Magnetoresistivity studies of Zr-M amorphous alloys (M=Ni, Co, and Fe): From superconductivity to ferromagnetism

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Magnetoresistivity studies of three-dimensional (3D) amorphous transition-metal alloys have demonstrated that although a qualitative understanding is possible, a quantitative agreement between experimental data and quantum theories of the conductivity is often difficult. This lack of a good quantitative comparison arises mainly from the failure to take into account all the different effects involved. In this work we present a comprehensive study of the magnetoresistivity of the Zr-M (M=Ni,Co,Fe) amorphous alloys and demonstrate concretely the importance of exchange enhancement for the nearly magnetic alloys and superconducting fluctuations for the superconducting alloys. By correctly taking into account both of these contributions, we achieve a quantitative description of the experimental results for the complete range of alloys, from superconducting to ferromagnetic.

I. INTRODUCTION

In recent years the low-temperature transport properties of amorphous metals have been the subject of much experimental and theoretical effort.[1-16] This interest comes mainly from the prediction of the quantum theories suggesting the presence of modifications, normally below 30 K, to the transport properties due to the structural disorder of these materials. It has been shown that this structural disorder has two major effects. The first is an increase in the localization of the electron wave function. If the disorder is large enough, a transition occurs from the metallic to the insulating state. The weak-localization[1] (WL) regime occurs if the disorder is not sufficient to give rise to completely localized states, but is high enough to influence the transport properties. The second consequence of disorder comes from the diffusive motion of the electrons. Because of the high resistivity found in amorphous metals (between 40 and 600 $\mu\Omega$ cm) each electron undergoes many more collisions compared to its motion in a crystalline environment. This increase in scattering results in some diminution in the effectiveness of the electronic screening, and hence increases the effective electron-electron (**e-e**) interactions.

Since both of these mechanisms are directly affected by a magnetic field, new contributions to the magnetoresistivity are found in such alloys. The change in resistivity observed is nonclassical since it is at least 10^4 to 10^5 times greater than what is predicted by an average elastic-scattering rate of $\tau_0 \sim 10^{-16}$ s.[14] However, since these two processes are affected in a different fashion by the magnetic field, the study of the magnetoresistivity can serve to distinguish them. Furthermore, it provides a unique way to obtain several important and fundamental transport parameters, such as the different relaxation times involved in the diffusive motion of the electron.

Previous studies have demonstrated that for two-dimensional (2D) samples the present theories can clearly account for the experimental data.[1,2,16] However, the behavior for 3D samples is not as well understood. There is also a certain confusion in the data analysis for 3D alloys, with some authors favoring the (**e-e**) term,[11-13] while others use mainly the WL equation to interpret their results.[3-10] Furthermore, in both theories several different theoretical expressions exist, with small but significant differences, thus adding to the confusion.

The reasons for several of these discrepancies are readily understood. First of all, the best 2D results were obtained on thin-film samples of simple metals, where transition metals appeared only as impurities.[1,2,16] Most 3D metal-metal samples, by contrast, are based on transition metals with only a few studies of simple non-transition-metal alloys. A second reason arises from the presence of superconductivity in many of the 3D systems.[17] Above T_c, the superconducting fluctuations influence the normal-state resistivity and are very sensitive to an applied magnetic field, giving rise to a positive magnetoresistivity. However, an expression for the precise field dependence of this term is not available for all conditions of magnetic field and temperature. Only at small fields and temperatures well above T_c is there a reliable estimate of the field dependence.[18] Experimentally, the variation in the resistivity in 3D samples are 2 or 3 orders of magnitude smaller than found in 2D samples, which serves to limit the field and temperature regions where precise measurements are obtainable. Finally, the lack of systematic data over a controllable range of superconducting and magnetic behavior has hindered the separation of different contributions making a consistent

analysis difficult.

In this paper we present results for three different Zr-based alloys: Zr-Ni, Zr-Co, and Zr-Fe, where by varying the relative composition it is possible to pass from the superconducting to the ferromagnetic regime. As a consequence, the relative importance of the superconducting and magnetic fluctuations have been distinguished.

II. MAGNETORESISTIVITY THEORIES

Three different mechanisms are known to contribute to the resistivity changes in the presence of a magnetic field in amorphous metals. The details of these theories, their origin, and their numerical evaluation can be found in the paper of Baxter *et al.*[19] In this section we present only the specific equations related to the present study.

A. Weak localization

The complete magnetoresistivity expression in the presence of spin-orbit interactions and Zeeman splitting was developed by Fukuyama and Hoshino[20] and can be written as

$$\frac{\Delta\rho_L}{\rho^2}=\frac{e^2}{2\pi^2\hbar}\left\{\left[\frac{eB}{\hbar}\right]^{1/2}\left[F_3\left[\frac{B^*_{\text{s.o.}}}{B}\right]-0.5\left[\frac{1}{\sqrt{1-\gamma}}\right]\left[F_3\left[\frac{t_+}{A}\right]-F_3\left[\frac{t_-}{A}\right]\right]\right]\right.$$
$$\left.-\frac{1}{\sqrt{D\tau_{\text{s.o.}}}}\left[\left[\frac{1}{\sqrt{1-\gamma}}\right](\sqrt{t_+}-\sqrt{t_-})-\sqrt{t+1}+\sqrt{t}\right]\right], \tag{1}$$

where

$$t=\tau_{\text{s.o.}}/4\tau_\phi\ ,$$
$$t_\pm=t+0.5(1\pm\sqrt{1-\gamma})\ ,$$
$$\gamma=[(g\mu_B B\tau_{\text{s.o.}})/2\hbar]^2\ ,$$
$$A=eDB\tau_{\text{s.o.}}/\hbar\ ,$$

and

$$B^*_{\text{s.o.}}=(\hbar/4eD)(\tau_\phi^{-1}+4\tau_{\text{s.o.}}^{-1})\ .$$

The function F_3 was calculated by Kawabata[21] and can be approximated[19] by

$$F_3(x)\approx 2(\sqrt{x+2}-\sqrt{x})-\left[\frac{1}{\sqrt{x+0.5}}+\frac{1}{\sqrt{x+1.5}}\right]$$
$$-\frac{1}{48(x+2.03)^{3/2}}\ . \tag{2}$$

τ_ϕ and $\tau_{\text{s.o.}}$ are, respectively, a phase-coherence (normally associated with inelastic scattering) and a spin-orbit lifetime.[22] In our analysis we used an isotropic lifetime as presented by Fukuyama and Hoshino[20] which means that

$$\tau_{\text{s.o.}}=\tau_{\text{s.o.}x}=\tau_{\text{s.o.}y}=\tau_{\text{s.o.}z}$$

compared to a global lifetime $\tau_{\text{s.o.}}=3\tau^T_{\text{s.o.}}$.[23]

As we have recently shown,[19,24–26] in nearly magnetic alloys the value of γ in (1) has to be replaced by an effective value $\gamma_{\text{eff}}=\gamma/(1-I)^2$, where $(1-I)^{-1}$ is the Stoner enhancement factor.

