Observation of magnetic excitons and spin waves in activation studies of a two-dimensional electron gas

A. Usher
Department of Physics, University of Exeter, Stocker Road, Exeter EX4 4QL, United Kingdom

R. J. Nicholas
Clarendon Laboratory, University of Oxford, Parks Road, Oxford OX1 3PU, United Kingdom

J. J. Harris and C. T. Foxon
Philips Research Laboratories, Cross Oak Lane, Redhill, Surrey RH1 5HA, United Kingdom

(Received 14 July 1989)

Measurements of activated conductivity for well-resolved Landau levels at filling factors $v=1$, 2, and 3 reveal a large exchange contribution to the energy gap, both at $v=1$ and 3 (the familiar exchange-enhanced g factor) and for the first time demonstrate a similar contribution at $v=2$. These results provide a measure of the large-wave-vector limits of the spin-wave and magnetic-exciton dispersion relations. The surprisingly strong magnetic field dependences of the measured energies are caused by the disorder present in the system.

I. INTRODUCTION

Quantization of the energy levels of a two-dimensional (2D) electron gas in high magnetic fields leads to the appearance of the quantum Hall effect,[1] while the high electron densities in such systems make electron-electron interactions significant. The elementary excitations of the strongly quantized system consist of magnetoplasmons (with $m=1$) and spin-waves (with $m=0$, $\Delta m_s=1$) with energies

$$\hbar\omega(k)=m\hbar\omega_c+\Delta m_s g^*\mu_B B+\Delta(k,B) , \tag{1}$$

where ω_c is the cyclotron frequency and $g^*\mu_B B$ is the "bare" spin energy. The term $\Delta(k,B)$ represents the contribution of the excitonic binding to the energy of the excitation,[2,3] and is caused by the electrons' Coulomb interactions. Optical excitations such as cyclotron resonance and electron spin resonance measure the $k=0$ magnetic exciton and leave the system in a nonequilibrium state. This is the result of Kohn's theorem,[4] which says that center-of-mass motion (with $k=0$) is unaffected by electron-electron interactions, so that $\Delta(k=0)=0$. There has recently been a report[5] of the measurement of the magnetic-excitation densities of states in a light-scattering experiment in which a considerable breakdown of wave-vector conservation has occurred.

In this paper we consider the measurement of the ionization energies ($k\to\infty$ limit) of the magnetic excitons for both magnetoplasmons and spin waves. This is done by measuring the energy gaps at the integer quantum Hall condition, from thermal activation of the diagonal conductivity σ_{xx}. Thermal activation probes the population of unbound electrons in thermodynamic equilibrium, when the Fermi energy lies in the gaps between Landau or spin-split levels. It is already well known that spin splittings are strongly affected by electron-electron exchange which gives rise to the exchange-enhanced g factor.[6,7] Enhancements of over ten times the original bare spin splitting[7] have been observed. For electrical conduction to occur, the excitation produced (in this case a spin wave) must be ionized; an activated conductivity experiment therefore probes the large-k limit of the excitation dispersion relation, which corresponds to a well-separated electron-hole pair (in a magnetic field, $k=r/l_c^2$, where l_c is the cyclotron radius). We now extend this concept to the quantum Hall condition at $vn_s h/eB=2$, where it will be seen that the exchange energy again makes a substantial contribution to the activation energies. The magnitude of the exchange energy is of order E_c, the Coulomb energy, where

$$E_C=\frac{e^2}{4\pi\epsilon_r\epsilon_0 l_c} , \tag{2}$$

which is about 170 K at 10 T.

II. EXPERIMENTAL DETAILS

The present communication reports activation measurements of the energy gaps at Landau-level occupancies, $v=1$, 2, and 3, performed on four high-mobility GaAs-$Ga_{0.66}Al_{0.34}As$ heterojunctions (grown at the Philips Research Laboratories, Redhill) and spanning the electron concentration range 3×10^{10} to 2.7×10^{11} cm^{-2}, with mobilities from 1.5×10^5 to 2.6×10^6 $cm^2/V\,s$.[8] Temperatures in the range 0.5 to 12 K were measured using carbon-glass or germanium resistance thermometry, taking care to ensure that thermal equilibrium was established before measurements were made. The heterojunctions have Hall bar geometry, and so the measured values of ρ_{xx} were converted to σ_{xx} using

$$\sigma_{xx}=\frac{\rho_{xx}}{\rho_{xx}^2+\rho_{xy}^2} , \tag{3}$$

where ρ_{xy} has its exact quantized values, h/ie^2, when the ρ_{xx} minima are in their activated regions.

