Magnetic breakdown and magnetoresistance oscillations in a periodically modulated two-dimensional electron gas

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We develop a semiclassical theory for the magnetoresistance oscillations recently observed in two-dimensional systems with periodically modulated potentials. We show that these oscillations may be thought of as oscillations in the probability of magnetic breakdown. Our theory demonstrates a connection between the magnitude of the oscillations and the size of the positive magnetoresistance at weak magnetic fields.

I. INTRODUCTION

Recent observations[1,2] of magnetoresistance oscillations in periodically modulated two-dimensional electron systems (2D ES) have generated much interest[3-7] (The literature on this phenomenon has recently been reviewed by Weiss.[8]) The oscillations are periodic in the ratio of the diameter of the cyclotron orbit to the modulation period. The origin of the oscillations has been understood since the original work.[1,2] The modulation potential causes the cyclotron orbit to drift in the direction perpendicular to the direction of modulation at a rate which depends on the position of the orbit center relative to the modulation potential. The rate of drift also depends on an average of the modulation potential over its orbit and this tends to be larger when the number of periods of modulation inside the orbit diameter is half an odd integer. The resulting magnetoresistance oscillations have been described both by a physically appealing semiclassical theory in which the oscillations are associated directly with oscillation in the average over orbit centers of the square of the drift velocity[3] and by more-detailed fully-quantum-mechanical theories.[5,6,4]

The modulated 2D ES's which show the resistance oscillations also show an unusually large positive magnetoresistance at fields weaker than those at which the oscilations first appear. The large positive magnetoresistance cannot be explained by existing theories of magnetotransport in modulated 2D systems which are valid only in the limit where the Landau level separation exceeds the disorder broadening of Landau levels. In this paper we report on a semiclassical theory of the novel magnetoresistance oscillations in which they appear as a consequence of oscillations in the probability of magnetic breakdown.[9] Our theory is an application of the ideas developed in describing magnetic breakdown in bulk metals to periodically modulated 2D ES's.

In the presence of a weak magnetic field applied perpendicular to the 2D ES (taken to lie in the $\hat{\mathbf{x}}$-$\hat{\mathbf{y}}$ plane) the dynamics of the electronic motion, when treated semiclassically, is governed by the following equation:

$$\hbar\frac{\partial \mathbf{k}}{\partial t} = -\frac{e}{c}\mathbf{v}_{\mathbf{k}}\times B\hat{\mathbf{z}} - e\mathbf{E} , \tag{1}$$

where $\mathbf{E}$ and $B\hat{\mathbf{z}}$ are the external electric and magnetic fields, respectively, and $\mathbf{v}_{\mathbf{k}}$ is the velocity expectation value of the electronic state defined by wave vector $\mathbf{k}$:

$$\mathbf{v}_{\mathbf{k}} = \frac{1}{\hbar}\nabla_{\mathbf{k}}\varepsilon(\mathbf{k}) , \tag{2}$$

where $\varepsilon(\mathbf{k})$ is the energy of the Bloch state at wave vector $\mathbf{k}$. In this paper we will restrict ourselves to the limit of weak modulation so that we can assume a nearly-free-electron description of the Bloch minibands in the modulated system and use an extended-zone scheme to level the electronic eigenstates. At zero electric field the electron trajectories in two-dimensional $\mathbf{k}$ space are thus defined by constant energy contours. Since we are interested in transport properties only trajectories at energies close to the Fermi energy, E_F, need to be considered. For the case of a periodic potential which depends only on the x coordinate, the trajectories at the Fermi energy are sketched[10] in Fig. 1. The corresponding electron tra-

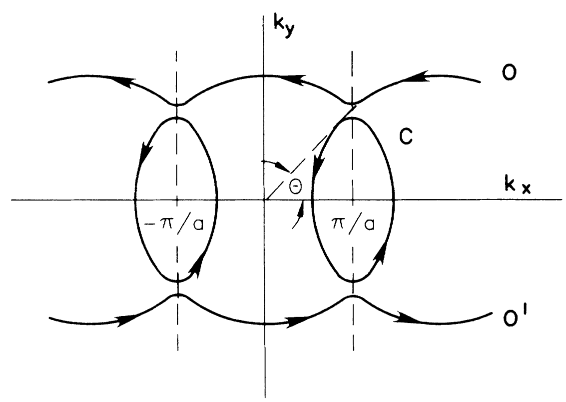

FIG. 1. Constant-energy contours at the Fermi energy for a modulated two-dimensional electron system shown in the extended minizone scheme. The open-orbit motion is along the $\hat{\mathbf{x}}$ direction in $\mathbf{k}$ space and along the $\hat{\mathbf{y}}$ direction in $\mathbf{r}$ space.

