Electron-phonon interactions in modulation-doped $Al_xGa_{1-x}As/GaAs$ heterojunctions

Y. Okuyama and N. Tokuda
Department of Engineering Science, Faculty of Engineering, Hokkaido University, Sapporo, Japan
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The mobilities and energy-loss rates of electrons in modulation-doped $Al_xGa_{1-x}As/GaAs$ heterojunctions at low temperatures are theoretically investigated based on the electron-phonon interactions. By using an extended Fang-Howard wave function it is shown that the experimental data on the mobilities and energy-loss rates of electrons are well explained in terms of the deformation-potential constant $D=8$ eV without screening and the piezoelectric constant $P=0.052$ with screening. As a result we can conclude that there are no discrepancies between the electron-phonon interactions in the $Al_xGa_{1-x}As/GaAs$ heterojunctions and the bulk GaAs.

I. INTRODUCTION

The electron mobilities and energy-loss rates in modulation-doped $Al_xGa_{1-x}As/GaAs$ heterojunctions at low temperatures are primarily understood based on the electron-phonon and/or electron-ionized impurity scatterings. The phonon modes interacting with two-dimensional (2D) electrons confined in a GaAs layer are considered to be the same as those of bulk GaAs. With regard to acoustic-deformation-potential coupling in the heterojunctions, however, there has been much controversy.[1-5] A number of studies of $Al_xGa_{1-x}As/GaAs$ heterojunctions have suggested that the deformation-potential constant D is $\sim 11-16$ eV,[1,2,4-9] considerably larger than the commonly accepted bulk GaAs value of $\sim 7-8$ eV.[10-12] Most studies of the deformation-potential coupling in heterojunctions have inferred a value for D from an analysis of the temperature dependence of mobility at low temperatures where no effective electron–optical-phonon scattering exists. The effect of ionized impurity scattering on the temperature dependence of mobility has been assumed to be negligibly small in samples with high electron concentration and high mobility at low temperatures. Recently the electron-energy-loss rates in the heterojunctions at low temperatures have also been used to determine D.[5,9] As the energy-loss rates are unaffected by elastic-scattering mechanisms such as ionized impurity scattering, and moreover, optical-phonon scattering is negligible at low temperatures ($\lesssim 40$ K), one would expect a more reliable value for D. The discrepancy concerned with the deformation-potential constant has not yet been resolved. The results of the analysis depend closely on the electron envelope function employed, the treatment of the screening factor, and the piezoelectric coupling, so that we must carefully proceed with calculations.

In this paper we determine D from an analysis of electron mobilities and energy-loss rates in modulation-doped $Al_xGa_{1-x}As/GaAs$ heterojunctions at low temperatures. We use as an electron envelope function the extended Fang-Howard variational wave function proposed by Ando,[13] since the energy-loss rates depend sensitively on the envelope function used,[5] and the extended Fang-Howard variational wave function is a good approximate function of the exact one.[13] As for screening, it is not clear whether the usual screening factor describes correctly real screening effects in the electron–acoustic-phonon interaction which is of short range.[14] We take into account the screening factor for the piezoelectric coupling which is of long range, while we use both the screened and unscreened theories for the deformation-potential coupling. We will decide from experiment whether the screening factor should be included or not.

In Sec. II we present the analytical expressions for the electron mobilities and the electron-energy-loss rates using the extended Fang-Howard variational wave function. In Sec. III we analyze the experimental data on the energy-loss rates and electron mobilities and determine the deformation-potential constant D. Discussions are also given in this section.

II. THEORY

A. Electronic states and screening factor

We employ the effective-mass approximation to electrons in modulation-doped $Al_xGa_{1-x}As/GaAs$ heterojunctions. The electronic state in the channel is characterized by subband index n and a 2D wave vector $\mathbf{k}=(k_x,k_y)$ along the heterointerface. The wave function and energy are given by

$$\Psi_{n,\mathbf{k}}(\mathbf{r},z)=\frac{1}{L}\zeta_n(z)\exp(i\mathbf{k}\cdot\mathbf{r}) , \quad (1)$$

$$E_n(\mathbf{k})=E_n+\hbar^2k^2/2m^* , \quad (2)$$

where $\zeta_n(z)$ is the envelope function corresponding to the nth energy level normal to the heterointerface (z direction), L^2 the normalization area, $\mathbf{r}$ a 2D position vector, m^* the effective mass in GaAs ($m^*=0.067m_0$), and $\hbar$ the reduced Planck constant. We assume that GaAs occupies the $z>0$ region and that $Al_xGa_{1-x}As$ occupies the $z<0$ region. $\zeta_n(z)$ and its corresponding energy eigenvalue E_n are obtained by self-consistently solving the