B. Electron-electron interactions

1. Interactions in the diffusion channel

Lee and Ramakrishnan[27] have proposed that the interaction in this channel can be expressed as

$$\frac{\Delta\rho_D}{\rho^2}=\frac{e^2}{4\pi^2\hbar}\left[\frac{k_BT}{2\hbar D}\right]^{1/2}\lambda_\sigma^{J=1}(F)g_3(\delta)\ , \tag{3}$$

where $\delta=g\mu_BB/k_BT$. $\lambda_\sigma^{J=1}(F)$ is a renormalized average over the Fermi surface of the static Coulomb interaction F for the state $J=1$ and is given by the expression:[28]

$$\lambda_\sigma^{J=1}(F)=-\frac{32}{3}\frac{1+3F/4-(1+F/2)^{3/2}}{F}\ . \tag{4}$$

A numerical form for the function $g_3(x)$ has been given by Ousset *et al.*[29] As for WL, in strongly paramagnetic alloys, the screening parameter $\lambda_\sigma^{J=1}(F)$ becomes the product of the screening and the Stoner values.[19,24,30]

2. Interaction in the Cooper channel

The variation of the resistivity with the magnetic field was calculated by Al'tshuler and Aronov[28] and by Isawa and Fukuyama.[31] The Isawa-Fukuyama expression can be written as

$$\frac{\delta\rho_C(B,T)}{\rho^2}=\frac{3e^2}{4\hbar}\left[\frac{eB}{\hbar}\right]^{1/2}\frac{k_BT}{4eDB}g(B,T)\phi_3(\nu,\gamma)\ , \tag{5}$$

with $\nu=4DeB\tau_i/\hbar$ and $\gamma=\hbar/2\pi k_BTr_i$. Since the function $g(B,T)$ depends on the magnetic field in an unknown way, an analysis of this interaction has to be restricted to low fields where it can be approximated by $\ln(T_0/T)$, where T_0 takes the value of the transition temperature in superconducting samples or the Fermi temperature ($T_F=\varepsilon_F e^{N(\varepsilon_F)g}/k_B$) in normal systems.[19]

C. Superconducting fluctuations

1. Aslamasov-Larkin contribution

The magnetoresistivity associated with that contribution has the form[32]

$$\frac{\Delta\rho_{AL}(B,T)}{\rho^2}=\frac{e^2}{4\hbar}\left[\frac{2k_BT}{\pi\hbar D}\right]^{1/2}\sum_{n=0}^{\infty}(n+1)[(\varepsilon+pn)^{-1/2}+(\varepsilon+pn+p)^{-1/2}-(\varepsilon+pn+p/2)^{-1/2}]-\frac{\delta\sigma_{AL}(T)}{\rho^2} \quad (6)$$

with $\varepsilon=\ln(T/T_c)+p/2$ and $p=\pi DeB/2k_BT$ with $\delta\sigma_{AL}(T)=-e^2/32\hbar\xi_0[(T-T_c)/T_c]^{-1/2}$ and ξ_0 given by[10] $\xi^2=\hbar/4eB_{c2}(0)$.

2. *Maki Thomson contribution*

Al'tshuler *et al.*[33] have predicted that, in the limit of low fields and sufficiently far from the transition temperature, the magnetic dependence can be expressed as

$$\frac{\delta\rho_{MT}(B,T)}{\rho^2}=\frac{e^2}{2\pi^2\hbar}\beta(B,T)\left[\frac{eB}{\hbar}\right]^{1/2}F_3\left[\frac{B_\phi}{B}\right] \quad (7)$$

with $B_\phi=4eD\hbar/\tau_\phi$ and where in those conditions[18] $\beta(B,T)$ and for limits has

$$\beta(T)=\begin{cases}\dfrac{\pi^2}{6\ln^2(T_c/T)} & \text{for } |\ln(T_c/T)|\gg 1 \quad (8a)\\ -\dfrac{\pi^2}{4\ln(T_c/T)} & \text{for } -\ln(T_c/T)\ll 1\ . \quad (8b)\end{cases}$$

III. EXPERIMENTAL DETAILS

Alloys pellets of the desired composition were prepared in an arc furnace under a titanium-gettered argon atmosphere. The purity of the constituents was 99.99% for the Zr and 99.999% for the Ni, Co, and Fe. Each pellet was subsequently melt spun under 15 kPa pressure of helium onto a copper wheel with a tangential velocity near 50 m/s. The amorphous ribbons have a typical width of 1 mm and a thickness around 15 μm. Their amorphicity was examined by detailed x-ray diffraction and the final composition homogeneity was verified by electron microprobe analysis on related samples.[36] The resistivity and density used in this work were those published by Altounian and Strom-Olsen[36] and by Batalla *et al.*[37]

Electrical contacts to the samples were made with very small aluminum wires (0.025 cm in diameter) that were ultrasonically bonded to the sample and to adjacent copper pads that served as terminals for external connections. We have found that such contacts produce less noise than pressure or conductive epoxy types and minimize recrystallization of the sample which occurs in arc welded or soldered contacts.[38]

Relative changes in sample resistance were measured with a four-terminal ac bridge at a frequency $\approx$150 Hz.[39] This bridge has an ultimate resolution of 2×10^{-7} Ω for a 1-Ω resistance. The ac current was limited to $\approx$5 mA resulting in a thermal dissipation in the sample around 25 μW, well below the value where sample heating becomes significant.