jectories in $\mathbf{r}$ space are equal to those in $\mathbf{k}$ space after a rotation by $\pi/2$ and multiplication by $l^2=\hbar c/eB$. (l is known as the magnetic length.) As illustrated in Fig. 1, in the limit of very weak magnetic fields the electrons move along two types of trajectories: open orbits, for which the real space motion is unbounded, and closed orbits. With increasing magnetic field this semiclassical description of electron dynamics fails. The description can be generalized, however, so that it captures much of the additional high-field physics in a physically appealing way. The generalization[9] is to acknowledge that electrons approaching Brillouin-zone edges have a finite probability for tunneling through the barrier and moving from one orbit to another. This effect is known as magnetic breakdown. It changes the topology of electron trajectories and has a drastic effect on all physical properties, especially the magnetoresistance.

The probability p of magnetic breakdown at a Brillouin-zone edge has been calculated using several different approaches which are valid either for the strong-field or the weak-field limits.[9,11] Amazingly, all approaches lead to the same expression in the nearly-free-electron limit:

$$p=\exp\left[-\frac{\pi V_0^2}{4\hbar\omega_c E_F \sin 2\theta}\right], \tag{3}$$

where V_0 is the width of the energy gap at the Brillouin-zone edge at zero magnetic field, $\omega_c=eB/m^*c$ denotes the cyclotron frequency, and 2θ is the scattering angle under Bragg reflection. (See Fig. 1.) Note that p goes to zero rapidly at weak fields, justifying the neglect of magnetic breakdown, and approaches 1 at strong fields. The crossover field is proportional to the square of the modulation potential and inversely proportional to the Fermi energy. For GaAs, modulation potentials on the order of 0.1 meV and Fermi energies on the order of 10 meV lead to crossover fields on the order of 0.1 T. This is precisely the range of field strengths[1,2] in which the magnetoresistance oscillations have been observed.

In Sec. II of this paper we apply the semiclassical approach to transport, allowing for the possibility of magnetic breakdown, to a nearly-free-electron 2D ES. For this purpose we adopt a relaxation-time approximation which allows us to use Chambers's convenient [12] solution to the Boltzmann equation. We find that this approach is able to explain the shape of the smooth background on top of which the magnetoresistance oscillations appear. In Sec. III we reexamine the expression for the breakdown probability in the strong-magnetic-field limit and find that, provided that the disorder potential is not too strong, p should be replaced by a breakdown probability which exhibits oscillations as a function of field strength. We are able to provide a criterion for the minimum field strength necessary for these oscillations to appear. Finally, in Sec. IV we return to a discussion of the magnetoresistance oscillations which are now seen to result from the oscillations in breakdown probability. We conclude in Sec. V with a brief summary of our results.

II. SEMICLASSICAL CONDUCTIVITY TENSOR

As a first approximation when studying electronic transport properties, all quantum phase coherence effects can be neglected. The electrons are considered to be classical particles with well-defined trajectories which satisfy the equation of motion, Eq. (1). Transport phenomena can be interpreted in terms of the electron distribution function $f(\mathbf{r},\mathbf{k})$ which satisfies Boltzmann's equations. If we also assume that the scattering can be treated in the uniform relaxation time approximation, linearizing the Boltzmann equation with respect to an external electric field leads to the following expression, due to Chambers,[12] for the conductivity tensor:

$$\sigma_{\alpha\beta}=-\frac{e^2}{2\pi^2}\int d\mathbf{k}\frac{df_0}{d\varepsilon(\mathbf{k})}v_\alpha(\mathbf{k},t_0)\times\int_{-\infty}^{t_0}v_\beta(\mathbf{k},t)\exp[(t-t_0)/\tau]dt\ , \tag{4}$$

where f_0 is the equilibrium Fermi-Dirac distribution function, τ is the relaxation time, and $\mathbf{v}(\mathbf{k},t)$ is the time-dependent velocity for a trajectory which at time t_0 is at wave vector $\mathbf{k}$. [The trajectories are calculated using Eq. (1) in the absence of an electric field.] At low temperatures we may replace df_0/dE by $-\delta(E-E_F)$ and consequently only constant energy trajectories at the Fermi energy contribute to transport phenomena.