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Schrödinger equation[15]

$$-\frac{\hbar^2}{2m^*}\frac{d^2}{dz^2}\zeta_n(z)+V(z)\zeta_n(z)=E_n\zeta_n(z)\ , \tag{3}$$

where $V(z)$ is the potential given by

$$V(z)=V_0\Theta(-z)-e\phi(z)+v_{xc}(z)\ , \tag{4}$$

and the electrostatic potential $\phi(z)$ satisfies the Poisson equation

$$\frac{d^2}{dz^2}\phi(z)=\frac{4\pi e^2}{\kappa_0}\left[\sum_{i=0}^{m} N_i\zeta_i(z)+N_A(z)-N_D(z)\right]\ . \tag{5}$$

Here V_0 is the conduction-band discontinuity ($V_0=300$ meV in the $Al_{0.3}Ga_{0.7}As/GaAs$ heterojunction), $\Theta(z)$ the step function, $v_{xc}(z)$ the local exchange-correlation potential, κ_0 the static dielectric constant ($\kappa_0=12.91$), e the elemental charge, and N_i the electron concentration in the ith subband. $N_A(z)$ and $N_D(z)$ are the acceptor and donor concentrations, defined as follows:

$$N_A(z)-N_D(z)=\begin{cases} N_{\rm depl}/z_d & (0<z<z_d) \\ -N_I & (-W_{\rm sp}-W_D<z<-W_{\rm sp}) \\ 0 & (\text{otherwise})\ , \end{cases} \tag{6}$$

where $N_{\rm depl}$ is the depletion charge concentration, z_d the depletion layer thickness, and N_I the ionized impurity density. $W_{\rm sp}$ and W_D are the spacer layer thickness and ionized impurity layer thickness, respectively. m in the summation represents the index of the highest subband occupied by electrons. In what follows, we assume that electrons populate only the lowest subband ($i=0$) and represent $\zeta_0(z)$ by $\zeta(z)$.

We use a variational wave function for $\zeta(z)$ and adopt the Hartree approximation in which $v_{xc}(z)$ can be neglected. In most previous studies the Fang-Howard variational wave function[16] has been used. This function works quite well for Si inversion layers.[15] However, in $Al_xGa_{1-x}As/GaAs$ heterojunctions the wave function has a nonvanishing amplitude in the $Al_xGa_{1-x}As$ barrier layer in contrast to the Si inversion layer. Taking into account this point, we adopt the extended Fang-Howard variational wave function proposed by Ando:[13]

$$\zeta(z)=\begin{cases} Bb^{1/2}(bz+\beta)\exp(-bz/2) & (z>0) \\ B'b'^{1/2}\exp(b'z/2) & (z<0)\ , \end{cases} \tag{7}$$

where b, b', B, B', and β are variational parameters. Among these parameters, B, B', and β are expressed in terms of b and b' through boundary conditions at heterointerface and the normalization of $\zeta(z)$. These conditions are given by

$$\begin{aligned} &B'b'^{1/2}=Bb^{1/2}\beta\ , \\ &B'b'^{3/2}/2=Bb^{3/2}(1-\beta/2)\ , \\ &B'^2+B^2(\beta^2+2\beta+2)=1\ . \end{aligned} \tag{8}$$

The subband energy is calculated as

$$\begin{aligned} E_0(b,b')=&-\frac{\hbar^2}{8m^*}[B^2b^2(\beta^2-2\beta-2)+B'^2b'^2]+V_0B'^2+\frac{4\pi e^2}{\kappa_0}N_{\rm depl}\left[\frac{B^2}{b}(\beta^2+4\beta+6)-\frac{B'^2}{b'}\right] \\ &+\frac{4\pi e^2}{\kappa_0}N_s\left[\frac{B^4}{4b}(2\beta^4+12\beta^3+34\beta^2+50\beta+33)+\frac{B'^4}{2b'}-\frac{B'^2}{b'}\right]+\frac{4\pi e^2}{\kappa_0}N_IB'^2\frac{1}{b'^2}(1-e^{-b'W_D})e^{-b'W_{\rm sp}}\ . \end{aligned} \tag{9}$$

The parameters b and b' are determined so as to minimize the total energy $E(b,b')$ numerically.[13] N_s is the 2D electron concentration.