Two different magnets were used: a normal electromagnet with a field up to 1 T and a 5 T superconducting magnet. The temperature was measured using calibrated carbon-glass resistors and was controlled by an automatic temperature controller. Measurements were taken from 1.5 to 300 K at fields up to 1 T (with the field perpendicular to the sample plane and the current direction) and at low temperature at fields up to 5 T for a number of characteristic samples.

IV. RESULTS

The results for the three alloy systems studied can be summarized in the following fashion.

A. Superconducting Zr-Ni and Zr-Co alloys

(i) The magnetoresistivity is positive for all samples at fields up to 5 T and for temperatures above 1.3 K.

(ii) As shown in Figs. 1(a) and 1(b) and 2(a) and 2(b) the field dependence starts as B^2 at low fields and tends to $\sqrt{B}$ at higher values. Furthermore, for the $Zr_{71}Ni_{29}$ and $Zr_{70}Co_{30}$, both of which have a similar T_c of $\sim$3.2 K, the magnetoresistivity data are practically identical but different from those for the two lower-T_c alloys. In particular at 5 K, $Zr_{50}Ni_{50}$ with a T_c of 1.4 K has a greater variation than $Zr_{53}Co_{47}$ for which $T_c=0.7$ K. However, as the temperature increases the resistivity variations crossover above 10 K indicating a change in the temperature dependence.

(iii) The resistivity variation at 1 T for several alloys is shown in a log-log plot versus the temperature in Fig. 3. The temperature variation of the magnetoresistivity is slower in the lower-T_c alloys.

Table I gives the values for $\Delta\rho/\rho^2$ at 6 K and 1 T for the different samples studied. We note that the overall change is more important for samples with higher Zr concentration. This behavior can be readily understood by the fact that for all our samples the superconducting temperature goes up with the Zr content, and thus at one particular temperature the effect of the superconducting fluctuations above T_c also increases with the Zr content.

B. Zr-Fe alloys

Only for the Zr-Fe system are we able to melt-spin samples from the superconducting ([Fe] < 30 at. %) to the ferromagnetic region ([Fe] > 38 at. %). The paramagnetic region between these two has been characterized by the presence of strong spin fluctuations[25,26] and the direct effect of exchange enhancement on the WL and e-e interaction contributions to the magnetoresistivity has been presented.[24] We summarize the magnetoresistivity as follows.

(i) As can be seen in Figs. 4(a) and 4(b) the magnetoresistivity for the superconducting range of alloys behaves exactly as for Zr-Ni and Zr-Co samples, i.e., a

positive magnetoresistivity (of the same order of magnitude) that decreases in relative value as we decrease the Zr content and thus the superconducting fluctuations. However, by comparing Figs. 4(a) and 4(b) it is evident that in the lower-T_c alloy the variation is smaller in magnitude and closer to a B^2 dependence.

(ii) As already pointed out,[24] this decrease in $\Delta\rho/\rho^2$ has a minimum at the concentration where superconductivity disappears: $Zr_{72}Fe_{28}$. For higher Fe concentrations the magnetoresistivity increases up to the concentration for ferromagnetism: $Zr_{62}Fe_{38}$. Throughout the

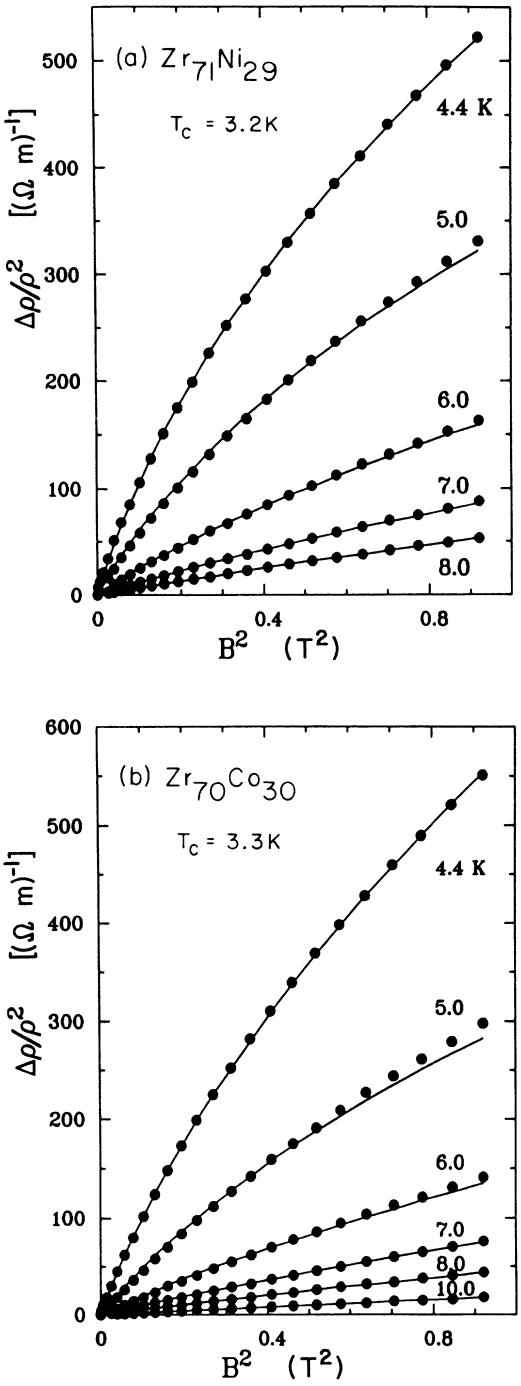

FIG. 1. Normalized magnetoresistivity, $\Delta\rho/\rho^2$, as a function of B^2 below 1 T for (a) $Zr_{71}Ni_{29}$, (b) $Zr_{70}Co_{30}$, both of which having a similar superconducting transition temperature ($T_c \sim 3.3$ K). The solid line represents our theoretical analysis of the data as described in Sec. V.

