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Quantum fluctuations and the onset of global superconductivity in disordered Josephson-junction arrays

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The measured normal-state resistance threshold for the onset of global superconductivity is calculated, in the mean-field approximation, for a Josephson-junction array in which the normal-state junction resistances and, hence, also the Josephson coupling energies, are weakly disordered. Such disorder increases the threshold by a nonuniversal factor which depends on the width of the junction conductance distribution and the lattice coordination number.

A number of recent experiments[1-3] on the onset of superconductivity in ultrathin granular films have focused on the precise determination of an apparently universal normal-state resistance criterion R_N^c for the occurrence of global superconductivity in the zero-temperature limit. Theoretically, the existence of such a criterion has been attributed to the destruction of global Cooper-pair phase coherence by quantum-mechanical phase fluctuations.[4] If a film is modeled as a regular two-dimensional array of individually superconducting granules, the theoretical threshold is universal,[5] that is, independent of the BCS energy gap Δ_0 in the small grain limit in which the mutual capacitance of neighboring grains is dominated by the nongeometrical virtual quasiparticle tunneling capacitance[6,7] ΔC required by causality.[8] A mean-field calculation[5] has yielded $R_N^c \simeq 5.7$ kΩ for a square array of identical Josephson junctions. Accounting for the correlated fluctuations of neighboring grains to first order in the inverse lattice coordination number and allowing for the frequency and phase dependence of ΔC have reduced this[9] to $R_N^c \simeq 3.8$ kΩ, in accord with the range of reported values that are found to vary from 4 to 6.5 kΩ between experimental groups.[1,2] But, while the theory has considered a lattice of identical, equally spaced grains, the experimental samples are most probably disordered. Indeed, such disorder may have contributed to the distribution[10] of the measured values of R_N^c.

The disorder present in granular films is difficult to control or to quantify. On the other hand, it has recently become possible to fabricate[11] Josephson-junction arrays consisting of grains sufficiently small that the quantum fluctuation effects of interest here are relevant. It is reasonable to expect that it will prove feasible to construct arrays in which disorder in the relevant junction parameter, the Josephson coupling energy, is deliberately introduced in known measure, as has already been done in the case of fabricated arrays in which disorder in plaquette areas has been introduced to study magnetic-field effects.[12] Arrays that have thus far been fabricated consist of grains sufficiently large that the capacitance determining the charging energy is the usual geometrical one,[13] C. Therefore, it is this nonuniversal case which will be treated here. It is planned to treat the more complicated problem of disorder in ΔC elsewhere.

The dynamics of such a two-dimensional array of granules, each described as zero temperature by BCS theory, is determined by the Lagrangian

$$L = \frac{C}{2}\sum_i V_i^2 + \sum_{\langle ij\rangle} J_{ij}\cos(\phi_i - \phi_j)\,, \tag{1}$$

where the time derivative of the common Cooper-pair phase of the electrons on the ith grain ϕ_i is given by the Josephson equation,

$$\dot{\phi}_i = \frac{2e}{\hbar}V_i\,, \tag{2}$$

with V_i the voltage on that grain. The Josephson coupling energy,

$$J_{ij} = \frac{h\Delta_0}{8e^2}\sigma_{ij}\,, \tag{3}$$

is determined by the normal-state conductance σ_{ij} of the tunnel junction formed by grains i and j. In Eq. (1), the double sum is restricted to nearest-neighbor grains. Planck's constant and the electron charge have been denoted by $h = 2\pi\hbar$ and $-e$, respectively.

The quantum-mechanical amplitude for the evolution of the array from one phase configuration at time zero to another at time T is

$$K(T) \propto \int D\phi \exp\left[\frac{i}{\hbar}\int_0^T L\,dt\right], \tag{4}$$

where the path-integral measure is $D\phi \equiv \prod_i D\phi_i$. For an array of identical junctions, $\sigma_{ij} \equiv \sigma_0$, $J_{ij} \equiv J_0$, the mean-field approximation leads, along the lines described, for example, in Refs. 5 and 9, to an effective single-grain Lagrangian,

$$L_1 = \frac{\hbar^2 C}{8e^2}\dot{\phi}_1^2 + z\mu J_0\cos\phi_1\,, \tag{5}$$

where z is the lattice coordination number and

$$\mu = \langle \cos\phi_1 \rangle_0 \quad (6)$$

is the ground-state order parameter, which is to be determined self-consistently. Introducing the dimensionless momentum canonically conjugate to ϕ_1, $p_1 = \hbar^{-1}\partial L/\partial\dot{\phi}_1$, the corresponding one-body Hamiltonian is $(4e^2/C)H_1$, where the reduced Hamiltonian,

$$H_1 = \frac{p_1^2}{2} - g\cos\phi_1 , \quad (7)$$

depends on the single parameter

$$g = z\mu\lambda_0 , \quad (8)$$

with

$$\lambda_0 = \frac{CJ_0}{4e^2} = \left(\frac{C\Delta_0}{8e^2}\right)\left(\frac{h\sigma_0}{4e^2}\right) , \quad (9)$$