In the absence of a periodic potential we can use the free-electron expression for the electron velocity, $\mathbf{v}(\mathbf{k},t)=\hbar\mathbf{k}(t)/m^*$. The position on the circular trajectory at the Fermi energy can then be specified by a polar angle so that

$$v_x=\frac{\hbar k_F}{m^*}\mu_x,\quad \mu_x\equiv\cos\varphi\ , \tag{5}$$

and

$$v_y=\frac{\hbar k_F}{m^*}\mu_y,\quad \mu_y\equiv\sin\varphi\ , \tag{6}$$

where $\mu_\alpha(\alpha=x,y)$ are the velocity components in units of Fermi velocity, $v_F=\hbar k_F/m^*$. (Isotropy is assumed in taking a circular Fermi line and in using a single effective mass.) Making use of the substitutions

$$\phi=\omega_c t,\quad d\mathbf{k}=k\,dk\,d\phi=\frac{m^*}{\hbar^2}dE\,d\phi \tag{7}$$

the components of the conductivity tensor, Eq. (4), can be expressed in terms of Chambers's path integrals I_β as

$$\frac{\sigma_{\alpha,\beta}}{\sigma_0}=\frac{1}{\pi\omega_c\tau}\int_0^{2\pi}\mu_\alpha(\phi)\exp(-\phi/\omega_c\tau)I_\beta(-\infty,\phi)d\phi\ . \tag{8}$$

Here,

$$I_\beta(\theta,\phi)=\int_\theta^\phi\mu_\beta(\phi')\exp(\phi'/\omega_c\tau)d\phi'\ , \tag{9}$$

$\sigma_0=e^2N\tau/m^*$ denotes the zero magnetic field conductivity, and N is the areal electron density. Evaluating these integrals yields the familiar Drude-Zener result for the conductivity tensor:

$$\sigma_{x,x}^{(0)}=\sigma_{y,y}^{(0)}=\sigma_0/(1+\omega_c^2\tau^2)\ , \tag{10}$$

$$\sigma_{x,y}^{(0)}=-\sigma_{y,x}^{(0)}=-\omega_c\tau\sigma_{x,x}^{(0)}\ . \tag{11}$$

The main consequence of a periodic potential is that it produces Bragg reflections whenever the trajectory reaches a Brillouin-zone edge. In order to be able to derive explicit expressions for the conductivity tensor, we limit ourselves to the case of a weak periodic potential,

$$V(x,y)=V_x\cos(K_x x)+V_y\cos(K_y y)\ . \tag{12}$$

It is assumed that the pseudopotential at the Fermi level is weak enough to permit neglect of Fermi line distortion away from zone edges. For sufficiently weak potentials we can assume that gaps appear only at the points illustrated in Fig. 2.

With these approximations the expressions, Eqs. (8) and (9), for the conductivity remain formally unchanged. The presence of a periodic potential affects only the integration path of $I_\beta(-\infty,\phi)$. The Fermi circle is divided into eight regions by its intersections with the Bragg lines which we retain. We specify the intersection points by a set of angles, $\{\theta_i\}$. For partial magnetic breakdown the integration paths are not uniquely defined; there are an infinite number of possible paths, each of which has a probability which depends on the breakdown probability, p. For example, an electron in the interval (θ_1,θ_8) has probability p, defined by Eq. (3), of having arrived there after tunneling from the open-orbit interval (θ_2,θ_1) and a probability $q=1-p$ of having arrived directly from the closed-orbit interval (θ_3,θ_2). Thus,

$$I_\beta^B(-\infty,\phi)=I_\beta(\theta_1^+,\phi)+pI_\beta^B(-\infty,\theta_1^-)+(1-p)I_\beta^B(-\infty,\theta_2^-)\exp(\delta/\omega_c\tau)\ , \tag{13}$$