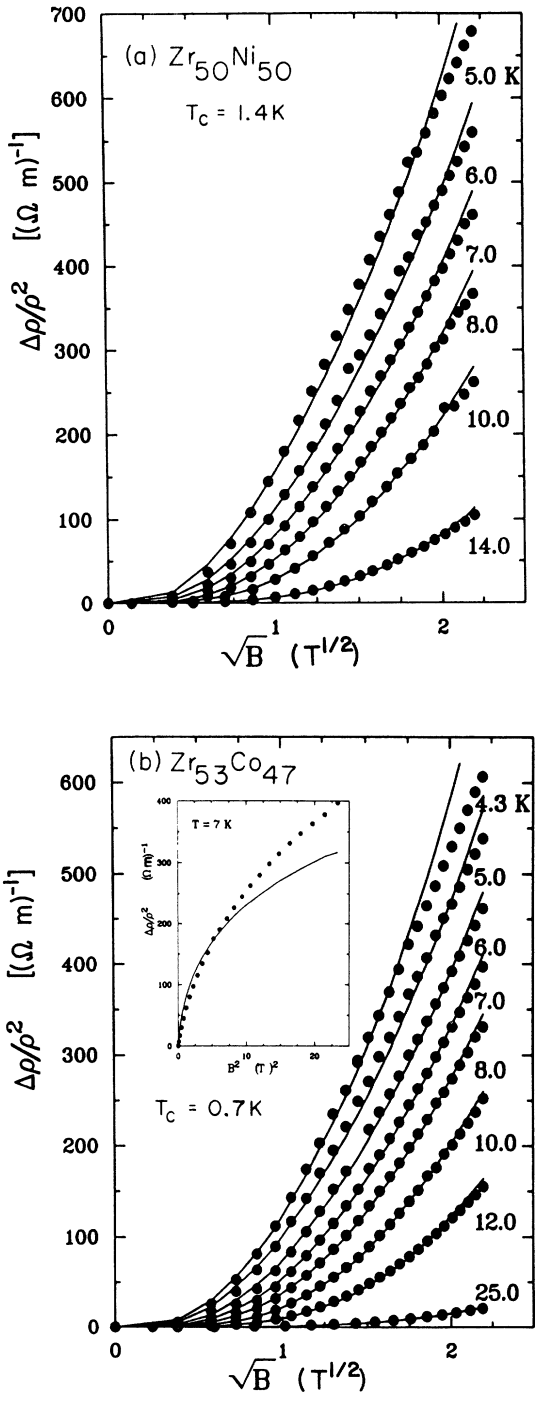

FIG. 2. Normalized magnetoresistivity, $\Delta\rho/\rho^2$, as a function of $\sqrt{B}$ for field up to 5 T for (a) $Zr_{50}Ni_{50}$ (T_c = 1.5 K) and (b) $Zr_{53}Co_{47}$ (T_c = 0.7 K). The solid line represent our theoretical analysis of the data. The insert in (b) shows the best fit of the data at 7 K, without using any exchange-enhancement factor (see Sec. V).

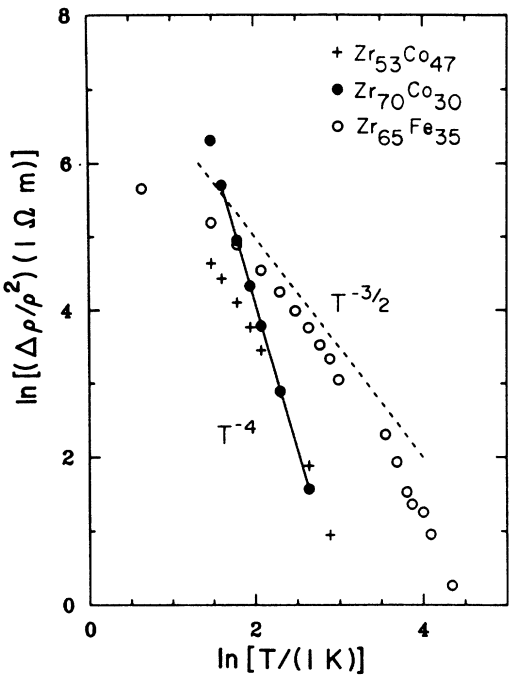

FIG. 3. Variation of $\Delta\rho/\rho^2(B,T)$ at 1 T with temperature in $Zr_{70}Co_{30}$ (●), $Zr_{53}Co_{47}$ (+), and $Zr_{65}Fe_{35}$ (○) (from Ref. 24). The solid line correspond to a T^{-4} variation and the dotted line to a $T^{-3/2}$ decrease.

range between these two concentrations the temperature variation of the magnetoresistivity slows systematically, being still measurable at 77 K for $Zr_{65}Fe_{35}$.

(iii) In the ferromagnetic samples the magnetoresistivity has a completely different character. In Fig. 5 we see that $\Delta\rho/\rho^2$ is negative at low fields, goes through a minimum, and starts a linear increase afterward, the position of this field minimum decreasing as the temperature increases. Furthermore, the magnetoresistivity varies much more slowly with temperature, being observable at temperatures well above liquid nitrogen. Finally, contrary to the paramagnetic samples, the magnetoresistivity shows a clear anisotropy in field. This is seen in Fig. 6, where the value of $\Delta\rho/\rho^2$ is presented as a function of the angle between the magnetic field and electric current direction. The relative change at high fields is the same for all angles but the low field behavior is different, being more negative when the current and field are perpendicular. This behavior has been observed in other amorphous magnetic samples and can be related to the anisotropy in the domain structure.[40,41] The important point is that the magnetoresistance for the ordered magnetic alloys is very distinctive. Hence, we can conclude that the magnetoresistance of the nearly magnetic alloys are not the result of magnetic inclusions arising from concentration variations in the samples. Moreover, the increase in the magnetoresistivity observed in the paramagnetic alloys can not be attributed directly to the presence of a local moment and direct magnetism or magnetic clustering.[24,1]

In Sec. V we analyze these results, including the effect of exchange enhancement in the nearly magnetic samples on the WL and e-e terms and also including the effect of the superconducting fluctuations above T_c for the superconducting alloys.

