Localization in a quantum Hall regime: Mixed short- and long-range scatterers

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The inverse localization length α in two-dimensional systems containing both short- and long-range scatterers is calculated numerically by a Thouless-number method in high magnetic fields. The strong localization in the case of long-range scatterers is reduced drastically by a small amount of short-range scatterers. The critical exponent s, defined by $\alpha \propto |E|^s$, where the energy E is measured from the center of the Landau level, is close to 2, independent of nature of scatterers for the lowest Landau level.

I. INTRODUCTION

One of the exciting phenomena observed in two-dimensional systems is the (integral) quantum Hall effect[1,2] in which the Hall conductivity is quantized into integer multiples of e^2/h. The quantum Hall effect requires the presence of both localized and extended states in strong magnetic fields and is closely related to the Anderson localization due to random potential fluctuations.[3] Several reviews have been published already.[4–6] The purpose of this paper is to present results of a numerical study on the localization in the presence of both short- and long-range scatterers in high magnetic fields.

The nature of the level broadening and transport depends strongly on the range of scattering potentials.[7] There are two length scales which characterize the electron motion in high magnetic fields, the de Broglie wavelength $\sim l/\sqrt{2N+1}$ and the cyclotron radius $\sim\sqrt{2N+1}l$, where N is the Landau-level index and l is the magnetic length defined by $l^2 = c\hbar/eH$ with H the magnetic field strength. When the range is much shorter than the de Broglie wavelength, the broadening is essentially lifetime broadening. When the range is much larger than the cyclotron radius, the broadening becomes of the inhomogeneous type. This difference manifests itself most strongly in cyclotron resonances:[7] In the case of short-range scatterers, optical transitions between all states in adjacent Landau levels become allowed. In the case of long-range scatterers, only the transition between states with nearly the same relative energy measured from the center of each broadened Landau level is allowed, since those states correspond to those with nearly the same position of the center of the cyclotron motion.

The potential range also plays an important role in localization. It has been shown previously that the localization becomes enhanced strongly in the case of long-range scatterers.[8,9] There has been a suggestion that the nature of the wave function of extended states at the center of the Landau level is also dependent on the range.[10–13] The strong dependence of localization on the potential range becomes evident if we compare the case of short-range scatterers with the long-range limit in which the range is much larger than the cyclotron radius.[14–17] In the long-range limit, the electron motion becomes classical and the center of the cyclotron motion moves along an equipotential line with velocity proportional to the local field. Therefore, the problem becomes equivalent with that of the percolation,[18] and states are all localized except just at the center of the Landau level corresponding to the percolation threshold. Even at the percolation threshold, states are localized although very weakly, because the velocity vanishes at saddle points of potentials where different equipotential lines cross. When the energy is very close to the threshold, however, quantum-mechanical effects set in and such a classical picture fails to be applicable. In the case of short-range scatterers, on the other hand, quantum effects play dominant roles at all energies.

Actual systems usually contain scatterers with different ranges. Therefore, the question arises whether the localization is dominated by short- or long-range scatterers. The purpose of this paper is to give an answer to this question. We use the Thouless-number method[19,20] in determining the inverse localization length in the lowest Landau level assuming scatterers with a δ potential and a long-range Gaussian potential, and study the relative importance of these two different scatterers. In Sec. II a brief review is given of the method of the calculation and results are presented in Sec. III. It will be demonstrated that the stronger localization in the case of long-range scatterers is drastically reduced by the inclusion of a small amount of short-range scatterers.

II. MODEL AND NUMERICAL METHOD

We consider a two-dimensional system with a finite size $(L\times L)$ in a strong magnetic field H. Scatterers are distributed randomly and their potentials are of a δ function or of a Gaussian form, i.e.,

$$V(\mathbf{r})=\sum_i V_i^S\delta(\mathbf{r}-\mathbf{r}_i^S)+\sum_i \frac{V_i^L}{\pi d^2}\exp[-(\mathbf{r}-\mathbf{r}_i^L)^2/d^2] , \tag{2.1}$$

where $\mathbf{r}_i^S$ denotes the position of the ith short-range scatterer, V_i^S $(=\pm V^S)$ its strength, and $\mathbf{r}_i^L$ and V_i^L