The static screening factor at finite temperature has been calculated to be[17]

$$S(q)=1+\frac{2\pi e^2F(q)\Pi(q)}{\kappa_0 q}\ , \tag{10}$$

where $F(q)$ is the form factor defined by

$$F(q)=\int_{-\infty}^{\infty}dz\int_{-\infty}^{\infty}dz'[\zeta(z)]^2[\zeta(z')]^2\exp(-q|z-z'|)\ . \tag{11}$$

Substituting (7) into (11), we have

$$\begin{aligned} F(q)=&\frac{B'^4b'}{b'+q}+2B^2B'^2bb'\frac{2b^2+2b\beta(b+q)+\beta^2(b+q)^2}{(b'+q)(b+q)^3} \\ &+\frac{B^4b}{2(b+q)^3}[2(\beta^4+4\beta^3+8\beta^2+8\beta+4)b^2+(4\beta^4+12\beta^3+18\beta^2+18\beta+9)qb+(2\beta^4+4\beta^3+6\beta^2+6\beta+3)q^2]\ . \end{aligned} \tag{12}$$

$\Pi(q)$ is the static polarizability function at finite temperature and is given by[18]

$$\Pi(q,T,E_F)=\int_0^{\infty}\frac{\Pi(q,0,\xi')}{4k_BT\cosh^2[(E_F-\xi')/2k_BT]}d\xi'\ , \tag{13}$$

where E_F is the Fermi energy, and k_B the Boltzmann constant. $\Pi(q,0,\xi')$ is the polarizability function at $T=0$ K,

$$\Pi(q,0,\xi')=\frac{m^*}{\pi\hbar^2}\{1-\Theta(q-2k_F)[1-(2k_F/q)^2]^{1/2}\}\ , \tag{14}$$

where k_F is the Fermi wave number [$k_F=(2m^*\xi')^{1/2}/\hbar$].

The phonons interacting with 2D electrons are considered to be those of bulk GaAs, because the experimental evidence of the interface phonon mode has never been found.

B. Electron mobility in $Al_xGa_{1-x}As/GaAs$ heterojunctions

It is known that the temperature dependence of electron mobility at low temperatures is dominated by phonon scattering, since ionized impurity scattering is independent of temperature in highly degenerate samples. In this paper we calculate only the temperature-dependent part of the electron mobility. The momentum relaxation time due to acoustic-phonon scattering is given by

$$\frac{1}{\tau(E)}=\sum_{\mathbf{k}'}W(\mathbf{k},\mathbf{k}')(1-\cos\alpha)\ , \tag{15}$$

where $W(\mathbf{k},\mathbf{k}')$ is the transition probability from state $\mathbf{k}$ to $\mathbf{k}'$ and α is the scattering angle. In the temperature range in which we analyze the electron mobility, acoustic-phonon scattering can be considered as elastic. We approximate the phonon-distribution function N_q by $k_BT/\hbar\omega_q$. Then $W(\mathbf{k},\mathbf{k}')$ is given by[19]

$$W(\mathbf{k},\mathbf{k}')=\frac{2k_BT}{\hbar^2}\int_{-\infty}^{\infty}dq_z\frac{|M(q)|^2}{\omega_qS^2(|\mathbf{k}-\mathbf{k}'|)}\times|I(q_z)|^2\delta(E_{\mathbf{k}}-E_{\mathbf{k}'})\ , \tag{16}$$

where q_z is the wave vector normal to the heterointerface and $|M(q)|$ is the 3D scattering matrix element. $I(q_z)$ is the overlap integral given by

$$I(q_z)=\int_{-\infty}^{\infty}\zeta(z)\exp(iq_zz)\zeta(z)dz\ . \tag{17}$$

Putting (7) into (17), we have

$$\begin{aligned}|I(q_z)|^2=&\frac{B'^2b'^2}{b'^2+q_z^2}\left[B'^2+2B^2b\frac{(\beta^2+2\beta+2)b^2-2bb'\beta(\beta+1)+\beta^2b'^2}{(b-b')^3}\right]\\&+\frac{B^2b^2}{b^2+q_z^2}\left[-2B'^2b'\frac{\beta^2b^2+2bb'(-\beta^2+\beta+1)+b'^2\beta(\beta-2)}{(b-b')^3}+B^2\beta^4\right]\\&+\frac{4B^2b^4}{(b^2+q_z^2)^2}\left[B'^2b'\frac{(1-2\beta)b+(2\beta-3)b'}{(b-b')^2}+B^2\beta^3\right]+\frac{4B^2b^6}{(b^2+q_z^2)^3}\left[-\frac{4B'^2b'}{b-b'}+B^2(2\beta^2+2\beta+1)\right].\end{aligned} \tag{18}$$