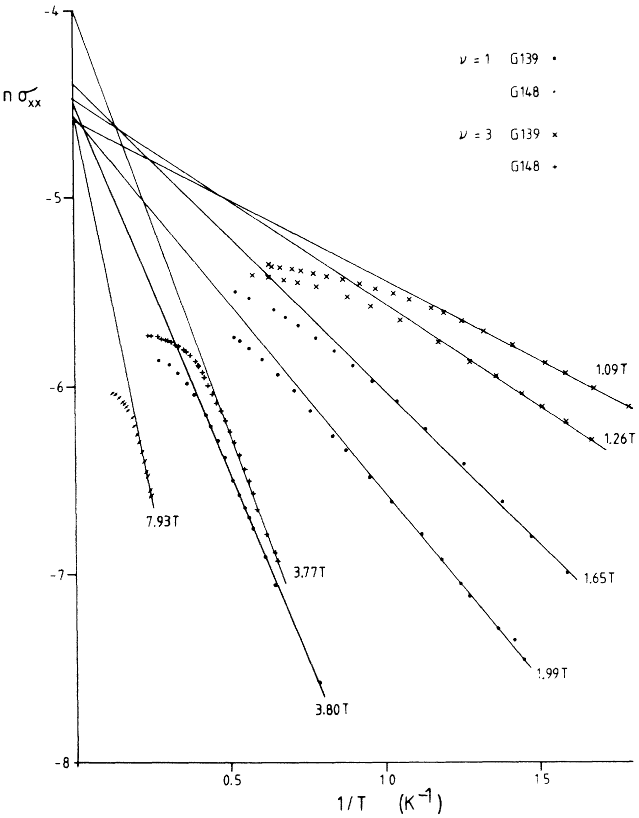

FIG. 1. Typical Arrhenius plots for $\upsilon=1$ and 3, for two samples (G139 and G148) at a variety of different electron concentrations and hence magnetic fields.

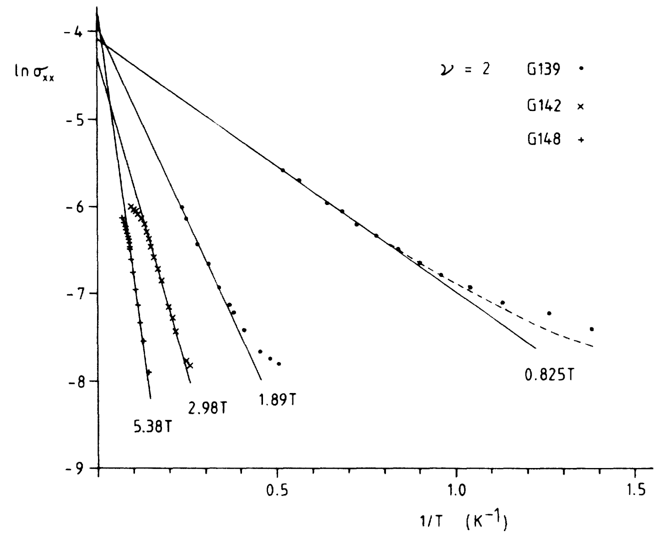

FIG. 2. Typical Arrhenius plots for $\upsilon=2$, for the three samples G139, G142, and G148. The dashed line shows the expected behavior when the Landau levels are broadened with a Gaussian form of half-width 3.4 K.

