H versus T, representing a transition between the ϵ_1 (electrons in the conduction band) and ϵ_3 (electrons hopping to empty impurity sites) conduction processes. Amitay and Pollak[36] and later Klein[37] attempted to measure the Hall coefficient in the hopping regime at low temperatures, but were only able to establish an upper bound,[38] which proved to be below the magnitude of the effect calculated by Holstein.[30] Sasaki[39] was the first to plot the Hall coefficient versus $T^{-1/4}$, but did not claim that $R_H(T)$ obeyed the VRH law. The first experimental results specifically showing the predicted[33] VRH temperature of $R_H(N,T)$ in the VRH regime were reported for uncompensated Si:As.[40] A more detailed discussion of these results is presented below. Of particular interest is the comparison of the results for barely insulting and barely metallic samples. The results for Si:As as $|n-n_c|\to 0$ seem to argue there is no vanishing of $n(T\to 0\text{ K})$ or $dN/d\mu$ [the thermodynamic density of states (DOS)] as $n\to n_c$, in agreement with Lee's prediction.[41]

In the semiconductor field it has been well known[42] that the Hall (correction) factor A ($R_H=A/ne$) depends on the type of scattering (phonon or impurity) mechanism. On the other hand, in ferromagnetic metals such as Fe (Ref. 43) and Ni the dominant extraordinary contribution to R_H is proportional to the magnetization and originates from the effect of the spin-orbit interaction.[44] There is recent evidence in paramagnetic amorphous alloys such as $Zr_{1-x}Fe_x$ (Ref. 45) that the Hall coefficient can be dominated by the extraordinary term. In this paper we suggest that spin-orbit contribution can make a significant correction to R_H near the MIT, even through the system is paramagnetic, albeit with a somewhat enhanced susceptibility. This correction is expected to be increasingly important as $n\to n_{c+}$ because of the rapidly increasing magnitude of the resistivity as the MIT is approached from the metallic side. It is argued the different reported scaling behaviors of R_H^{-1} can be explained as resulting from this spin-orbit contribution to R_H, which is most important near n_c.

In Sec. II the background for the Hall effect near the MIT is presented. Experimental procedures and samples are discussed in Sec. III. The main experimental results are given in Sec. IV. These results are considered and discussed in Sec. V, which is followed by the principal conclusions.

II. BACKGROUND

A. Scaling behavior of $\sigma(n,\ T\to 0\text{ K})$ near the MIT

Since the scaling-theory predictions of Abrahams *et al.*[1] for $\sigma(n,\ T=0\text{ K})$ there have been many experimental reports[2-14] showing $\sigma(n,\ T\to 0\text{ K})\propto(n/n_c-1)^{\mu}$ as $n\to n_{c+}$. Weakly compensated Si:P,[2] SiAs,[3,4] Si:P+As,[5] Si:Sb,[6] and Ge:As (Ref. 7) all exhibit μ close to $\frac{1}{2}$, with Si:P particularly carefully studied very close to n_c (to within 0.1% of n_c) by Paalanen *et* +*al.*[2] Ge:Sb has exhibited exponents between $\frac{1}{2}$ and 1 and has also shown significant apparent compensation dependence.[46] Recent studies of compensated n-type Si by Hirsch *et al.*[47] yield $\mu\sim 1.0$. The amorphous Si-metal and Ge-metal alloys[10-14] all show $\mu\sim 1$, in agreement with the scaling-theory prediction.[1] The $\mu\sim\frac{1}{2}$ result for the uncompensated n-type Si and Ge case is thought to arise from electron-electron interactions, although this has not been firmly established.

The temperature dependence of $\sigma(n>n_c,T)$ at sufficiently low temperatures has been shown[2] to be of the form $\delta\sigma(T)=\sigma(T)-\sigma(0)=m(n)T^{1/2}$ resulting from e-e interactions in the presence of strong impurity scattering.[48] $m(n)$ results from both Hartree and exchange terms, but very close to n_c, $m(n)\propto[D(n)]^{-1/2}$, where $D(n)$ is the diffusion coefficient, $m(n)$ increases rapidly as $n\to n_{c+}$ since $D(n,\ T=0\text{ K})$ is related to $\sigma(n,\ T=0\text{ K})$ by the Einstein relationship

$$\sigma(n,\ T=0\text{ K})=e^2(dN/d\mu)D(n,\ T=0\text{ K})\ , \tag{1}$$

where $dN/d\mu$ is the thermodynamic DOS and purportedly varies slowly and smoothly in the vicinity of n_c according to Lee.[41] Accordingly, $\sigma(n,\ T=0\text{ K})$ and $D(n,\ T=0\text{ K})$ should exhibit scaling behavior with the same exponent μ, which is just the prediction of weak-localization theory. This has not yet been verified by experiment. The scaling behavior of the ESR linewidth for Si:As (Ref. 49) has suggested a larger exponent for D than for σ, but the analysis is complicated by the fact that the linewidth does not go to zero at n_c and there may be a second contribution to the linewidth not related to carrier diffusion for barely metallic samples.

On the insulating side of n_c, $\sigma_{\rm dc}(N<n_c,\ T=0\text{ K})=0$. At finite, but at sufficiently low temperatures, $\sigma(N,T)$ is dominated by VRH conduction of the form

$$\sigma(N,T)=\sigma_0(N,T)e^{-(T_0/T)^p}\ , \tag{2}$$

where $p=\frac{1}{4}$ for the Mott case[15] and $p=\frac{1}{2}$ for the Efros-Shklovskii case.[16] The characteristic temperature T_0 scales to zero as $N\to n_{c-}$. as has been well documented for Si:As (Ref. 29) and other semiconductors. The behavior of VRH in the critical region as $N\to n_{c-}$ has been reviewed by one of us.[50] Sufficiently close to n_c $[1-N/n_c<0.12$ for Si:As (Ref. 29)] the VRH is of the Mott type at available temperatures for reasons discussed previously.[4] When the Hall data in the VRH regime are discussed in Sec. IV, it is very important to compare the $R_H(N,T,H)$ results with the $\sigma(N,T,H)$ results under identical conditions since the temperature dependence of $R_H(N,T,H)$ is much less than for $\sigma(N,T,H)$ at low fields for barely insulating samples.

B. Hall effect and magnetoresistance behavior

In this paper we consider the magnetic field H applied normal to the van der Pauw disk along the z axis and the current density j along the x axis. We measure the Hall voltage V_H (Hall field $E_y=\rho_{yx}j_x)=I_xR_y$ and thereby determine the Hall resistivity $\rho_H=\rho_{yx}$ given by

$$\rho_{yx}=\frac{\sigma_{xy}}{\sigma_{xx}\sigma_{yy}+\sigma_{xy}^2}\sim\frac{\sigma_{xy}}{\sigma_{xx}^2}\ , \tag{3}$$

where $\sigma_{xx}=\sigma_{yy}$ and at all experimental fields the Hall angle ($\tan\theta_H=\rho_{yx}/\rho_{xx}=\sigma_{xy}/\sigma_{xx}$) is small and $\sigma_{xy}^2<0.01\sigma_{xx}^2$, allowing neglect of σ_{xy}^2 in the denominator. Both σ_{xy} and σ_{xx} are a function of donor density N_D, temperature T, and the magnetic field H. Because the Hall angle is small, one obtains $\rho_{xx}(H,T)/\rho_{xx}(H=0,T)\approx\sigma_{xx}(H=0,T)/\sigma_{xx}(H,T)$, and the Hall coefficient $R_H(n,H,T)=\rho_{yx}(n,H,T)/H$ can be written as

$$R_H(n,H,T)\approx\frac{1}{H}\frac{\sigma_{xy}(n,T,H)}{[\sigma_{xx}(n,\ T=0\ \mathrm{K},H)]^2}\times\left[\frac{\rho_{xx}(n,H,T)}{\rho_{xx}(n,H=0,\ T)}\right]^2 . \tag{4}$$

At low fields $\sigma_{xy}\propto H$, but Eq. (4) illustrates the fact that R_H can exhibit a field dependence because of the transverse magnetoresistance (MR), which becomes more important as $n\to n_{c+}$. MR studies of Si:As (Refs. 51 and 52) have demonstrated that

$$\rho_{xx}(n,T,H)/\rho_{xx}(n,\ T,\ H=0)=\kappa(n,H,T)$$

increases significantly as $n\to n_{c+}$. At sufficiently low temperatures such that $kT\ll g\mu_BH$, one finds

$$\sigma_{xx}(n,T,H)=\sigma_{xx}(n,H)[1+m_{xx}(n,H)T^{1/2}-b_{xx}(n)H^{1/2}]\ ,$$

in agreement with MR theory[53] featuring e-e interaction. By analogy, we assume

$$\sigma_{xy}(n,T,H)=H\sigma'_{xy}(n)[1+m_{xy}(n,H)T^{1/2}-b_{xy}(n)g(H,T)]\ ,$$

where $g(H,T)\propto H^{1/2}$ for $kT\ll g\mu_BH$ and $g(H,T)\propto H^2/(kT)^{3/2}$ for $kT\gg g\mu_BH$. Inserting $\sigma_{xy}(n,T,H)$ and $\sigma_{xx}(n,\ T,\ H=0)$ into Eq. (4), one obtains

$$R_H(n,H,T)\simeq R_H(n,\ H=0,\ T=0\ \mathrm{K})[1+m'_H(n,H)T^{1/2}-b_{xy}(n)g(H,T)[\kappa(n,H,T)]^2\ , \tag{5}$$

where $m'_H(n,H)=m_{xy}(n,H)-2m_{xx}(n,\ H=0)$ and $R_H(n,\ H=0,\ T=0\ \mathrm{K})=\sigma'_{xy}(n)/\sigma_{xx}(n,\ H=0)]^2$. The experimental objective is to determine $R_H(n,\ H=0,\ T=0\ \mathrm{K})$ by the extrapolation of data for $R_H(n,H,T)$ as both T and H approach zero. Because the Hall voltage $V_H\propto H$ in the low-field limit, one rapidly loses sensitivity at low fields and errors became significant for $H\lesssim0.5$ T. It was also found for the barely metallic samples studied that, for $T<2$ K, $R_H(n,H,T)$ exhibited a $T^{1/2}$ dependence. As a result it was decided to fit the Hall-coefficient expression to the empirical expression

$$R_H(n,H,T)=R_H(n,\ H,\ T=0\ \mathrm{K})[1+m_H(n,H)T^{1/2}]\ . \tag{6}$$

The MR correction $\kappa(n,H,T)$ reaches its maximum value at $T=0$ K and is effectively at its maximum for $kT\ll g\mu_BH$. Previous Si:As MR results[51,52] have suggested $\kappa(n,\ H,\ T=0\ \mathrm{K})$ of the form $\kappa(n,\ H,\ T=0\ \mathrm{K})=[1+A(n)H^{1/2}]\exp[f(n)H^2]$, where $\exp[f(n)H^2]$ was explained[51] as resulting from magnetic tuning of n_c with H. In the lower-field range, where $f(n)H^2\ll1$, one finds

$$R(n,\ H,\ T=0\ \mathrm{K})\simeq R(n,\ H=0,\ T=0\ \mathrm{K})\times\{1+[2A(n)-b_{xy}(n)]H^{1/2}+[A(n)]^2H+2f(n)H^2\}\ .$$

Tousson and Ovadyahu[54] have stressed, from their In_2O_{3-x} Hall-effect results, the importance of making Hall measurements at very low magnetic fields, where their In_2O_{3-x} samples show a negligible temperature dependence. We have discussed the temperature and field requirements previously.[28] To summarize: (1) The temperature T must be low enough to get into the $T^{1/2}$ region which for Si:As with $n/n_c\geq1.028$ requires $T<2$ K$\sim0.01T_F$; (2) the magnetic field H must be small enough such that the magnetic length $L_H=(\hbar c/eH)^{1/2}$ must be sufficiently greater than the correlation length $\xi(n)=\xi_0(n/n_c-1)^{-\nu}$. Furthermore, the magnetic tuning of the correlation length[51,52] must be negligible, i.e., $\delta[n/n_c(H)-1]$ must be small. From a pragmatic standpoint it is sufficient that the magnetic field is low enough so that the Hall voltage V_H is strictly linear with H. It is worth emphasizing that among the standard n-type semiconductor MIT systems (Ge, Si, GaAs, InP, InSb), Si:As has the highest degeneracy temperature T_F and smallest ξ_0 (largest n_c) and it is easiest to satisfy the T and H requirements for Si:As when compared to the others that have been studied near the MIT.

On the insulating side of the MIT, one combines the predictions of Gruenewald *et al.*[33] ($\ln[\mu_H(N,T)]\propto-\frac{3}{8}(T_0/T)^{1/4}$), with the Mott prediction,[15] ($\ln[\sigma(n,T)]\propto-(T_0,T)^{1/4}$), and obtains from $\mu_H=R_H\sigma$ the theoretical prediction $\ln[R_H(n,T)]\propto\frac{5}{8}(T_0/T)^{1/4}=(T_{0H}/T)^{1/4}$. It is important to emphasize that the prediction of the Gruenewald *et al.* is based only on the percolation among three site clusters. Taking account of possible N, H, and T dependencies of the prefactors and N and H dependencies of the characteristic temperatures T_0 and T_{0H}, we have fitted our data to the empirical expression

$$R_H(N,H,T)=R_0(N,H,T)e^{[T_{0H}(N,H)/T]^{1/4}}\ . \tag{7}$$

This expression is only expected to be valid very near n_c ($0.88<N/n_c<0.99$) at temperatures below a characteristic temperature and at magnetic fields below a critical value that depends on N/n_c.