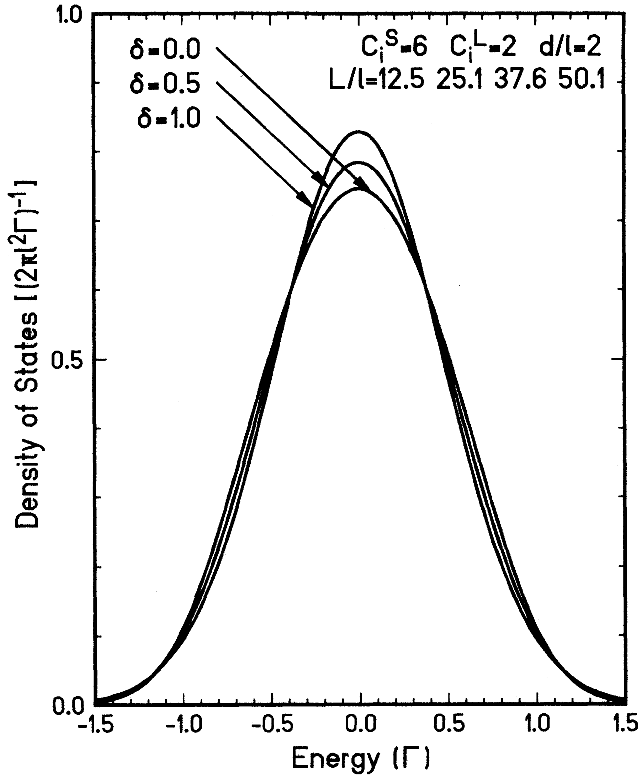

FIG. 1. Calculated density of states of the lowest Landau level for $\delta=0$ (short range), $\delta=0.5$ (mixed), and $\delta=1$ (long range). The energy is measured from the center of the Landau level and normalized by the broadening Γ calculated in the self-consistent Born approximation. The potential range is $d/l=2$ and the concentration for short- and long-range scatterers is $c_i^S=6$ and $c_i^L=2$, respectively. The curves are obtained by a smooth interpolation of calculated density-of-states histograms for energy width 0.05Γ. Statistical errors of the histograms are of the order of the width of the lines.

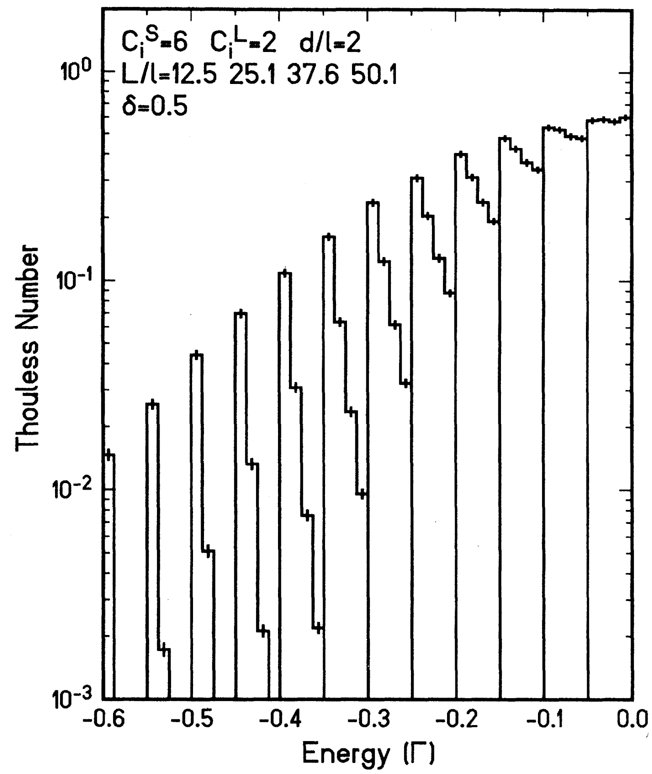

FIG. 2. Calculated Thouless number as a function of energy for $\delta=0.5$. Within each energy interval with width 0.05Γ, the Thouless number is plotted with increasing system size L. Only the low-energy half is shown because of the symmetry about the center of the Landau level.

($=\pm V^L$) are the corresponding quantities for long-range scatterers. The magnetic field is assumed to be so strong that matrix elements of the potential between different Landau levels are neglected. Equal amounts of attractive and repulsive scatterers are assumed, which enables us to use a symmetry relation about the center of each Landau level, and confine ourselves to the lowest Landau level. We shall exclusively consider the case of high concentrations of weak short- and long-range scatterers and will not treat the case of abrupt and very strong potentials, for which complex scattering dynamics was studied quite recently.[21]

In the self-consistent Born approximation, the broadening of the Landau level Γ is given by

$$\Gamma^2=\Gamma_S^2+\Gamma_L^2 ,$$
$$\Gamma_S^2=\frac{2n_i^S|V^S|^2}{\pi l^2}, \quad \Gamma_L^2=\frac{2n_i^L|V^L|^2}{\pi l^2}\frac{1}{1+(d/l)^2} , \tag{2.2}$$

where n_i^S and n_i^L are the concentration of short- and long-range scatterers per unit area, respectively. We introduce a parameter δ characterizing the strength of long-range scatterers such that $\Gamma_S^2=(1-\delta)\Gamma^2$ and $\Gamma_L^2=\delta\Gamma^2$.

