

Supercurrent tunneling between conventional and unconventional superconductors: A Ginzburg-Landau approach

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We investigate the Josephson tunneling between a conventional and an unconventional superconductor via a Ginzburg-Landau theory. This approach allows us to write down the general form of the Josephson coupling between the two superconductors, and to see which terms are forbidden or allowed by spatial symmetries. The time-reversal symmetry is also considered. We discuss the current-phase relationships, magnetic, and ac effects if we just include this direct coupling to the unconventional superconductor. In addition we consider the Josephson coupling between two short-coherence-length superconductors, extending the work of Deutscher and Müller (DM) to a finite-current calculation. We find that the critical current is suppressed below the DM value due to the fact that the coupling between the two superconductors across the junction depends on the phase difference and hence the current itself. Finally we investigate the possibility of the proximity effect, in particular the possibility that the conventional-type pairing is induced and hence coexists with the unconventional pairing near the junction. This would give the dominant contribution to the tunneling current if the direct tunneling to the unconventional pairs are suppressed for some reason. We point out that there is no possibility of dissipationless tunneling above the transition temperature of the unconventional superconductor. Even in the case in which the unconventional superconductor is below its transition temperature, we find that, for the possibility of a dissipationless current, it is crucial to have a coupling between the induced s wave and the unconventional superconductor that depends on their phase difference, which allows the conversion of the supercurrent from one type to the other. The behavior of this current, in particular as a function of temperature, is discussed. We also discuss the magnetic and time-dependent effects of the junction in the presence of this proximity effect. We see that, while some of these remain unaffected, some, in particular the time-dependent processes, are affected in a rather nontrivial manner.

I. INTRODUCTION

Heavy-fermion superconductivity^{1,2} and the more recently discovered high-temperature superconductivity³ have stimulated studies in unconventional superconductivity. Besides the possible nonphonon mechanism for the superconductivity itself, there is suggestive evidence that in these materials the electrons condense into an unconventional pairing state, with the gap function of a lower symmetry than that of the underlying lattice.⁴ Various efforts have been made on both how to distinguish experimentally such a nontrivial pairing state from the conventional (“ s -wave”) superconductivity, and how to identify which state, allowed by the symmetry classifications,⁵⁻⁸ actually exist. In the present paper, we would like to expand on one of the observables, namely the Josephson tunneling of electron pairs.

Pals, von Harington, and van Maaren⁹ have suggested that there should be no pair tunneling between a singlet and a triplet superconductor. However, later it was shown that this is not true when spin-active barriers are considered.^{10,11} The nonconservation of (pseudo)-spin, i.e., that the total spin S of a Cooper pair is not a good quantum number in the presence of such a junction, al-

lows Josephson coupling between a singlet and triplet superconductor. By the same argument (when the crystal symmetry is not considered), since the total angular momentum L of a Cooper pair is not conserved in the presence of the junction, the consideration of L does not forbid coupling between superconductors of different L states.

It is clear from the last paragraph that a useful consideration is that of the symmetry operations that leave the junction invariant. Only these operations can allow us to assign “good” quantum numbers for the tunneling junction and thus consider the possible Josephson tunneling between them.

This idea has been discussed by Kurkijarvi¹² and by Rainer and Sauls¹¹ in the quasiclassical formalism. In the present paper we expand on this idea using the phenomenological Ginzburg-Landau (GL) approach which allows us to easily write down the possible Josephson couplings, and to deduce the corresponding current-phase relationships.

The Ginzburg-Landau approach for superconductivity in the bulk material¹³ is well known. The main idea is to write down the most general terms of the free energy allowed by the underlying symmetry. Such a consideration

for superconductivity leads to the following well-known expression [$\psi = \Delta(\mathbf{r}) \exp[i\phi(\mathbf{r})$]:

$$F = \int \left[a |\psi(\mathbf{r})|^2 + b |\psi(\mathbf{r})|^4 + \frac{K}{2} |\nabla \psi(\mathbf{r})|^2 \right] d\mathbf{r}, \quad (1.1)$$

where $\psi(\mathbf{r})$ is the relevant order parameter, a , b , K are phenomenological coefficients, a is negative (positive) for the temperature T below (above) the critical temperature T_c . Similar considerations allows us to easily write down the possible (Josephson-type) coupling term between two (conventional) superconductors connected by a junction (taken to be the plane at $x = 0$)¹⁴⁻¹⁶

$$F_J = -t [\psi_L^*(x=0^-) \psi_R(x=0^+) + \text{c.c.}] \\ = -2t \Delta_L(x=0^-) \Delta_R(x=0^+) \cos(\phi_R - \phi_L), \quad (1.2)$$

where Δ_L , Δ_R are the gap parameters, and ϕ_L and ϕ_R , their phases on the left and right of the junction, respectively (see Fig. 1). Minimization of the total free energy then gives us the current-phase relationships. We shall consider other possible terms in F_J in Sec. II. A term of the form given in (1.2) gives rise to the well-known Josephson tunneling effects between two ordinary s -wave superconductors. In Sec. III A we discuss some special effects if the coherence length(s) of the superconductor(s) involved is short. The coupling (1.2) also leads to the proximity effect, and perhaps also the controversial proximity-induced Josephson tunneling¹⁷⁻²⁴ (PIJT). This arises when, say, the left, strong, superconductor is already in the superconducting state whereas the right side is not. Thus, in Eq. (1.1) $a_L < 0$ and $a_R > 0$. It is energetically favorable to have Δ_R induced in the vicinity of the junction, despite the fact that in the bulk there is no superconductivity allowed by itself. We shall discuss this in Sec. III B.

Even more interesting is the possibility when the s -wave superconductor on the left induces an s -wave pairing in the right superconductor, which has its instability in some other pairing state which may also couple to the left, via terms also of form (1.2) (or its generalization, see below). We find that in this case, the supercurrent can tunnel *both* by the induced term and the direct coupling between the two superconductors; with the former transformed into a supercurrent solely carried the non-conventional pairs away from the junction. Crucial in this respect are the free-energy terms that couple the induced s wave and the unconventional pairs which depend on their phase difference. Thus, the phase angle of the in-

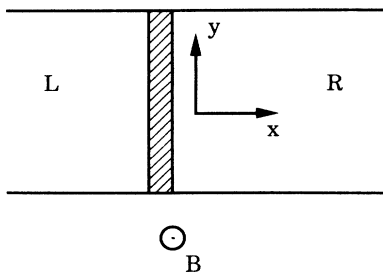


FIG. 1. The configurations of the Josephson junction. The magnetic field will be introduced in Sec. IV.

duced s wave will be a function of that of the p - or d -wave pairs. Depending on the temperature, the signs and magnitudes of the various coefficients, the two types of tunneling can assist or compete with each other. These shall be discussed in Secs. III C and III D.

In Secs. IV and V we turn to the magnetic-field effects and the ac effects. We find that a possible SQUID arrangement (suggested first by Geshkenbein and Larkin^{25,26}) can be an unambiguous test for odd-parity triplet pairing, though with some complication in the presence of the PIJT mentioned. In some particular orientation, where by symmetry considerations some of the Josephson tunneling terms vanish [see (2.3) below] we shall see that this test fails. In this latter case there are characteristic differences from the conventional case, such as in the field dependences of maximum supercurrent through a planar junction, which are not complicated by the presence of the PIJT. However, for the time-dependent effects, we shall see that there are substantial complications in the presence of PIJT. The fact that there are two superconducting order parameters leads to complications in the dynamics of the junction.

The use of Josephson tunneling as a probe, however, is not without its disadvantages. It is, in particular, a probe of the order parameter only at the vicinity of the junction. Phenomena such as surface pair breaking and textures which may complicate the interpretation of a Josephson experiment are discussed in the final section.

II. SYMMETRY CONSIDERATIONS

In this section we use symmetry considerations to deduce the possible terms of the Ginzburg-Landau free energy which couple the order parameter at the vicinity of the junction and do involve the phase difference between the two order parameters. Using this we discuss the possible current-phase relationships for the *direct* Josephson tunneling between a conventional (L) and an unconventional (R) superconductor. Both spatial and time reversal symmetries will be considered.

For definiteness we always consider the unconventional pairing, if any, with a particular fixed orientation, and write the gap matrix as

$$\Delta_{\sigma\sigma'}(\mathbf{k}) = \psi_R(i\sigma_y) f(\mathbf{k}) \quad (2.1a)$$

or

$$\Delta_{\sigma\sigma'}(\mathbf{k}) = \psi_R(i\mathbf{d}(\mathbf{k}) \cdot \boldsymbol{\sigma}) \sigma_y \quad (2.1b)$$

for singlet and triplet pairing, respectively, for the order parameter on the right [i.e., we consider given $f(\mathbf{k})$ or $\mathbf{d}(\mathbf{k})$] and analogously for the order parameter on the left. When convenient we shall write $\psi_{R,L} = \Delta_{R,L} e^{i\phi_{R,L}}$ and shall call ϕ_R, ϕ_L the phases of the order parameters. Angular momentum is not a good quantum number in the presence of the crystal symmetry. We shall loosely refer to (2.1a) as s or d wave according to whether $f(\mathbf{k})$ has the same symmetry as the crystal or not, and refer to (2.1b) as p wave.

Since the free energy must be invariant under global gauge transformation, the tunneling terms can only involve $\psi_R^* \psi_L$, or its integral powers and their complex

conjugates. Thus, in general,

$$F_J = F_J(\phi)$$

with

$$F_J(\phi) = F_J(\phi + 2\pi), \quad (2.2a)$$

where $\phi = \phi_R - \phi_L$, and hence by $J = (2/\hbar)(\partial F_J/\partial \phi)$, where J is the current,

$$J(\phi) = J(\phi + 2\pi) \quad (2.2b)$$

and is periodic in ϕ with period 2π .