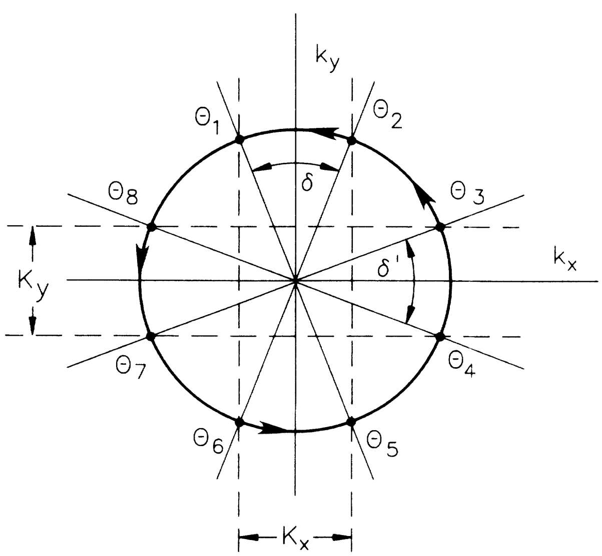

FIG. 2. Points along the Fermi circle where Bragg reflection occurs with probability $q=1-p$ and magnetic breakdown occurs with probability p.

where $\delta=\theta_1-\theta_2$ and I_β^B is the generalized Chambers path integral which includes the possibility of magnetic breakdown. (The upper index on θ, when present, implies that a positive or negative infinitesimal is to be added to the angle.) Similarly, an electron arriving at θ_1^- has probability p of having arrived at this point after tunneling through the barrier at θ_2 and moving along the interval (θ_2,θ_1) and probability of $1-p$ of arriving at this point after having been Bragg reflected along the open orbit at θ_2 at least once. It follows that

$$I_\beta^B(-\infty,\theta_1^-)=I_\beta(\theta_2,\theta_1)+pI_\beta^B(-\infty,\theta_2^-)+(1-p)\exp(-\delta/\omega_c\tau)I_\beta^B(-\infty,\theta_1^-)\ . \tag{14}$$

The exponential factor in Eq. (14) accounts for the probability of an electron being scattered while traversing the interval (θ_2,θ_1) once. Equation (14) can be solved to express $I_\beta^B(-\infty,\theta_1^-)$ in terms of $I_\beta^B(-\infty,\theta_2^-)$ with the result

$$I_\beta^B(-\infty,\theta_1^-)=\frac{I_\beta(\theta_2,\theta_1)+pI_\beta^B(-\infty,\theta_2^-)}{1-(1-p)\exp(-\delta/\omega_c\tau)}\ . \tag{15}$$

Repeating this argument along the Fermi circle and taking account of the periodicity of the velocities with respect to angle allows us to obtain explicit, if unwieldy, expressions for the generalized Chambers's integrals in terms of the breakdown probabilities and ideal integrals along finite segments. These may be inserted into the expression for the conductivity, Eq. (8), to obtain an expression in which the conductivity tensor is expressed as the sum of the Drude-Zener part and a contribution, $\Delta\sigma_{\alpha,\beta}$ due to the presence of the weak periodic potential. The derivation of the expressions for $\Delta\sigma_{\alpha,\beta}$ is outlined in the Appendix. Here we quote only the results for the limit, of relevance to present experiments, where δ and $\delta'=\theta_3-\theta_4$ (see Fig. 2) are much smaller than π:

$$\Delta\sigma_{x,x}=\frac{8}{\pi}\frac{\omega_c\tau\sigma_0}{(1+\omega_c^2\tau^2)^2}[\omega_c^2\tau^2 Q'(\delta')\sin^2(\delta'/2)-Q(\delta)\sin^2(\delta/2)]\ , \tag{16}$$

$$\Delta\sigma_{y,y}=\frac{8}{\pi}\frac{\omega_c\tau\sigma_0}{(1+\omega_c^2\tau^2)^2}[\omega_c^2\tau^2 Q(\delta)\sin^2(\delta/2)-Q'(\delta')\sin^2(\delta'/2)]\ , \tag{17}$$

$$\Delta\sigma_{x,y}=-\Delta\sigma_{y,x}=\frac{8}{\pi}\frac{\omega_c^2\tau^2\sigma_0}{(1+\omega_c^2\tau^2)^2}[Q(\delta)\sin^2(\delta/2)+Q(\delta')\sin^2(\delta'/2)]\ . \tag{18}$$