C. Theoretical predictions for $R_H(n, T\rightarrow 0\text{ K})$ near the MIT

Weak-localization- (WL-) theory predictions by Fukuyama[17] and by Shapiro and Abrahams[18] for σ_{xy} find $\sigma_{xy}\propto(n/n_c-1)^{2\nu}$, while $\sigma_{xx}\propto(n/n_c-1)^{\nu}$, where ν is the localization-length exponent. As a result, WL theory predicts no scaling of $R_H(n, T\rightarrow 0\text{ K})$ as $n\rightarrow n_{c+}$. On the other hand, calculations by Alt'shuler *et al.*[19] have obtained $(\delta\sigma_{xy})_{\text{int}}=0$ due to *e-e* interactions due to the so-called g_1 and g_3 processes. However, as has been emphasized by Fukuyama,[55] the g_2 and g_4 processes have not yet been calculated; hence the result $(\delta\sigma_{xy})_{\text{int}}=0$ should not necessarily be considered a general result. These two different results can be summarized as

$$(\delta R_H/R_H)_{\text{WL}}=\delta\sigma_{xy}/\sigma_{xy}-2\,\delta\sigma_{xx}/\sigma_{xx}=0\ , \tag{8a}$$

$$(\delta R_H/R_H)_{\text{int}}=-2\,\delta\sigma_{xx}/\sigma_{xx}=2\,\delta R/R\ , \tag{8b}$$

where $\delta X=X-X_0$ for $X=\sigma_{xx}$, σ_{xy}, R_H, and R (R is the sample resistance), with X_0 the unperturbed value in the absence of WL or *e-e* interactions. It is useful to define the quantity $\gamma=\delta(\ln R_H)/\delta\ln R=(\delta R_H/R_H)/(\delta R/R)$.

Two groups[56,57] have studied the dependence of γ on $R_\square$ for the two-dimensional (2D) electron gas in Si inversion layers. They find $\gamma\sim 2$ as $R_\square\rightarrow 0$ and γ decreases approximately linearly with increasing $R_\square$. (Bishop *et al.*[56] find $\gamma\sim 0.4$ at $R_\square=12\text{ k}\Omega$.) These results are in good agreement with the dominance of *e-e* interactions at small $R_\square$ and the increasing importance of WL with increasing $R_\square$. Ordinarily one expects $0<\gamma<2$ when potential scattering is dominant; however, γ can be larger than 2 when spin-orbit (s.o.) scattering is very important, and this has been reported for quasi-2D electrons in (*p*-type-InSb/*i*-GaAs) by Kichigin *et al.*[58] Although spin-orbit scattering is not expected to be dominant for most *n*-type semiconductors (3D case) near the MIT, there is good reason to believe that the s.o. interaction can make an important contribution to the Hall coefficient and this will be discussed below. The result in Eq. (8b) was originally derived for the 2D case, but has been claimed[20] to be valid for the 3D case. The experimental situation for the 3D case near the MIT is not yet well understood. Tousson and Ovadyahu found $\gamma\sim 0$ for their In_2O_{3-x} films. Preliminary results[28] for barely metallic Si:As samples yielded $0.4<\gamma<0.6$ with no particular magnetic field or donor-density dependence.

Another theoretical approach that takes into account many-body interactions for good metals has been Fermi-liquid theory. Kohno and Yamada[59] have derived a general expression for the Hall coefficient including many-body effects based on Fermi-liquid theory. The result is supposed to be exact with respect to the most singular terms concerning quasiparticle damping. These authors find $\sigma_{xy}\propto\gamma_{\mathbf{p}}^2$ and $\sigma_{xx}\propto 1/\gamma_{\mathbf{p}}$, where $\gamma_{\mathbf{p}}$ is the quasiparticle damping rate for a quasiparticle of momentum $\mathbf{p}$. As a result, R_H remains finite and there is no predicted scaling of R_H as $\gamma_{\mathbf{p}}\rightarrow 0$ basically a result similar to WL results for R_H. There has been recent interest[60,61] in applying Fermi-liquid theory to disordered systems near the MIT. However, recent Si:P experimental results for the spin susceptibility[62] and specific heat[63] in the dilution-refrigerator temperature range have demonstrated the importance of localized moments for barely metallic samples, and these results are not in good agreement with Fermi-liquid theory, and the relevance of Fermi-liquid-theory predictions for transport near n_c is certainly in doubt.

D. Spin-orbit contribution to the Hall effect near n_c

It has been well known since the classic paper of Karplus and Luttinger[44] that the spin-orbit contribution is responsible for the extraordinary or anomalous contribution to the Hall effect observed[43] in ferromagnetic metals. In doped semiconductors near the MIT, the magnetization is very weak because of a small spin susceptibility that is only slightly enhanced over the Pauli susceptibility on the metallic side of the MIT. As a result, it would be surprising to expect an anomalous contribution to R_H from the spin-orbit interaction for doped semiconductors near n_c, even though it is to be noted that Nozières and Lewiner,[64] using a two-band model, developed the theory of the anomalous Hall effect for semiconductors. However, Trudeau *et al.*[45] recently discovered that, in the paramagnetic amorphous alloy $Zr_{1-x}Fe$, $R_H(T)$ is closely related to the valence (spin) susceptibility $\chi_v(T)$. These authors note the anomalous contribution is enhanced in the paramagnetic regime because of the large value of the electrical resistivity ($R_{H\,\text{anom}}\propto\rho^2\chi_v$, $\rho\sim 1.7\times 10^{-4}\ \Omega\,\text{cm}$ for $x\sim 0.35$). For the doped semiconductors that are barely metallic the spin susceptibility is several orders of magnitude smaller, but, on the other hand, the resistivity $\rho(n)$, because it is scaling toward infinity as $n\rightarrow n_{c+}$, can easily be 2 orders of magnitude larger close to n_c. Thus it appears necessary to seriously consider the spin-orbit contribution to $R_H(n)$ as $n\rightarrow n_{c+}$.

At the present time there is no satisfactory theory of the spin-orbit contribution to R_H near n_c, even though there have been theoretical efforts[65,66] to consider the effects of the s.o. coupling on the disordered electron gas. Lacking on appropriate theory, we will use the expression given by Trudeau[45] based on the earlier work of Berger,[67] namely

$$R_H\simeq R_0+(2e^2/\mu_0\hbar g\mu_B)\lambda_{\text{s.o.}}\rho_{xx}^2\chi_v\ , \tag{9}$$

where R_0 is the Hall coefficient in the absence of the s.o. interaction, χ_v is the spin susceptibility of the valence electrons which corresponds to the impurity-band susceptibility in the MIT case, and $\lambda_{\text{s.o.}}$ is a measure of the strength of the s.o. interaction, which in the present case corresponds to the impurity s.o. interaction. In the ferromagnetic metals[44] and also in the paramagnetic alloys $Zn_{1-x}Fe_x$[45] the important contribution to $\lambda_{\text{s.o.}}$ arises from *d*-band states and $\lambda_{\text{s.o.}}$ involves orbit matrix elements $|\langle i|\mathbf{L}|n\rangle|$ and an energy denominator, which is characteristic of the *d*-band splitting for Fe. The ground impurity states in *n*-type semiconductors are predominantly s-like and it would, at first sight, appear that there would be no contribution for s-like states. It has recently been shown[68] that a new contribution can arise from the gradient of the impurity s.o. interaction and that this contribution will be most important for s-like orbital states. This leads to $\lambda_{\mathrm{s.o.}}$ in Eq. (9) being replaced by a s.o. frequency $\Omega_{\mathrm{s.o.}} \propto \langle \partial E_{ix}/\partial x \rangle$, where $\partial E_{ix}/\partial x$ is one diagonal component of the field-gradient tensor. The most important qualitative aspects of Eq. (9) are (1) this s.o. correction term is only important for large ρ_{xx} and becomes increasingly important as the MIT is approached from the metallic side; χ_v is very small for MIT systems, even with an enhanced susceptibility near n_c, but this is counterbalanced by the large increase in ρ_{zz} as $n \rightarrow n_c+$; (2) $\lambda_{\mathrm{s.o.}}$ is likely to be large for high-Z impurities. These factors make it likely that $R_H(n)$ can show scaling behavior for high-Z impurity MIT systems arising from the second term in Eq. (9).

III. EXPERIMENTAL PROCEDURES

The initial data were obtained in an electromagnet at the University of Rochester (UR) at fields between -1 and $+1$ T in the temperature range 1.4–300 K. After initial superconducting-solenoid (SS) runs at fields up to 11.5 T at UR, the importance of achieving lower temperatures made it advantageous to carry out ^{3}He-refrigerator experiments in the 15 T SS at the Francis Bitter National Magnet Laboratory (FBNML) at the Massachusetts Institutes of Technology. Experiments at the FBNML were carried out between 4.2 and 0.5 K. One advantage of the UR electromagnet data was the possibility of field reversal permitting an experimental check of Hall-voltage (V_H) offsets for the van der Pauw–geometry samples. Field reversal was not possible when employing the superconducting solenoids.

A. Samples

Most of the data reported came from samples prepared from an uncompensated Si:As ingot 35 cm long and 5.0 cm in diameter, Czochralski grown along a $\langle 111 \rangle$ crystal axes which was purchased from MACOM. Earlier transport studies[3,4] of Si:As near the MIT were made on samples prepared from this same ingot. The As concentration varied from 5.0×10^{18} cm^{-3} at the seed end to 11×10^{18} cm^{-3} at the end, with the critical density N_c ($8.5 < N_c < 8.6\times10^{18}$ cm^{-3}) near the center of the ingot. Several more metallic individual Si:As wafers were obtained from MACOM in order to obtain a greater range of $n/n_c - 1$ on the metallic side of the MIT. The van der Pauw–geometry disk samples (6 and 3 mm in diameter) were prepared for Hall measurements as described previously.[28,40] The larger 6-mm-diam sample allowed the placement of the welded 0.002-in. $Au_{0.98}Sn_{0.02}$ wire leads relatively closer to the sample edges, thus reducing systematic errors in the measurement of the resistivity and Hall coefficient. However, the larger-diameter samples are more susceptible to doping inhomogeneities. The "cloverleaf" geometry was employed on some samples because it reduces the systematic errors due to the contact placement and it also reduces errors due to doping inhomogeneities. The four "cloverleaf" slots, approximately 90° apart, were cut with a diamond-coated wafering saw.

B. Concentration determination

Because we are concerned with the critical behavior of the Hall coefficient $R_H(n, H=0, T\rightarrow 0\text{ K})$ as $n \rightarrow n_c$, an independent determination of the concentration scale is required and one cannot employ the result $R_H = A(n,T)/ne$ as a measure of the room-temperature (RT) carrier density and the As donor density N_D. There is now evidence[69] that the Hall correction factor $A(n,T)$ may be donor specific and is larger for Si:As than for Si:P.[70] The most sensitive determination of the relative concentrations of the samples is obtained from the ratio $r_0 = \rho(4.2\text{ K})/\rho(\text{RT}) \approx R(4.2\text{ K})/R(\text{RT})$ measured accurately on each sample. This ratio varies by a factor of 7 for $7.7 < N < 10.3\times10^{18}$ cm^{-3}, whereas $\rho(\text{RT})$ changes by only 20% in the same range. The absolute scale r_0 versus N_D for Si:As was determined using our r_0 and $\rho(\text{RT})$ values and the $\rho(\text{RT})$-versus-N_D scale determined for Si:As determined by Newman *et al.*[69] using the neutron-activation-analysis method. For the three most metallic samples at 17.9×10^{18}, 20.9×10^{18}, and 32.8×10^{18} cm^{-3}, respectively, the r_0 approach had become much less sensitive and the concentrations were determined using mostly the $\rho(\text{RT})$ values, but also employing the Hall results as a check, noting that the Hall factor $A(n)$ approaches 0.87 in the strongly degenerate limit.[71] The r_0-versus-N_D scale is shown in Fig. 1, and represents an extension of that shown earlier.[4] The values of r_0 and $\rho(\text{RT})$ for the samples measured in this study are listed in Table I.

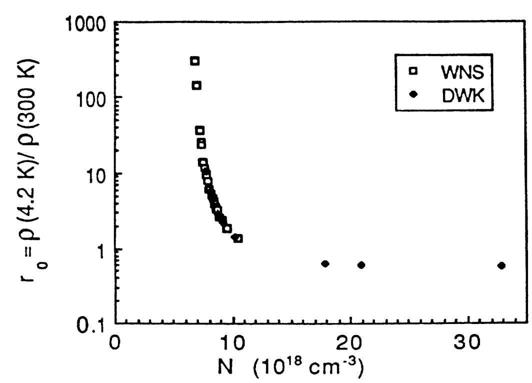

FIG. 1. $r_0 = \rho(4.2\text{ K})/\rho(300\text{ K})$ vs donor density for uncompensated Si:As, □, determined by Shafarman (WNS) (Ph.D. thesis, University of Rochester); ◇, determined by Koon (DWK) (Ph.D. thesis, University of Rochester).

