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Bogoliubov quasiparticles, spinons, and spin-charge decoupling in superconductors

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We show that, in the presence of Coulomb interactions, the charge of a quasiparticle in a fully gapped superconductor is a sharp quantum observable. It follows that the familiar Bogoliubov quasiparticles are neutral, spin-$\frac{1}{2}$ fermions. Since the current-carrying collective modes are spinless, charge and spin are carried by separate excitations in a superconductor. The quantum numbers of these quasiparticles are identical to those of a spinon in a "resonating-valence-bond" (RVB) superconductor. This supports the notion that a RVB superconductor is simply a strong-coupling version of a BCS superconductor. We describe an experiment that would directly verify the separation of charge and spin in a bulk superconductor.

The low-lying excited states of a quantum many-body system can often be described as a dilute gas of weakly interacting quasiparticles. The charge, spin, and even the statistics of these quasiparticles, however, need not be simply related to those of the bare particles. Indeed, it is sometimes not clear whether a well-defined charge and spin can be associated with the quasiparticles at all. These properties are "sharp quantum observables" if a measurement using a suitably gentle probe (e.g., a long-wavelength, low-frequency external field) produces a well-defined, reproducible result. This idea is a familiar one outside of condensed-matter physics: for example, even an electron has a well-defined charge only if it is probed on large length scales compared with its Compton wavelength and low energies compared with its rest mass.

The charge and spin of a quasiparticle are sharp quantum observables if a gap exists in the charge and spin-excitation spectra of states with far separated, localized quasiparticles.[1] Such states may be obtained by creating particle-antiparticle pairs, pulling the quasiparticles far apart, and then tying them down in some gentle way (so that we may do experiments on one particle at a time). At frequencies well below the charge (spin) gap, the state of the system will, to lowest order, be unchanged by its interaction with a probe, and the measured charge (spin) will be equal to its expectation value in the unperturbed quasiparticle state. Moreover, the fluctuations in the charge and spin can be made arbitrarily small by making the probe increasingly "gentle."

Note that a gap in the spectrum makes the definition of quasiparticle states rather straightforward, and simplifies the determination of their quantum numbers. Although a gap is *sufficient* to discuss quasiparticles, it may not be *necessary;* a normal Fermi liquid is well described (by definition) as a weakly interacting gas of gapless quasiparticles. In the absence of a gap, however, the notion of a quasiparticle becomes somewhat subtle, and it is not obvious what its charge should be,[2] nor even whether charge is a sharp quantum observable.

These subtleties associated with the determination of quasiparticle quantum numbers arise in the case of one-dimensional commensurate charge-density-wave systems, whose quasiparticles are solitons with unusual quantum numbers. For commensurability two (as in polyacetylene) they have reversed charge-spin relations,[3] i.e., there are neutral, spin-$\frac{1}{2}$ quasiparticles and charge e, spinless quasiparticles. For higher commensurabilities, fractional[4] charges are possible. In all these cases, the presence of a gap guarantees a well-defined charge and spin for the quasiparticles.

While the charge and spin of the quasiparticles in polyacetylene can be precisely defined, their statistics cannot, since the notion of "exchanging" particles is ill defined in one dimension. In two dimensions, however, one expects that quasiparticles will belong to specific representations of the braid group,[5] while in three and higher dimensions quasiparticles will belong to particular representations of the permutation group. Since the absence of Lorentz invariance typically prevents us from applying the spin-statistics theorem to condensed-matter systems, quasiparticle statistics cannot in general be inferred from their charge and spin, and must be computed explicitly. In the fractional quantum Hall effect, for example, it is believed that quasiparticles are fractionally charged and have fractional statistics.[6] As in the one-dimensional case, the existence of a gap in the excitation spectrum is sufficient to ensure both (a) that the quasiparticle is a well-defined object and (b) that its charge, spin, and statistics are sharp observables.