In these equations,

$$Q(\delta)=\frac{1}{1-(1-p)\exp(-\delta/\omega_c\tau)}-1 \tag{19}$$

and $Q'(\delta')$ is defined similarly with both the angle and the breakdown probability in the latter case referring to the Bragg line associated with the y-dependent term in the potential. Note that Q and Q', respectively, approach the Bragg reflection probabilties $q=1-p$ and $q'=1-p'$ in the strong magnetic field. These equations accurately approximate the exact results given in the Appendix over the entire range of magnetic field for parameters appropriate to the samples in which magnetoresistance oscillations have been observed. We will return to these results after examining the high field limit of the expression, Eq. (3), for p.

III. BRAGG REFLECTION PROBABILITY

The probability of magnetic breakdown approaches unity as the ratio of the zero-field electron gap, V_0, to $\hbar\omega_c$ approaches the ratio of E_F to V_0. As we remark below, the magnetoresistance oscillations will occur only when $V_0/\hbar\omega_c$ is not too much larger than 1. In this limit the periodic potential can be treated as a small perturbation to the Hamiltonian for a free electron in a magnetic field. For simplicity we consider a potential of the form

$$V(\mathbf{r})=V_x\cos(K_x x) , \tag{20}$$

where $K_x=2\pi/a$. In the Landau gauge, $\mathbf{A}\equiv(-By,0,0)$, the eigenfunctions of the unperturbed Hamiltonian,

$$H_0=\frac{1}{2m^*}\left[\mathbf{p}+\frac{e}{c}\mathbf{A}\right]^2 , \tag{21}$$

may be written as the product of a plane wave in the x coordinate and a harmonic oscillator eigenfunction of the y coordinate:

$$\psi_{Y,n}(\mathbf{r})=\frac{1}{\sqrt{2\pi}l}\exp(iYx/l^2)\phi_n(y-Y) . \tag{22}$$

The eigenvalues are degenerate with respect to Y and form Landau levels at energies

$$\varepsilon_n=\hbar\omega_c(n+\tfrac{1}{2}) . \tag{23}$$

For large n this wave function provides the quantum description of a classical cyclotron orbit of radius $R_n=k_n l^2$, where $\hbar^2k_n^2/2m^*=\varepsilon_n$. The y coordinate of the orbit center is located at Y while the z coordinate is quantum mechanically uncertain.

The perturbing potential, Eq. (20), has the effect of inducing transitions which change the x component of the wave vector by K_x and hence the y component of the orbit center by $K_x l^2$. These transitions are the strong-field limits of the Bragg reflections discussed in the previous section. The probability per unit time for Bragg reflections can be calculated using the golden rule

$$w=\frac{4\pi}{\hbar}\int_{-\infty}^{\infty}dE\sum_{n'}|V_{n,n'}|^2\rho(E)\rho(E+(n'-n)\hbar\omega_c) , \tag{24}$$

where we have added the rates of orbit shifts to the right and to the left and we will use a Lorentzian to approximate the spectral density, $\rho(E)$, of a broadened Landau level

$$\rho(E)=\frac{1}{\pi}\frac{\Gamma}{(E^2+\Gamma^2)} . \tag{25}$$

We have averaged the transition rate over energies in the broadened Landau level to take the limit where a semiclassical analysis is valid. In Eq. (24), $V_{n,n'}$ is the matrix element for orbit center transitions:[13]

$$|V_{n,n'}|^2=(V_x/2)^2\frac{n!}{n'!}(K_x^2l^2/2)^{n'-n}\times\exp(-K_x^2l^2/2)[L_n^{n'-n}(K_x^2l^2/2)]^2 . \tag{26}$$

In the limit where $n\gg R_n/a\gg ak_F\gg 1$ this reduces to

$$|V_{n,n'}|^2=\frac{1}{\pi}\frac{V_x^2a}{8\pi R_F}\{1+\cos[4\pi R_F/a-\pi/4-(n'-n)\pi/2]\} . \tag{27}$$

Here, R_F is the classical cyclotron orbit radius and k_F the free-electron Fermi radius corresponding to energy ε_F. We use this result to explain the magnetoresistance oscillations.