The most general form of the tunneling term in the Ginzburg-Landau free energy, a real quantity, is of the form

$$F_J = -t_1 \psi_L^* \psi_R + \text{c.c.} - t_2 \psi_L^* \psi_R^2 + \text{c.c.} - \dots, \quad (2.3a)$$

where, in general, t_1, t_2, \dots , can be *complex*; and even when real, not necessarily positive or negative. All terms are up to factors of Δ_R^2 and Δ_L^2 which amounts to temperature dependent t_1, t_2, \dots . Omitting these factors (2.3a) reads

$$\begin{aligned} F_J = & -\Delta_R \Delta_L [\text{Re}(t_1) \cos \phi - \text{Im}(t_1) \sin \phi] \\ & - \Delta_R^2 \Delta_L^2 [\text{Re}(t_2) \cos 2\phi - \text{Im}(t_2) \sin 2\phi] - \dots \end{aligned} \quad (2.3b)$$

When $f(\mathbf{k})$ and $d(\mathbf{k})$ are real up to an overall phase factor (independent of \mathbf{k}), then the time-reversal symmetry operation transforms the states back to themselves, except with the phase factor $\phi_{R,L} \rightarrow -\phi_{R,L}$ (see the Appendix). In this case then, since the free energy must be invariant under such an operation, and therefore it must be even in the phase difference $\phi \equiv \phi_R - \phi_L$:

$$F_J(\phi) = F_J(-\phi). \quad (2.4a)$$

In this case then, all t_1, t_2, \dots , are real and we have

$$J(\phi) = -J(-\phi). \quad (2.4b)$$

Similar conclusions apply when the state is complex, but when there exists a spatial symmetry operation, say S , which when combined with the time reversal, does leave the states invariant, then, in this case, we have the relation (see the Appendix)

$$F_J(\phi) = F_J(-\phi + \alpha), \quad (2.5a)$$

$$J(\phi) = -J(-\phi + \alpha), \quad (2.5b)$$

where α is some constant depending on S . By redefining the phase of ψ_R [by multiplying $f(\mathbf{k})$ or $d(\mathbf{k})$ by a complex number of unit magnitude], we again obtain (2.4), and hence, in this new definition, t_1, t_2, \dots , is again real to all orders.

In most cases, however, such an operation S does not exist. In those cases (2.3) is the most general form (we can, of course, get rid of one of them, say, $\text{Im}t_1$, by redefining the phase of Δ_R), and the energy of junction is not necessarily even, and the current not necessarily odd

in ϕ .^{25,26}

Thus, note that time-reversal consideration *alone* does *not* forbid tunneling among s , p , or d waves (see also the Appendix).

Next we consider spatial symmetries. For this we can just concentrate on the unconventional superconductor. Since our probe (L) is assumed to be s wave and impurities in the probe introduce mixing among the Bloch waves, for practical purposes the underlying crystal symmetries are irrelevant for our probe, or any s wave superconductor. Since the systems (the potentially unconventional superconductors) involved are believed to have strong crystal-field effects, we shall only consider symmetries of the underlying crystals. The operations of particular interest belong to the subset that (i) leave the junction unchanged, and for the moment we also confine ourselves to those which (ii) leave the state invariant, i.e., leave the order parameters invariant up to a phase factor. In the case where the operations satisfying the former requirement does not exist, then there is no restriction, and all the terms listed in (2.3) can exist.

When a crystal spatial symmetry that leaves the junction invariant does exist, we have to find whether any such operation changes the order parameter only up to a phase factor. This can be done easily by going through the symmetry group of the crystal and the group representation of the relevant order parameter and the character table. (We shall defer the case where the operations map the order parameter into different configurations.) If all such operations leave the order parameter invariant then, again, no restrictions will apply. However, occasionally some operations change the phase of, say ψ_R but not ψ_L , and thus forbid the term t_1 . For example, when there is an operation that changes the sign (i.e., add π to the phase factor) of one of the order parameters, say R , but not the other (L), then, in this case, F_J is invariant under $\phi \rightarrow \phi + \pi$,

$$\begin{aligned} F_J(\phi) &= F_J(\phi + \pi), \\ J(\phi) &= J(\phi + \pi) \end{aligned} \quad (2.6)$$

since the symmetry operations amount to $\phi_R \rightarrow \phi_R + \pi, \phi_L \rightarrow \phi_L$. The energy and current are periodic in ϕ with period π and all t_j with j odd vanish. Explicit examples include, for illustration, polar phase $\hat{z}k_z$ with the tunneling plane normal in the \hat{x} direction for any crystal with reflection symmetry about the x - y plane or x - z plane; the state $(k_x + ik_y)^2$ with the tunneling plane normal along \hat{x} when \hat{z} is a fourfold rotation axis. The A phase, $d(\mathbf{k}) = \hat{d}(\hat{m} + i\hat{n}) \cdot \hat{k}$ with $\hat{l} \equiv \hat{m} \times \hat{n}$ parallel to the boundary, with a reflection plane of symmetry perpendicular to \hat{l} , and \hat{d} lies in (but not perpendicular to) this symmetry plane.

The extension of these discussions to other cases is obvious. For example, when there exists a threefold rotation axis along the interface normal (in suitably oriented hexagonal or cubic crystal), and if the state involved transforms into itself up to a factor of $e^{\pm i(2\pi/3)}$, then F_J must be invariant under $\phi \rightarrow \phi \pm (2\pi/3)$:

$$F_j(\phi) = F_j \left[\phi \pm \frac{2\pi}{3} \right], \quad (2.7)$$

$$J(\phi) = J \left[\phi \pm \frac{2\pi}{3} \right],$$

and all t_j with j not a multiple of 3 will be excluded. The energy and current are periodic in ϕ with period $2\pi/3$. Explicit examples included the state

$$(\hat{z}k_y + k_z\hat{y}) + e^{i(2\pi/3)}(\hat{x}k_z + \hat{z}k_x) + e^{-i(2\pi/3)}(\hat{x}k_y + \hat{y}k_x)$$

for tunneling along the cubic diagonal $\hat{x} + \hat{y} + \hat{z}$. Similarly, when the analogous fourfold rotation of the above exists, then

$$F_j(\phi) = F_j \left[\phi \pm \frac{\pi}{2} \right], \quad (2.8)$$

$$J(\phi) = J \left[\phi \pm \frac{\pi}{2} \right],$$

and the energy and current are periodic with period $\pi/2$. Examples include an A phase with \hat{l} and \hat{d} along the surface normal which is a fourfold rotation axis. In this case all t_j with j not a multiple of 4 vanish.

Before proceeding further we make contact with the alternative way of representing the order parameter, namely as a tensor A in spin and orbit space for p and orbit space alone for d wave. In the case of tunneling, the normal to the tunnel junction provides a spatial vector. If there is no spin-orbit coupling, the spin index of the order parameter of a p wave can only be contracted with itself. Thus, there is no invariant first order in A , accordingly t_1 vanishes identically.⁹ If there is spin-orbit coupling, one cannot distinguish spin and orbit space by symmetry. Furthermore, spatial indices can be contracted with the surface normal. A p behaves identically, as far as symmetry is concerned, with a corresponding d wave with spin vectors replaced by the appropriate spatial vectors^{10,11} (with the exceptions of improper rotations, which give a sign difference). The results discussed above for t_j can also be obtained by this consideration.

A similar argument applies to the case with both L and R unconventional; for example, if both L and R are odd under a reflection symmetry of the junction, $\phi_{R,L} \rightarrow \phi_{R,L} + \pi$, then there is no restriction on (2.3), whereas if one of them is odd and the other even, (2.6) applies.

So far we have confined ourselves to the case that the symmetry operations leave the states invariant. For those operations which do not, they simply give us a relation between the physics of two (inequivalent) junctions (cf., transport across the ^3He A - B interface).²⁷ Since a group contains all inverses of its elements and the products of any two of them, any restriction on a given junction by these symmetries has already been included by the consideration of some other symmetries above. By the same reasoning, the consideration of the combination of time reversal and various spatial symmetries will be contained in the particular one combination that does

leave the state invariant [i.e., (2.5)] if we have used an odd number of these, or just a restriction that we have already obtained by spatial symmetries alone if we have used an even number of these operations. We have thus exhausted all restrictions.

Though a symmetry operation of the crystal that does not leave the state invariant does not put restriction on the current-phase relationship on a given junction but relates the physics of different junctions, can occasionally give some interesting results. An interesting example has been considered by Geshkenbein and Larkin.^{25,26} Two junctions, inversion related, with one phase odd and the other even in inversion, the energy of the junctions are related by, for the same ϕ ,

$$F_1(\phi) = F_2(\pi + \phi), \quad (2.9)$$

whereas we can always choose F to be minimum at $\phi = 0$ for one of the junctions, using the same definition for $\Delta_{\sigma\sigma}$, it will be minimum at $\phi = \pi$ for the other.

For the case with the even phase (singlets) the same consideration leads to

$$F_1(\phi) = F_2(\phi). \quad (2.10)$$

An interesting consequence of the difference between (2.9) and (2.10) will be explored in Sec. IV.