If the Landau levels are assumed to be discrete energy levels, and the energy gap ΔE satisfies $\Delta E \gg k_B T$, then σ_{xx} is given by

$$\sigma_{xx}=\sigma_0 \exp\left[-\frac{\Delta E}{2k_B T}\right], \tag{4}$$

where the prefactor σ_0 has been related to some minimum metallic conductivity[9–12] which is thought to be independent of temperature. The main objective of this paper is to discuss the energy gaps ΔE; however, the values of σ_0 as deduced from Arrhenius plots of $\ln(\sigma_{xx})$ versus 1/T, will be discussed in Sec. V. The data presented below are initially analyzed in terms of this simplified model which is found to be a good approximation at higher magnetic fields. A more careful analysis involving three possible models for the density of states in a disorder-broadened Landau level is given in Sec. V.

Typical Arrhenius plots obtained at $\upsilon=1$, 2, and 3 are shown in Figs. 1 and 2. All the plots have substantial straight-line regions, but curve away from the straight line at high temperatures when the ρ_{xx} minimum is poorly resolved and conduction is no longer activated. In addition, some curvature is evident at the low-temperature extremes of some of the plots. This is conventionally attributed to the onset of hopping; however, we will show in Sec. V that many aspects of this behavior can be attributed to the finite-level width.

III. MEASUREMENT OF THE EXCHANGE-ENHANCED g FACTOR AT $\upsilon=1$ AND 3

The energy gaps calculated from the Arrhenius plots for $\upsilon=1$ and 3, such as those in fig. 1, are plotted against magnetic field in Fig. 3. The energies for both occupancies lie on the same straight line, of slope 4.9 $\pm$0.2 K/T. Using $\Delta E=g^{**}\mu_B B$, this slope gives an enhanced g factor of 7.3, almost 20 times its bare value, 0.44, as measured by electron spin resonance (ESR) in bulk GaAs.[13] The enhancement factor is even greater compared with bare values found in heterojunctions.[14] This is the largest value of g^{**} so far reported in GaAs, which emphasizes the low level of disorder in these samples. Nicholas *et al.*[7] report a value of 6.23, but their 2D EG, with a mobility $\mu=4.5\times10^5$ cm^2/V s at $n=8.1\times10^{10}$ cm^{-2}, has somewhat more disorder than those in the present study. The effect of disorder on g^{**} is discussed in Sec. V in relation to the level populations and the spin-wave dispersion curve.

The origin of the exchange enhancement is the electron-electron interaction in the system, as calculated by Ando and Uemura,[15] who derived the formula

$$g^{**}\mu_B B=g^{*}\mu_B B+\sum_q \sum_{N'} \frac{V(q)}{\epsilon(q,0)}[J_{NN'}(q)]^2(n_{N'\uparrow}-n_{N'\downarrow}) \tag{5}$$

in which $V(q)$ is the Fourier transform of the Coulomb interaction, $\epsilon(q,0)$ is the static dielectric function of the electrons, and $n_{N'\uparrow}$, and $n_{N'\downarrow}$ are the populations of the two spin-split N'th Landau levels. $J_{NN'}$ is related to the wave-function overlap between electrons in the Nth and

N'th Landau levels. Equation (5) represents the energy required to excite a well-separated electron-hole pair with total spin 0 and is the large-k limit of the spin-wave dispersion relation show schematically in the inset of Fig. 3.

In the high-field case studied here we may ignore Landau-level overlap and set $N=N'$. The magnitude of the exchange energy may then be expected to be dominated by the spin-population difference and $J_{NN}(q)$. This should lead to a significant difference between different Landau levels. Furthermore, the density of states may also be broadened differently for different levels. In practice, however, the results shown in Fig. 3 suggest that the exchange enhancement is the same in the first two Landau levels, and is determined only by the total field. This appears to be rather different to recent calculations of Haldane and Rezayi,[16] who have calculated the pair correlation functions at short range for a system of six electrons in the $N=0$ and $N=1$ Landau levels. They find that in this regime the wave-function overlap is considerably smaller in the higher level.

Finally we note that the energy gaps of Fig. 3 are approximately linear in B. Since the Coulomb interaction gives a $B^{1/2}$ dependence, our observations suggest that there is an additional B dependence caused by other factors. A strong possibility is the overlap of levels in the presence of disorder giving a nonsaturated population difference between levels, particularly at low fields. This would be in agreement with activation,[17–19] capacitance,[20] specific-heat,[21] and magnetic susceptibility[22] studies of the Landau-level density of states. These conclude that there is a significant tail to the density of states which can be accounted for by the addition of a constant background to a series of Gaussian peaks. Such a background may be explained by assuming that the 2D EG contains long-range spatial inhomogeneities.[23,24]

Disorder with a length scale comparable to the magnetic length will also have a direct effect on $J_{NN'}$ effectively screening electron-hole pairs which are separated by a distance greater than the disorder length. This is discussed further in Sec. V.