TABLE I. Si:As samples. (LN denotes liquid nitrogen and RT denotes room temperature.)

Sample no.	r_0	N (10^{18} cm^{-3})	ρ_{RT}^{c} (mΩ cm)	R_1/R_2 ($H=0$, $T=4.2$ K)	Measurements
1 ("A4-2-X-1")[a]	10.94	7.77	8.13	0.92[d]	^{3}He, ^{4}He, LN,RT
2 ("B10-X-2")	5.78	8.12	7.90	1.21[d]	^{3}He, ^{4}He, LN, RT
3 ("B10-X-3")	5.21	8.21	7.84	1.54[d]	^{3}He, ^{4}He, LN,RT
4 ("B13-3-X-1")[a]	4.47	8.36	7.75	0.78[d]	^{3}He, ^{4}He, LN, RT
5 ("C9-X-2")	2.94	8.78	7.85	0.36[e]	^{3}He, ^{4}He, LN, RT
6 ("D9-X-1")[b]	2.34	9.14	7.33	1.04[e]	^{3}He, ^{4}He, LN, RT
7 ("D9-X-2")[b]	2.11	9.30	7.08	0.80[e]	^{3}He, ^{4}He, LN, RT
8 ("F6-X-1")	1.47	10.2	6.84	2.37[e]	^{3}He, ^{4}He, LN, RT
9 ("B1-Y-1")	0.64	17.9	4.22	1.002[f]	^{4}He, LN, RT
10 ("A1-Y-2")	0.60	20.9	3.44	1.13[f]	^{4}He, LN, RT
11 ("C1-Y-1")	0.58	32.8	2.30	1.35[f]	^{4}He, LN,RT

[a] $\frac{1}{8}$-in.-diam disk.
[b] $\frac{1}{4}$-in.-diam disk with "cloverleaf" structure.
[c] ρ_{RT} values obtained for the van der Pauw disk did not always agree with comparable ρ_{RT} values for the bars discussed in Ref. 4. ρ_{RT} values have not been used in determining N, except for the three most concentrated samples.
[d] Some magnetic field and temperature dependencies of R_1/R_2: at 4.2 K, less than 5% variation for $0<H<14$ T; at 0.52 K up to 100% variation at highest fields for some samples.
[e] Negligible magnetic field and temperature dependencies ($<5\%$) over entire range of H and T.
[f] Only studied between 4.2 and 1.4 K for $-1<H<1$ T; no appreciable dependence of R_1/R_2 on H and T.

C. Measurement circuits and instrumentation

The six different van der Pauw[72] configurations[73] are shown in Fig. 2(a) and yield resistances R_1, R_2, R_3, R_4, R_5, and R_6, where $R_j=V_j/I_j$. The Hall resistances R_5 and R_6 corresponds to the voltage leads on opposite corners of the square (180° apart on our circular disks). van der Pauw[72] has found the resistivity ρ and the Hall resistivity ρ_H given by

$$\rho=\frac{\pi t}{\ln 2}\frac{R_1+R_2}{2}f\left[\frac{R_1}{R_2}\right], \tag{10a}$$

$$\rho_H=\frac{R_5+R_6}{2}t, \tag{10b}$$

where t is the sample thickness (which must be constant) and $f(R_1/R_2)$ is the asymmetry-ratio correction given for small x by[72]

$$f(x)=1-\frac{\ln 2}{2}x^2-\left[\frac{(\ln 2)^2}{4}-\frac{(\ln 2)^3}{12}\right]x^4+\cdots, \tag{11}$$

where $x=(R_1-R_2)/(R_1+R_2)$. The Hall coefficient is obtained from $R_H=10^4\rho_H/B$, where ρ_H is in units of Ω cm, R_H is in units of cm^3/C, and B is the magnetic induction in teslas (T). All values of ρ_H, and consequently of R_H, have been found to be negative, consistent with negatively charged carriers in an impurity band. The measurement of six different resistances yields some redundancy and provides an additional check on the homogeneity of the doping. R_1/R_2-1 is an indirect measure of anisotropic doping variations. Because of re-

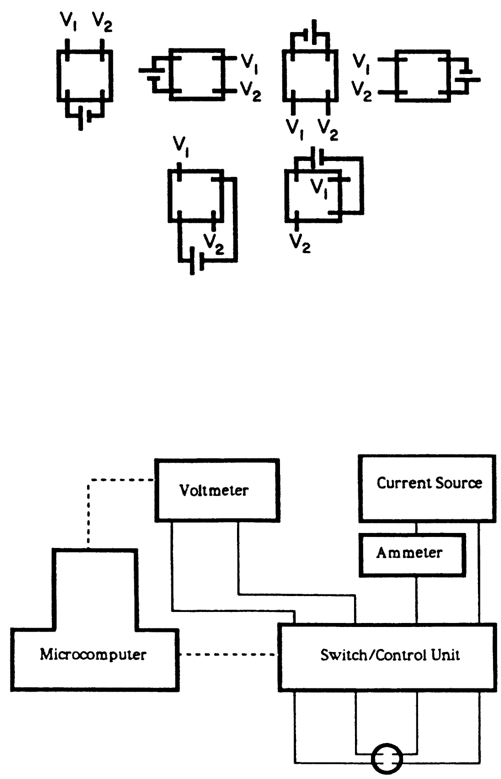

FIG. 2. (a) van der Pauw configurations (after Ref. 74) with the current leads hooked to the electromagnetic field (EMF) source; $V1$ and $V2$ are used to measure the voltage drop; (b) configuration of various instruments, microcomputer, and switch-control unit. The dashed lines represent IEEE-488 buses.

ciprocity one has $R_1=R_3$, $R_2=R_4$, and $R_5(B=0)=R_1-R_2=-R_6(B=0)$, where $|R_5(B=0)|=0$ results for a perfectly homogeneous sample due to voltage offset resulting from imperfect placement of the leads. Measurement of the ratio R_1/R_2 (or R_3/R_4) gives some information on the doping homogeneity of the individual sample. The results for R_1/R_2 at $T=4.2$ K and zero magnetic field are listed in Table I. The R_1/R_2 values range between 0.36 and 2.37. In general, the metallic samples have a very small or negligible dependence of R_1/R_2 on temperature or magnetic field. Some of the insulating samples have an appreciable variation of R_1/R_2 with temperature and magnetic field. In general, the H dependence of R_1/R_2 is largest at the lowest temperatures (0.5 K). Some portion of R_1/R_2-1 is just a geometric factor resulting from the position of the leads, although this is expected to be small from van der Pauw's error analysis.[72] While one might have anticipated larger deviations of R_1/R_2 from unity for the more insulating samples (larger r_0 values), the data do not support that expected trend.

A block diagram of the circuits and instruments employed is shown in Fig. 2(b). The IBM PC/XT microcomputer (with an IEEE interface board) controlled both the Keithley 181 nanovoltmeter and the Hewlett-Packard HP-3488 A switch-control unit featuring a 4×4 matrix switching module. The PC/XT connected the four sample leads to the appropriate current-voltage leads for each configuration [see Fig. 2(a)] to be measured. For each configuration the current and voltage leads are reversed to avoid thermal-gradient effects and R_j is the average of V_j/I_j with the current flow in the two directions. The number of readings obtained for each configuration depended on the desired variance, which was set with the PC/XT software. The current ($3<I<100$ μA was typically stable to 100 ppm and the noise voltage per reading was of order 30 nV. For R_5 and R_6, 75 readings were typically required, allowing us to measure ρ_H as low as 1 $\mu\Omega$ cm, corresponding to a Hall coefficient R_H as small as 0.01 cm^3/C for a field of 1 T. This corresponds to an electron density of 6×10^{20} cm^{-3}, or nearly 2 orders of magnitude greater than n_c. Even with some reduction in the sensitivity at the lowest currents employed at the lowest temperatures to avoid sample heating and non-Ohmic effects, the sensitivity was adequate to measure R_H to better than $\pm5\%$.

Errors introduced because the sample leads are not precisely at the edge of the disk of diameter D have been considered by van der Pauw.[72] For leads in from the edge a distance d, the error are, respectively, $\Delta\rho/\rho\simeq-d^2/(2\ln2)D^2$ and $\Delta R_H/R_H\simeq-2d/\pi D$. For a typical realistic case $d\sim0.25$ mm and $d=6$ mm, one finds a systematic error of 2.5% for R_H and 0.1% for ρ. An analysis of the "cloverleaf"-geometry samples has been considered by Koon.[74]

D. Cryogenic rigs

For measurements in the Varian Associates electromagnet (UR) the samples were heat-sunk to a Cu block and electrically insulated from the Cu with a thin Mylar film. The Cu block could be heated with a noninductively wound heating coil and was situated inside a vacuum-tight can which could be filled with ^{4}He exchange gas. A calibrated (between 1.5 and 100 K) germanium resistor thermally sunk to the Cu block was utilized to monitor the sample temperature. Temperature between (1) 4.2 and 50 K were obtained with liquid ^{4}He as the refrigerant, (2) 55 and 77 K were obtained with pumped liquid N_2, and (3) 77 and 100 K with the heater using liquid N_2. The magnetic field was oriented to normal to the disks to within $\pm1°$ by rotating the electromagnet.

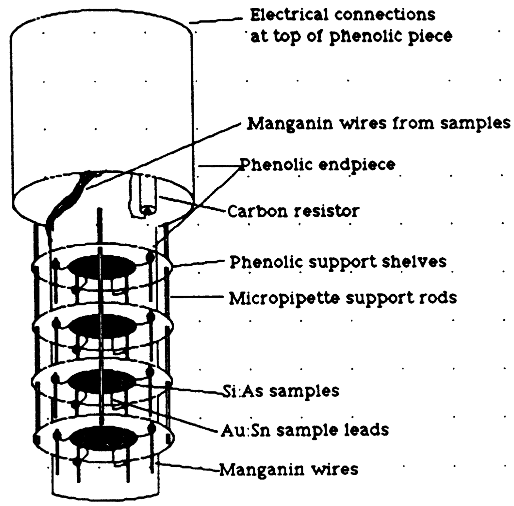

FIG. 3. Sample holder for cryogenic rig employed for superconducting-solenoid runs at the Francis Bitter National Magnet Laboratory and at the University of Rochester.

A second type of low-temperature inset was employed for the solenoid experiments. No copper block could be employed because of eddy-current heating. The samples were mounted on the holder shown in Fig. 3, which is at the bottom of the cryogenic insert which fit inside the ^{3}He-refrigerator Dewar at the FBNML. The samples reached thermal equilibrium with the ^{3}He exchange gas above 1.3 K and with the ^{3}He liquid below 1.3 K. Monitoring the sample temperature with the resistance $\rho(T)$, it took about approximately 5 min to react at thermal equilibrium above the ^{4}He λ point and 1 min below the λ point, with equilibration time decreasing as the pressure above the ^{3}He liquid was lowered. Equal intervals in $1/T$ were selected to permit calculation of the logarithmic derivative, as discussed previously.[4]

IV. EXPERIMENTAL RESULTS

In Table I are listed the eleven Si:As samples, four insulating and seven metallic, with their values of $\rho(\mathrm{RT})$, $r_0=\rho(4.2\ \mathrm{K})/\rho(\mathrm{RT})$, R_1/R_2 at 4.2 K, and $H=0$ T, and their donor densities as already described in Sec. III B. All but the three most concentrated samples (17.9×10^{18}, 20.9×10^{18}, and 32.8×10^{18} cm^{-3}) were studied at the FBNML in a superconducting solenoid at fields up to 15

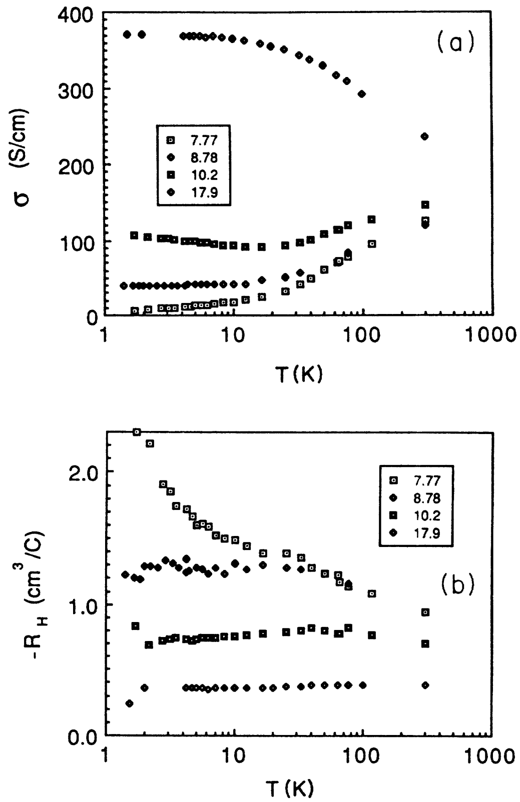

FIG. 4. (a) Conductivity and (b) Hall-coefficient data for three metallic and one insulating Si:As samples between 1.4 and 300 K. The Hall data were taken at $H=1$ T.