So far we have discussed the tunneling coupling completely phenomenologically. One may, of course, resort to microscopic theory. We shall confine ourselves to the simplest calculation, which views the Josephson coupling as the ‘‘anomalous’’ term in the perturbation theory of the one-particle tunneling Hamiltonian²⁸

$$H_1 = \sum_{k,p} T_{kp} c_k^\dagger c_p + \text{c.c.}$$

Here and below we shall simplify our notation by dropping all spin indices. The lowest-order Josephson coupling arising from the terms in the second-order perturbation theory:

$$- \sum_m T_{k,p} T_{-k,-p} c_k^\dagger c_p \frac{|m\rangle\langle m|}{E_m} c_{-k}^\dagger c_{-p} + \text{c.c.},$$

where m is an intermediate state and E_m its energy over the ground state. It can easily be checked that this term vanishes when the appropriate symmetries discussed above (2.6) exist. In this case there is no contribution from second-order perturbation theory and t_1 vanishes. One may easily convince oneself that t_j arises as the $2j$ th-(and possibly higher) order perturbation theory of the tunneling Hamiltonian and can, in general, exist unless symmetry forbidden. One should notice that by this argument we see that when there is no special symmetry restriction, in general,

$$t_1 \Delta_R \Delta_L \gg t_2 \Delta_R^2 \Delta_L^2 \gg \dots$$

Some comments on the present experimental status are in order. For the heavy-fermion superconductors coherent tunneling has been observed in CeCu_2Si_2 - Al junctions, with the critical current J_c comparable to that expected from two s -wave superconductors assuming a

Bardeen-Cooper-Schrieffer (BCS) gap magnitude for CeCu_2Si_2 .²⁹ The UBe_{13} experiments are the ones which produced the so far controversial proximity effects as mentioned previously, whereas for UPt_3 only null effects have been found.²⁹ Microscopic theory has shown that J_c for s - p coupling, when allowed in the lowest order [see (2.3)], is expected to be much smaller. Hence, unless CeCu_2Si_2 has a gap magnitude substantially larger than the BCS value, we can conclude that it is an s or d -wave superconductor. For the UBe_{13} , it is claimed that there is a Josephson current, increasing with decreasing temperature, even when the temperature is above the UBe_{13} transition temperature,¹⁷ and this tunneling current goes through a maximum at that transition temperature and then decreases with decreasing temperature.¹⁸ This was interpreted by the original authors as tunneling via the proximity-induced s -wave order parameter, which is suppressed by the developing p wave below the transition temperature of UBe_{13} . We shall point out that the necessity that the tunneling current has to pass from the induced order parameter to something else nondissipatively is left out in these references. A more careful analysis leads to behavior at odds with the experiment. Further work, especially on single crystals, will be needed. The null effect in UPt_3 or a very small J_c is, of course, suggestive of a nontrivial, especially a p -wave, pairing.

The situation for the high- T_c materials is just as inconclusive. Deutscher and Müller,³⁰ Estève *et al.*,^{31(a)} as well as Kuznik *et al.*^{31(b)} have suggested that the Josephson I-V characteristics and Shapiro steps observed are due to coupling *internal* to the polycrystal, and, as it will be clear below, this gives no information on the pairing state of the superconductors (in contrast to some other published reports). However, Tsai *et al.*³² have maintained that their measurement, especially J_c as a function of temperature, is due to coupling between their Nb probe and the 1:2:3 compound. They suggested an s -wave pairing to explain their result, but as we have seen, their experiment only suggests that the lowest-order coupling exists. Note also that due to the short-coherence length of the material, the critical current J_c for a spin-active barrier if the 1:2:3 is a triplet, is expected to be comparable to that if it is a singlet.

III. ORDER PARAMETER AND TUNNELING CURRENT

In this section we study the detailed consequences of two principal concerns of this paper: the short-coherence length and the symmetry-determined form of the Josephson free energy as discussed in Sec. II. We shall first consider the case of two s -wave superconductors (Sec. III A) where we shall investigate some consequences of a short-coherence length. The rest of the subsections are concerned with the proximity effect. We first consider the case where only the left superconductor is below the critical temperature (Sec. III B). We discuss the proximity-induced superconductivity for $x > 0$, in particular, the possibility (or rather the absence) of a proximity-induced

Josephson current. In Sec. III C we discuss the Josephson coupling between the s -wave superconductor and an odd-parity (p) superconductor, in particular, the indirect coupling via the induced s wave which is also present on the right due to the s wave on the left. In Sec. III D we discuss the corresponding cases for the nontrivial even-parity (d -wave) symmetry.

A. The case of short-coherence length

In this section we consider the coupling between two conventional, albeit small, coherence-length superconductors. Deutscher and Müller³⁰ (DM) have argued that the suppression of the order parameter near the surface is important in the case of short-coherence length and obtained a rapid fall in the order-parameter temperature dependence near the junction, especially near T_c . Our discussion below is an extension of this in that the further suppression of the order parameter near the surface by the current is explicitly included.

We shall consider the case where the two superconductors on the two sides of the junctions are identical. The free energy is

$$F = F_L + F_R + F_J + F_S, \quad (3.1)$$

where F_L and F_R are the bulk free energies of the superconductor on the left and right, respectively [Eq. (1.1)]. The Josephson coupling F_J is as in Eq. (1.2). Here we introduce the pair-breaking term

$$F_S = w [|\psi_L(0)|^2 + |\psi_R(0)|^2]. \quad (3.2)$$

F_J and F_S together can be considered as symmetric and antisymmetric combinations $t_{\pm} |\psi_L(0) \pm \psi_R(0)|^2$, where $w = t_+ + t_-$, $t = t_- - t_+$. If $t_+ = 0$ then the order parameter at zero current that minimizes the free energy is constant in space. In order that the order parameter is suppressed rather than enhanced at the interface, $t_+ \geq 0$ and hence $w \geq t$. As is obvious from (1.2) one can always redefine phase angles such that $t \geq 0$. Thus, we shall consider $w \geq t > 0$ in the following. The order parameter for the left can be obtained from the right by symmetry, and we shall thus concentrate on the right and drop subscripts R . Minimization of the free energy gives

$$2K\Delta^2 \frac{d\phi}{dx} = \text{const} = -j, \quad (3.3a)$$

$$K\Delta^2 \frac{d\phi}{dx} \Big|_{x=0} + 2t\Delta(0)^2 \sin\phi = 0, \quad (3.3b)$$

$$-|a|\Delta + 2b\Delta^3 + \frac{K}{2}\Delta \left[\frac{d\phi}{dx} \right]^2 - \frac{K}{2} \left[\frac{d^2\Delta}{dx^2} \right] = 0, \quad (3.3c)$$

$$-K \frac{d\Delta}{dx} \Big|_{x=0} + 2(w - t \cos\phi)\Delta(0) = 0, \quad (3.3d)$$

where $\phi \equiv [\phi_L(0) - \phi_R(0)]$. Equation (3.3d) can be written as

$$\frac{1}{\Delta} \frac{d\Delta}{dx} = \frac{2(w - t \cos\phi)}{K}, \quad (3.4)$$

which defines the extrapolation length in the spirit of Eq.

(1) of DM, including the effect of the current. Note that in our case this length depends on ϕ and hence the current. For convenience we rescale the variables using the equilibrium gap and the coherence length, thus defining

$$\Delta(x) = \left[\frac{a}{2b} \right]^{1/2} f(x), \quad (3.5)$$

$$y = x / \xi(T) = \left[\frac{2a}{K} \right]^{1/2} x. \quad (3.6)$$

Substituting (3.3a) into (3.3c), the resulting equation can be integrated to give

$$\left[\frac{df}{dy} \right] = \frac{1}{2f} (f_\infty^2 - f^2)(f^2 + 2f_\infty^2 - 2)^{1/2}, \quad (3.7)$$

where f_∞ is the gap in the bulk in the presence of j , satisfying

$$(j/j_0)^2 = f_\infty^4 (1 - f_\infty^2) \quad (3.8)$$

and $j_0 \equiv 2\xi a \Delta_0^2$.

Eliminating $\cos\phi$ via (3.3b) in (3.3d), we have, at $x=0$,

$$\left. \frac{df}{dy} \right|_0 = \left[\frac{2}{aK} \right]^{1/2} \left[w - t \left[1 - \frac{(j/j_{c0})^2}{f^4} \right]^{1/2} \right] f \Big|_0. \quad (3.9)$$

Here j_{c0} is the critical Josephson current *without* gap suppression at the junction, i.e., $j_{c0} = 2t \Delta_0^2$. (See below for the choice of the sign in form of t .)

To find the critical current one has to find the maximum j such that the simultaneous solution to (3.7) and (3.9) becomes impossible for higher values of j . If the square roots are well defined, then (3.7) and (3.9) together determine $f(0)$, and substituting back to (3.7) allows one to obtain $f(y)$ for all y . Thus, the condition for the existence of the solution is just that the square roots are well defined. Notice that if there were no junction, (3.7) still would apply and determine the critical current in the bulk corresponding to $f_\infty^2 = \frac{2}{3}$ and $(j/j_0)^2 = \frac{4}{27}$. Since we are interested in the case where the critical current j_c is determined by the junction, we assume that t is small enough that $(j_c/j_0)^2 \ll \frac{4}{27}$. Thus, $f_\infty \approx 1$ for our purposes. $f(0)$ thus satisfies

$$\sqrt{aK/2} [1 - f(0)^2] = \left[w - t \left[1 - \frac{(j/j_{c0})^2}{f(0)^4} \right]^{1/2} \right] f(0). \quad (3.10)$$

Notice that, as remarked, at $j=0$, w suppresses $f(0)$, and $t (> 0)$ has the opposite effect. Increasing j decreases the effective t in (3.10) and thus suppresses $f(0)$. These arguments justify the choice of the minus sign in front of t in Eq. (3.9).

First we consider the $j=0$ case. Equation (3.10) reads

$$\sqrt{aK/2} (1 - f^2) = (w - t)f. \quad (3.11)$$

To see the physics better, we shall consider only the two

limits (i) $w - t \gg \sqrt{aK}$, here

$$f(0) \approx \frac{\sqrt{aK/2}}{w - t} \ll 1, \quad (3.12a)$$

and (ii) $w - t < w \ll \sqrt{aK}$, here

$$f(0) \approx 1. \quad (3.12b)$$

These reproduce DM's result in our language. Case (i) is appropriate when one is very close to T_c , the temperature range being more significant when the (zero-temperature) coherence length is short and the pair-breaking $(w - t)$ is large. In this case the order parameter near the interface will be severely suppressed. In the opposite limit this suppression can be ignored.