using Eq. (3). Evaluating the ground-state eigenvalue of the pendulum Hamiltonian H_1, using the Feynman-Hellman theorem to obtain the order parameter,

$$\mu = 2g - 7g^3 + \cdots , \quad (10)$$

and imposing self-consistency via Eq. (8), gives a threshold coupling

$$\lambda_0^c = \frac{1}{2z} , \quad (11)$$

and, according to Eq. (9), a threshold resistance criterion

$$R_N^c = (\sigma_0^c)^{-1} = \left(\frac{C\Delta_0 z}{4e^2}\right)\left(\frac{h}{4e^2}\right) . \quad (12)$$

The Hamiltonian approach just described is, according to fundamental quantum-mechanical principles,[14] equivalent to evaluating the one-body version of Eq. (4) for the single-grain propagator and extracting the ground-state energy using the Feynman-Kac formula,

$$E_G = i\hbar \lim_{T\to\infty} \frac{\ln K}{T} . \quad (13)$$

We now use this correspondence between the Hamiltonian and path-integral approaches to calculate the conductance threshold for a disordered array in which σ_{ij}, which now varies from junction to junction, is distributed according to a sharp Gaussian probability density,

$$P(\sigma_{ij}) = \frac{1}{\sqrt{2\pi}\Delta\sigma}\exp[-(\sigma_{ij}-\sigma_0)^2/2(\Delta\sigma)^2] , \quad (14)$$

with $\delta \equiv \Delta\sigma/\sigma_0 \ll 1$. We consider two approaches, which yield the same final result. One employs the familiar replica trick and the other, which we treat first, is a perturbative calculation to $O(\delta^2)$, based on the smallness of δ.

From Eqs. (1), (4), and (13), we must work with the disorder-averaged value of the logarithm of the propagator

$$K(\tau) \propto \int D\phi \exp\left[-\int_0^{\tau_m} d\tau \left(\tfrac{1}{2}\sum_i \dot{\phi}_i^2 - \sum_{\langle ij\rangle}\lambda_{ij}\cos(\phi_i-\phi_j)\right)\right] , \quad (15)$$

where the time variable has been scaled by $\hbar C/4e^2$ and analytically continued to the imaginary axis. In Eq. (15), we have redefined $\dot{\phi}_i \equiv \partial\phi_i/\partial\tau$ and $\tau_m = 4e^2 iT/\hbar C$. The coupling strength $\lambda_{ij} = CJ_{ij}/4e^2$ is proportional, according to Eq. (3), to σ_{ij},

$$\lambda_{ij} = \left(\frac{C\Delta_0}{8e^2}\right)\left(\frac{h}{4e^2}\right)\sigma_{ij} . \quad (16)$$

The width of the λ_{ij} distribution, about a mean value λ_0, is

$$[\langle(\lambda_{ij}-\lambda_0)^2\rangle]^{1/2} \equiv \Delta\lambda = \left(\frac{C\Delta_0}{8e^2}\right)\left(\frac{h}{4e^2}\right)\Delta\sigma , \quad (17)$$

where $\langle\ \rangle$ denotes a disorder average with respect to Eq. (14).

Expanding the potential-energy term in Eq. (15) to second order in the fluctuations in the Josephson coupling strength, performing the disorder average, and reexponentiating the result, we obtain

$$\langle \ln K \rangle = \ln \int D\phi \exp\left(-\int_0^{\tau_m} d\tau \tilde{L}\right) + \text{const} , \quad (18)$$

where $\tilde{L}$, the effective Lagrangian, is defined by

$$\int_0^{\tau_m} d\tau \tilde{L} = \int_0^{\tau_m} d\tau \tfrac{1}{2}\sum_i \dot{\phi}_i^2 - \lambda_0\int_0^{\tau_m} d\tau \sum_{\langle ij\rangle}\cos(\phi_i-\phi_j) - \frac{(\Delta\lambda)^2}{2}\int_0^{\tau_m} d\tau \int_0^{\tau_m} d\tau' \sum_{\langle ij\rangle}\cos[\phi_i(\tau)-\phi_j(\tau)]\cos[\phi_i(\tau')-\phi_j(\tau')] . \quad (19)$$

In the notation of Eq. (6), the mean-field average can be introduced into Eq. (19) by

$$\cos(\phi_i - \phi_j) \simeq \cos\phi_i\langle\cos\phi_j\rangle_0 + \sin\phi_i\langle\sin\phi_j\rangle_0 = \mu\cos\phi_i . \quad (20)$$

Consequently, the one-body mean-field version of $\tilde{L}$, $\tilde{L}_1$ is defined by

$$\int_0^{\tau_m} d\tau \tilde{L}_1 = \tfrac{1}{2}\int_0^{\tau_m} d\tau \dot{\phi}_1^2 - \mu\lambda_0 z \int_0^{\tau_m} d\tau \cos\phi_1 - \frac{\mu^2(\Delta\lambda)^2 z}{2}\int_0^{\tau_m} d\tau \int_0^{\tau_m} d\tau' \cos\phi_1(\tau)\cos\phi_1(\tau') . \tag{21}$$