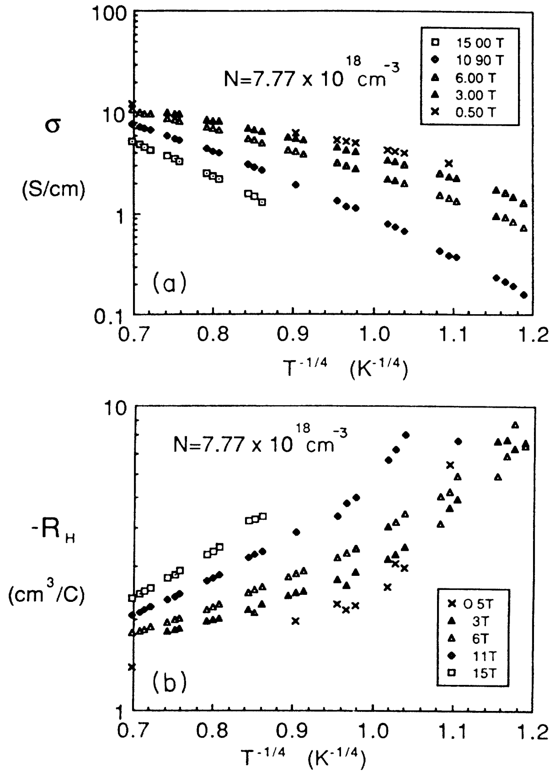

FIG. 5. (a) Conductivity and (b) Hall-coefficient data for the 7.77×10^{18}-cm^{-3} sample vs $T^{-1/4}$ for various magnetic fields for $0.5<T<4.2$ K.

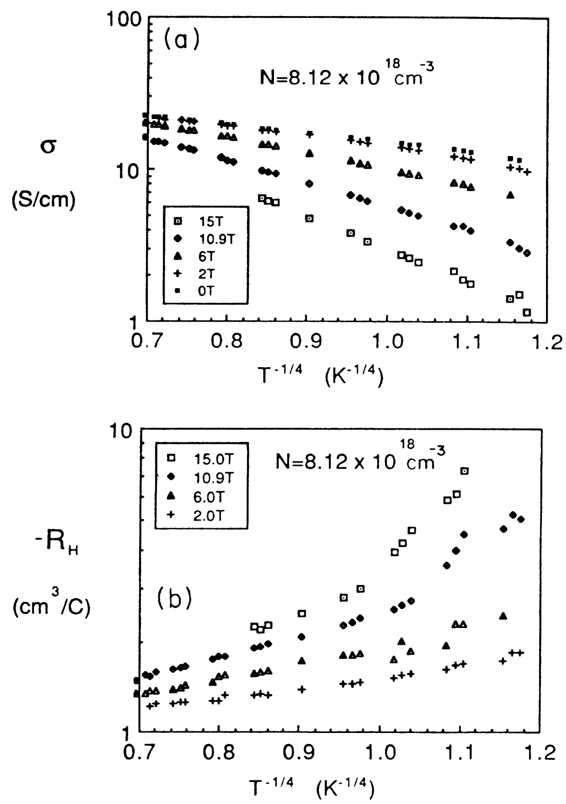

FIG. 6. (a) Conductivity and (b) Hall-coefficient data for the 8.12×10^{18}-cm^{-3} sample vs $T^{-1/4}$ for various magnetic fields for $0.5<T<4.2$ K.

T in the temperature range $0.5<T<4.2$ K. The latter three were only studied in the electromagnet ($-1<H<1$ T) for $1.4<T<300$ K.

Figure 4 gives an overall view of the temperature dependence of three metallic samples and one insulating sample for both $\sigma(T)$ and $R_H(T)$ between 1.4 and 300 K. The insulating sample shows a continuous decrease in $\sigma(T)$ and increase in $R_H(T)$ as the temperature is lowered. The most metallic sample ($n\sim2n_c$) shows a continuous increase in $\sigma(T)$ (but with a substantial flattening as $T\rightarrow0$ K) and a nearly-temperature-independent value of R_H, which is the smallest of the four samples. The 10.2×10^{18}-cm^{-3} metallic sample shows a minimum in $\sigma(T)$ between 10 and 20 K, but R_H shows a much smaller T-dependent variation. In fact, for all four of these samples the temperature variation of $R_H(T)$ is much less than that of $\sigma(T)$, and we need to look more closely at the temperature dependencies of $\sigma(T)$ and $R_H(T)$ as $T\rightarrow0$ K.

A. Insulating samples—low-temperature behavior

$\sigma(T)$ and $R_H(T)$ for the 7.77×10^{18}-, 8.12×10^{18}-, and 8.21×10^{18}-cm^{-3} Si:As samples are shown in Figs.5–7 (the data for the 8.36×10^{1}-cm^3 sample have been given earlier[40]). The following trends are noted in all of these figures. Both $\sigma(T)$ and $R_H(T)$ exhibit a reasonable fit to

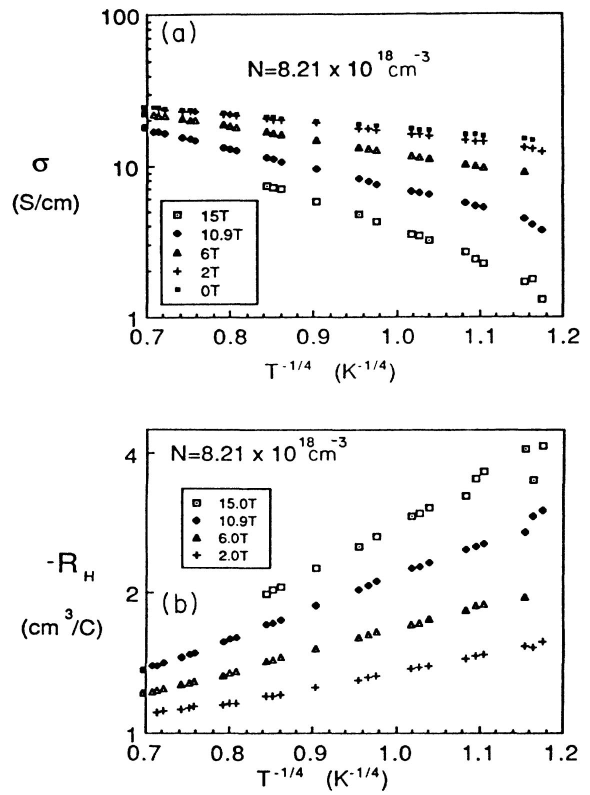

FIG. 7. (a) Conductivity and (b) Hall-coefficient data for the 8.21×10^{18}-cm^{-3} sample vs $T^{-1/4}$ for various magnetic fields for $0.5<T<4.2$ K.

the Mott VRH law when plotted versus $T^{-1/4}$, although the Hall data show more scatter because of the small Hall voltages, particularly at low fields. Some of the Hall data at the higher fields (15 and 10.9 T) show a distinct upturn at larger values of $T^{-1/4}$, which may represent a change from Mott hopping to Efros-Schklovskii VRH or may represent a magnetoresistance contribution, or a contribution that correlates with an appreciable change in R_1/R_2 at high magnetic fields and low temperatures. The slopes of $\ln\sigma$ or $\ln R_H$ with $T^{-1/4}$ increase substantially with increasing magnetic field, representing increases in the characteristic temperatures T_0 and T_{0H} with H. In general, the temperature variation of R_H is less than that of $\sigma(T)$. As an example from Fig. 6, $\sigma(T)$ at $H=6$ T varies by a factor of 3, while $R_H(T)$ at $H=6$ T varies by only a factor of 1.6 over the same temperature interval. From our previous analysis,[4] the best overall fit to the data is for the Mott exponent $p=\frac{1}{4}$ and a temperature-independent prefactor $\sigma_0(N,H)$. The uncertainties in the analysis of the Hall-coefficient data are much larger, but also seem to be best fitted by a temperature-independent prefactor $R_0(N,H)$ [see Eq. (7)]. The smaller temperature dependence of $R_H(N,H,T)$ relative to $\sigma(N,H,T)$ in the VRH regime results from the smaller value of $T_{0H}(N,H)$ relative $T_0(N,H)$, in qualitative agreement with the theoretical prediction of Gruenewald *et al.*[33] As will be further discussed below, T_{0H} mimics the behavior of the Mott characteristic temperature in terms of the density and field dependencies, but is smaller by an approximately constant factor. Recently, similar Hall data for *n*-type compensated CdSe in the VRH regime have been reported by Roy *et al.*[75]

From the fitting of the VRH data in these figures to Eqs. (2) and (7), the parameters $\sigma_0(N,H)$, $T_0(N,H)$, $R_0(N,H)$, and $T_{0H}(N,H)$ have been determined and are given in Table II. The behavior of these parameters with donor density and magnetic field will be discussed in Sec. V.

B. Metallic samples—low-temperature results

It has been stressed in Sec. II of the importance of obtaining $R_H(n,H,T)$ in the limit as both H and T ap-

TABLE II. Hall parameters for insulating samples. (*, no temperature sweep at this magnetic field; —, parameter could not be experimentally determined.)

N (10^{18} cm^{-3})		H (T) 0.0	0.5	2.0	3.0	6.0	10.9	15.0
7.77	σ_0 (S/cm)	*	123	*	176	300	980	1670
	T_0 (K)	*	—	*	210	520	2250	4600
	R_0 (cm^3/C)	*	—	*	0.46	0.36	0.23	0.16
	T_{0H} (K)	*	—	*	11	28	95	220
8.12	σ_0 (S/cm)	58	*	72	*	110	180	390
	T_0 (K)	3.2	*	7.3	*	33	147	560
	R_0 (cm^3/C)	—	*	0.65	*	0.57	0.46	0.32
	T_{0H} (K)	—	*	0.52	*	2.1	8.1	27
8.21	σ_0 (S/cm)	53	*	65	*	94	140	280
	T_0 (K)	1.4	*	3.4	*	18	80	330
	R_0 (cm^3/C)	—	*	0.64	*	0.56	0.45	0.33
	T_{0H} (K)	—	*	0.33	*	1.4	6.3	21
8.36	σ_0 (S/cm)	*	51	*	69	84	130	200
	T_0 (K)	*	0.9	*	4.0	12	64	200
	R_0 (cm^3/C)	*	0.72	*	0.67	0.62	0.49	0.42
	T_{0H} (K)	*	0.09	*	0.44	1.1	4.9	12

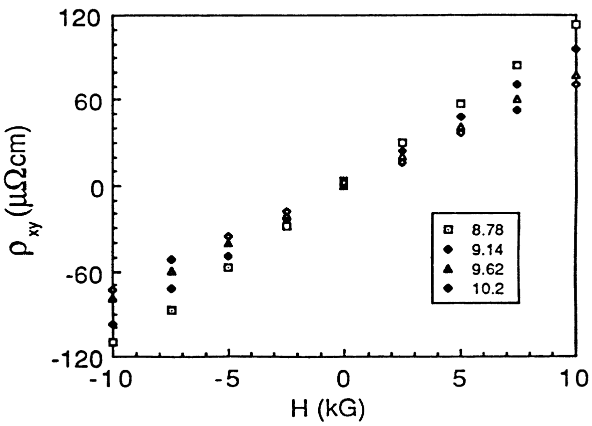

FIG. 8. Hall resistivity ρ_{xy} vs H at $T=1.4$ K for the metallic samples 8.78×10^{18}, 9.14×10^{18}, 9.62×10^{18}, and 10.2×10^{18} cm^{-3}.

proach zero in order to avoid magnetoresistance contributions to R_H and to avoid magnetic field tuning of $n_c(H)$ and thereby obtain the correct behavior of $R_H(n, H\rightarrow0, T\rightarrow0\ \text{K})$ as $n\rightarrow n_{c+}$. Figure 8 shows the Hall resistivity ρ_{xy} versus H for four metallic samples taken at 1.4 K in a UR electromagnet. For all four samples ρ_{xy} exhibits excellent linearity in the field range $-10<H<10$ kG. These data show a slope difference of less than 40% between the 8.78×10^{18}- and 10.2×10^{18}-cm^{-3} samples, thereby suggesting there is no significant scaling of R_H since n/n_c-1 is, respectively, 0.027 and 0.19 for these two samples. Similar data taken at 4.2 K showed the same excellent linearity and indicated only a very small temperature dependence of ρ_{xy}. In experiments at the FBNML we were unable to carefully explore the field dependence in the low-field region and there may have been errors in R_H. because of uncertainties in the field for $H<1$ T.

Figures 9–12 show $\sigma(T)$ and $R_H(T)$ plotted versus $T^{1/2}$, the predicted temperature dependence [see Eq. (6)] at sufficiently low temperatures. It is apparent in Fig. 9 that the 8.78×10^{18}-cm^{-3} sample shows deviations from $T^{1/2}$ behavior for $T>2$ K, whereas the 9.30×10^{18}-cm^{-3} sample yields a reasonable fit to $T^{1/2}$ behavior at the two lower fields up to 4.2 K, but not at the higher fields, where $n_c(H)$ tuning and magnetoresistance corrections may have become significant. One observes a significant field dependence of $R_H(n,H,T)$ that can be associated with the magnetoresistance contribution in Eqs. (4) and (5). It is also worth mentioning that the 0.5-T data in Fig. 12 for the 10.2×10^{18}-cm^{-2} sample shows significant scatter, but is only slightly smaller ($\sim12\%$) than R_H obtained from the slope in Fig. 8 for the 10.2×10^{18}-cm^{-3} sample at $T\sim1.4$ K. Even though there were problems with occasional points at low fields ($H<1$ T) taken in the superconducting solenoid, the extrapolated values of $R_H(T, H\rightarrow0)$ obtained from $R_H(T,H)$-versus-H plots were in very satisfactory agreement with the low-field electromagnetic results for $-1<H<1$ T. From the $R_H(T,H)$ data in Figs. 9–12 the extrapolated values $R_H(T=0, \text{K}\,H)$ have been obtained (see Table III) and these results are plotted versus H in Fig. 16(a).