Now we go to the case with a current. The presence of j changes the relative phase ϕ and hence the slope of the magnitude of the order parameter [see (3.4)], and hence the magnitude of $f(0)$ itself is current dependent. As remarked, a current depresses the effective t and hence $f(0)$ [see (3.10)]. These statements are self-consistent since a decreasing $f(0)$ makes the square root in (3.10) smaller and decreases the effective t . At the critical current the square root vanishes, i.e.,

$$j_c = j_{c0} f(0)^2, \quad (3.13)$$

and (3.10) becomes

$$\sqrt{aK/2} [1 - f(0)^2] = w f(0), \quad (3.14)$$

and hence for case (i)

$$f(0) \approx \frac{\sqrt{aK/2}}{w}, \quad (3.15a)$$

whereas for case (ii)

$$f(0) \approx 1. \quad (3.15b)$$

Notice that $f(0)$ is suppressed below its zero-current value in case (i), whereas it is unaffected in case (ii). The critical current is therefore, for case (i),

$$\begin{aligned} j_c &= j_{c0} \frac{aK}{2w^2} \\ &= 2t \Delta_0^2 [f(0)|_{j=0}]^2 \left[\frac{w - t}{w} \right]^2, \end{aligned} \quad (3.16)$$

i.e., it is reduced from the value $2t \Delta_0^2$ by two factors, one at zero current, and one extra factor in the presence of the current. Notice that since both Δ_0^2 and $f(0)^2$ are linear in $T - T_c$ [see (3.15a)], $j_c \propto (1 - T/T_c)^2$ near T_c . For case (ii),

$$j_c \approx j_{c0} = 2t \Delta_0^2 \quad (3.17)$$

and is just the standard value.

Thus, we have extended DM's result: For the short-coherence-length superconductors, the critical tunneling current is reduced due to the suppression of the gap near the interface. This magnitude of the gap is further reduced in the presence of a current. For the long-coherence superconductors these considerations are not relevant.

B. The S - N case

Here we discuss the case where the order parameter on the right is induced by the proximity effect, and discuss the controversial “proximity-induced Josephson tunneling effect.” Our viewpoints are similar to a very recent paper by Geshkenbein and Sokol,²² and we shall only make some further comments.

The relevant terms of the GL free energy are as those in the last subsection except now $a_R > 0$ and we can ignore the b_R term. The continuity equation of the current requires

$$\frac{2}{\hbar} K_R \Delta_R^2 \nabla \phi_R = j_s = \text{const} . \quad (3.18)$$

It is intuitively clear that, due to $a_R > 0$, Δ_R must decay to zero as x increases. However, if j_s is nonzero, this causes problem since then $\nabla \phi_R$ has to increase to infinity to conserve the current. This solution is simply not physical. This is just the representation of the fact that for an equilibrium state (obtained by the minimization of the free energy, and hence no dissipation via evolving to a higher entropy state) the only current that can flow in the system is a supercurrent. Since $\Delta_R \rightarrow 0$ as $x \rightarrow \infty$ this is simply impossible unless $j_s = 0$ (and $\phi_R = \phi_L$).

Thuneberg and Ambegaokar²¹ (TA) considered a different configuration; they consider a slab, of thickness D , of the “weak superconductor” which we have called R , in between the “strong superconductor” L and a normal conductor N for $x > D$. Here D is chosen to be much smaller than the coherence length of R , defined to be $\xi_R = (2a_R/K_R)^{1/2}$. Hence, we can ignore the variation of the magnitude of Δ_R . They consider the conductivity of N to be infinite, but putting this at a finite value does not affect their microscopic calculation. In this case, the free energy of the system $L + R$ is

$$F = F_L - 2t \Delta_L \Delta_R \cos(\phi_R - \phi_L) + D \left[a_R \Delta_R^2 + \frac{K_R}{2} \Delta_R^2 (\nabla \phi_R)^2 \right], \quad (3.19)$$

where F_L is the free energy of the bulk superconductor at the left. In this case we are able to “minimize F under the constraint of a constant current j ,” by using the Lagrange multiplier and transforming to

$$G = F - \frac{\hbar}{2} \phi(D) j . \quad (3.20)$$

Notice, however, that in this case PIJT is allowed because there is no difficulty of the constraint of a constant current, since we are only imposing it in the normal region, N , and can be done by a suitable voltage. All the current that is passing into the region R is a dissipationless supercurrent, and thus can be obtained by minimization of a free energy. [Thus, this is an extra feature of the simplified model, and if one wants to discuss the case where R extends to infinity (a distance much larger than ξ), one should not impose “the constraint of a constant current” (cf., Ref. 20)].

The above scenario should be compared with the case

where R extends all the way to ∞ , where the (dissipationless) critical current defined in the last paragraph is zero.

In this case when the current is nonzero one necessarily has an electric field or chemical potential gradient in order that a normal current can flow, and these fields have nontrivial effects on the superconducting order parameter. Since dissipation inevitably occurs this cannot be discussed just within the *equilibrium* GL theory, in contrast to what was asserted in Refs. 17 and 20. To discuss the problem, one has to include dissipative terms in the theory (which includes the charge imbalance). In the quasi-one-dimensional case this has been done in Refs. 22 and 33. Note that a finite voltage is needed for a nonzero current.^{19,22,33} In fact, whether a steady state exists is a nontrivial question.

Geshkenbein and Sokol²² further claim that the pseudo-Shapiro steps, of $V = h v / 2e$ observed in the experiment,¹⁷ can be explained in their theory. We shall not go into that here.

C. The s - p case

Now we turn to a Josephson junction between two superconductors, where the left one is the conventional pairing but the right is not.³⁴ We shall, in particular, consider the complication that arises due to the proximity-induced s -wave parameter on the right-hand side of the junction. In principle the p/d wave on the right can also induce p/d -wave parameters on the left. In view of the fact that experimentally we usually use a stronger (higher- T_c) s -wave superconductor as a probe, we only consider the effect mentioned above. (In the p -wave case where the lowest-order coupling [t_1 terms in (2.3)] exists, the effect can simply be obtained by in the lowest order, $L \rightarrow R$, $p \rightarrow s$, and $s \rightarrow p$; that, however, is not true for d , see below.) The situation is schematically shown in Fig. 2. We shall deduce below the various properties of the critical supercurrent, in particular, its temperature dependence.

We shall first consider the case for the junction between an s -wave superconductor on the left with a p -wave one on the right. We ignore for simplicity the effect on Δ_L due to the junction, and simply take $\phi_L(x=0)=0$. When the t_1 term in (2.3) is allowed, the free energy is

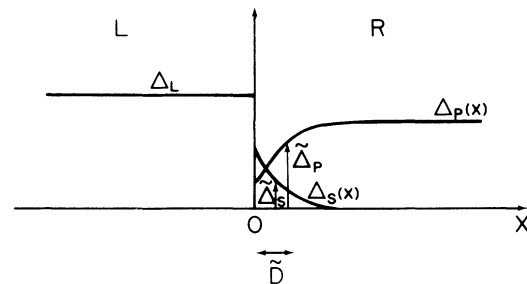


FIG. 2. Schematic diagram for the L - p junction with a proximity-induced s -wave order parameter.

$$\begin{aligned}
F = & -2t_p \Delta_L \Delta_p \cos \phi_p - 2t_s \Delta_L \Delta_s \cos \phi_s + w_s \Delta_s^2 + w_p \Delta_p^2 \\
& + \int_0^\infty dx \left[a_p \Delta_p^2 + b_p \Delta_p^4 + \frac{K_p}{2} [(\nabla \Delta_p)^2 + \Delta_p^2 (\nabla \phi_p)^2] + a_s \Delta_s^2 + \frac{K_s}{2} [(\nabla \Delta_s)^2 + \Delta_s^2 (\nabla \phi_s)^2] \right. \\
& \left. + \gamma_1 \Delta_p^2 \Delta_s^2 + \gamma_2 \Delta_p^2 \Delta_s^2 \cos[2(\phi_p - \phi_s)] \right], \tag{3.21}
\end{aligned}$$

where the terms not under the integral sign are for $x=0$, and we have assumed that the phase p is real [i.e., $\mathbf{d}(\mathbf{k})$ in (2.1a) can be chosen to be real] and the γ_2 term is even in $\phi_p - \phi_s$ (since usually for complex p states there exist either fourfold or threefold rotations in which Δ_p changes by $e^{\pm i(\pi/2)}$ and $e^{\pm i(2\pi/3)}$, respectively, and γ_1, γ_2 terms would be forbidden). Note that parity forbids terms of odd power in Δ_p in the bulk. Moreover, we require that the bulk has a second-order transition into the p state but not the s state; accordingly $a_p < 0$, $a_s > 0$, and $\gamma_1 > |\gamma_2|$, and a similar argument implies $w_s > 0$, otherwise an s layer will appear automatically even when $t_s = 0$.

We can study the behavior by minimization of F under the variation of Δ_s , Δ_p , ϕ_s , and ϕ_p , under the constraint that $\delta\phi_p(+\infty) \rightarrow 0$. We get, at $x=0^+$,

$$0 = 2w_s \Delta_s - 2t_s \Delta_L \cos \phi_s - K_s \nabla \Delta_s, \tag{3.22}$$

$$0 = 2w_p \Delta_p - 2t_p \Delta_L \cos \phi_p - K_p \nabla \Delta_p, \tag{3.23}$$

$$0 = 2t_s \Delta_L \Delta_s \sin \phi_s - K_s \Delta_s^2 \nabla \phi_s, \tag{3.24}$$

$$0 = 2t_p \Delta_L \Delta_p \sin \phi_p - K_p \Delta_p^2 \nabla \phi_p, \tag{3.25}$$

and in the bulk

$$0 = -\nabla(K_s \Delta_s^2 \nabla \phi_s) + 2\gamma_2 \Delta_p^2 \Delta_s^2 \sin[2(\phi_p - \phi_s)], \tag{3.26}$$

$$0 = -\nabla(K_p \Delta_p^2 \nabla \phi_p) - 2\gamma_2 \Delta_p^2 \Delta_s^2 \sin[2(\phi_p - \phi_s)], \tag{3.27}$$

$$\begin{aligned}
0 = & 2a_s \Delta_s - K_s \nabla^2 \Delta_s + 2\gamma_1 \Delta_s \Delta_p^2 \\
& + 2\gamma_2 \Delta_s \Delta_p^2 \cos[2(\phi_p - \phi_s)], \tag{3.28}
\end{aligned}$$

$$\begin{aligned}
0 = & 2a_p \Delta_p + 4b_p \Delta_p^3 - K_p \nabla^2 \Delta_p + 2\gamma_1 \Delta_s^2 \Delta_p \\
& + 2\gamma_2 \Delta_s^2 \Delta_p \cos[2(\phi_p - \phi_s)]. \tag{3.29}
\end{aligned}$$

We note that, if in the bulk terms we introduce the vector potential \mathbf{A} by using the rule $\nabla \psi \rightarrow (\nabla - 2ie \mathbf{A}/\hbar c) \psi$, and vary with respect to \mathbf{A} , we obtain $-(eJ/c)$ which defines the number current J . In this case we have

$$J = \frac{2}{\hbar} (K_s \Delta_s^2 \nabla \phi_s + K_p \Delta_p^2 \nabla \phi_p). \tag{3.30}$$

An inspection of (3.24)–(3.27) suggests that we define

$$J_s = \frac{2}{\hbar} K_s \Delta_s^2 \nabla \phi_s, \tag{3.31}$$

$$J_p = \frac{2}{\hbar} K_p \Delta_p^2 \nabla \phi_p, \tag{3.32}$$

and call them currents carried by the s wave and the p wave, respectively. Equations (3.24) and (3.25) are then

the continuity equations at the tunneling junction, each tunneling current has the usual sine dependence. Equations (3.26) and (3.27), however, tell us that there is conversion of one type of current to the other due to the bulk term in F involving $\phi_p - \phi_s$, with the total current J being conserved:

$$\nabla \cdot \mathbf{J} = 0. \tag{3.33}$$

This feature is true for all forms of GL free energy since the analogous equations (3.26) and (3.27) are obtained by variation with respect to the phase, and the GL free energy must depend on the phase difference alone: these current conversion terms always occur in pairs of opposite signs. Thus, while γ_1 always suppresses the PIJT, γ_2 , when the phase angles are right, always assists (and is essential for) it.