The last term in Eq. (21) gives a shift in the large-τ_m behavior of the propagator or, alternatively, a change in the ground-state energy. Evaluating the change shows that effect of the disorder average is simply to replace λ_δ^2 by $\lambda_\delta^2 + (\Delta\lambda)^2/z$. Self-consistency then implies that the mean-field threshold coupling is now no longer given by Eq. (11) but by

$$1 = 2z\left[(\lambda_\delta)^2 + \frac{(\Delta\lambda)^2}{z}\right]^{1/2} \tag{22}$$

or

$$\lambda_\delta \simeq \frac{1}{2z}\left[1 - \frac{(\Delta\lambda)^2}{2z(\lambda_\delta)^2}\right] . \tag{23}$$

Thus, to $O(\delta^2)$,

$$\lambda_\delta = \frac{1}{2z}\left[1 - \frac{\delta^2}{2z}\right] . \tag{24}$$

Recall that $\delta = \Delta\sigma/\sigma_0 \ll 1$ measures the strength of the disorder.

As an alternative to the perturbative approach, we can avoid the necessity of reexponentiating by keeping the computation in the exponent throughout and performing the disorder average using the replica trick,

$$\langle \ln K \rangle = \lim_{n\to 0} \frac{1}{n} \ln\langle K^n \rangle . \tag{25}$$

Introducing a replica index α, the disorder-averaged replicated propagator is, from Eqs. (14) and (15),

$$\langle K^n \rangle \sim \int D\phi \exp\left[-\int_0^{\tau_m} d\tau \sum_\alpha \left(\tfrac{1}{2}\sum_i (\dot{\phi}_i^\alpha)^2 - \lambda_0 \sum_{\langle ij\rangle} \cos(\phi_i^\alpha - \phi_j^\alpha)\right)\right] \exp\left[\frac{(\Delta\lambda)^2}{2}\sum_{\langle ij\rangle}\left(\int_0^{\tau_m} d\tau \sum_\alpha \cos(\phi_i^\alpha - \phi_j^\alpha)\right)^2\right] , \tag{26}$$

where $D\phi \equiv \prod_\alpha D\phi^\alpha$. Making the mean-field approximation, as in the disorder-free case, we obtain as a one-body effective Lagrangian per replica $\tilde{L}_1$ of Eq. (21). The ground-state energy is then simply proportional to n and the limit indicated in Eq. (25) can be taken with no difficulty. The resulting self-consistency condition is, consequently, exactly the same as Eq. (22), tending to confirm the reexponentiation performed in the perturbative approach.

To establish the observational consequences of Eq. (24), the result must be recast in terms of the measured value of the normal-state sheet-resistance, $R_\square$. The effect of disorder on $R_\square$, as described by a network of resistors of resistance $R = \sigma^1$, where σ is randomly distributed according to Eq. (14) about the mean value $\sigma_0 = R_0^{-1}$, has been calculated[15,16] for the honeycomb, square, and hexagonal lattices, corresponding to $z = 3$, 4, and 6, respectively. The result of the calculation for all three of these cases is expressed by

$$R_\square = R_\square^0\left[1 + \frac{2}{z}\delta^2\right] , \tag{27}$$

where $R_\square^0 = \sqrt{3}R_0$, R_0, and $R_0/\sqrt{3}$ for $z = 3$, 4, and 6, respectively.

Combining Eqs. (9) and (24), the critical value of R_0 is

$$R_0^c = \left[\frac{C\Delta_0 z}{4e^2}\right]\left[\frac{h}{4e^2}\right]\left[1 + \frac{\delta^2}{2z}\right] , \tag{28}$$

so that disorder increases the measured critical sheet resistance $R_\square^c$ by a factor

$$f = 1 + \frac{5\delta^2}{2z} . \tag{29}$$

This mean-field prediction for the increase in the threshold can be checked by fabricating weakly disordered Josephson-junction arrays of the appropriate coordination number.

In summary, we have shown that disorder in the Josephson coupling energies increases the normal-state sheet resistance threshold for the onset of global superconductivity in an array of Josephson junctions. This arises as a consequence of two independent effects. One, as quantatively given by Eq. (27), is a purely normal-state effect true for any weakly disordered resistance network: the measured normal-state sheet resistance is greater than that corresponding to the inverse of the mean local conductance. The other effect, the main result of this paper, is expressed through Eq. (24), which lowers the minimum mean local conductance that is required for the occurrence of global Copper-pair phase ordering. The sign of this latter effect is an immediate consequence of the convexity of the ground-state energy: Any weak perturbation, including disorder, which does not have a diagonal ground-state matrix element lowers the ground-state energy.

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