Noting that for this potential four Bragg planes are encountered per cyclotron orbit, we see that the Bragg reflection probability $q=1-p$ is given by

$$q=2\pi w/4\omega_c . \tag{28}$$

For the limit of weak magnetic fields, $\hbar\omega_c\ll\Gamma$, the sum over n' in Eq. (24) may be converted to an integral and w may be evaluated analytically. The following result for the reflection probability is obtained:

$$q^C=\frac{\pi V_x^2}{8\hbar\omega_c E_F\sin(\delta/2)} , \tag{29}$$

where $\sin(\delta/2)=K_x/2k_F$. Comparing with Eq. (3) we see that this result is simply the strong-field limit of the classical probability. We will interpolate between the weak-field expression, Eq. (3), and the results obtained in this section by taking

$$p\equiv 1-q\simeq\exp(-2\pi w/4\omega_c) . \tag{30}$$

In the strong-field limit the sum over n' in Eq. (24) is dominated by the $n'=n$ term. In the quantum limit where only the $n'=n$ term is retained we obtain

$$q^Q=\frac{V_x^2\cos^2(2\pi R_F/a-\pi/4)}{8\Gamma E_F\sin(\delta/2)} . \tag{31}$$

In this limit the breakdown probability oscillates with period $2R_F/a$. The physics behind this oscillation is just that described in the Introduction. We see in the next section that it is the oscillation in the breakdown probability which is responsible for the magnetoresistance oscillations seen experimentally. In closing this section we remark that in the limit of weak modulations potentials, contributions to the modulation potential depending on the x and y coordinates can be treated separately so that our results can be applied to the more general case of Eq. (12) as well.

IV. RESISTANCE OSCILLATIONS

In this section we will restrict our attention to the case where the modulation potential depends only on the x coordinate. Inverting the conductivity tensor [Eqs. (16)–18)] we find that for $\delta \ll \pi$

$$\rho_{yy} = \rho_0 \equiv 1/\sigma_0 , \quad (32)$$

$$\rho_{xy} = -\rho_{yx} = \omega_c \tau \rho_0 = B/(ecN) , \quad (33)$$

and

$$\rho_{xx} = \rho_0 \frac{1+\omega_c^2\tau^2 C(\delta)}{1-C(\delta)} , \quad (34)$$

where

$$C(\delta) = \frac{8}{\pi} \frac{\omega_c \tau}{1+\omega_c^2\tau^2} Q(\delta)\sin^2(\delta/2) \quad (35)$$

and $Q(\delta)$ is defined in terms of the breakdown probability in Eq. (19). Note that only the xx component of the resistivity tensor is altered by the modulation potential in our theory. (As emphasized by Gerhardts *et al.*,[6,5] the small oscillations which occur in ρ_{yy} can only be explained by a fully quantum theory.) The results obtained for ρ_{xx} using Eqs. (34), (35), and (30) are illustrated in Fig. 3. In the strong-field limit we are able to compare our theoretical results with those of earlier workers.[5,4,6,3] We find that up to first order in $(V_0/E_F)^2$

$$\Delta\rho_{xx} \equiv \frac{\rho_{xx}-\rho_0}{\rho_0} = \frac{8}{\pi}\omega_c \tau q^{\mathcal{Q}} \sin^2(\delta/2) . \quad (36)$$

Choosing $2\Gamma = \hbar/\tau$ this result may be seen to be identical to that obtained by Beenakker[3] and to the limit reached by the quantum theories when $k_B T \gg \Gamma$. We note that the relative magnitude of the resistance oscillations is proportional to the square of both the strength of the modulation potential and the relaxation time or Landau level width. It is worth remarking that in the low-temperature limit, $k_B T \ll \Gamma$, to which the semiclassical approaches are not applicable, the quantum theories[5,6,4] imply relative magnitudes for the oscillations which are proportional to $(V_x \tau)^1$. This regime has not yet been systematically studied experimentally and a study of the crossover between these limits will provide a demanding quantitative test of the quantum theories.