The inverse localization length is determined by the system-size dependence of the Thouless number $g(L)$, which is defined as the ratio of the shifts ΔE of individual energy levels due to a change in boundary conditions to

TABLE I. Sample numbers used in the Thouless-number study. Actual sample numbers are twice those shown below because of the symmetry about the center of the Landau level.

δ	0.0	0.2	0.5	0.8	1.0
$L/l=12.5$	10 240	10 240	10 240	10 240	10 240
$L/l=25.1$	2240	2240	2240	2240	2240
$L/l=37.6$	960	960	926	960	960
$L/l=50.1$	640	640	640	640	640

the level separation $[L^2 D(E)]^{-1}$ with $D(E)$ the density of states per unit area. The calculation of the density of states and the Thouless number goes exactly in the same way as that described previously.[8] The number of eigenvalues, to which the density of states is proportional, is counted in each energy interval and the results are accumulated for all the samples with different system sizes. The Thouless number is determined by the geometric mean of energy shifts of individual energy levels for a given energy interval.

III. RESULTS

Numerical calculations are performed for the range $d/l=2$ and the impurity concentration $c_i^S=2\pi l^2 n_i^S=6$ and $c_i^L=2\pi l^2 n_i^L=2$ in systems with size $L/l=12.5$, 25.1, 37.6, and 50.1. The numbers of samples are listed in Table I. The actual sample number is twice as large as those given in the table because of the symmetry about the center chosen at $E=0$.

Figure 1 gives the density of states for $\delta=0$ (short range), 0.5 (mixed), and 1 (long range). The density of states for $\delta=0.5$ is very close to the average of those for $\delta=0$ and $\delta=1$, as might be expected. For $\delta=0$, it agrees with the exact result.[22,23]

Examples of calculated system-size dependence of the Thouless number are given in Fig. 2 for $\delta=0.5$. By fitting these to the expression $g(L)\propto\exp[-\alpha(E)L]$, we can determine the inverse localization length $\alpha(E)$. The results are given in Fig. 3. Within statistical errors, the results are all expressed well by the expression $\alpha(E)=A(\delta)|E/\Gamma|^s$ with $s\approx 2$ in agreement with the previous results.[8,9]

The coefficient $A(\delta)$ for $s=2$, obtained by a least-squares fit, is shown in Fig. 4. It can be concluded that short-range scatterers are much more important in determining α than long-range scatterers, although the localization becomes stronger for long-range scatterers. As a matter of fact, near $\delta=1$, the inverse localization length drops very rapidly when δ becomes smaller than unity, meaning that the presence of a small amount of short-range scatterers greatly reduces the localization effect. This fact may be understood by the difference in the nature of the localization between the cases of long- and short-range scatterers. In the case of long-range scatterers, the center of the cyclotron orbit moves along a certain equipotential line, i.e., the electron is confined along the equipotential line. The presence of short-range scatterers causes jumps of electrons from one equipotential line to another with equal energy (their distance is of the order of the cyclotron radius), thus tending to reduce the localization effect.

The dependence of the coefficient $A(\delta)$ and that of the peak value of the diagonal conductivity σ_{xx} are strongly correlated to each other. The peak σ_{xx}, proportional to