Since the current J in (3.30) is independent of x , and since $a_s > 0$, $\Delta_s \rightarrow 0$ as $x \rightarrow \infty$, the s -wave current which passes by tunneling into $x > 0$ region must gradually convert into p -wave current. By the same token, the presence of current J not only fixes the $\phi_s(0^+)$ by continuity, but ϕ_s and ϕ_p together, i.e., for practical purposes at equilibrium, ϕ_s is a function of ϕ_p (or ϕ_s, ϕ_p are functions, probably multivalued, of J).

Now we turn to the behavior of Δ_s . From (3.28) and the inequalities $a_s > 0$ and $\gamma_1 > |\gamma_2|$ mentioned, we see that $\nabla^2 \Delta_s$ is positive definite. This statement is entirely general in the case where the free energy depending on Δ_s only has terms at least in Δ_s^2 : the terms other than $\nabla^2 \Delta_s$ which enter in (3.28) are proportional to Δ_s and have the coefficients twice those of Δ_s^2 in F , and by the condition that there is no second-order instability of Δ_s , this is positive definite. Then for a physically meaningful solution $\nabla \Delta_s < 0$ at $x=0$, we must have, by (3.22)

$$2t_s \Delta_L \cos \phi_s > 2w_s \Delta_s. \tag{3.34}$$

Similar arguments as just given for the bulk guarantee the generality of this type of condition: since the surface form involving the right side of the junction alone forbids the automatic appearance of Δ_s (here $w_s > 0$), the coupling to the left (here t_s) has to be of sufficient magnitude and the right sign for $|\Delta_s|$ to appear at $x > 0$. This situation should again be compared with that of the superconducting-normal case discussed in the last subsection (see Ref. 21). Ψ_s is again restricted to the right-hand portion of the complex plane (if $t_s > 0$) and passes through $\Delta_s = 0$, where the phase angle ϕ_s is actually undefined.

The various coefficients in (3.21) lead to a rich set of phenomena. We shall consider them in a detail below. Consider first the particular case $\gamma_2 = 0$, here the situa-

tion is much simpler, in that the phases of ϕ_s and ϕ_p have no direct coupling. Equations (3.26) and (3.24) then immediately imply ϕ_s is a constant and no current is carried by the proximity-induced s pairs. ϕ_s is either 0 or π , according to [by inspecting 3.21] whether $t_s > 0$ or < 0 , respectively. This is a trivial case where, in equilibrium, $\phi_s = \phi_s(\phi_p) = 0$ or π .

When $\gamma_2 \neq 0$ complex competitions between various terms occur, we shall only briefly indicate the behavior for the dissipationless tunneling current. Due to the large number of parameters, studying the properties of the general solution of the set of the equations (3.22)–(3.29) is quite a formidable task. Therefore, we shall study a highly simplified model (in the spirit of the TA model for PIJT above). We first consider the special case $t_p = 0$, as would be the case of a spin-inactive barrier, and for simplicity drop w_s and w_p . From the previous discussion, we know that the tunneling current must be first to the proximity-induced s wave, and then converted into the p -wave current via the phase-coupling term involving γ_2 . We assume an effective length, \bar{D} , in which the proximity-induced s -wave order parameter exists and thus one in which the conversion of the current takes place. We take the effective order parameter in this region as $\bar{\Delta}_s, \bar{\phi}_s, \bar{\Delta}_p, \bar{\phi}_p$. The effective free energy is

$$\begin{aligned} \bar{F} = & -2t_s \Delta_L \bar{\Delta}_s \cos \bar{\phi}_s \\ & + \bar{D} \{ a_p \bar{\Delta}_p^2 + b_p \bar{\Delta}_p^4 + a_s \bar{\Delta}_s^2 + \gamma_1 \bar{\Delta}_p^2 \bar{\Delta}_s^2 \\ & + \gamma_2 \bar{\Delta}_p^2 \bar{\Delta}_s^2 \cos[2(\bar{\phi}_p - \bar{\phi}_s)] \} . \end{aligned} \quad (3.35)$$

Minimization with respect to $\bar{\Delta}_s$ and $\bar{\phi}_s$ give the magnitude of $\bar{\Delta}_s$ and the conservation of current [cf. (3.26) and (3.27)]

$$\bar{\Delta}_s = \frac{t_s \Delta_L}{\bar{D}} \frac{\cos \bar{\phi}_s}{\{ a_s + [\gamma_1 + \gamma_2 \cos(2\bar{\phi}_{ps})] \bar{\Delta}_p^2 \}} , \quad (3.36)$$

$$J = 4t_s \Delta_L \bar{\Delta}_s \sin(\bar{\phi}_s) = -4\bar{D} \gamma_2 \bar{\Delta}_p^2 \bar{\Delta}_s^2 \sin(2\bar{\phi}_{ps}) , \quad (3.37)$$

where we have defined $\bar{\phi}_{ps} = \bar{\phi}_p - \bar{\phi}_s$. [In reality $\Delta_p = \Delta_p(x)$, $\phi_p = \phi_p(x)$ are given by the minimization of the total energy, as in (3.22)–(3.29), and $\Delta_p(x \rightarrow \infty)$ should approach the bulk value: this is not represented by (3.35) and therefore we should not minimize (3.35) with respect to $\bar{\Delta}_p$ and $\bar{\phi}_p$. Everything will be in terms of these two variables, and the solution is representative of the essential physics if the maximum current of the junction is controlled by those in (3.37) (a reasonable assumption) and when one is not interested in the true current-phase relationship $J(\phi_p)$] Combining (3.36) and (3.37), one can express $\bar{\phi}_s$ in terms of $\bar{\phi}_{ps}$

$$\tan(\bar{\phi}_s) = - \frac{\gamma_2 \bar{\Delta}_p^2 \sin(2\bar{\phi}_{ps})}{\{ a_s + [\gamma_1 + \gamma_2 \cos(2\bar{\phi}_{ps})] \bar{\Delta}_p^2 \}} , \quad (3.38)$$

and so

$$J = - \frac{4t_s^2 \Delta_L^2}{\bar{D}} \frac{\gamma_2 \bar{\Delta}_p^2 \sin(2\phi_{ps})}{\{ a_s + [\gamma_1 + \gamma_2 \cos(2\bar{\phi}_{ps})] \bar{\Delta}_p^2 \} + \gamma_2^2 \bar{\Delta}_p^4 \sin^2(2\phi_{ps})} . \quad (3.39)$$

Defining the dimensionless quantity j by

$$J = \frac{4t_s^2 \Delta_L^2}{a_s \bar{D}} j . \quad (3.40a)$$

Thus,

$$\begin{aligned} j = & j(\phi_{ps}, z, r) \\ = & - \frac{rz^2 \sin(2\bar{\phi}_{ps})}{\{ 1 + [1 + r \cos(2\bar{\phi}_{ps})] z^2 \}^2 + r^2 z^4 \sin^2(2\bar{\phi}_{ps})} , \end{aligned} \quad (3.40b)$$

where the dimensionless variables r, z are defined by

$$r \equiv \frac{\gamma_2}{\gamma_1} \quad (|r| < 1) , \quad (3.41a)$$

$$z^2 \equiv \frac{\gamma_1 \bar{\Delta}_p^2}{a_s} \equiv \bar{\gamma}_1 \bar{\Delta}_p^2 \quad (0 \leq z) . \quad (3.41b)$$

One sees that $j(\phi, z, r) = j(\phi + \pi/2, z, -r)$. Thus, the

critical current is a function of z and $|r|$ only. We shall consider $r \geq 0$ in the following. For a given set of parameters and a given temperature (fixed $\bar{\Delta}_p^2$ and hence z), the dimensionless critical current is given by $j_c = j_c(\phi_c, z; r)$ where $\partial j_c / \partial \bar{\phi}|_{\phi=\phi_c} = 0$. After some algebra we find

$$\cos(2\phi_c) = \frac{2rz(1+z)}{[1+2z+(1+r^2)z^2]} .$$

The behavior of j_c as a function of z [and hence approximately $\propto (1 - T/T_p)^{1/2}$] is sketched in Fig. 3. At $z = 0$ ($\bar{\Delta}_p = 0$), $j_c = 0$ since the bulk right-hand side cannot carry any supercurrent. Thus, j_c first rises as T is lowered through T_p , the critical temperature of the p -wave superconductor [initially roughly $\propto z^2$ as the p -wave gap develops, see (3.37)]. It then goes through a maximum and then decreases due to the fact that $\bar{\Delta}_p$ suppresses the magnitude of the induced s -wave order parameter (3.36) (cf., Ref. 18). The position of this maximum (z_m), being a function of r , is as plotted in Fig. 4 and the value of the maximum current j_m , again a function of r only, is plot-

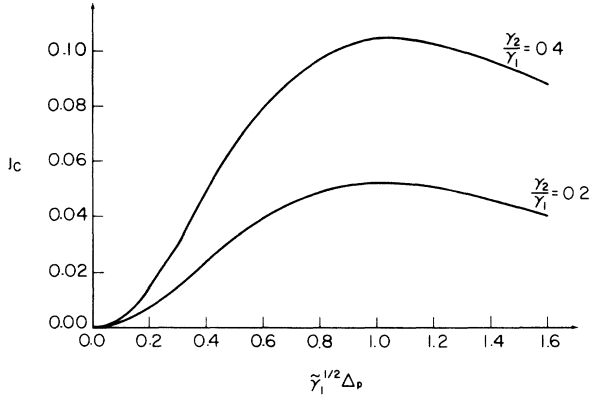


FIG. 3. The dimensionless supercurrent j_s vs the dimensionless gap, for various parameters of r .

ted in Fig. 5. As discussed previously, $j_m = 0$ at $\gamma_2 = 0$ and rises rapidly as $|\gamma_2| \rightarrow \gamma_1$.