IV. EXCHANGE ENHANCEMENT OF THE LANDAU-LEVEL SPLITTING AT $v=2$

Figure 4 shows a plot of ΔE versus B for the energy gap between the ($N=0$, $\downarrow$) level and the ($N=1$, $\uparrow$) level. Neglecting the Coulomb interaction, this energy would simply be the single-particle energy $\Delta E=\hbar\omega_c-g^*\mu_B B$ (where g^* is the bare g factor, and $\hbar\omega_c \gg g^*\mu_B B$). This is the dashed line in the figure: except at very low fields, all the $v=2$ data lie well above this line, demonstrating clearly the importance of the exchange interaction at $v=2$. The exchange contribution, measured from the difference in slope between the activated energy gaps and the single-particle energy, is shown in the lower part of the figure, and at higher fields tends to 7 ± 1 K/T,

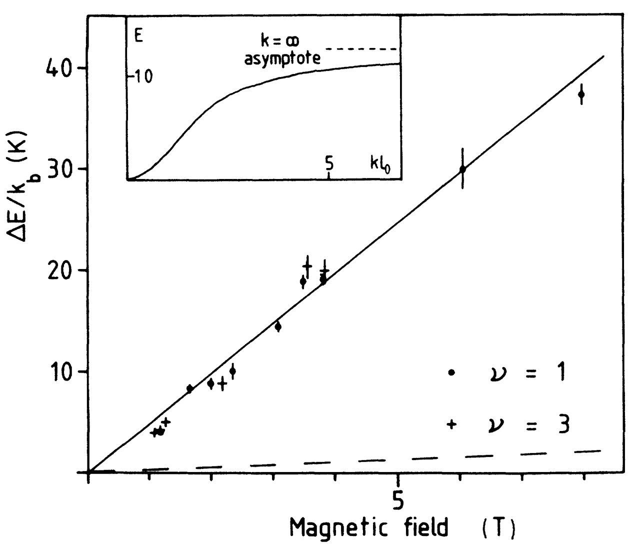

FIG. 3. A plot of ΔE vs B obtained from the activation measurements at $v=1$ and 3. The dashed line shows the bare Zeeman splitting. The inset is the dispersion relation for spin waves at this occupancy, with the energy in units of the Coulomb energy E_C.