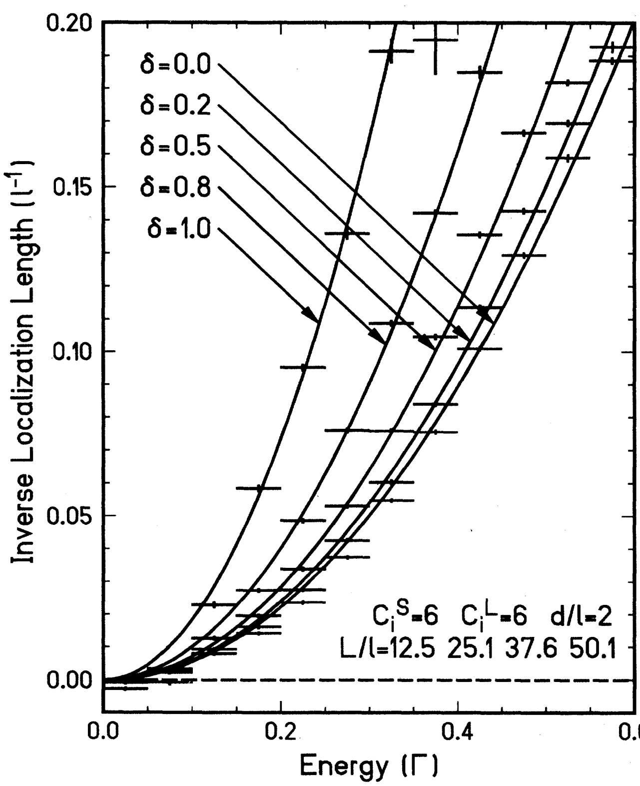

FIG. 3. The inverse localization length as a function of energy for the lowest Landau level determined by the system-size dependence of the Thouless number. The solid lines represent $\alpha(E)=A(\delta)|E|^s$ with $s=2$ fitted to the data. Only the high-energy half is shown because of the symmetry about the center of the Landau level.

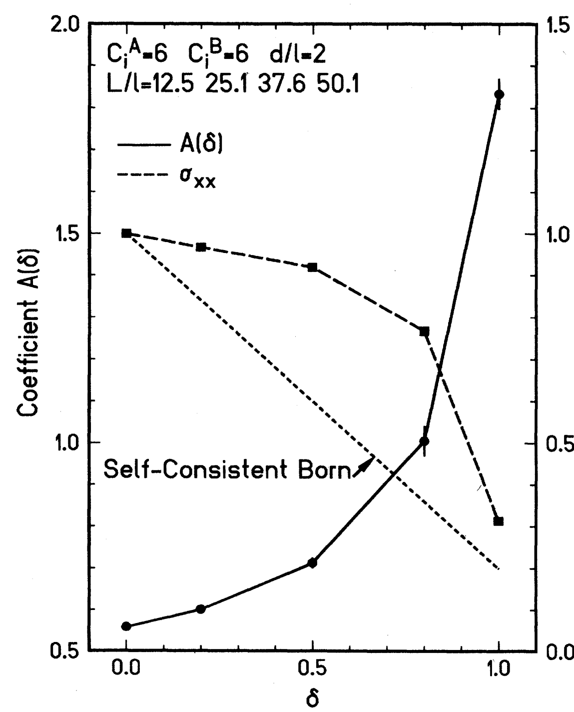

FIG. 4. The coefficient $A(\delta)$ as a function of δ. The peak value of the diagonal conductivity σ_{xx} estimated from the Thouless number at the center of the Landau level is also shown together with that calculated in the self-consistent Born approximation. The peak of σ_{xx} is normalized to unity at $\delta=0$.

the Thouless number $g(L)$ at the band center, is also plotted as a function of δ in the figure. It is normalized to $e^2/2\pi^2\hbar$ at $\delta=0$, which is the prediction of the self-consistent Born approximation.[7] There have been several suggestions that the actual peak value is slightly larger.[8,24–26] With increasing δ, the peak value decreases in qualitative agreement with the prediction of the self-consistent Born approximation, given by

$$\sigma_{xx}^{\text{peak}}=\frac{e^2}{2\pi^2\hbar}\left[1-\delta+\frac{\delta}{1+(d/l)^2}\right] . \tag{3.1}$$

Quantitatively, however, the dependence is quite different. This suggests that the self-consistent Born approximation becomes poorer in the case of long-range scatterers.[8,17]

IV. CONCLUDING REMARKS

We have presented results of a numerical study on the localization in the lowest Landau level in high magnetic fields. It has been shown that scatterers with short-range potentials are dominant in determining the localization. This is consistent with the behavior of the diagonal conductivity σ_{xx}. The critical exponent s for the energy dependence of the inverse localization length is close to 2 independent of scattering potentials. However, the present numerical method cannot give results accurate enough to determine s in a more immediate vicinity of the center of Landau levels where the localization length becomes much larger than the maximum system size ($\sim$5000 Å at $H\sim 10$ T).

Another known powerful method is the finite-size scaling method first applied successfully by MacKinnon and Kramer in the absence of a magnetic field.[27] This method has been applied for the case $\delta=0.2$ (the case $\delta=0$ and 1 has been reported already[9]). It has turned out that except in the narrow energy range $|E|\lesssim 0.05\Gamma$ the results are the same as those of the Thouless-number method.

This critical exponent s may be obtained experimentally if we assume that the temperature dependence of the conductivities is solely determined by the effective system size given by inelastic diffusion length and if we know the temperature dependence of inelastic scattering time. Recently, Wei *et al.*[28] reported $s\sim 1.2$ for both the lowest ($N=0$) and the first excited Landau level ($N=1$) in an $In_xGa_{1-x}As/InP$ heterostructure, and Wakabayashi, Yamane, and Kawaji[29] gave $s\sim 2$ for $N=0$ and a larger value for $N=1$ in a Si inversion layer. The origin of this discrepancy is not known.

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