We go on to briefly discuss the case with $t_p \neq 0$. Assuming that the lowest-order coupling is allowed, the free energy acquires an extra term $-2t_p \Delta_L \tilde{\Delta}_p \sin(\tilde{\phi}_p)$ and the total current is given by

$$J = 4t_s \Delta_L \tilde{\Delta}_s \sin(\tilde{\phi}_s) + 4t_p \Delta_L \tilde{\Delta}_p \sin(\tilde{\phi}_p). \quad (3.42)$$

$$J = -\frac{4t_s^2 \Delta_L^2}{a_s \tilde{D}} \frac{rz^2 \sin(2\tilde{\phi}_{ps})}{\{1 + [1 + r \cos(2\tilde{\phi}_{ps})]z^2\}^2 + r^2 z^4 \sin^2(2\tilde{\phi}_{ps})} + 4t_p \Delta_L \tilde{\Delta}_p \frac{\sin(\tilde{\phi}_{ps}) \{1 + [1 + r \cos(2\tilde{\phi}_{ps})]z^2\} - rz^2 \sin(2\tilde{\phi}_{ps}) \cos(\tilde{\phi}_{ps})}{(\{1 + [1 + r \cos(2\tilde{\phi}_{ps})]z^2\}^2 + r^2 z^4 \sin^2(2\tilde{\phi}_{ps}))^{1/2}} \quad (3.43)$$

which is much more complicated than before. It is out of the scope of the present paper to investigate in detail the behavior of (3.43), but a number of points are clear by observing this equation: (i) As a function of temperature, again $J = 0$ for a temperature above T_p . Just below T_p , since the direct tunneling via the p wave is $\propto x$ whereas to the s wave it is $\propto z^2$, the former dominates and the critical current rises as $(1 - T/T_p)^{1/2}$. Depending on the parameters it may then be taken over by the s -wave tunneling at intermediate temperatures. At $T \ll T_p$ the induced s wave is suppressed and finally it is essentially via the p wave alone. (ii) J is a function of t_s^2 and t_p . If both are nonzero, for junctions of different transmittivity (with t_s, t_p scaled by the same factor), the J are essentially entirely different functions of $\tilde{\phi}_{ps}$ [and therefore so is $J(\phi)$]. The phase at which J is a maximum (and hence J_c) is a complicated function of t_s^2 and t_p . This should be contrasted with the case where J scales as $t_s^2(t_p)$ if only $t_s(t_p)$ exists (but not both), and junctions of different transmittivity have the same $J(\phi)$ except up to an overall scale factor.

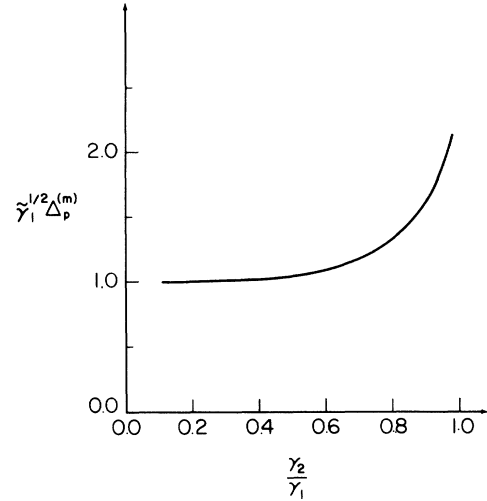


FIG. 4. The value of the gap at which the current is a maximum as a function of z for a given value of r .

The minimization of \tilde{F} with respect to $\tilde{\Delta}_s$ and $\tilde{\phi}_s$ give (3.36) and the second equality of (3.37) and hence (3.38) as before. Using $\tilde{\phi}_p = \tilde{\phi}_{ps} + \tilde{\phi}_s$, and after some algebra, one finds

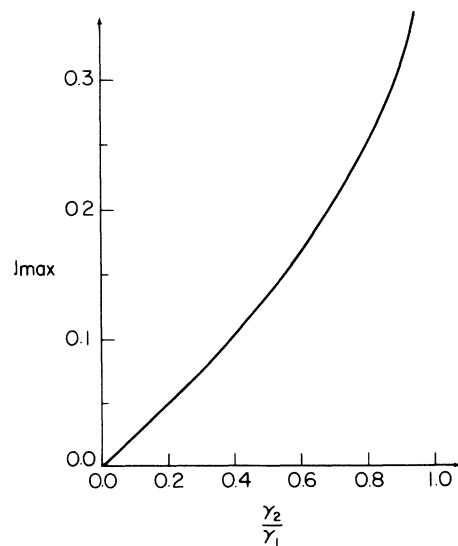


FIG. 5. Maximum possible current for a given parameter r .

We expect most of the essential behavior is represented correctly qualitatively by the model above, but the universal behaviors as a function of γ_2/γ_1 may not hold. The above discussions are in direct contrast with that of Ref. 18 (which left out γ_2). We thus do not support their interpretation of the observation of proximity-induced Josephson effects in their experiment (see also Sec. V).

D. The s - d case

We now proceed to the case where the unconventional superconductor has an instability into a d wave. In this case there are many more possibilities. For example if the d wave has a twofold rotation axis or reflection plane with which the order parameter changes sign (e.g., $k_x^2 - k_y^2$ in tetragonal symmetry), then the general form of the free energy is the same as the p -wave one shown explicitly in (3.21), except $p \rightarrow d$. Similar arguments can easily be extended to other cases (keeping in mind that in the p -wave case one always has an odd operation, namely the inversion). In the last mentioned example, if the direct pair tunneling into the d wave is of the same order as that of the one into the p wave, it is virtually indistinguishable from the p -wave case. (This would be the case if the barrier is strongly pair breaking, or very spin active). The discussion on the p -wave case carries over without changes.

There is one particular case where the tunneling would behave rather differently, namely when the d wave has the symmetry $\sim 2k_z^2 - k_x^2 - k_y^2$. i.e., Γ_3^+ (using the notation of Refs. 5, 6, and 8) of cubic crystals or Y_2^0 in spherical systems. In this case the bulk term $\gamma_3 \Delta_s \Delta_d^3 \cos(\phi_s - \phi_d)$ is allowed, and the s wave is induced by the presence of the d wave^{35,36} (independent of the sign of the coefficient of this term and whether the pairing interaction in the s channel is repulsive or attractive). Moreover, complicated gradient terms of the form $K_{sd} \nabla \psi_s^* \nabla \psi_d + \text{c.c.}$ may also arise.³⁷ It is difficult to discuss in detail the behavior of J_c as a function of temperature, as it depends on the relative signs as well as the magnitude of the various coefficients. The effects of the signs can best be illustrated by considering the minimization of energy. $t_{s,d} > (<) 0$ favors $\phi_s, \phi_d = 0(\pi)$, whereas $\gamma_3 > (<) 0$ favors $\phi_s - \phi_d = \pi(0)$. Thus, the presence of γ_3 may or may not help the existence of Δ_s . Anyway since Δ_s induced in the bulk (and extends to infinity), it opens up the possibility that the critical current always increases as the temperature is lowered through T_d even when $t_d = 0$. This behavior of $J_c(T)$ is similar to that between two s -wave superconductors (cf., Sec. V).

IV. MAGNETIC EFFECTS

Now we turn to the magnetic effects. An interesting experiment is suggested by Geshkenbein and Larkin:²⁵ take a single crystal of the exotic superconductor, R , and make two Josephson junctions with parallel orientations with an ordinary superconductor wire, connected to form a closed loop and thus forming a SQUID (Fig. 6). They suggest that the maximum allowed critical current will be

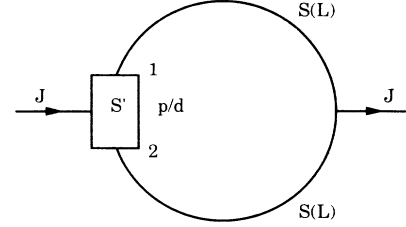


FIG. 6. The experimental arrangement suggested by Geshkenbein and Larkin. $S'(R)$ is the superconductor to be investigated. Here S' stands for the induced s -wave.

at integral or half-integral flux quantum through the loop, depending on whether the superconductivity involved in R is even or odd under parity.