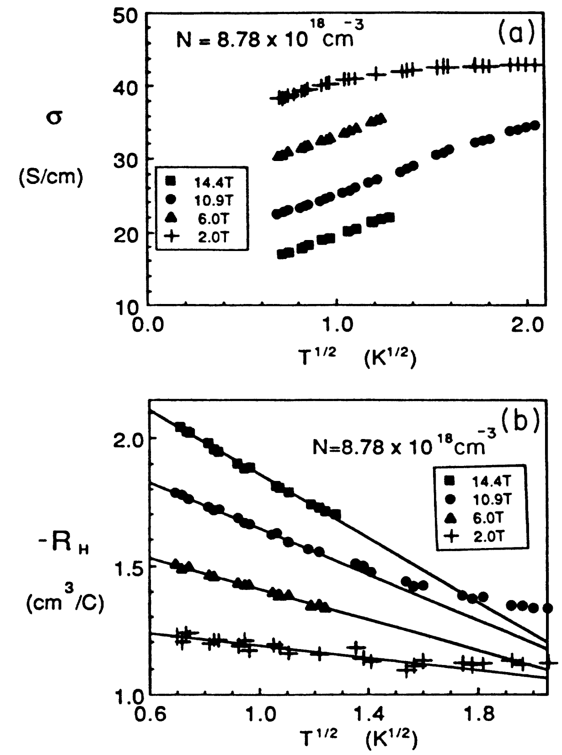

FIG. 9. (a) Conductivity and (b) Hall-coefficient data for the 8.78×10^{18}-cm^{-3} sample vs $T^{-1/2}$ at various magnetic fields for $0.5<T<4.2$ K.

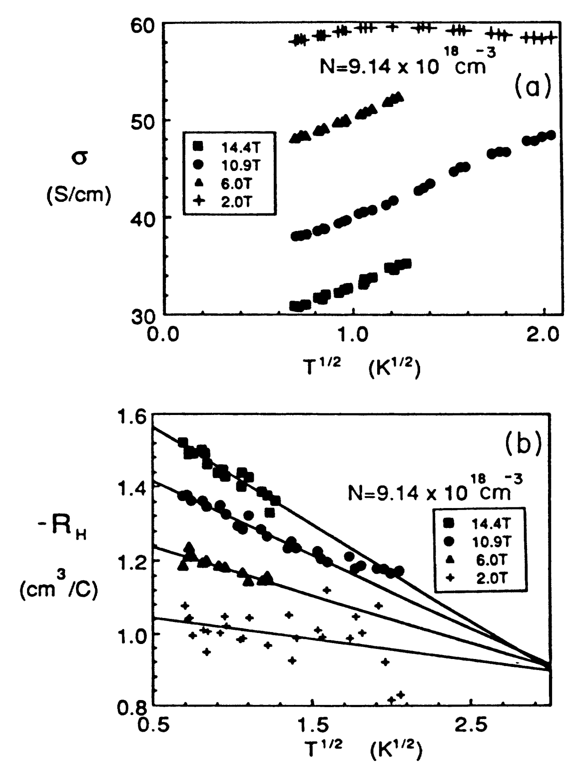

FIG. 10. (a) Conductivity and (b) Hall-coefficient data for the 9.14×10^{18}-cm^{-3} sample vs $T^{1/2}$ at various magnetic fields for $0.5<T<4.2$ K.

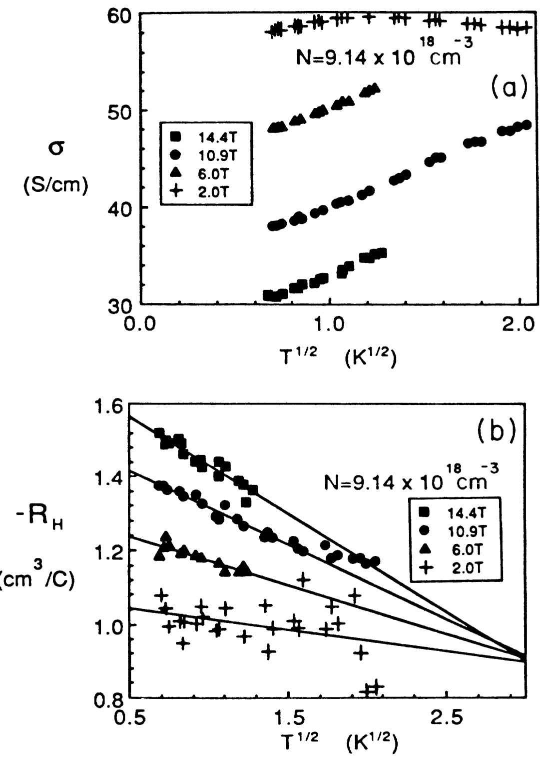

FIG. 11. (a) Conductivity and (b) Hall coefficient data for the 9.30×10^{18}-cm^{-3} sample vs $T^{1/2}$ at various magnetic fields for $0.5<T<4.2$ K.

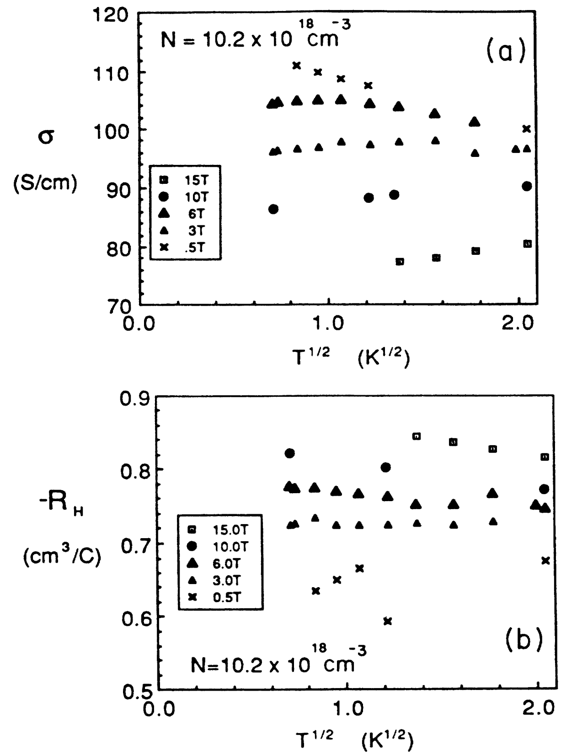

FIG. 12. (a) Conductivity and (b) Hall-coefficient data for the 10.2×10^{18}-cm^{-3} sample vs $T^{1/2}$ for various magnetic fields for $0.5<T<4.2$ K.

TABLE III. Hall parameters for metallic samples. (*, no temperature sweep at this magnetic field; —, parameter could not be experimentally determined.) (Units: σ, in S/cm; R_H, in cm^3/C; m_{xx}, m_H, and m_{xy} in K$^{-1/2}$.)

		H (T)						
N (10^{18} cm^{-3})		0	0.5	2.0	3.0	6.0	10.9[a]	14.35[b]
8.78	σ (T=0 K)	40	*	33	*	24	16	10.6
	m_{xx}	0.012	*	0.22	*	0.40	0.59	0.85
	R_H (T=0 K)	—	*	1.31	*	1.71	2.1	2.5
	m_H	—	*	−0.09	*	−0.18	−0.22	−0.25
	m_{xy}	—	*	0.35	*	0.62	0.86	1.45
9.14	σ(T=0 K)	68	*	56	*	43	32	25
	m_{xx}	−0.057	*	0.067	*	0.18	0.25	0.32
	R_H (T=0 K)	—	*	1.17	*	1.31	1.52	1.70
	m_H	—	*	−0.054	*	−0.10	−0.13	−0.16
	m_{xy}	—	*	0.080	*	0.26	0.37	0.48
9.30	σ (T=0 K)	87.3	*	79	*	67	57	48
	m_{xx}	−0.080	*	−0.004	*	0.047	0.069	0.092
	R_H (T=0 K)	—	*	0.86	*	0.99	1.13	1.20
	m_H	—	*	0.011	*	−0.032	−0.068	−0.074
	m_{xy}	—	*	0.010	*	0.062	0.070	0.11
10.2	σ (T=0 K)	120	118	*	103	93	84	71
	m_{xx}	−0.085	−0.076	*	0.016	0.047	0.035	0.066
	R_H (T=0 K)	—	0.61	*	0.73	0.79	0.85	0.90
	m_H	—	0.043	*	−0.001	−0.024	−0.043	−0.047
	m_{xy}	—	−0.11	*	0.031	0.070	0.027	0.085

[a] H at 10.0 T for the 10.2×10^{18}-cm^{-3} samples.
[b] H at 15.0 T for 9.30×10^{18}- and 10.2×10^{18}-cm^{-3} samples.

TABLE IV. Results for more metallic samples. (* denotes values that could not be determined from experimental data.)

N (10^{18} cm^{-3})	σ (N, $T=0$ K, $H=0$) (S/cm)	m (K$^{-1/2}$)	R_{H0} (cm^3/c)	m_H (K$^{-1/2}$)
17.9	370	−0.006	0.35	0.008
20.9	480	0.0013	*	*
32.8	750	0.003	0.17	−0.007

The $\sigma(n,T,H)$ data shown in Figs. 9–12 are similar to those reported previously.[28,29,50,51] Beyond a critical field the slope $m_{xx}(n,H)$ becomes less sensitive to the magnetic field and is positive and increases with field; whereas at zero field $m_{xx}(n)$ changes sign with density near $n/n_c \sim 1.04$, and again near $n/n_c \sim 2.4$, at sufficiently high fields $m_{xx}(n,H)$ is always positive and increases substantially as $n \rightarrow n_{c+}$. The values of $m_{xx}(n,H)$ and $\sigma_{xx}(n, H, T=0\text{ K})$ obtained from Figs. 9–12 are given in Table III. The more limited results for the three most metallic samples are shown in Table IV.

Tousson and Ovadyahu[53] have stressed the importance of making the Hall measurements at very low fields. Their results suggest that $R_H(H,T)$ becomes temperature independent at sufficiently small fields. Our own results are qualitatively consistent with the In_2O_{3-x} results in that $m_{xx}(n,H)$ [see Eq. (6)] becomes small as $H \rightarrow 0$, as seen in Table III. In particular, m_H appears to remain negative as $H \rightarrow 0$ for the 8.78 sample, while for the 10.2 sample m_H changes sign near $H \simeq 3$ T and becomes slightly positive at $H \sim 0.5$ T, although the error bars are large for the latter data. Although most of the Hall data came from temperature sweeps at constant field, we also took data versus magnetic field at fixed temperatures ($T \sim 0.5$, 2, and 4.2 K). Figure 13 shows fields sweeps at 4.2 and 0.48 K for the three metallic samples (8.78×10^{18}, 9.30×10^{18}, and 10.2×10^{18} cm^{-3}) of the Hall resistivity ρ_{zy}. One notes upward curvature from the expected $\rho_{xy} \propto R_H H$ results due to the magnetoresistance contribution that is most pronounced for the 8.78×10^{18}-cm^{-3} sample ($n/n_c - 1 \sim 0.028$), which is closest to n_c. For the 9.30×10^{18}- and 10.2×10^{18}-cm^{-3} samples one has nearly linear behavior of ρ_{xy} for $H<2$ T and $H<3$ T, respectively. On the other hand, the 8.78×10^{18}-cm^{-3} sample already exhibits an approximate 15% nonlinearity at $H \sim 2$ T, as seen from Fig. 13. It is possible the 8.78×10^{18}-cm^{-3} sample shows some up-turn or nonlinearity because of the larger anisotropy ($R_1/R_2=0.36$), but we emphasize that this sample shows magnetoresistance behavior in very good agreement with earlier results[51,52] on bar samples. The same is true of the temperature dependence, as determined by the magnitude of $m_H(n,H)$. Figure 13 clearly shows the largest temperature dependence for the 8.78×10^{18}-cm^{-3} sample and that the temperature dependencies increase with increasing field. Figure 13, along with the result in Table III, clearly suggests that as $n \rightarrow n_{c+}$ it is necessary to go to smaller values of both H and T to obtain the correct asymptotic values of $\rho_{xy}(H \rightarrow 0, T \rightarrow 0\text{ K})$ and $R_H(H \rightarrow 0, T \rightarrow 0\text{ K})$. It has been asserted[73] that this is only possible with dilution-refrigerator measurements. The present data for Si:As for $n/n_c - 1 \geq 0.028$ suggest $T \sim 0.5$ K is adequate to obtain the correct asymptotic value of $R_H(H \rightarrow 0, T \rightarrow 0\text{ K})$. If one were to approach n_c much more closely, as for Si:P (Ref. 2) $[(n/n_c - 1)_{\min} \approx 0.001]$, then measurements to very much lower temperatures would be required.