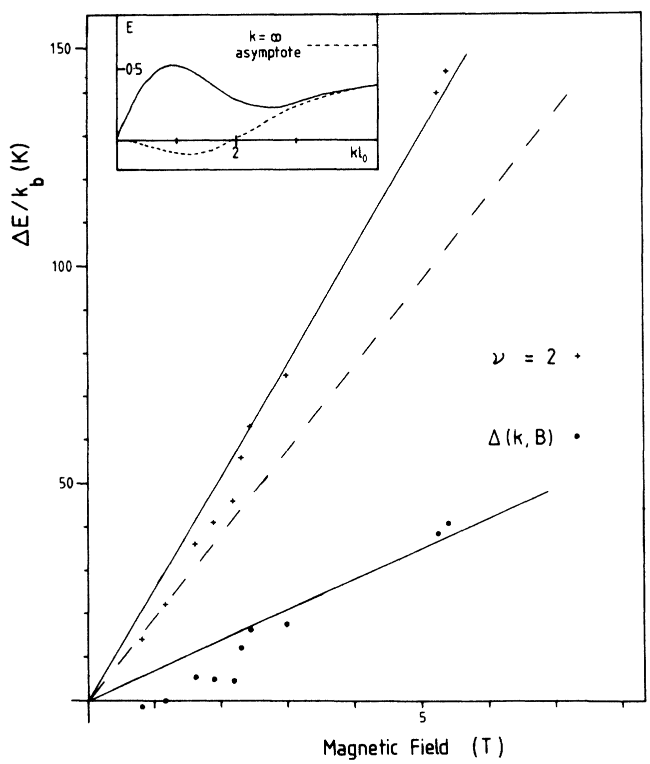

FIG. 4. ΔE vs B for the $v=2$ activation data (+). The dashed line is $\Delta E=\hbar\omega_c-g^*\mu_B B$, the single-particle energy separation in the absence of the exchange interaction. The dots show the exchange contribution $\Delta(k,B)$ obtained by subtracting this energy from the experimental data. The inset shows the singlet (solid line) and triplet (dashed line) magnetic exciton dispersion relations for $v=2$, in units of E_C.

significantly larger than the exchange contribution to the spin splitting at $\upsilon=1$ and 3.

Other authors[19] have noted that there can be some discrepancy between the activation and single-particle gaps, but have not directly associated the difference with the Coulomb interaction. Recently, Pinczuk *et al.*[5] have reported the observation of the magnetoplasma density of states in light-scattering experiments, but do not have a clear explanation of the mechanism for the breaking of wave-vector conservation.

Kallin and Halperin[2,3] have calculated the dispersion relations for the elementary neutral excitations of 2D electrons at $\upsilon=1$ and 2, with and without a reversal of the electron's spin. Following the discussion at the beginning of this communication, the energies measured above at $\upsilon=1$ and 2 should be the large-wave-vector asymptotes of the $(N=0,\uparrow)\rightarrow(N=0,\downarrow)$ (spin-wave) and the $(N=0,\downarrow)\rightarrow(N=1,\uparrow)$ (spin-reversed magnetic exciton) dispersion curves. Kallin and Halperin calculate these asymptotes to be $(\pi/2)^{1/2}e^2/4\pi\epsilon_0\epsilon_r l_c=68B^{1/2}$ kelvin for $\upsilon=1$, and half that value for $\upsilon=2$. These were calculated for a perfect 2D system and we would expect that inclusion of the finite z extent, as first done by Ando and Uemura,[15] will reduce the Coulomb interactions by about 50%, as has also been found for the fractional quantum Hall ground state.[25,26] Another important factor, as already mentioned above, is the influence of disorder.

The data shown in Fig. 4 give an exchange energy of about 40 K at 5.5 T, approximately 50% of the value calculated by Kallin and Halperin for the perfect 2D system at $\upsilon=2$, and thus in good agreement with theory, given the finite z extent. The more rapid decrease at lower fields (linear as opposed to $B^{1/2}$) is attributed to the effects of disorder. These may enter in two distinct ways: firstly, the density-of-state broadening will reduce the Landau-level population difference of the ground state of the system, thus reducing the exchange interactions, by analogy with Eq. (5); secondly, the disorder will break up the exchange interactions at longer range. From the estimates given below (and from other measurements on heterojunctions[17–22]) we do not expect much Landau-level overlap above 1 T. However, the disorder also reduces the ionization energy. Excited magnetic exciton states with radii $r>l_d$, where l_d characterizes the length scale of the disorder fluctuations, will be effectively ionized. This effect will be field dependent, since the important length scale in the calculations is the cyclotron radius, and so the magnetic excitons are larger at lower fields, sensing the disorder more. This is equivalent to defining a critical ionization length of order l_d, $\propto kl_c^2$, which gives a critical value of $k\propto l_d/l_c^2$. Thus the ionization energy falls faster than the expected $B^{1/2}$ dependence from the Coulomb energy.

The ideal dispersion curve is shown in the inset of Fig. 4. The turning points have been identified by Pinczuk *et al.*[5] in an elastic light-scattering experiment. Assuming a reduction due to the finite z extent of the order of 50%, they find quantitative agreement with the calculations of Kallin and Halperin.[2,3]

At $\upsilon=1$ and 3 the discrepancy is much larger, with the measured values being about 20% of the predictions for spin-wave excitations. In this case the energy splittings are substantially smaller (typically 10–20 K) and we expect there to be a significant overlap of the two adjacent spin states. An overlap of about 30% of the total states would be sufficient to reduce the exchange terms by a further factor of 3, through the term $(n_{N'\uparrow}-n_{N'\downarrow})$ in Eq. (5). This is consistent with reasonable values of the level broadening, of order 10 K.[17–22]

Thus we may account for the magnitude and B dependence of the energy gaps using the theory of Kallin and Halperin as a starting point and considering the effects of disorder on both the population difference $(n_{N'\uparrow}-n_{N'\downarrow})$ and the wave vector of the ionized electron-hole pair.