V. DISCUSSION

In the following analysis we take into account the sum of all the contributions listed in Sec. II. To first order all these effects are considered independent[35] so that

$$\Delta\rho(B,T)=\Delta\rho_L(B,T)+\Delta\rho_C(B,T)+\Delta\rho_D(B,T)+\Delta\rho_{AL}(B,T)+\Delta\rho_{MT}(B,T)\ .$$

A. $Zr_{71}Ni_{29}$, $Zr_{70}Co_{30}$ an $Zr_{80}Fe_{20}$–high-T_c alloys

An initial analysis was undertaken on the higher-T_c superconducting samples (Zr-rich samples) using all the contributions with the exception of the exchange enhancement of the WL and e-e interaction in the diffusion channel (IDC) terms. Since the expression for $\beta(T,B)$ given by Larkin for the Maki-Thomson contribution is only valid for $B<1$ T at 4.2 K in samples with a $T_c\sim 3$ K, only the magnetoresistivity data below fields of 1 T were analyzed using the calculated value of β with τ_i as the only fitting parameter. Values for $\tau_{s.o.}$ and D found in paramagnetic ZrFe alloys[24] were used allowing for some variation (less than 5%) to achieve the best fit.

Results of this analysis for the three highest-T_c samples are shown as the solid lines in Figs. 1(a), 1(b), and 4(a); the values of τ_i (4.2 K) are presented in Table I. We conclude (i) that the present theories can explain nicely

TABLE I. Resistivity (Refs. 36 and 37), superconducting transition temperature (Refs. 36 and 37), $\Delta\rho/\rho^2$ (6 K, 1 T), inelastic lifetime τ_i (4.2 K), and Stoner enhancement factor $(1-I)^{-1}$ in several superconducting Zr-Ni, Zr-Co, and Zr-Fe amorphous alloys.

Sample	ρ ($\mu\Omega$m)	T_c (K)	$\Delta\rho/\rho^2$ (6 K, 1 T) $(\Omega\,m)^{-1}$	τ_i (4.2 K) (10^{-11} s)	$(1-I)^{-1}$	$(1-I)^{-1}$[a]
$Zr_{71}Ni_{29}$	1.69	3.24	165	2.9		1.7
$Zr_{50}Ni_{50}$	1.84	1.4	95	5.8	2.4	2.6
$Zr_{70}Co_{30}$	1.73	3.29	150	2.7		1.9
$Zr_{53}Co_{47}$	1.76	0.70	67	4.3	2.57	3.7
$Zr_{80}Fe_{20}$	1.62	3.30	125	1.7		2.0
$Zr_{72}Fe_{28}$	1.67	0.6	35	1.0	3.1	3.8

[a]References 36 and 37.

the observed changes in the resistivity and (ii) that the prediction form superconducting fluctuation theories are accurate for these ranges of temperatures and fields. Figures 7(a) and 7(b) show a breakdown of the individual contributions for $Zr_{71}Ni_{29}$ at 4.2 and 12 K, respectively. In this sample the two superconducting fluctuation pro-

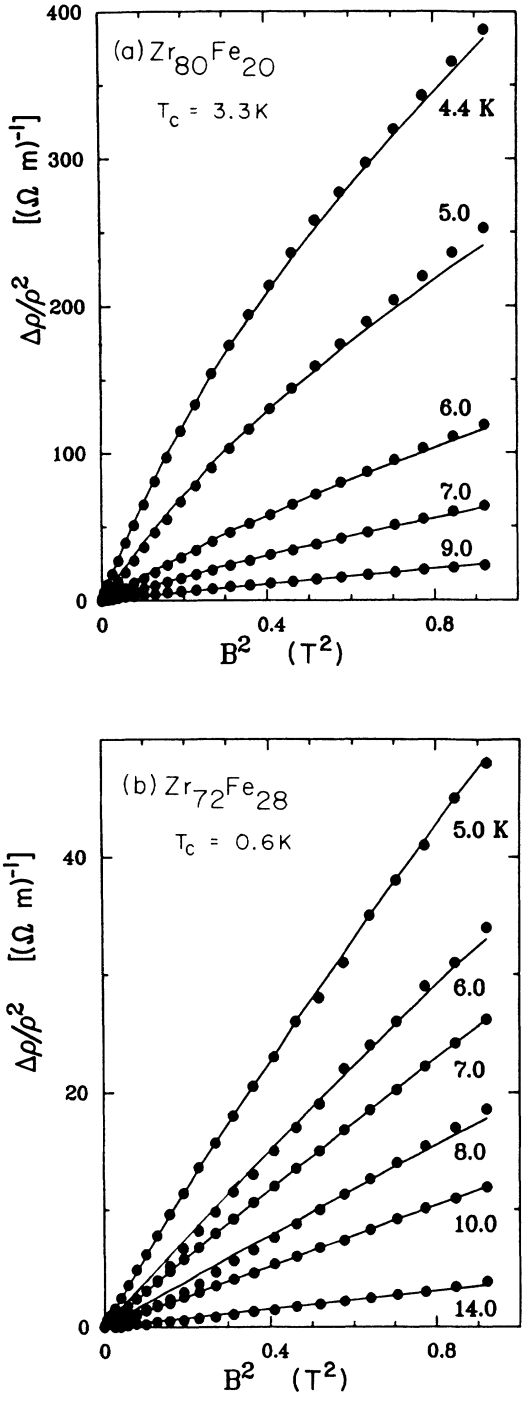

FIG. 4. Normalized magnetoresistivity $\Delta\rho/\rho^2$, as a function of B^2 below 1 T for (a) $Zr_{80}Fe_{20}$ (T_c=3.3 K) and (b) $Zr_{72}Fe_{28}$ (T_c=0.6 K). The solid line represent our theoretical analysis of the data as described in Sec. V.