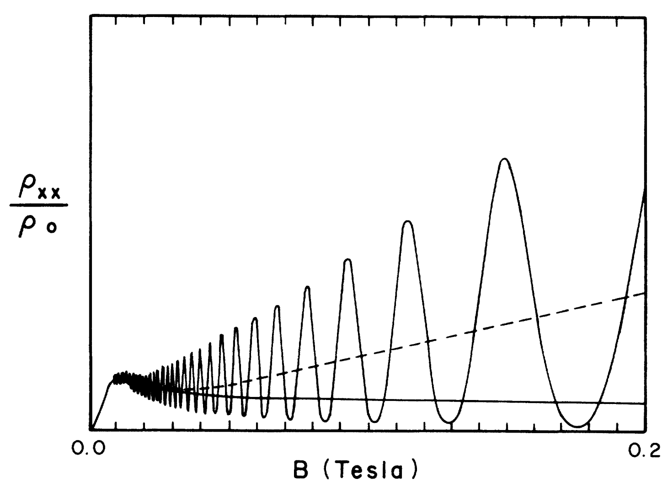

FIG. 3. Magnetoresistance for current flow in the direction of modulation. The solid lines show the results obtained using the classical and quantum expressions for the breakdown probability. The dashed line shows the effect of using the quantum expression for the breakdown probability when the oscillating term is neglected. The electron density and the modulation period for this calculation were taken from a sample studied by Gerhardts *et al.* (Ref. 1); $\lambda_F = 2i/k_F = 44.88$ nm, $E_F = 10.98$ meV, and $a = 382$ nm. We evaluated Eq. (24) using $\tau/\tau_0 = 2$, where τ_0 is the lifetime associated with the Landau level width, to account approximately for the importance of vertex corrections to the transport relaxation time. The results shown in this figure were calculated using $V_x = 0.27$ meV and $v_F \tau = 12$ μm and are in excellent agreement with experiment.

Previous theories fail when the probability of magnetic breakdown is not close to 1 and cannot treat the limit of weak magnetic fields. The modulated samples all show a strong positive magnetoresistance at weak magnetic fields. This behavior is characteristic of systems with open orbits and is associated with an increase in the probability that electrons moving along open-orbit trajectories in **k** space will reach a Bragg line and be Bragg reflected before they are scattered. On the basis of our theory we are able to predict that the size of this positive magnetoresistance will be larger for samples with higher mobility and hence longer scattering times. Since the probability of magnetic breakdown is smaller for a stronger modulation potential, we are also able to predict that the weak-field magnetoresistance will be larger and persist to stronger magnetic fields when the modulation potential is stronger. Furthermore, since the magnetoresistance oscillations will occur only when the dominant electron trajectory is a simple cyclotron orbit, they will not occur until the probability of magnetic breakdown becomes appreciable. Comparing with Eq. (3) we are able to predict that the resistance oscillations begin when

$$\hbar\omega_c \sim V_0^2/[E_F \sin(2\theta)] . \quad (37)$$

Our thoeretical results actually show the oscillations persisting to somewhat lower fields than found experimentally. We attribute the differences to the effects of Bragg reflection at the Bragg lines we have neglected, which must occur at sufficiently low fields and, possibly, to inhomogeneity in the 2D ES samples.

We close with a final observation concerning the resistance oscillations. The presence of the $\pi(n-n')/2$ factor in Eq. (27), which is of quantum origin, implies that the magnetoresistance oscillations will be averaged out when inter-Landau-level transitions become of comparable importance to intra-Landau-level transitions. Thus the oscillations can only occur for $\Gamma \leq \hbar\omega_c$. This requirement is similar to the requirement for the occurrence of Shubnikov–de Haas oscillations. On the other hand, while Shubnikov–de Haas oscillations will persist only up to temperatures satisfying $k_B T \sim \hbar\omega_c$, the magnetoresistance oscillations in modulated systems will persist as long as the range of cyclotron orbit diameters which occur for kinetic energies within $k_B T$ of the Fermi level is small compared to the modulation period. It follows that the modulation-induced magnetoresistance oscillations will persist until $k_B T \sim (\hbar\omega_c)(k_F a) \gg \hbar\omega_c$.