V. LANDAU-LEVEL BROADENING

The energies in Fig. 4, and to a lesser extent those in Fig. 3, fall below the straight lines at very low fields. This behavior is caused by broadening of the Landau levels. An estimate of the Landau-level widths at $\upsilon=1,3$ and $\upsilon=2$ may be obtained by extrapolating from these data to zero field; the full widths obtained by this method are $\Gamma=3$ K for $\upsilon=1,3$ and $\Gamma=10$ K for $\upsilon=2$.

A more thorough analysis of these Arrhenius plots allows us to derive both the width and the approximate shape of the Landau levels. It should be emphasized that the width and shape deduced will be those of the extended state region, in contrast to those measured in Refs. 17–22. According to theories of localization, the extended states only account for a few percent of the total number of states, and lie close to the center of the levels.

If the upper Landau level has a density of states $D(E)$, then the number of electrons populating the upper level is given by

$$n=\int_{E_F}^{\infty}f(E)D(E)dE\ , \tag{6}$$

where $f(E)$ is the Fermi-Dirac distribution function. If we assume that $\sigma\propto n$, then this equation reduces to Eq. (4) when $D(E)$ is a δ function and the energy gap $\Delta E\gg k_BT$.

The $\upsilon=2$ and 3 activation measurements discussed above have been reanalyzed using three models for $D(E)$. The $\upsilon=1$ data are omitted from this analysis because, with the exception of one point, all the data in Fig. 4 lie close to the straight line. The models used for the shape of $D(E)$ are rectangular, Gaussian, and Lorentzian: the Lorentzian distribution has a large number of states in its tails, and represents an approximation to the Gaussian plus constant background $D(E)$ which fits the experiments of Refs. 17–22 and is caused by the occurrence of long-range inhomogeneities in the 2D EG.[23,24] The Gaussian distribution is obtained from self-consistent treatments of short-range scattering, with the inclusion of higher-order scattering processes.[27] Finally, the rectangular distribution is suggested by theories of localization which show that extended states occupy a narrow band at the center of the total $D(E)$, with a sharp cutoff at the "mobility edge."

The results of these analyses are as follows: the Gaussian and rectangular models both fit the Arrhenius plots for the $\upsilon=2$ and $\upsilon=3$ data very well. In particular, they produce a low-temperature curvature similar to, or slightly less than, that of the data. A typical fit is shown in Fig. 2. The Lorentzian model produces too much low-temperature curvature and may therefore be discarded. The average values deduced for $\Gamma_{1/2}$ at $\upsilon=2$ and 3 are shown in Table I; the difference in the broadening obtained from the two models is related to the different definitions of half-width in the two cases. The Gaussian distribution has tails outside the region $E=\Delta E/2\pm\Gamma_{1/2}$, whereas the rectangular one does not. Much more significant is the large difference in broadening for the two occupancies; at $\upsilon=2$ the widths are 3–5 times larger than at $\upsilon=3$. We attribute this to the suppression of screening at complete filling of separate Landau levels. In contrast, the spin-split levels are thought to have considerable overlap, as discussed above, and the screening therefore remains effective. This idea was first introduced to explain a filling-factor dependence of the cyclotron resonance linewidth[28]; the samples studied here have a strong peak in the cyclotron-resonance linewidth at $\upsilon=2$ which can be over five times larger than at $\upsilon=1$, but have a less pronounced difference for $\upsilon=2$ and $\upsilon=3$.[29] The origin of the large linewidth at $\upsilon=2$ lies in the inability of a completely filled Landau level to respond to local potential fluctuations which are small compared with the energy splitting. For the spin-split case, the splitting is smaller and the overlap of adjacent levels leaves the electrons free to screen the local disorder potential. Finally, it should be noted that there appears to be a significant increase in $\Gamma_{1/2}$ with magnetic field.