A conventional, fully gapped singlet superconductor possesses a gap to both spin and charge excitations, so that the charge and spin of the quasiparticles of such a system are sharp quantum observables at energies below the plasma frequency and the quasiparticle gap. We show below that the familiar Bogoliubov quasiparticles of a fully gapped superconductor in three dimensions are in fact neutral, spin-$\frac{1}{2}$ particles, i.e., have reversed charge-spin relations. This is the main result of this paper. Thus when an electron (carrying both spin and charge) is injected into a bulk superconductor, it is converted (in a time of order the greater of $\hbar/\Delta$ and ω_p^{-1}) into a spin-$\frac{1}{2}$ quasiparticle, which carries spin (and spin current) but neither charge nor charge current. This is a consequence of the perfect screening of charge and of the Meissner effect in a bulk superconductor. The charge of the injected electron resides in a distortion of the condensate at the surface of the superconductor; charge and current are necessarily confined to within a Thomas-Fermi screening length and a London penetration length of the surface, respectively. Thus spin may flow by the motion of quasiparticles through the bulk of a superconductor, but charge current is carried by collective excitations and can only be found at the surface. Such a spatial separation of charge and spin currents should be experimentally detectable, as described below. This phenomenon of spin-charge decoupling in a superconductor should be contrasted with the quite different case of a normal metal. Although the quasiparticles of a metal are accompanied by no net charge because of the perfect screening in a metal, and can therefore be considered in some sense "neutral," their charge is not a sharp quantum observable since a metal is gapless. Despite the absence of charge density associated with these Landau quasiparticles, they may carry charge currents as well as spin currents through the bulk of a metal.

Considerable recent interest has focused on the nature of quasiparticles in resonating-valence-bond (RVB) states, i.e., hypothetical disordered ground states of frustrated quantum antiferromagnets. Anderson[7] argued that the quasiparticles in a gapless RVB state should be neutral, spin-$\frac{1}{2}$ fermions, dubbed "spinons." Kivelson, Rokhsar, and Sethna[8] (KRS) then noted the similarities between an RVB insulator having a spin gap (a "short-range" RVB state) and polyacetylene, and identified spinons with the spin-soliton excitations of polyacetylene. Upon doping, it is expected that the resulting short-range RVB states are charge-$2e$ superconductors.[9] It has been suggested[10] that various superconducting RVB states discussed in the literature (both s wave and non-s-wave) are simply strong coupling (i.e., small Cooper pair[11]) versions of the more familiar BCS superconductor. As such, the "RVB" and "BCS" superconductors should not have dramatically different properties, and in particular they should possess the same quasiparticles. The fact that the hypothesized RVB "spinons" share the same quantum numbers with the quasiparticles of a conventional superconductor lends further credence to this equivalence of superconducting RVB and BCS states. This work is an extension of previous work by one of us (Ref. 12) and is similar in spirit to ideas discussed in Ref. 13.

MICROSCOPIC PICTURE OF A QUASIPARTICLE

Consider first the distribution of charge in a piece of bulk superconductor at zero temperature. By virtue of charge neutrality enforced by the long-range Coulomb interaction, there is no average charge density and no local charge fluctuations. (Of course, a state with a well-defined superconducting order parameter necessarily has global charge fluctuation.) Any experiment done with a small energy scale compared to the plasma frequency $\hbar\omega_p$ will see a uniformly neutral system. This is precisely the same sense in which a semiconductor (or for that matter the vacuum) has no local charge fluctuations. In a superconductor with widely separated quasiparticles, the expectation value of the charge density integrated over a large region containing each quasiparticle must vanish due to the perfect screening of the Coulomb field. At a microscopic level this screening will occur over a Thomas-Fermi screening length, $l_{\rm TF}$. Beyond this length, we expect Friedel oscillations (with no integrated charge) to extend out to a distance of order ξ_0 due to the existence of an almost sharp Fermi surface. For distances greater than ξ_0, the charge density will be exponentially small.