Here we examine whether the PIJT, discussed in Sec. III, if present, will complicate the picture. If the superconductivity in R is a conventional s wave, then no further discussion is necessary. For the p - or d -wave cases we note that, provided the two junctions 1 and 2 are of a distance which is large compared with the coherence length of the induced s -wave part, then we can apply what we learned in the last section, namely that the phase of the induced s wave, ϕ_s , will be a functional of the current passing through the junction, or, equivalently, $\phi_p - \phi_L$ or $\phi_d - \phi_L$:

$$\phi_s^{(1,2)} = \phi_s^{(1,2)}(\phi_R - \phi_L),$$

where $R = p$ or d and ϕ_L is the phase of the gap at the other side of junction. The total current J in Fig. 6 is given by

$$J = J_1 + J_2, \quad (4.1)$$

where J_1 and J_2 are, respectively, the current through junctions 1 and 2. Allowing for a different maximum critical current through the two (due to the difference in areas of the junctions, see below)

$$J_1 = J_{1m} f_1(\phi_{R1} - \phi_{L1}), \quad (4.2)$$

$$J_2 = J_{2m} f_2(\phi_{R2} - \phi_{L2}), \quad (4.3)$$

where J_{1m} and J_{2m} are the amplitude of J_1 and J_2 (i.e., $\max|J_{1,2}| = J_{1m,2m}$, $\max|f_1| = \max|f_2| = 1$). Now junctions 1 and 2 are related by parity (see below) and therefore for a given argument ϕ of the functions $f_{1,2}$

$$\begin{aligned} f_1(\phi) &= f_2(\phi) \quad \text{if } R \text{ even} \\ &= f_2(\phi + \pi) \quad \text{if } R \text{ odd} . \end{aligned} \quad (4.4)$$

If there is no texture inside R , then we have, by standard arguments,

$$(\phi_{R2} - \phi_{L2}) - (\phi_{R1} - \phi_{L1}) = 2\pi \frac{\Phi}{\Phi_0}, \quad (4.5)$$

where Φ is the flux linking the SQUID and Φ_0 (superconducting) unit flux quanta. Now assume that the lowest-order coupling between Δ_R and Δ_s is allowed in (2.3), then $f_{1,2}$ will be periodic in 2π . Since

$$J = J_{1m} f_1(\phi_{R1} - \phi_{L1}) + J_{2m} f_1 \left[\phi_{R1} - \phi_{L1} + 2\pi \frac{\Phi}{\Phi_0} \right] \quad (4.6)$$

if R is even, it is obvious that J can achieve $J_{1m} + J_{2m}$, the maximum possible value, when Φ/Φ_0 is an integer, whereas since

$$J = J_{1m} f_1(\phi_{R1} - \phi_{L1}) + J_{2m} f_1 \left[\phi_{R1} - \phi_{L1} + 2\pi \frac{\Phi}{\Phi_0} + \pi \right] \quad (4.7)$$

if R is odd, the maximum J can be achieved if Φ/Φ_0 is a half-integer. Thus, the suggestion by Geshkenbein and Larkin still holds in this case. Note, however, since there is nothing that tells us that $f_1(\phi) = -f_1(\phi + \pi)$ (and, in fact, this relation is simply false in the presence of PIJT), there is no general argument as to where the minima of J are, except that they must be separated by integral flux quanta. Note that at the flux where the maximum current can be achieved, the phases of the induced s waves are equal. For example, for the $R = p$ case

$$\begin{aligned} \phi_s^{(2)} &= \phi_s^{(2)}(\phi_{R2} - \phi_{L2}) = \phi_s^{(1)}(\phi_{R2} - \phi_{L2} + \pi) \\ &= \phi_s^{(1)}(\phi_{R1} - \phi_{L1}) = \phi_s^{(1)}, \end{aligned}$$

the second equality is again due to the fact that the two junctions are parity related [reversing the sign of t_p in (3.21)].

The above discussion is true only when the two junctions are related by parity except for the possible difference in area, i.e., the coefficients t should be the same, or else the argument leading to (4.4) fails. (Recall the end of Sec. III C.)

Now we turn to the case where the lowest-order term in (2.3) is forbidden but t_2 is allowed. Equations (4.1)–(4.5) hold without modifications, but now f_1 has period π , and by (4.4) and therefore

$$f_1(\phi) = f_1(\phi + \pi) = f_1(\phi + \pi) = f_2(\phi),$$

independent of whether the state R is odd or even under parity. Examining (4.6) and (4.7) again in this case shows that the maximum currents will be attained at both integral and half-integral flux quanta. They would be entirely indistinguishable.

This result is actually not surprising since, when t_1 is forbidden but t_2 is allowed, there exists a symmetry operation that leaves a given junction invariant and makes the R order parameter flip sign. Combining with the parity operation gives us an operation that map junctions 1 to 2 and vice versa, which is odd (even) if the state is even (odd). This operation can well play the role of parity in the discussion of Geshbenkein and Larkin. Therefore, the two types of superconductivity become indistinguishable by the suggested experiment (but then we know that it must be in either the p or d , but not s state).

Next we discuss the single-slit-interference pattern which occurs when a magnetic field on the z direction is applied to the junction is depicted in Fig. 1. It is well

known that the maximum for the case where both superconductors are s wave the current as a function of flux Φ threading the junction shows a single-slit-diffraction pattern, with zeroes at Φ/Φ_0 being nonzero integers. First we consider the case without PIJT, then, if the lowest-order term in (2.3) is allowed, we can easily see that the standard arguments go through yielding an identical pattern as discussed above. However, if t_1 is forbidden and t_2 is allowed, the current-phase relationship has half the ordinary period and results in a pattern of half size (as a function of Φ/Φ_0).

Now we consider whether any complication arises when the PIJT is included. For this we first notice that formally the problem is solved by minimizing the free energy afresh under the appropriate external magnetic field. The equations are, for example, in the case of coupling to a p wave where t_1 term is allowed, just the set (3.22)–(3.28) with the placement $\nabla\phi \rightarrow \nabla\phi + (2e/\hbar c)\mathbf{A}$ (and $\phi_{s,p} \rightarrow \phi_{s,p} - \phi_L$ for $x=0^+$). Note now, in general, all variables are functions of x, y and we have a genuine two-dimensional (2d) problem. These equations should be combined with the Maxwell equation which reads, in this case,

$$\begin{aligned} \nabla \times (\nabla \times \mathbf{A}) &= \frac{4\pi e}{c\hbar} \left[2K_p \Delta_p^2 \left[\nabla\phi_p - \frac{2e}{\hbar c} \mathbf{A} \right] \right. \\ &\quad \left. + 2K_s \Delta_s^2 \left[\nabla\phi_s - \frac{23}{\hbar c} \mathbf{A} \right] \right]. \quad (4.8) \end{aligned}$$

The standard textbook argument for the ‘‘single-slit-diffraction pattern’’¹⁶ makes use of the simplification by first ignoring the tunneling current. Thus, one just solves for a Meissner-effect problem and substitutes the phase found in the current-phase relation and sum over y . Notice, however, that one cannot make the analogous simplification here, for if we ignore the tunneling current, in particular that of the induced s wave, and thus ignore all second-order terms in t_s , an examination of (3.22)–(3.29) shows that ϕ_s is not a function of ϕ_p anymore, it becomes completely arbitrary. This is not surprising since the equation that fixes ϕ_s is from the minimization of the free energy with respect to ϕ_s , which is simply the equation for the current of the s pairs. Thus, we really have to minimize the free energy to second order in t_s (and first order in t_p) to get the diffraction pattern.

Note when we do this the solution we get would be rather different by patching together solutions from Sec. III with the appropriate $\phi_p(x \rightarrow \infty, y)$: the current and gradient energy across these ‘‘strips’’ cannot be ignored.

However, we can reasonably guess that due to the fact that (3.22)–(3.29) are invariant under $\phi_p \rightarrow \phi_p + 2\pi$, $\phi_s \rightarrow \phi_s$, then the solution is expected to have a periodicity in y in some distance Y . This distance Y can be seen from (4.8) at $x \rightarrow +\infty$, where the Meissner effect has shielded off the magnetic fields and³⁸

$$\frac{\partial \phi_p(x \rightarrow +\infty, y)}{\partial y} = \frac{2e}{\hbar c} A_y(+\infty)$$

and by a similar system for ϕ_L , that the current is periodic in y with period Y , where Y is such that a flux unit is enclosed.

Similarly, if the lowest-order term t_1 is forbidden, then the equations repeat themselves at $\phi_p \rightarrow \phi_p + \pi$, $\phi_s \rightarrow \phi_s$; the distance we need to go for the periodicity is half of that above, i.e., the interference pattern still shrinks laterally by half.

V. TIME-DEPENDENT EFFECTS

We now turn briefly to a discussion of the dynamics of the Josephson junctions [again between a conventional (L) and an unconventional (R) superconductor], namely (i) the internal oscillation about an equilibrium state, i.e., the plasma oscillation, and (ii) the oscillation of the current under an externally applied voltage difference, i.e., the ac Josephson effect. If there is just a pure p or d wave in the unconventional superconductor, then the answers are trivial and can be directly read off from the current-phase relationships. If t_1 in (2.3) exists and dominates, then $J = J_0 \sin\phi$; we just have the ordinary plasma frequency $[\omega_J = (2eJ_0/\hbar C)^{1/2}]$, where C is the capacitance of the junction] and Shapiro steps ($\hbar v/2e$). If $t_j, j=1, \dots, n-1$ vanishes but t_n exists, then $J = J_0 \sin(n\phi)$, using identical reasoning to the standard arguments that give a plasma frequency $n^{1/2}$ times and a Shapiro step n^{-1} times the standard value.

Now we consider the effect due to the presence of the proximity-induced s order parameter in the unconventional (p - or d -wave) superconductor. We shall only describe the scenario, relegating the details to a later publication. As mentioned before, we can just discuss the s - p case without loss in generality (except for the case where the d wave induces an s wave as discussed in Sec. III D).

Recall that in the right-hand side superconductor the p wave coexists with the proximity-induced s -wave order parameter in a length scale of \bar{D} . During a time-dependent process the two order parameters are usually tossed out of equilibrium from each other. Besides the usual relaxation processes, the supercurrent conversion term involving γ_2 in (3.21) also couples the dynamics of the two order parameters. Due to these couplings, one expects that (i) for the oscillations about an equilibrium state, there is a mode for the internal oscillation between the s - and p -wave order parameters, (as a generalization of the bulk modes of Ref. 35), in addition to the original plasmon mode corresponding to the relative oscillation between the phases on the opposite sides of the junction. These two modes are coupled, among other things, via the γ_2 term, and (ii) for the ac Josephson effect, due to the existence of the two order parameters on the right superconductor, the dependence of the tunneling currents on their phase angles, and the γ_2 coupling between them, their phase angles relative to that on the left, in general, cannot be both linear in time with the same coefficient ($2eV/\hbar$). Frequency of the current and hence the Shapiro steps are expected to deviate from the standard value.