The broadening is also partly responsible for the low-temperature curvature of some of the Arrhenius plots of Figs. 1 and 2. At low temperatures, electrons are thermally excited predominantly to the tails of the upper Landau level, and so the apparent activation energy is lower than that obtained from the higher temperature data.

An alternative mechanism which is often invoked to explain low-temperature curvature is that of variable range hopping, which Ono[30] has predicted to have the temperature dependence

$$\sigma \propto \frac{1}{T}\exp[(-T_0/T)^{1/2}] , \quad (7)$$

where T_0 depends on the density of states at the Fermi energy, and on the magnetic length. However, Ebert *et al.*[31] have made a thorough investigation of this effect at $\upsilon=3$, and find that although their low-temperature data fits Ono's formula, the resulting value of T_0 gives a density of states at the Fermi energy 36 times larger than that in zero field. Since, at $\upsilon=3$, the Fermi energy is in the gap between spin-split levels, this value is clearly unrealistic. The data presented in this paper do not probe this temperature region as closely, but would produce similar values for T_0, with a considerable sample- and electron concentration dependence, if analyzed in the same way. The experiments of Ebert *et al.*, and the alternative analysis of the low-temperature curvature in terms of Landau-level broadening given above, suggest that the observation of true hopping conduction may be more difficult than previously assumed.

TABLE I. The fitted "density-of-states" half-widths.

Minimum	Gaussian	Rectangular
$\upsilon=2$	4.8 K	6.7 K
$\upsilon=3$	0.95 K	2.3 K

The values of σ_0 deduced from the Arrhenius plots of Figs. 1 and 2 show considerable scatter. In general, these were found to be grouped around $(1-2)e^2/h$ with a scatter of typically a factor of 3. Illumination of the sample produced significant variations in the values of σ_0 obtained, presumably because of the change in disorder and homogeneity associated with persistent photoconductivity. Earlier measurements of the conductivity minima in silicon devices[9–12] attributed the value of σ_0 to the minimum metallic conductivity, but found a considerable spread of values of around 2 orders of magnitude. However, it has recently been suggested that this number is accurately quantized for the fractional quantum Hall case in $GaAs/Ga_{1-x}Al_xAs$ heterojunctions.[32] The analysis given at the beginning of this section suggests that σ_0 will be significantly affected by the density of states when the level broadening becomes comparable with ΔE. In addition, σ_0 is an inherently more difficult quantity to measure than ΔE; small deviations from thermal equilibrium have a marked effect on σ_0 while influencing ΔE only slightly.

VI. CONCLUSIONS

We report an enhanced g factor of 7.3, from activation experiments at Landau-level occupancies $\upsilon=1$ and 3, in $GaAs/Ga_{1-x}Al_xAs$ heterojunctions. An even larger exchange contribution to the Landau-level separation is observed at $\upsilon=2$. These energies correspond to the large wave-vector ionization limits of the collective excitations of the system, which are spin waves and magnetic excitons, respectively. Some discrepancies in the magnitude and magnetic field dependences of the ionization energies compared with simple theory are attributed to residual level overlap and disorder, with a further reduction due to the finite z extent of the electrons' wave functions.

The finite width of the extended state region of each level leads to anomalously low activation energies at low fields. Analysis of the results in this regime suggests that the extended states occupy a region, of the width of a few kelvins, at the center of each Landau level, and that few extended states occur outside this region.