In a singlet superconductor, there is a gap in the spin excitations spectrum Δ_S, and the system possesses a finite spin-spin correlation length given by the superconducting correlation length ξ_0. The spin of the quasiparticle is therefore a nonfluctuating quantity on energy scales less than Δ_S. Since a quasiparticle is essentially an unpaired electron immersed in an otherwise singlet-paired environment,[12] we conclude (see also below) that any experiment will observe a single spin-$\frac{1}{2}$ in the vicinity of the quasiparticle (i.e., the spin-flip neutron-scattering oscillator strength for energies less than $2\Delta_0$ will correspond to precisely spin-$\frac{1}{2}$). Injecting or removing a single electron from a superconductor changes the spin of the sample by $\frac{1}{2}$, so that the quasiparticle number will generally change by one. The charge of the sample will also change by $\pm e$ in this process. Where does the charge go? When a quasiparticle diffuses to within a Thomas-Fermi screening length of the surface, it acquires a charge ($-|e|$ with probability u_k^2, and $+|e|$ with probability v_k^2, where u_k and v_k are the familiar BCS parameters) with the result that it is possible to remove an electron or hole from the system. As long as the quasiparticle is more than a screening length from the surface, however, *there is no charge movement associated with the motion of the quasiparticle.*

In order to have a gauge invariant description of the quasiparticle, it is well known that backflow effects must be included.[14,12] In the absence of long-range Coulomb interactions, one finds a collective current (of order ev_F/ξ^3) passing through the center of the quasiparticle and returning at large distances such that the current density falls off as a dipole field, $\mathbf{J}(R)\sim 1/R^3$. The Bogoliubov quasiparticle can therefore be thought of as a Cooper-pair roton with a spin trapped inside. In the presence of Coulomb interactions, however, this current generates a magnetic field which is forbidden by the Meissner effect. The resulting backflow field is then compressed to within a London penetration depth about the quasiparticle, and the quasiparticle has quite a complicated structure on scales smaller than λ. When observed with long-wavelength, low-energy probes, however, the quasiparticle has precisely zero net charge and carries no current.

The operator that creates a Bogoliubov quasiparticle is essentially a superposition of dressed electron and hole creation operators. Thus, at least in three dimensions, it is clear from the commutation relations of these operators that the spin-$\frac{1}{2}$, neutral quasiparticles of a superconductor are fermions. (Even in two dimensions, where statistics are properly understood in terms of representations of the braid group[5] rather than the permutation group, we believe that the quasiparticles remain fermions. It is conceivable, however, that the complicated distortion of both the condensate and the electromagnetic field surrounding the quasiparticle could alter this result in two dimensions.)

EXPERIMENTAL CONSEQUENCES

The suggestion that spin and charge are carried by separate excitations in a superconductor is not simply a question of semantics. The separation of spin and charge can be experimentally verified (or disproven) by injecting spin-polarized electrons[15] into a superconductor and observing the resulting distribution of charge and spin currents, as follows.

Construct a metal-oxide-superconductor-oxide-metal sandwich in which the metals are itinerant ferromagnets, so that spin-polarized electrons can be injected or extracted from the superconductor. Let the width of the superconductor be large compared with the London penetration depth λ, but narrow enough so that spin-flip scattering is negligible in the time it takes for a quasiparticle to diffuse across the sample.[16] Place a voltage V across the sandwich such that $2\Delta_0 > eV > \Delta_0$. Thus, a voltage great enough for particle injection can appear across the first oxide barrier, but not the second. In order for a steady-state current to flow, the injected quasiparticles must diffuse across the sample and exit on the other side. Since only spin up quasiparticles are injected, they cannot annihilate one another; only spin-flip scattering allows the quasiparticle number (i.e., the spin) to decay. (If a paramagnetic current is injected, spin up and down quasiparticles will recombine in a time measured by branch imbalance experiments.) So long as the current flowing through the superconductor is less than the critical current, it will be entirely confined to a surface layer of thickness λ. A bulk spin current will also flow, carried by the spin-polarized quasiparticles diffusing across the sample. Thus, separation of charge and spin will be evident in the spatial separation of the charge and spin currents. An experiment which measured the spin current in the bulk would verify this separation of charge and spin. Of course, it is difficult to probe the spin current in the bulk of the superconductor. In principle it should be observable as a current dependent contribution to the spin-flip neutron scattering or in μSR. There are probably less clumsy experimental methods for observing this or related effects which would demonstrate the spatial separation of the charge and spin currents.