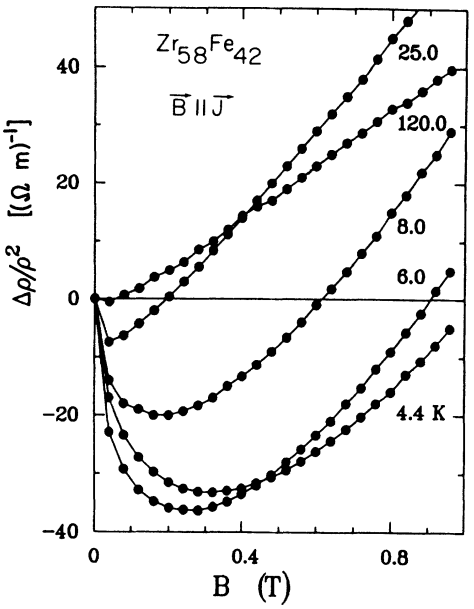

FIG. 5. Normalized magnetoresistivity $\Delta\rho/\rho^2$ plotted as a function of the magnetic field B in ferromagnetic $Zr_{58}Fe_{42}$.

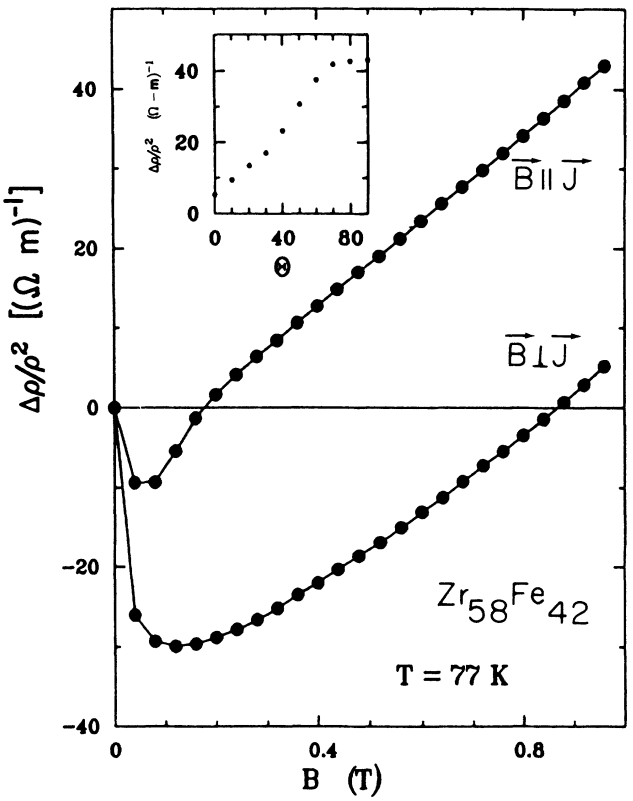

FIG. 6. Difference in resistivity variation $\Delta\rho/\rho^2(B,T)$ in ferromagnetic $Zr_{58}Fe_{42}$ for the case where $I \| B$ and $I \perp B$. The insert shows the value at 1 T as a function of the angle Θ between B and I.

cesses dominate the magnetoresistivity with WL contributing only 7% of the total change at 4.2 K and 1 T. The two e-e interactions are negligible at these temperatures (the absolute values for the Cooper channel term are presented, the contribution being negative due to the sign of the electron coupling constant). These results clearly show the importance of the Aslamasov-Larkin process to the magnetoresistivity of superconductors close to T_c. Well above T_c (12 K) only the WL and the Maki-Thomson contributions are non-negligible.

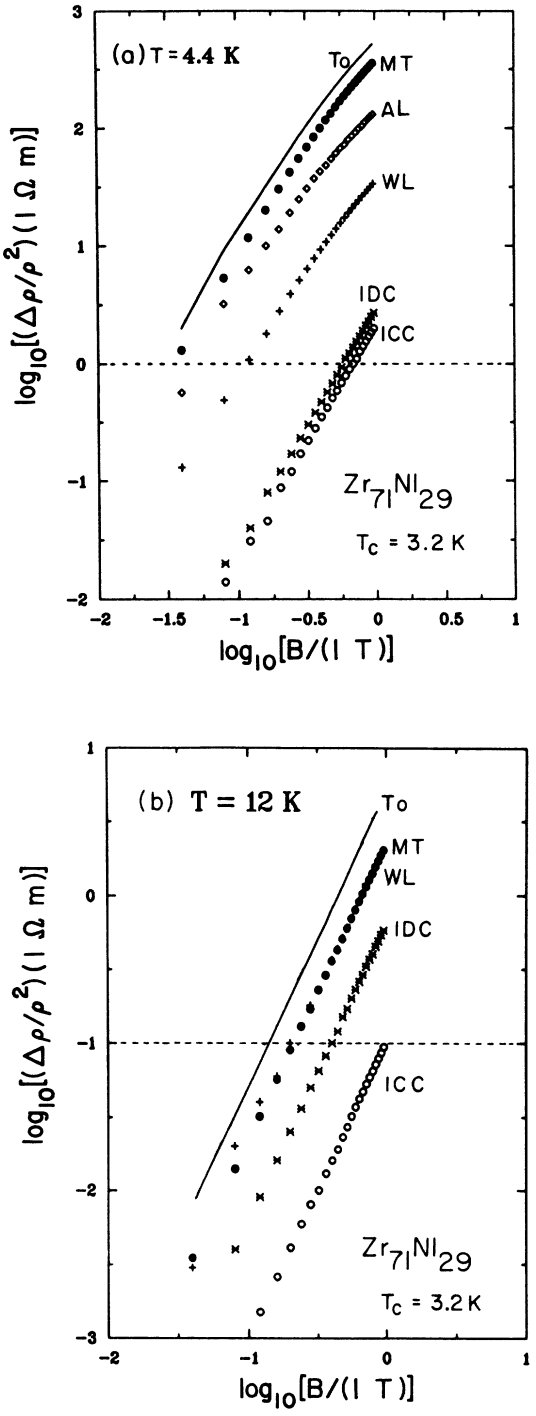

FIG. 7. Values for the different contributions to $\Delta\rho/\rho^2$ between 0 and 1 T in $Zr_{71}Ni_{29}$ (T_c = 3.24 K) at (a) 4.2 K and (b) 12 K. Maki-Thomson contribution (MT); Aslamasov-Larkin contribution (AL); weak localization (WL); electron-electron interaction in the diffusion channel e-e (IDC); electron-electron interaction in the Cooper channel e-e (ICC); sum of all contribution (TO).