[1]K. von Klitzing, G. Dorda, and M. Pepper, Phys. Rev. Lett. **45**, 494 (1980).
[2]C. Kallin and B. I. Halperin, Phys. Rev. B **30**, 5655 (1984).
[3]C. Kallin and B. I. Halperin, Phys. Rev. B **31**, 3635 (1985).
[4]W. Kohn, Phys. Rev. **123**, 1242 (1961).
[5]A. Pinczuk, J. P. Valladares, D. Heiman, A. C. Gossard, J. H. English, C. W. Tu, L. Pfeiffer, and K. West, Phys. Rev. Lett. **61**, 2701 (1988).
[6]K. von Klitzing and Th. Englert, Surf. Sci. **73**, 70 (1978).
[7]R. J. Nicholas, R. J. Haug, K. von Klitzing, and G. Weimann, Phys. Rev. B **37**, 1294 (1988).
[8]A. Usher, R. J. Nicholas, J. J. Harris, and C. T. Foxon, in *Proceedings of the 19th International Conference on the Physics of Semiconductors, Warsaw, 1988,* edited by W. Zawadzki (Polish Academy of Sciences, Warsaw, 1989), p. 151.
[9]R. J. Nicholas, R. A. Stradling, and R. J. Tidey, Solid State Commun. **23**, 341 (1977).
[10]S. Kawajii and J. Wakabayashi, Soild State Commun. **22**, 87 (1977).
[11]M. Pepper, J. Phys. C **10**, L173 (1977).
[12]R. J. Nicholas, R. A. Stradling, S. Askenazy, P. Perrier, and J. C. Portal, Surf. Sci. **73**, 106 (1978).
[13]C. Weisbuch and G. Hermann, Phys. Rev. B **15**, 823 (1977).
[14]M. Dobers, K. von Klitzing, and G. Weimann, Phys. Rev. B **38**, 5453 (1988).
[15]T. Ando and Y. Uemura, J. Phys. Soc. Jpn. **37**, 1044 (1974).
[16]F. D. M. Haldane and E. H. Rezayi, Phys. Rev. Lett. **60**, 956 (1988).
[17]E. Stahl, D. Weiss, G. Weimann, K. von Klitzing, and K. Ploog, J. Phys. C **18**, 783 (1985); H. P. Wei, A. M. Chang, D. C. Tsui, and M. Razeghi, Phys. Rev. B **32**, 7016 (1985).
[18]M. G. Gavrilov and I. V. Kukushkin, Pis'ma Zh. Eksp. Teor. Fiz. **43**, 79 (1986) [JETP Lett. **43**, 103 (1986)].
[19]D. Weiss, K. von Klitzing, and V. Mosser, in *Two-Dimensional Systems: Physics and New Devices,* Vol. 67 of *Springer Series in Solid State Sciences,* edited by G. Bauer, F. Kuchar, and H. Heinrich (Springer-Verlag, Berlin, 1986), p. 204.
[20]T. P. Smith, B. B. Goldberg, P. J. Stiles, and M. Heiblum, Phys. Rev. B **32**, 2696 (1985).
[21]E. Gornik, R. Lassnig, G. Strasser, H. L. Störmer, A. C. Gossard, and W. Wiegmann, Phys. Rev. Lett. **54**, 1820 (1985).
[22]J. P. Eisenstein, H. L. Störmer, V. Narayanamurti, A. Y. Cho, A. C. Gossard, and C. W. Tu, Phys. Rev. Lett. **55**, 875 (1985).
[23]R. R. Gerhardts and V. Gudmundsson, Phys. Rev. B **34**, 2999 (1986).
[24]V. Gudmundsson and R. R. Gerhardts, Phys. Rev. B **35**, 8005 (1987).
[25]F. C. Zhang and S. Das Sarma, Phys. Rev. B **33**, 2903 (1986).
[26]D. Yoshioka, J. Phys. Soc. Jpn. **55**, 885 (1986).
[27]T. Ando, J. Phys. Soc. Jpn. **37**, 622 (1974).
[28]Th. Englert, J. C. Maan, Ch. Uihlein, D. C. Tsui, and A. C. Gossard, Solid State Commun. **46**, 545 (1983).
[29]R. J. Nicholas, M. A. Hopkins, D. J. Barnes, M. A. Brummell, H. Sigg, D. Heitmann, K. Ensslin, J. J. Harris, C. T. Foxon, and G. Weimann, Phys. Rev. B **39**, 10 955 (1989).
[30]Y. Ono, J. Phys. Soc. Jpn. **51**, 237 (1982).
[31]G. Ebert, K. von Klitzing, C. Probst, E. Schuberth, K. Ploog, and G. Weimann, Solid State Commun. **45**, 625 (1983).
[32]R. G. Clark, J. R. Mallett, S R Haynes, J. J. Harris, and C. T. Foxon, Phys. Rev. Lett. **60**, 1747 (1988).