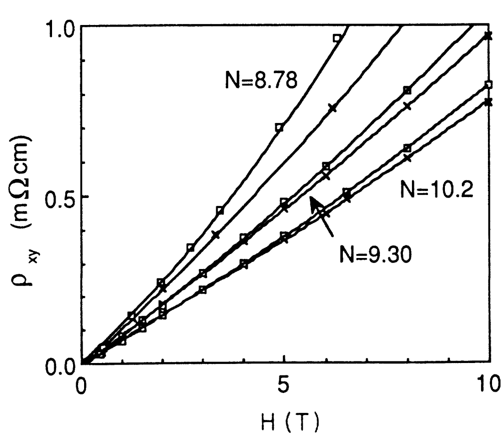

FIG. 13. Hall resistivity ρ_{xy} vs magnetic field H for the $8.78\times10^{10^{18}}$-, 9.30×10^{18}-, and 10.2×10^{18}-cm^{-3} metallic samples at $T=4.2$ K (×) and $T=0.48$ K (□).

V. DISCUSSION

The VRH results on the insulating side of n_c will be considered first, followed by a discussion of the metallic results. Finally, a comparison of $R_H(N \rightarrow n_{c-}, T \rightarrow 0\text{ K})$ will be made with $R_H(n \rightarrow n_{c+}, T \rightarrow 0\text{ K})$ to ascertain what can be said about R_H at $n - n_c$.

A. VRH-regime Hall results

Shown in Fig. 14(a) values of the characteristic Hall temperature $T_{0H}(N,H)$ versus the Mott temperature $T_0(N,H)$ for the four insulating samples at a variety of magnetic fields (see Table II). The data indicate an approximately linear relation between T_{0H} and T_0, with T_{0H} approximately an order of magnitude smaller than T_0. Nevertheless, the ratio $T_{0H}(N,H)/T_0(N,H)$ is not a fixed constant and is a function of both N and H, as has been discussed previously[40] [the field dependence of $(T_{0H}/T_0)^{1/4}$ was shown in Fig. 3 of Ref. 40]. The variation in $(T_{0H}/T_0)^{1/4}$ can be as much as 15% between 2–3 and 15 T. Figure 14(b) shows the extrapolated zero-field

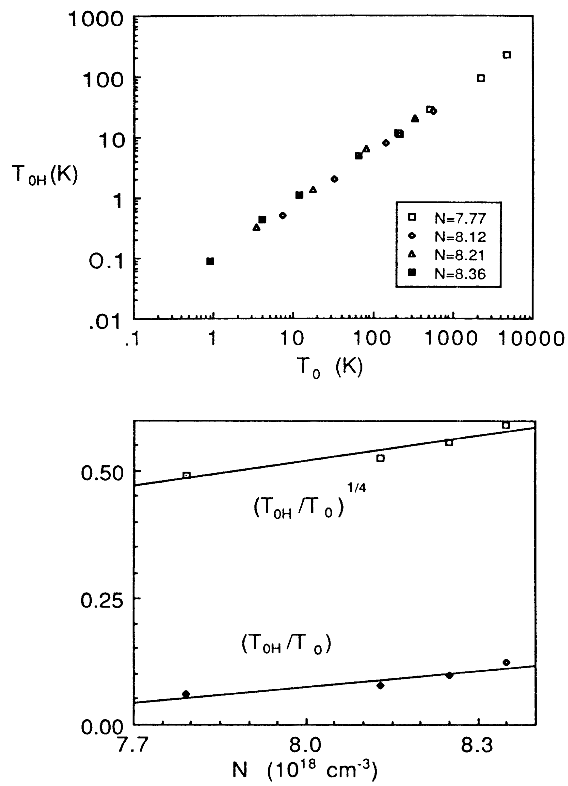

FIG. 14. (a) T_{0H} vs T_0 for the four insulating samples at various magnetic fields (see Table II); (b) the zero-field extrapolations of T_{0H}/T_0 and $(T_{0H}/T_0)^{1/4}$ vs donor density.

ratios T_{0H}/T_0 and $(T_{0H}/T_0)^{1/4}$ versus donor density. T_{0H}/T_0 increases from 0.06 to 0.11 from the 7.7×10^{18}- to the 8.36×10^{18}-cm^{-3} sample, while $(T_{0H}/T)^{1/4}$ increases from 0.49 to 0.59. If one extrapolates this latter result to n_c ($n_c \sim 8.55\times10^{18}$ cm^{-3}), one obtains $\lim_{H=0,N\to n_{c-}}(T_{0H}/T_0)^{1/4}=0.63\pm0.02$.

The theoretical prediction by Gruenewald *et al.*[33] is $(T_{0H}/T_0)^{1/4}=\frac{5}{8}$ or $T_{0H}/T_0=0.15$. The theory makes no prediction of how the ratio should depend on N and H. From Fig. 14(a) (or Table II) one sees T_{0H} and T_0 vary by more than 1000, while the ratio varies by less than a factor of 2. The agreement of the experimental ratio at $H=0$ and n_c with the theoretical prediction is surprising and perhaps fortuitous. The theoretical calculation might not be expected to be valid for barely insulating samples where *e-e* interactions, correlated multiple-electron hopping, and the Coulomb-gap problem might be important. However, these features would be expected to be important for the $\sigma(N<n_c, T, H=0)$ in the critical regime where the data[4,29,49] now strongly suggest that Mott VRH is dominant at easily obtainable temperatures. The reasons for the dominance of Mott VRH in the critical regime have been discussed in detail elsewhere.[49] It seems reasonable to suppose that these same reasons are applicable in explaining the VRH behavior of $R_H(n<n_c, T, H\to0)$ in the critical regime.

One cannot overemphasize the point that experimentally we have not been able to accurately establish that both $\sigma(n<n_c, T)$ and $R_H(N<n_c, T)$ have exactly the same VRH exponent $p\sim\frac{1}{4}$, nor have we been able to experimentally establish that the prefactors $\sigma_0(N<n_c, T)$ and $R_0(N<n_c, T)$ have the same temperature dependence or lack of temperature dependence. The variation of $R_H(N,T)$ with T is too small to accurately independently determine ρ_H and $R_0(N,T)$. We have employed $p_H=p=\frac{1}{4}$ in obtaining the parameters T_{0H} and T_0, which is certainly plausible and is strongly suggested by the theory.[33] Considering the small variations in $(T_{0H}/T_0)^{1/4}$ with N and H, it does not seem worth addressing physical reasons for these variations, since if p_H was slightly different from p, this would also change the experimental ratio T_{0H}/T_0. Suffice it to say that data seem to provide strong support for the prediction of Gruenewald *et al.*[33] that $(T_{0H}/T_0)^{1/4}\sim\frac{5}{8}$, but it seems unwise to attribute much significance to the slow variation of $(T_{0H}/T_0)^{1/4}$ with N and H.

The results differ in detail from those obtained for compensated n-type Cd:Se by Roy *et al.*[75] Their samples show a negative magnetoresistance in the low-field range and the Mott characteristic temperature $T_0(H)$ decreases with increasing magnetic field (the opposite of the Si:As results discussed above), which is consistent with the negative magnetoresistance. Their parameter $K_H(H)$ $K_H(H)=[T_{0H}(H)/T_0(H)]^{1/4}$) increases with increasing magnetic field ($0.3<H<1.0$ T), whereas out ratio, $[T_{0H}(H)/T_0(H)]^{1/4}$, decreases with increasing magnetic field. These two sets of results can be viewed as qualitatively consistent with each other if for SiAs $n_c(H)$ increases with H, while, for Cd:Se, $n_c(H)$ decreases with H.

Figure 15 shows $R_0(N,H)$ versus H for the four insulating samples. There is an increase of $R_0(N,H)$ as $H\to0$. There is also an increase in R_0 $(N, H\to0)$ as $N\to n_{c-}$. An extrapolation of $R_0(N, H\to0)$ to n_{c-} yields $R_0(N=n_c, H\to0)\sim0.73\pm0.02$ cm^3/C. From Table II and Fig. 14(a) we see that $T_{0H}\sim0.09$ K at $H\sim0.5$ T for the 8.36×10^{18}-cm^3 sample where $1-N/n_c=0.022$. As $N\to n_c$ previous results[4,29] have demonstrated that $T_0(N, H=0)$ scales to zero as $[\xi(N)]^{-3}$, where $\xi(N)$ is the localization length. MR studies[51] have shown a rapid increase in $T_0(N,H)$ with H and Table II shows the same rapid increase of both

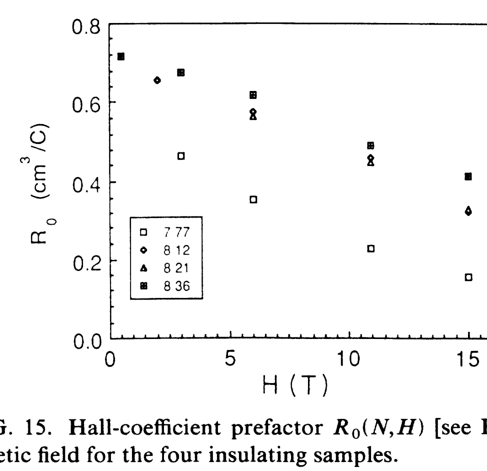

FIG. 15. Hall-coefficient prefactor $R_0(N,H)$ [see Eq. (7)] vs magnetic field for the four insulating samples.

$T_0(N,H)$ and $T_0(N,H)$ with H. As discussed earlier,[51] the rapid increase of $T_0(N,H)$ is not yet well understood; however, it is clear that as $N \to n_{c-}$ the magnetic length L_H [$L_H = (\hbar c/eH)^{1/2}$] becomes much less than the zero-field localization length, even at low fields. Since the magnetic tuning problem of how $n_c(H)$ varies with H is not yet well understood, it is more difficult to say how the field-dependent correlation length $\xi(N,H)$ varies with H. Based on the data in Table II, it seems plausible that $T_{0H}(N,H)$ may approach a finite but small value as $N \to n_{c-}(H=0)$. However, this characteristic temperature $T_{0H}(N \to n_{c-}(H=0))$ will be so small at $H \le 0.5$ T that the VRH temperature dependence of $R_H(N,H,T)$ in Eq. (7) will not be observable in the available temperature range. In this sense the prefactor $R_0(N \to n_{c-}, H \to 0)$ represents the Hall coefficient right at the MIT and should be compared with the metallic sample result for $R_H(n > n_c, H \to 0, T \to 0 \text{ K})$ as $n \to n_{c+}$.

B. Metallic regime Hall results

The low-temperature Hall results presented in Sec. IV B demonstrate it is possible to extract the zero-field, zero-temperature value $R_H(n, H=0, T=0 \text{ K})$ from the $R_H(n,H,T)$ data. Figure 16(a) shows the values of $R_H(n, H, T=0 \text{ K})$ versus H for the four metallic samples closest to the transition. $R_H(n, H, T=0 \text{ K})$ seems to show an approximate linear dependence on H with a slope that increases as $n \to n_{c+}$. In Sec. II the expression

$$R_H(n,H,T=0\text{ K}) = R_H(n,H=0,T=0\text{ K}) \times \{1 + [2A(n) - b_{xy}(n)]H^{1/2} + A^2(n)H + 2f(n)H^2\} ,$$

where the parameters $A(n)$ and $f(n)$ have been determined from MR studies.[50,51] The data in Fig. 16(a) might seem to suggest $2A(n) - b_{xy}(n) \ll A(n)$ and that $2f(n)H^2$ is negligible in this field range. However, the number of points and their experimental accuracy are insufficient to rule out a small $H^{1/2}$ term. Furthermore, one cannot explain the slope and field variation with just the A^2H term. The $A(n)$ have been determined from Si:As MR measurements.[52] The A^2H term is not only a factor of 3–4 too small, but also does not vary rapidly enough with density n. The addition of the $[2A - b_{xy}H^{1/2} + 2f(n)H^2]$ term, which can appear approximately linear over much of the field interval for appropriate values of $f(n)$ and $2A - b_{xy}$, improves the results substantially. $f(n)$ has been determined previously from MR measurements on bar samples;[51,52] however, it should be recognized the current distribution in the van der Pauw disk samples is more complicated than in bar samples and may not lead to the same ratio $\rho_{xx}(n, H, T \to 0 \text{ K})/\rho_{xx}(n, H=0, T \to 0 \text{ K}) = \varkappa(n, H, T \to 0 \text{ K})$. The Hall data only extend to 0.5 K, whereas the MR data were taken down to 0.05 K. This also means the $b_{xy}g(H,T)$ term in $\sigma_{xy}(n,T,H)$ is only marginally into the low-temperature regime where $g(H,T) \propto H^{1/2}$ since at very low fields ($H \sim 0.5$ T) $g\mu_B H$ is comparable to kT. Our numerical analysis yields good agreement