B. $Ni_{50}Zr_{50}$, $Co_{47}Zr_{53}$, and $Fe_{28}Zr_{72}$–low-T_c alloys

The data analysis for the samples with small values of T_c ($\sim$1 K) is more complicated. Since T_c is smaller, the highest field at 4.2 K for which Larkin's expression should be valid is $\sim$5 T. Thus only the WL, Maki-Thomson, and the Aslamasov-Larkin contributions should play a measurable role in the magnetoresistivity. In a first instance, the analysis was based only on these three processes. For that situation the best fits that can be obtained are similar to the one presented in the insert of Fig. 2(b). Due to the particular curvature of the theoretical expression, these three contributions alone are not able to explain the observed changes in the resistivity. It should be stressed here that this behavior cannot be due to a weakness in the Maki-Thomson theory, since only the ratio B/T is important to determine its range of validity. Furthermore, the analysis of the low-field data for $Zr_{72}Fe_{28}$ gives a similar conclusion.

In previous superconductivity studies[36,37] on similar Zr-based metallic glasses some spin fluctuations effect on the critical temperature has been observed. Consequently, we decided to examine the effect of such fluctuations on the results by incorporating a Stoner enhancement factor in the spin splitting terms of the WL and the IDC e-e processes.[19,24] The data for these alloys were reanalyzed using a temperature-independent Stoner factor $(1-I)^{-1}$. The analysis were done with $(1-I)^{-1}$ as a free parameter along with the phase-coherence lifetime τ_ϕ at the lowest temperature; at higher temperature $(1-I)^{-1}$ was fixed and only τ_ϕ allowed to vary. The theoretical curves obtained in this way are presented in Figs. 2(a), 2(b), and 4(b) for the Ni, Co, and Fe samples, respectively, and the obtained values for τ_ϕ (4.2 K) and $(1-I)^{-1}$ are given in Table I. This analysis confirms that the inclusion of exchange enhancement due to spin fluctuations is able to remove the discrepancies between our data and the present magnetoresistivity theories.

For the $Zr_{53}Co_{47}$ sample, in Figs. 8(a) and 8(b) we present the importance of each contribution to the total magnetoresistivity at 4.3 and 12 K, respectively. At 4.2 K and at high fields the two dominant effects are the enhanced WL and IDC e-e interactions. The superconducting contributions are much less important than in the higher-T_c alloys with the Maki-Thomson contributing only 20% of the total magnetoresistivity and the Aslamasov-Larkin being negligible at 5 T. At 12 K the Maki-Thomson process also becomes negligible, with the WL being more important than the IDC e-e contribution. However, at still higher temperatures the IDC e-e contribution will dominate since it has a slower temperature variation.

As for the case of the previous samples the value of D

and $\tau_{s.o.}$ were taken near the values found in the paramagnetic ZrFe alloys. In all cases best fits were found with the diffusion coefficient and the spin-orbit lifetime no more than 25% away from the original values.

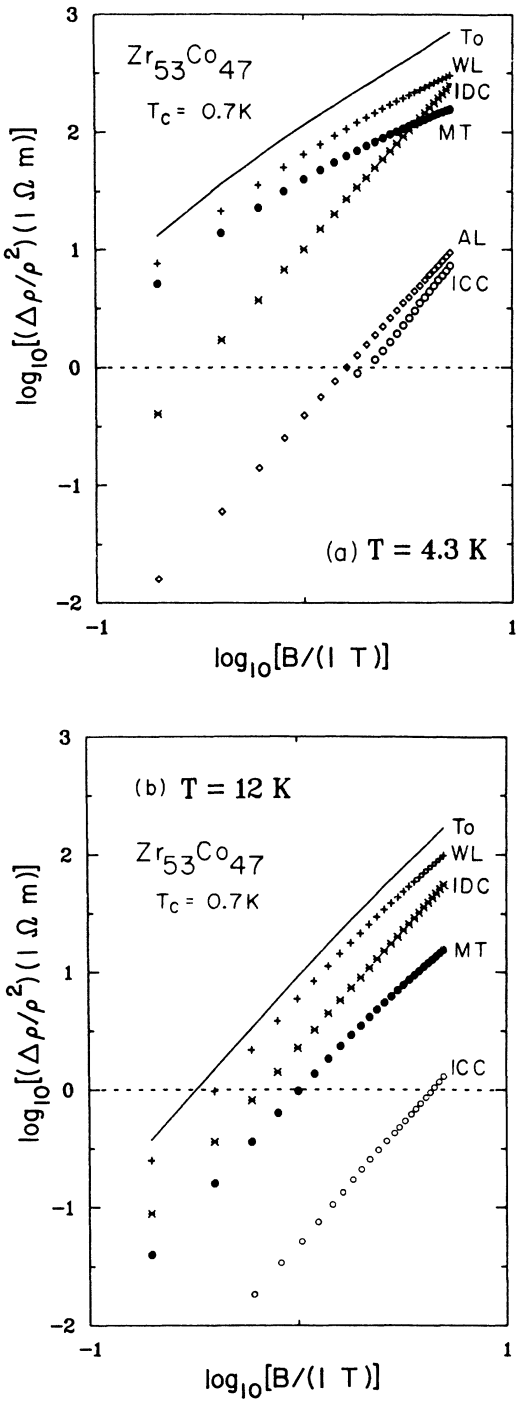

FIG. 8. Values for the different contributions to $\Delta\rho/\rho^2$ between 0 and 5 T in $Zr_{53}Co_{47}$ (T_c=7 K) at (a) 4.2 K and (b) 12 K. Maki-Thomson contribution (MT); Aslamasov-Larkin contribution (AL); weak localization (WL); electron-electron interaction in the diffusion channel e-e (IDC); electron-electron interaction in the Cooper channel e-e (ICC); sum of all contribution (TO).

C. Parameter analysis

We now examine the values of the different transport parameters found in our alloys.