We close this section by estimating the importance of the γ_2 coupling which can be measured by the coupling energy per unit junction area, namely $\gamma_2 \bar{\Delta}_p^2 \bar{\Delta}_s^2 \bar{D}$ (in the

notation of Sec. III). This should be compared with the ordinary Josephson coupling energy, i.e., the t_p or t_d terms in (3.21). All these can be converted into units of a current (by multiplication of $2e/\hbar$). For heavy fermions, assuming \bar{D} is of the order of a coherence length, γ_2 of the same order as the coefficient of the quartic term in a BCS theory, and assuming $\bar{\Delta}_s \sim 1$ K, one obtains the enormous value corresponding to a critical current of order 10^{10} mA/cm² (compared to a $J_c \sim 10^6$ mA/cm² of the experiment of Ref. 18). Thus, the presence of the proximity-induced s -wave order parameter is potentially important for modifying the dynamics of the Josephson junctions.

In view of the discussion above and in Sec. III C, and given that the experiment on UBe₁₃ in Ref. 18 finds ordinary Shapiro steps, we do not find the suggestion of proximity-induced Josephson tunneling from their probe to the superconducting UBe₁₃ there plausible.

VI. CONCLUSIONS

We would like to conclude by making some cautionary remarks on using the Josephson tunneling as a probe of unconventional superconductivity. It is well known that, as in ³He, the unconventional pairing suffers from “depairing” effects due to surfaces, thus diminishing the critical current. Moreover, the “perpendicular” part of the order parameter would be more strongly suppressed than the “parallel” part.³⁹ This, however, does not cause problems directly if the order parameter keeps its orientation, because if the bulk state is such that t_1 is allowed, then it must be even under all symmetry operations that leave the junction invariant. The depaired state near the interface will likewise have this property. Thus, if t_1 is forbidden for the depaired state near the surface it will be so also for the bulk state.

The depairing, however, can cause an indirect problem. It is also well known in ³He- A , that if we have the order parameter in the bulk with the \hat{l} vector parallel to a surface, due to the strong depairing that would occur if \hat{l} were to keep the same direction also near the surface, \hat{l} instead forms a “texture” so that it becomes perpendicular to the surface.⁴⁰ Similar scenarios may or may not occur in the crystal. In a crystal there are strong anisotropies, it may happen that reorientation is never energetically favorable; however, in some directions they always occur, at least near T_c . Some examples will clarify this point.⁴¹ Consider for definiteness a tetragonal crystal. According to the theory as outlined in Refs. 4–8, if the superconducting state belongs to the one-dimensional representations, then barring accidental cases where some other representations also have a T_c close to the original one, only the order parameter of the one of the highest T_c needs to be considered (cf., Ref. 36). Thus, there is no possibility of a texture, be the order parameter an s , p , or d wave. However, if the order parameter belongs to a representation of dimensionality two or greater, rotations among equivalent directions are always possible near T_c . Whether it is still advantageous to produce such a “texture” depends on the magnitude of the quartic (or higher) terms, the surface depairing, and the gradient energies.

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APPENDIX

In this appendix we consider the symmetry operations, in particular the time-reversal symmetry, in more detail. In particular, we emphasize that Josephson tunneling between say, the s and p waves is not forbidden by time-reversal symmetry considerations alone (cf., Refs. 5 and 42).

The time-reversal-symmetry operator can be represented by the antiunitary operator θ where⁴³

$$\theta a_{k\sigma} \theta^{-1} = e^{i(\pi/2)} e^{i\delta_\sigma} a_{-k-\sigma} \quad (\text{A1})$$

with $\delta_\sigma = \sigma(\pi/2)$. The gap transforms in the same way as the anomalous average, $\langle \psi | a_{k\sigma} a_{-k\sigma'} | \psi \rangle$, which, in the time-reversed systems, simply reads $\langle \theta \psi | a_{k\sigma} a_{-k\sigma'} | \theta \psi \rangle$. One may simply verify that for singlet

$$\Delta_{\sigma\sigma'}^\theta(\hat{\mathbf{k}}) = [\Delta_{\sigma\sigma'}(\hat{\mathbf{k}})]^* , \quad (\text{A2})$$

whereas for triplet

$$\begin{aligned} \Delta_{\uparrow\downarrow}^\theta(\hat{\mathbf{k}}) &= -[\Delta_{\uparrow\downarrow}(\hat{\mathbf{k}})]^* , \\ \Delta_{\sigma\sigma}^\theta(\hat{\mathbf{k}}) &= -[\Delta_{-\sigma-\sigma}(\hat{\mathbf{k}})]^* , \end{aligned} \quad (\text{A3})$$

or, using the representation as in (2.1),

$$\Delta_{\sigma\sigma'}^\theta(\hat{\mathbf{k}}) = |\Delta| e^{-i\phi(i\sigma_y)} f(\mathbf{k})^* , \quad (\text{A4})$$

$$\Delta_{\sigma\sigma'}^\theta(\hat{\mathbf{k}}) = |\Delta| e^{-i\phi[id(\hat{\mathbf{k}})^* \cdot \sigma] \sigma_y} . \quad (\text{A5})$$

When the phase is real up to an overall phase factor, we can always choose $f(k)$, $\mathbf{d}(k)$ to be real, and the time reversal thus just reverses the value of the phase angle. In this case we have (2.4). Thus, despite the apparent sign difference between (A2) and (A3), Josephson coupling between the phases is allowed (cf., Ueda and Rice, Ref. 5).

When $f(\hat{\mathbf{k}})$ or $\mathbf{d}(\hat{\mathbf{k}})$ is complex then time reversal maps the pairs into a different state, for example, the angular momentum of the axial phase is reversed. Thus, time reversal alone does not relate $F(\phi)$ for different ϕ 's. A relation exists only when there is another symmetry operation of the junction to bring the state back to the original one, up to some phase factor, say α . In this case we obtain (2.5).

As an illustration for the discussion in the last paragraph we consider tunneling between an s wave and an axial $\mathbf{d}(\mathbf{k}) = \mathbf{d}(\hat{\mathbf{m}} + i\hat{\mathbf{n}}) \cdot \hat{\mathbf{k}}$, as sketched in Figs. 7(a) and 7(b). Here we take $\hat{\mathbf{l}}, \hat{\mathbf{m}}, \hat{\mathbf{n}}$ to be along crystal symmetry axes; in particular, we assume there is a twofold rotation symmetry axis, and since inversion symmetry always exists in the systems in which we are interested, reflection symmetry occurs about the plane perpendicular to $\hat{\mathbf{n}}$ in

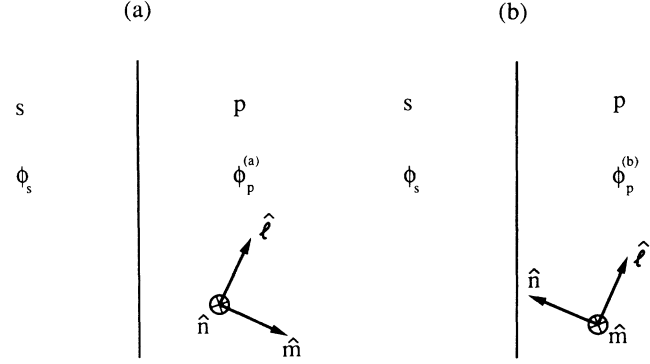


FIG. 7. The two configurations discussed in the Appendix.

Fig. 7(a) [i.e., $\hat{\mathbf{m}}$ in Fig. 7(b)]. The combined symmetry operation mentioned in the last paragraph is the product of the time reversal and this reflection symmetry. Time reversal, by (A5), keeps $\hat{\mathbf{d}}$ unchanged but changes the sign of $\hat{\mathbf{n}}$ and $\hat{\mathbf{l}}$, and ϕ . To decide the effect of the reflection, we use the fact that it is a parity operation followed by a π rotation about the normal of the reflection plane. The former is

$$P^\dagger \psi_\sigma(\mathbf{r}) P = \psi_\sigma(-\mathbf{r})$$

and we simply have

$$\Delta_{\sigma\sigma'}^P(\mathbf{r}, \mathbf{k}) = \Delta_{\sigma\sigma'}(-\mathbf{r}, -\mathbf{k}) , \quad (\text{A6})$$

where \mathbf{r} is the center-of-mass coordinate of the pair. Thus, parity keeps the s wave invariant and for the axial phase its effect can be represented by $\hat{\mathbf{m}} \rightarrow -\hat{\mathbf{m}}$, $\hat{\mathbf{n}} \rightarrow -\hat{\mathbf{n}}$ and keeps $\hat{\mathbf{l}}$, $\hat{\mathbf{d}}$, and ϕ_p unchanged. The effect of rotation is simply included by rotating everything. One finds, from considering the combined operations,

$$F(\phi_p^{(a)} - \phi_s) = F(\phi_s - \phi_p^{(a)}) \quad (\text{A7a})$$

or

$$F(\phi_p^{(b)} - \phi_s) = F(\phi_s - \phi_p^{(b)} + \pi) \quad (\text{A7b})$$

for \mathbf{d} perpendicular to the plane of the paper, and

$$F(\phi_p^{(a)} - \phi_s) = F(\phi_s - \phi_p^{(a)} + \pi) , \quad (\text{A8a})$$

$$F(\phi_p^{(b)} - \phi_s) = F(\phi_s - \phi_p^{(b)}) \quad (\text{A8b})$$

for \mathbf{d} in the plane of the paper. These are special cases of (2.4) and (2.5). They agree with each other since (a) and (b) represent the same state if

$$\phi_p^{(a)} = -\frac{\pi}{2} + \phi_p^{(b)} .$$

According to these equations, (a) and (b) are the “convenient” choices so that the energy is even in ϕ , depending on whether \mathbf{d} lies perpendicular to or in the plane of the paper, respectively. In both cases, the lowest-order Josephson coupling, i.e., t_1 in (2.3), is allowed, t_1 being real for (A7a) and (A8b), and purely imaginary for (A7b) and (A8a).

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