V. SUMMARY AND CONCLUSIONS

We have developed a semiclassical theory of the magnetoresistance oscillations in weakly periodically modulated two-dimensional electron systems. Our picture starts from a semiclassical description of electron dynamics in the Bloch minizones created by the modulation. Unlike earlier theories, our approach is able to treat the weak-field limit. We predict that the positive magnetoresistances seen at weak fields in modulated samples will strengthen with improvements in sample mobility and with strengthening modulation and that the minimum field at which the oscillations will occur will increase with modulation potential strength and disorder. We point that, unlike the case of Shubnikov–de Haas oscillations, the modulation-induced oscillations are much more easily destroyed by increasing disorder than by increasing temperature. Our theory is readily applied to the case where the weak modulation occurs in both directions.

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APPENDIX

Following the procedure outline in Sec. II we find for the components of the conductivity tensor

$$\begin{aligned}
\frac{\pi\omega_c\tau\sigma_{\alpha\beta}}{2\sigma_0} = & \int_{\theta_1}^{\theta_8} d\varphi\, \mu_\alpha(\varphi) l^{-\varphi/\omega_c\tau} \left\{ -I_\beta(\varphi,\theta_8) + \frac{1}{1+PP'\exp(-\pi/\omega_c\tau)} \right. \\
& \left. \times [R I_\beta(\theta_2,\theta_1) + PR' I_\beta(\theta_4,\theta_3) + I_\beta(\theta_1,\theta_8) + P I_\beta(\theta_3,\theta_2)] \right\} \\
& + \int_{\theta_3}^{\theta_2} d\varphi\, \mu_\alpha(\varphi) l^{-\varphi/\omega_c\tau} \left\{ -I_\beta(\varphi,\theta_2) + \frac{1}{1+PP'\exp(-\pi/\omega_c\tau)} \right. \\
& \left. \times [R' I_\beta(\theta_4,\theta_3) + P'R I_\beta(\theta_6,\theta_5) + I_\beta(\theta_3,\theta_2) + P' I_\beta(\theta_5,\theta_4)] \right\} \\
& + \int_{\theta_4}^{\theta_3} d\varphi\, \mu_\alpha(\varphi) l^{-\varphi/\omega_c\tau)} \left\{ -I_\beta(\varphi,\theta_3) + \frac{R'}{p'} I_\beta(\theta_4,\theta_3) + \frac{R'}{1+PP'\exp(-\pi/\omega_c\tau)} \right. \\
& \left. \times [R' I_\beta(\theta_4,\theta_3) + P'R I_\beta(\theta_6,\theta_5) + I_\beta(\theta_3,\theta_2) + P' I_\beta(\theta_5,\theta_4)] \right\} \\
& + \int_{\theta_2}^{\theta_1} d\varphi\, \mu_\alpha(\varphi) l^{-\varphi/\omega_c\tau} \left\{ -I_\beta(\varphi,\theta_1) \frac{R}{p} I_\beta(\theta_2,\theta_1) + \frac{R}{1+PP'\exp(-\pi/\omega_c\tau)} \right. \\
& \left. \times [R I_\beta(\theta_6,\theta_5) + PR' I_\beta(\theta_8,\theta_7) + I_\beta(\theta_5,\theta_4) + P I_\beta(\theta_7,\theta_6)] \right\} ,
\end{aligned} \tag{A1}$$

where

$$R = \frac{p}{1-(1-p)\exp(-\delta/\omega_c\tau)}, \quad P = pR + (1-p)\exp(\delta/\omega_c\tau) ,$$

and the quantities R', P' have the same form but p and δ must be replaced by p' and δ', respectively. In the case of complete breakdown $p = R = P = p' = R' = P' = 1$ and the Drude-Zener expressions [Eqs. (10) and (11)] can be obtained. In the limit of small angles δ and δ' ($\delta, \delta' \ll \pi$) the dominant corrections due to the effect of Bragg reflection originate in those terms that are multiplied by the ratio R/p or R'/p', and we get

$$\Delta\sigma_{\alpha\beta} = \frac{2\sigma_0}{\pi\omega_c\tau} \left[\left[\frac{R}{p} - 1 \right] I_\beta(\theta_2,\theta_1) \int_{\theta_2}^{\theta_1} \mu_\alpha(\varphi) l^{-\varphi/\omega_c\tau} d\varphi + \left[\frac{R'}{p'} - 1 \right] I_\beta(\theta_4,\theta_3) \int_{\theta_4}^{\theta_3} \mu_\alpha(\varphi) l^{-\varphi/\omega_c\tau} d\varphi \right] . \tag{A2}$$

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