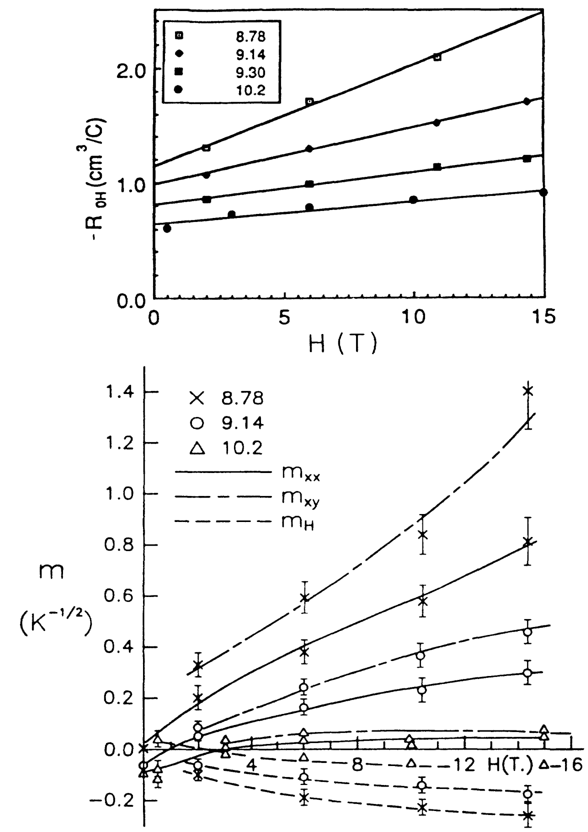

FIG. 16. (a) Extrapolated zero-temperature Hall coefficient $R_0(N,H)$ [see Eq. (7)] vs magnetic field for four metallic samples; (b) the values of m_{xx}, m_H, and m_{xy} vs magnetic field for the 8.78×10^{18}-, 9.14×10^{18}-, and 10.2×10^{18}-cm^{-3} metallic samples.

with the data in Fig. 16(a) for $f(n)$ values about 30–40% smaller than those found for the bar samples at lower temperatures and for $2A(n) - b_{xy}(n)$ in the range $(0.025–0.055)T^{-1/2}$. The coefficient $2A - b_{xy}$ is certainly smaller than A, but the $H^{1/2}$ term in $R_H(n, H, T=0 \text{ K})$ is only a few percent at 1 T and should not lead to significant errors in the determinaton of $R_H(n, H \to 0, T=0 \text{ K})$. It would be desirable to have more points in the low-field range to more accurately determine the magnitude of the $H^{1/2}$ term, but this would only be possible at much lower temperatures where $kT \ll g\mu_B H$. We should emphasize the MR corrections to $R_H(n,H,T)$ are larger when $kT \ll g\mu_B H$ than in the range $kT \sim g\mu_B H$, so that the present data describe the situation satisfactorily in our view.

Figure 16(b) shows $m_{xx}(n,H)$, $m_H(n,H)$, and $m_{xy}(n,H)$ versus H using for the latter the relationship $m_{xy}(n,H) = m_H(n,H) + 2m_{xx}(n,H)$, as discussed in Sec. II. One observes that $m_{xx}(n,H)$ becomes more positive with increasing field for all four samples, although $m_{xx}(n, H=0)$ is negative for three samples. On the other hand, $m_H(n,H)$ becomes more negative with H, but $m_H(n, H \to 0)$ is positive for the 9.30 and 10.2 samples. The quantity $m_{xy}(n,H)$ is positive for all samples (except for the 10.2 sample at 0.5 T) and also increases with field. An alternative way of addressing this question is to plot the quantity $\delta R_H/R_H$ versus $\delta\sigma_{xx}/\sigma_{xx}$, where $\delta R_H=R_H(n,H,T)-R_H(n,H,T=0\text{ K})$ and $\delta\sigma_{xx}=\sigma_{xx}(n,H,T)-\sigma_{xx}(n,H,T=0\text{ K})$. Such a plot yields a slope γ (see Sec. II), which is in the range $0.4<\gamma<0.6$. The expression in Sec. II yield $\gamma=-m_H/m_{xx}=2-m_{xy}/m_{xx}$ and one obtains an average over the different samples and fields $\langle m_{xy}/m_{xx}\rangle\sim 1.5$. The theoretical prediction by Alt'shuler *et al.*[19] is $\delta\sigma_{xy}=0$ due to *e-e* interactions; however, this calculation only considers the g_1 and g_3 processes. There is not enough known about $m_{xy}(n,H)$ for other MIT systems at the present time to reach any general conclusions on the behavior of the temperature dependence $\sigma_{xy}(n,H,T)$. There is certainly a need for additional theoretical consideration of $m_{xy}(n,H)$.

The scaling behavior of R_H^{-1} for Ge:Sb,[22] Si:As,[28] and a reanalysis of some earlier Si:P data[35] is shown in Fig. 17. The data for $Kr_{1-x}Bi_x$ (Ref. 23) and *a*-Si:Pt (Ref. 24) also exhibits scaling behavior similar to that Ge:Sb,[22] although the latter yields a larger yields a larger exponent μ_H. The errors for our Si:As $R_H(n,H\rightarrow 0,T\rightarrow 0\text{ K})$ values are definitely less than 10% and one is forced to conclude that any scaling behavior for Si:As (and also Si:P) is very small and would have to be characterized by a very much smaller exponent, if any at all. When this data was first discussed,[28] we noted that it was the higher-*Z* impurity MIT systems (Sb, Bi, and Pt) that exhibit scaling behavior and suggested this might result from the impurity-s.o. interaction. At that time we were unaware of the important role of the s.o. interaction for the extraordinary Hall effect for ferromagnetic metals[43,44] and to R_H as described in Eq. (9), for the amorphous paramagnetic alloy $Zr_{1-x}Fe_x$.[45] Le us now discuss the nature of the second term in Eq. (9) near the MIT for the *n*-type semiconductors.

As already noted, the spin susceptibility is very small for the *n*-type semiconductor systems [$\chi_v\sim 10^{-7}$ cgs units for Si:P (Ref. 74)] near n_c. However, $\rho_{xx}(n,T\rightarrow 0\text{ K})$ becomes much larger as $n\rightarrow n_{c+}$, although it may not truly diverge in a finite magnetic field when s.o. interactions are important. Perhaps the most important difference between the present MIT systems, at least for the *n*-type semiconductors, is the orbital character of the states near the Fermi level that are responsible for the conduction. For the ferromagnetic metals[43] and for the paramagnetic alloys like $Zr_{1-x}Fe_x$,[45] this orbital character is *d*-like, whereas for the impurity band near n_c the character is predominantly *s*-like, even though in individual Bloch states near the minimum of one of the degenerate conduction-band valleys is predominantly *p*-like. Thus, one might expect the orbital matrix element $\langle\Psi_j|\mathbf{L}|\Psi_i\rangle$ to be negligible. However, it has been demonstrated[68] there is a s.o. contribution resulting from the internal electric field gradient that is actually largest for *s*-like orbitals because the field gradient is largest close to the impurity nuclei. In Eq. (9) the quantity $\lambda_{\text{s.o.}}$, which is a perturbation-theory result between the *d* bands, is replaced by the s.o. frequency $\Omega_{\text{s.o.}}=(\mu_B/m^*c)\langle\partial E_{ix}/\partial x\rangle$. The average $\langle\partial E_{ix}/\partial x\rangle\propto eZ_{\text{eff}}|\Psi_d(n,r=0)|^2$, where $|\Psi_d(n,r=0)|^2$ is the probability the electron is at the impurity nucleus, and $Z_{\text{eff}}=Z_i-Z_h$ is the difference between the *Z* of the impurity and the host. The largest values of $\Omega_{\text{s.o.}}$ are found for the Ge:Sb and the smallest for Si:P, where the s.o. contribution is negligible. The contribution to R_H depends on the quantity $\Omega_{\text{s.o.}}\tau$, where τ is the elastic scattering time. The calculations[68] suggest that there can be a very significant contribution to R_H from the s.o. interaction for *n*-type Ge:Sb and we suggest that the "scaling" observed for R_H^{-1} results in a term like the second term in Eq. (9) and can, in principle, arise from a combination of the host and impurity s.o. interactions. However, given the theoretical predictions[64,65] for $\sigma_{xx}(n,H)\propto(e^2/\hbar)[\xi^{-1}(n)+\frac{3}{2}L_{\text{s.o.}}^{-1}+L_H^{-1}]$, where $\xi(n)$ is the correlation length and $L_{\text{s.o.}}$ ($L_{\text{s.o.}}=\sqrt{D\tau_{\text{s.o.}}}$) is the characteristic s.o. length, then ρ_{xx} ($\rho_{xx}\propto\sigma_x^{-1}$ at low fields) stops increasing when $\xi(n)$ approaches $L_{\text{s.o.}}$ or L_H, whichever is smaller. In this case if one approaches n_{c+} closely enough, one should see a flattening of R_H^{-1} and the end of scaling. For *n*-type Si the s.o. lengths $L_{\text{s.o.}}$ are $560l$, $160l$, and $32l$ for Si:P, Si:As, and Si:Sb, respectively, as inferred from ESR linewidth measurements[49] for barely metallic samples where l is the elastic mean free path. The situation is less clear for *n*-type Ge, but one might expect a very short value of $L_{\text{s.o.}}$ for Ge:Sb.

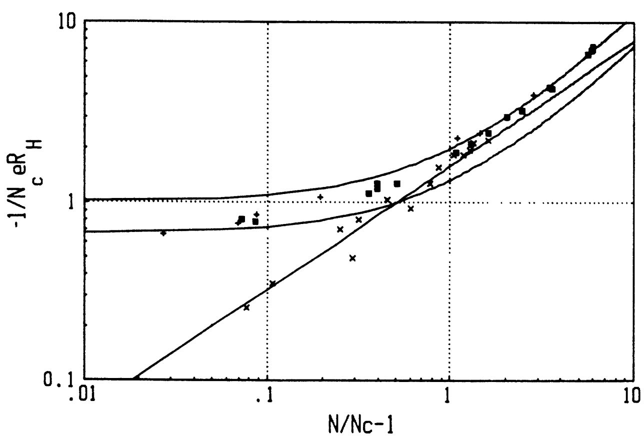

FIG. 17. Normalized reciprocal Hall coefficient $1/N_ceR_H$ [where $R_H=R_H(N,T\rightarrow 0\text{ K},H\rightarrow 0)$] vs reduced donor density for Ge:Sb (×), SiAs (+), and Si:P (■).

C. Comparison between metallic and insulating Hall coefficients as n_c is approached

Despite earlier reports[22–25] of scaling behavior of R_H^{-1} as $n\rightarrow n_{c+}$, the evidence presented above for Si:As, and also inferred for Si:P, suggests that R_H^{-1} for these smaller-*Z* impurity MIT systems does not exhibit critical behavior, implying that the number of carriers remains finite as $n\rightarrow n_{c+}$. From the insulating side for the transition, the extrapolation of the four insulating Si:As samples yields a value of the prefactor [see Eq. (7)] of $R_0(N=n_c,H\rightarrow 0)\sim 0.73\pm 0.02$ cm^3/C. This is 32% smaller than the $R_H(n=1.027n_c,H\rightarrow 0,T\rightarrow 0\text{ K})=1.08\pm 0.05$ cm/C value found for the least metallic sample. While these results are nearly consistent with one another and suggest that $R_H(n,H\rightarrow 0,T<0.01T_F)$ varies smoothly through the transition, the 32% difference between the two numbers is outside the stated errors and may reflect systematic discrepancies from several sources. The exponent p_H may, in fact, not be equal to the Mott exponent p, and as a result the parameters T_{0H} an $R(N, H \rightarrow 0)$ may require correction. Data over a much larger temperature range would certainly improve the determination of these parameters, although one might need temperatures that are 2 orders of magnitude lower for the barely insulating samples to obtain enough variation in $R_H(N, H \rightarrow 0, T)$ with T. It is possible, even if it seems unlikely, that $n_c(H)$ tuning with H is important at low fields ($H \leq 2$ T) and this has not been properly accounted for very close to n_c. We have suggested the field is low enough when $\rho_{xy} \propto H$, and our electromagnetic results have accurately confirmed this linear behavior for $-1 < H < 1$ T. Nevertheless, the requirement that one keeps $L_H \gg \xi(n,H)$ as $n \rightarrow n_c(H)$ is a very stringent condition, and the lack of understanding of $n_c(H)$ tuning at low fields only compounds the problem. In our extrapolation we have employed $n_c(H=0) = 8.55 \times 10^{18}$ cm^{-3}. A significantly larger value of $n_c(1 < H < 2$ T) would improve the agreement.

At the present time the data on both sides of the transition for Si:As seem to be consistent with one another. Very close to n_c, $R_H(n, H \rightarrow 0, T)$ slowly increases with decreasing temperature for both metallic and insulating samples. However, this temperature dependence decreases with decreasing magnetic field analogous to the In_2O_{3-x} case of Tousson and Ovadyahu.[53] The data in the VRH regime are consistent with the theoretical prediction of Gruenewald *et al.*;[33] however, measurements over a much larger temperature range are required to conclusively experimentally establish that $p_H = \frac{1}{4}$.