(i) Diffusion coefficient D: D in all samples was taken $\sim 5\times10^{-5}$ m^2 s^{-1}. Other workers have found or used a similar coefficient in other transition-metal systems, such as YAl,[10] CuZr,[8] and CuTi.[11] This value is also in agreement with the one obtained through the alloy density of states using the Einstein relation, $\sigma = e^2 D N(\varepsilon_F)$ and also through the derivative of the upper critical field in superconductors[42] given by $D = 4k_B T/\pi e B'_{c2}$. Both methods yield an lower and upper bound for D of 3 and 7×10^{-5} m^2 s^{-1}, respectively. However, a free-electron approach, such as that of Bieri *et al.*[3] or Li and Rapp[6] produces a significantly higher value ($\sim 2\times10^{-4}$ m^2 s^{-1}) since it neglects the d electrons contribution. Their overestimate of D will also affect the complete analysis, since a greater D value diminishes the importance of the Zeeman splitting in the weak-localization theory and reduces all the electron-electron interaction contributions.[19]

(ii) Spin-orbit lifetime $\tau_{s.o.}$: The value found for the spin-orbit lifetime is $\tau_{s.o.} \sim 4\times10^{-13}$ s in all our alloys. This value corresponds to scattering in a single direction. If we consider an isotropic medium the total scattering time should be one-third or $\sim 1.3\times10^{-13}$ s, which is of the same magnitude as found in magnetoresistivity measurements in other Zr-based alloys[3,4,8,9] [one should note that in some of these studies the value obtained for $\tau_{s.o.}$ has to be divided by 1.5 (Ref. 43)]. The fact that for all three of our alloys no significant difference was found confirms the assumption that most of the spin-orbit scattering is due to the presence of the Zr atoms in accord also with the upper-critical-field studies of Wong *et al.*,[44] where they found a spin-orbit scattering time between 4×10^{-14} and 2×10^{-13} s in various Zr alloys.

(iii) Stoner factor $(1-I)^{-1}$: The values for $(1-I)^{-1}$ (Table I) in the paramagnetic alloys and lower-T_c superconductors compare favorably to those obtained from susceptibility analyses.[36,37] These results demonstrate the importance of exchange enhancement in the various quantum-correction theories and furthermore, support our empirical approach for taking this enhancement into account.

(iv) Phase coherence lifetime τ_ϕ: As mentioned in the introduction τ_ϕ is a fundamental parameter that can be obtained from magnetoresistivity studies. Its absolute value and temperature dependence can give us a better understanding of the dominant relaxation mechanisms. The temperature variation of τ_ϕ for all samples can be found in Fig. 9. This figure presents a more complex temperature dependence then previously observed. Below 10 K the temperature variation closely follows a $T^{\sim -2}$ temperature dependence. However, at higher temperature the coherence lifetime decreases more rapidly. A $T^{\sim -2}$ variation has also been observed in paramagnetic Zr-Fe (Ref. 24) and in other studies[3-7,15] and is normally associated with the electron-phonon relaxation process. Takayama[45] has calculated that $\tau_{e\text{-ph}}$ should be given by

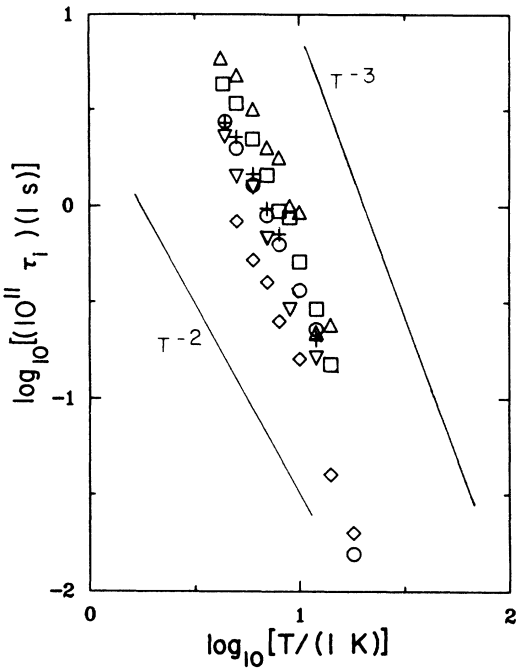

FIG. 9. Variation of the dephasing lifetime τ_ϕ with the temperature in our studies superconductor Zr-based alloys. The solid lines correspond to a T^{-2} and a T^{-3} decrease.

$$\tau_{e\text{-ph}}^{-1}=\frac{2\pi^2\lambda k_B}{3Dm\Theta_D}T^2-3\pi\ln 2\frac{k_BT}{(3Dm)^2}\ , \tag{9}$$

where λ is a constant $\sim O(1)$. At temperature above 4.2 K the first term should dominate and for Zr-based alloys would give a $\tau_\phi\sim 1.25\times 10^{-10}T^{-2}$, corresponding to a value $\sim 10^{-11}$ s at 4.2 K, which agrees with our values and indicates, at least between 4.2 and 10 K, that probably most of the dephasing processes arise from electron-phonon interactions. Above 10 K it is still not clear if the deviation from the Takayama theory is due to the activation of other processes or to temperature-dependent corrections in the electron-phonon processes.

VI. CONCLUSION

We have shown that in the case of 3D transition-metal alloys it is possible to obtain a good quantitative understanding of the magnetoresistivity data. To do his, however, it is necessary to take into account all the processes that affect the low-temperature resistivity: weak localization, the two electron-electron interactions processes, and the Maki-Thomson and Aslamasov-Larkin superconducting fluctuations contributions. We have demonstrated that, in the limits set by Larkin, the sum of the Maki-Thomson and Aslamasov-Larkin terms associated with weak-localization theory can explain the magnetoresistivity data of superconductors not too far from T_c. In addition, we have been able to describe the effect of exchange enhancement on the magnetoresistance in enhanced paramagnetic samples. The values found for the Stoner enhancement factor from the magnetoresistivity analysis are in good agreement with the ones obtained from direct magnetic measurements.

With the inclusion of the terms describing the superconducting and the magnetic fluctuations we describe completely the field and temperature dependence of the magnetoresistivity. No arbitrary scale factors are required in contrast to previous analyses.[3,5,7] The correct inclusion of the different contributions provides a clear understanding of the 3D, and probably even the 2D, magnetoresistivity in the presence of transition metals. To extend this understanding, further theoretical as well as experimental effort is required on superconducting alloys very close to T_c and also on normal alloys at very low temperatures and high magnetic fields.

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