FORMAL DISCUSSION

A formal demonstration of the separation of charge and spin can be obtained by studying the behavior of various correlation functions. Let $S(\mathbf{q},\omega)$ be the imaginary part of the spin-spin response function (which is measured in a spin-flip neutron-scattering experiment), and let $\sigma(\mathbf{q},\omega)$ be the current-current response function (i.e., the optical conductivity). We also define

$$S_2(\mathbf{q},\omega)=\mathrm{Im}\int_0^\infty dt\int d\mathbf{R}\,d\mathbf{r}_1\,d\mathbf{r}_2 f(r_1)f(r_2)e^{i\omega t-i\mathbf{R}\cdot\mathbf{q}}\langle[S^-(\mathbf{R}+\mathbf{r}_1,t)S^-(\mathbf{R}-\mathbf{r}_1,t),S^+(\mathbf{r}_2,0)S^+(-\mathbf{r}_2,0)]\rangle\,,$$

where $f(r)$ is an arbitrary short-range form factor. S measures the spectrum of spin-one excitations in the system, S_2 measures the spectrum of spin-two excitations, and σ measures the spectrum of current-carrying states, i.e., the charge spectrum. Since we are interested in probing the long-wavelength properties of the system, we consider the $\mathbf{q}\to\mathbf{0}$ properties of these response functions. S and S_2 can be calculated in the framework of the BCS theory, and we expect them to be qualitatively unchanged by the long-range Coulomb interaction since they do not involve charge excitations.

Near $\hbar\omega=2\Delta_0$, the functional form of $S(\mathbf{q},\omega)$ corresponds to a two-particle threshold [i.e., $S(\mathbf{0},\omega)\sim(\hbar\omega-2\Delta_0)^{-1/2}\Theta(\hbar\omega-2\Delta_0)$ where Θ is the Heavyside function]; furthermore, S_2 vanishes near $\hbar\omega=2\Delta_0$ but has a multiparticle threshold at $\hbar\omega=4\Delta_0$ [i.e., $S_2(\mathbf{0},\omega)\propto\Theta(\hbar\omega-4\Delta_0)$]. From this we infer that the lowest energy spin-1 states consist of two quasiparticles, the lowest energy spin-2 states consist of four quasiparticles, etc. Thus a quasiparticle picture implies that each quasiparticle carries spin-$\frac{1}{2}$. While this conclusion follows directly from the approximate quasiparticle creation operators of BCS theory, the preceding discussion in terms of response functions is presumably more general.

Similarly, one may study the quasiparticle charge by measuring the oscillator strength associated with the quasiparticle pair threshold in σ. Note that in a direct-gap semiconductor (or in the Dirac vacuum), $\sigma(\mathbf{0},\omega)$ displays a threshold at the quasiparticle pair creation energy. This feature is in dramatic contrast with the case of a clean superconductor[17] at zero temperature: there is no oscillator strength associated with pair production because the δ function at $\omega=0$ saturates the sum rule. This confirms that the quasiparticles are neutral.

In a *dirty* superconductor, however, the oscillator strength near the quasiparticle threshold does not vanish—does this imply that the charge of the quasiparticle changes with the addition of disorder? The answer is that while a vanishing oscillator strength implies neutral quasiparticles, the converse is not necessarily true. This is best seen by considering the low-lying excitations of a large-U (Mott) insulator, which are purely magnetic excitations and certainly qualify as "neutral." In a clean Mott insulator, their neutrality is reflected in the absence of optical absorption at energies less than U. The addition of disorder, however, transfers oscillator strength to these neutral excitations because light couples to the slight spatial variation of the virtual charge fluctuations present in the insulator, as we discuss elsewhere.[18] [Even in the disordered case, however, it is easy to see that no current is transported by the spin excitations: in a toroidal geometry, the dependence of the energy of a Mott insulator on the flux through the torus is exponentially small in the size of the donut, since transporting charge around the loop is an order $t(t/U)^L$ process.]

In summary, we have shown that the quasiparticles in a bulk superconductor are neutral, spin-$\frac{1}{2}$ excitations. This observation leads to detectable consequences in that the charge and spin currents are carried by separate excitations and can be spatially separated. Finally, we have noted that these quasiparticles have a natural analog in the spinons in a doped short-range RVB state. This observation lends further support to the interpretation[10] of the doped short-range RVB state as a strong coupling (but otherwise conventional) fully gapped superconductor.

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