VI. CONCLUSIONS

The Hall measurements for Si:As have given the first experimental evidence that the Hall coefficient in the hopping regime exhibits VRH behavior that as $N \rightarrow n_{c-}$ appears to be in good agreement with the theoretical prediction of Gruenewald, even though it was not possible to accurately, independently determine the VRH exponent p_H for $R_H(T)$. The metallic Si:As samples show a smoothly varying $R_H(n, H \rightarrow 0, T \rightarrow 0$ K) as $n \rightarrow n_{c+}$, contrary to some other MIT systems, but in agreement with the weak-localization predictions of Fukuyama[17] and of Shapiro and Abrahams.[18] The temperature dependence of $R_H(n,T)$ of the metallic samples yields the result $m_{xy}/m_{xx} \approx 1.5$ at sufficiently low temperatures into the $T^{1/2}$ regime and this corresponds to the quantity $\gamma = -(\delta R_H / R_H)/(\delta\sigma_{xx}/\sigma_{xx}) \simeq 0.5$. This result is characteristic of potential scattering and is intermediate between the weak-localization prediction ($\gamma = 0$) and the current *e-e* interaction prediction.

An explanation is offered for differing critical behavior of $R_H(n, T \rightarrow 0$ K$, H \rightarrow 0)$ of different MIT systems which depends on the impurity spin-orbit interaction that has been well known for ferromagnetic metals and has recently been found to be important for the amorphous paramagnetic alloy $Zr_{1-x}Fe_x$.[45] This spin-orbit contribution is proportional to $\rho_{xx}^2 \chi \lambda_{\mathrm{s.o.}}$ and is important for large-Z impurity MIT spin systems near n_c because of the large, diverging increase in ρ_{xx} as $n \rightarrow n_{c+}$, despite the very small values of the spin susceptibility of these MIT systems. For the usual case (ferromagnetic metals), where the spin-orbit mechanism has contributed to R_H, the orbital character has been *d*-like, whereas the orbital character of the impurity MIT systems is predominantly *s*-like, and a new type[68] of spin-orbit contribution to R_H is required that emphasizes the role of the *s* component to the wave functions near n_c.

ACKNOWLEDGMENTS

We wish to acknowledge technical assistance from B. Brandt and L. Rubin of the Francis Bitter National Magnet Laboratory (Cambridge, MA) and H. R. Hart of General Electric Research and Development Center (Schenectady, NY). We also thank M. Hirsch and D. Holcomb for useful discussions and M. Migliuolo and V. Zarifis for experimental assistance. This work was supported in part by the U.S. National Science Foundation under Grant No. DMR-83-06106.

*Present address: Department of Physics, Saint Lawrence University, Canton, NY 13617.

[1]E. Abrahams, P. W. Anderson, D. C. Licciardello, and T. V. Ramakrishnan, Phys. Rev. Lett. **42**, 673 (1979).

[2]M. Paalanen, T. F. Rosenbaum, G. A. Thomas, and R. N. Bhatt, Phys. Rev. Lett. **48**, 1284 (1982); G. A. Thomas, M. Paalanen, and T. F. Rosenbaum, Phys. Rev. B **27**, 3897 (1983).

[3]P. F. Newman and D. F. Holcomb, Phys. Rev. B **28**, 628 (1983).

[4]W. N. Shafarman, D. W. Koon, and T. G. Castner, Phys. Rev. B **40**, 1216 (1989).

[5]P. F. Newman and D. F. Holcomb, Phys. Rev. Lett. **51**, 2144 (1983).

[6]A. P. Long and M. Pepper, J. Phys. C **17**, L425 (1984); Solid-State Electron. **28**, 61 (1985).

[7]A. N. Ionov, M. N. Matveev, and I. S. Shlimak, Fiz. Tekh. Poluprovdn. **20**, 1083 (1986) [Sov. Phys.—Semicond. **20**, 682 (1986)].

[8]G. A. Thomas, Y. Ootuka, S. Katsumoto, S. Kobayashi, and W. Sasaki, Phys. Rev. B **25**, 4288 (1982).

[9]Y. Ootuka, H. Masuoka, and S. Kabayashi, in *Anderson Localization*, edited by T. Ando and H. Fukuyama (Springer-Verlag, Berlin, 1988), p. 40.

[10]Y. Ootuka, S. Kobayashi, S. Ikehata, W. Sasaki, and J. Kondo, Solid State Commun. **30**, (1979); N. Nishia, T. Furubayashi, M. Yamaguchi, K. Morigaki, and H. Ishimoto, Solid-State Electron. **28**, 81 (1985).

[11]D. J. Bishop, E. G. Spencer, J. P. Garno, and R. C. Dynes, Bull. Am. Phys. Soc. **29**, 343 (1984).

[12]B. W. Dodson, W. L. McMillan, J. M. Mochel, and R. C. Dynes, Phys. Rev. Lett. **46**, 46 (1981).

[13]D. J. Bishop, E. G. Spencer, and R. C. Dynes, Solid-State Electron. **28**, 73 (1985).
[14]S. Yoshizumi, D. Mael, T. H. Geballe, and R. L. Greene, in *Localization and Metal-Insulator Transition,* edited by H. Fritzshe and D. Adler (Plenum, New York, 1985), p. 77.
[15]N. F. Mott, J. Non-Cryst. Solids **1**, 1 (1969).
[16]A. L. Efros and B. I. Shklovskii, J. Phys. C **8**, L49 (1975).
[17]H. Fukuyama, J. Phys. Soc. Jpn. **49**, 644 (1980).
[18]B. Shapiro and E. Abrahams, Phys. Rev. B **24**, 4025 (1981).
[19]B. L. Alt'shuler, D. Khmelnitzkii, A. I. Larkin, and P. A. Lee, Phys. Rev. B **22**, 5142 (1980).
[20]B. L. Alt'shuler and A. G. Aronov, in *Electron-Electron Interactions in Disordered Systems,* edited by A. L. Efros and M. Pollak (North-Holland, New York, 1985), p. 1.
[21]D. J. Bergman and D. Stroud, Phys. Rev. B **32**, 6067 (1985).
[22]S. B. Field and T. F. Rosenbaum, Phys. Rev. Lett. **55**, 522 (1985).
[23]M. Rohde and H. Micklitz, Phys. Rev. B **36**, 7572 (1987).
[24]P. Löbl, M. Esquerra, F. Küb, E. Lüscher, G. Fritsch, M. von Allmen, U. Kambli, and A. Schulte, Solid State Commun. **64**, 1107 (1987).
[25]J. Stankiewicz, S. von Molnar, and F. Holtzberg, J. Magn. Magn. Mater. **54-57**, 1217 (1986).
[26]U. Dai. A. Palevski, and G. Deutscher, Phys. Rev. B **36**, 790 (1987).
[27]E. Tousson and Z. Ovadyadu, Solid State Commun. **60**, 407 (1986).
[28]D. W. Koon and T. G. Castner, Phys. Rev. Lett. **60**, 1755 (1988).
[29]W. N. Shafarman and T. G. Castner, Phys. Rev. B **33**, 3570 (1986).
[30]T. Holstein, Phys. Rev. **124**, 1329 (1961).
[31]H. Böttger and V. V. Bryksin, Phys. Status Solidi B **80**, 569 (1977).
[32]L. Friedman and M. Pollak, Philos. Mag. B **38**, 173 (1978); **44**, 487 (1981).
[33]M. Gruenewald, H. Mueller, P. Thomas, and D. Wuertz, Solid State Commun. **38**, 1011 (1981).
[34]H. Fritzsche, Phys. Rev. **99**, 406 (1955).
[35]C. Yamanouchi, K. Mizuguchi, and W. Sasaki, J. Phys. Soc. Jpn. **22**, 859 (1967).
[36]M. J. F. LeHir, J. Phys. (Paris) **28**, 563 (1967).
[37]M. Amitay and M. Pollak, J. Phys. Soc. Jpn. Suppl. **21**, 549 (1966).
[38]R. S. Klein, Phys. Rev. B **31**, 2014 (1985).
[39]W. Sasaki, Philos. Mag. B **52**, 427 (1985).
[40]D. W. Koon and T. G. Castner, Solid State Commun. **64**, 11 (1987).
[41]P. A. Lee, Phys. Rev. B **26**, 5882 (1982).
[42]F. J. Blatt, in *Solid State Physics,* edited by F. Seitz and D. Turnbull (Academic, New York, 1957), Vol. 4, p. 241–242.
[43]See the review by E. M. Pugh and N. Rostoker, Rev. Mod. Phys. **25**, 151 (1953).
[44]R. Karplus and J. M. Luttinger, Phys. Rev. **95**, 1154 (1954).
[45]M. Trudeau, R. W. Cochrane, D. V. Baxter, J. O. Strom-Olsen, and W. B. Muir, Phys. Rev. B **37**, 4499 (1988).
[46]G. A. Thomas, Y. Ootuka, S. Katsumoto, S. Kobayashi, and W. Sasaki, Phys. Rev. B **25**, 4288 (1982).
[47]M. J. Hirsch, U. Thomanschefsky, and D. F. Holcomb, Phys. Rev. B **37**, 8257 (1988).
[48]B. L. Alt'shuler and A. G. Aronov, Zh. Eksp. Teor. Fiz. **77**, 2028 (1979) [Sov. Phys.—JETP **50**, 968 (1979)].
[49]V. Zarifis and T. G. Castner, Phys. Rev. B **36**, 6198 (1987).
[50]T. G. Castner, in *Hopping Transport in Solids,* edited by M. Pollak and B. I. Shklovskii (North-Holland, New York, 1989).
[51]W. N. Shafarman, T. G. Castner, J. S. Brooks, K. P. Martin, and M. J. Naughton, Phys. Rev. Lett. **56**, 980 (1986).
[52]T. G. Castner, W. N. Shafarman, and D. Koon, Philos. Mag. B **56**, 805 (1987).
[53]P. A. Lee and T. V. Ramakrishnan, Phys. Rev. B **26**, 4009 (1982).
[54]E. Tousson and Z. Ovadyahu, Phys. Rev. B **38**, 12 290 (1988).
[55]H. Fukuyama, in *Electron-Electron Interactions in Disordered Systems,* edited by A. Efros and M. Pollak (North-Holland, New York, 1985), p. 196.
[56]D. J. Bishop, D. S. Tsui, and R. C. Dynes, Phys. Rev. Lett. **46**, 36 (1981).
[57]M. J. Uren, R. A. Davies, and M. Pepper, J. Phys. C **13**, L985 (1980).
[58]D. A. Kichigin, V. T. Igumenov, O. A. Mironov, and S. V. Chistyakov, Pis'ma Zh. Eksp. Teor. Fiz. **36**, 121 (1982) [JETP Lett. **36**, 151 (1982)].
[59]H. Kohno and K. Yamada, Prog. Theor. Phys. **80**, 623 (1988).
[60]C. Castellani and C. DiCastro, Phys. Rev. B **34**, 5935 (1986).
[61]C. Castellani, G. Kotliar, and P. A. Lee, Phys. Rev. Lett. **59**, 323 (1987).
[62]M. A. Paalanen, S. Sachdev, R. N. Bhatt, and A. E. Ruckenstein, Phys. Rev. Lett. **57**, 2061 (1986).
[63]M. A. Paalanen, J. E. Graebner, R. N. Bhatt, and S. Sachdev, Phys. Rev. Lett. **61**, 451 (1988).
[64]P. Nozières and C. Lewiner, J. Phys. (Paris) **34**, 901 (1973).
[65]M. Kaveh and N. F. Mott, Philos. Mag. Lett. **56**, 97 (1987).
[66]D. Schmeltzer and M. Kaveh, Phys. Rev. B **36**, 6698 (1987).
[67]L. Berger, Phys. Rev. B **2**, 4559 (1970).
[68]T. G. Castner (unpublished).
[69]P. F. Newman, M. J. Hirsch, and D. F. Holcomb, J. Appl. Phys. **58**, 3779 (1985).
[70]J. A. del Alamo and R. M. Swanson, J. Appl. Phys. **57**, 2314 (1985).
[71]J. D. Quirt and J. R. Marko, Phys. Rev. B **7**, 3842 (1973).
[72]L. J. van der Pauw, Phillips Res. Rep. **13**, 1 (1958).
[73]P. M. Hemenger, Rev. Sci. Instrum. **44**, 6 (1973).
[74]D. W. Koon, Rev. Sci. Instrum. **60**, 271 (1989).
[75]A. Roy, M. Levy, X. M. Guo, M. P. Sarachik, R. Ledesma, and L. L. Issacs, Phys. Rev. B **39**, 10 185 (1989).

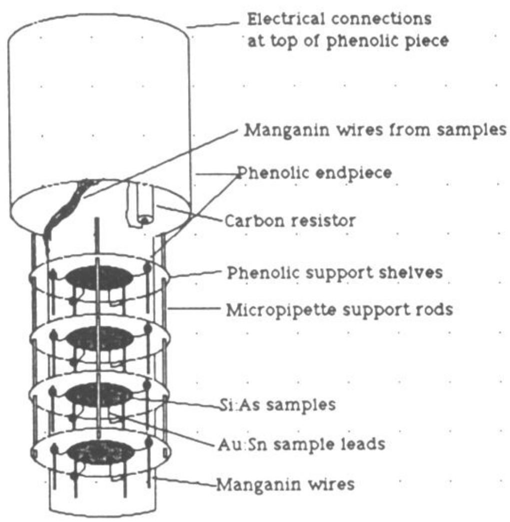

FIG. 3. Sample holder for cryogenic rig employed for superconducting-solenoid runs at the Francis Bitter National Magnet Laboratory and at the University of Rochester.