The scattering matrix elements for the deformation-potential and piezoelectric coupling are, respectively, given by

$$|M(q)|^2=\frac{D^2\hbar\omega_q}{2c_lL^3}\ , \tag{19}$$

and

$$|M(q)|^2=\frac{2\pi e^2P^2\hbar\omega_q}{\kappa_0q^2L^3}\ , \tag{20}$$

where c_l is the longitudinal elastic constant, P the piezoelectric constant, and L^3 the sample volume. The momentum relaxation time due to the deformation-potential coupling is given by

$$\frac{1}{\tau_{\rm def}(E)}=\frac{m^*D^2k_BT}{4\pi\hbar^3c_l}\left[B'^4b'+\frac{B^4b}{2}(2\beta^4+4\beta^3+6\beta^2+6\beta+3)\right]\int_0^{2\pi}d\alpha(1-\cos\alpha)\frac{1}{S^2(2(2m^*E/\hbar^2)^{1/2}\sin(\alpha/2))}\ , \tag{21}$$

while for the piezoelectric coupling

$$\frac{1}{\tau_{\rm piez}(E)}=\frac{m^*e^2P^2k_BT}{\hbar^3\kappa_0}\int_0^{2\pi}d\alpha(1-\cos\alpha)\frac{A(2(2m^*E/\hbar^2)^{1/2}\sin(\alpha/2))}{S^2(2(2m^*E/\hbar^2)^{1/2}\sin(\alpha/2))}\ , \tag{22}$$

where

$$\begin{aligned}A(q)=&\left[\frac{B'^4b'}{q(b'+q)}+\frac{B^4b}{2q(b+q)^3}[(2\beta^4+8\beta^3+16\beta^2+16\beta+8)b^2\right.\\&+(4\beta^4+12\beta^3+18\beta^2+18\beta+9)qb+(2\beta^4+4\beta^3+6\beta^2+6\beta+3)q^2]\\&+\frac{2B^2B'^2bb'}{q}\left[\frac{(\beta^2+2\beta+2)b^2-2bb'\beta(\beta+1)+\beta^2b'^2}{(b'+q)(b-b')^3}-\frac{\beta^2b^2+2bb'(-\beta^2+\beta+1)+b'^2\beta(\beta-2)}{(b+q)(b-b')^3}\right.\\&\left.\left.+\frac{(q+2b)[(1-2\beta)b+(2\beta-3)b']}{(b+q)^2(b-b')^2}-\frac{8b^2+9bq+3q^2}{(b+q)^3(b-b')}\right]\right].\end{aligned} \tag{23}$$

Then the relaxation time due to acoustic-phonon scatterings is given by

$$\frac{1}{\tau_{ac}(E)}=\frac{1}{\tau_{def}(E)}+\frac{1}{\tau_{piez}(E)} . \tag{24}$$

The acoustic-phonon-limited mobility is given by

$$\mu_{ac}=\frac{e}{m^*}\langle\tau_{ac}\rangle , \tag{25}$$

where

$$\langle\tau_{ac}\rangle=\int_0^\infty \tau_{ac}(E)E\frac{\partial f(E)}{\partial E}dE \Big/ \int_0^\infty E\frac{\partial f(E)}{\partial E}dE . \tag{26}$$

$f(E)$ is the Fermi-Dirac distribution function. As seen from (21) and (22) the temperature-dependent part of the inverse mobility is expressed as follows:

$$\mu_{ac}^{-1}=\alpha T . \tag{27}$$

We will discuss the temperature coefficient α in the next section.

C. Electron-energy-loss rates in $Al_xGa_{1-x}As/GaAs$ heterojunctions

The energy-loss rate is unaffected by elastic-scattering mechanisms such as ionized impurity scattering. Since at low temperatures below 40 K the contribution from optical-phonon scattering is negligible, important scattering mechanisms are due to acoustic-phonon scatterings. Furthermore, the effect of the piezoelectric coupling is only about 10% of that of the deformation-potential coupling, so that we can expect to determine D correctly. The electron-energy-loss rate is given by[5]

$$\langle\partial E/\partial t\rangle=-(1/N_e)\sum_{\mathbf{q}}\hbar\omega_q\partial N_q/\partial t , \tag{28}$$

where N_e is the number of total electrons, and $\hbar\omega_q$ is the energy of a phonon with wave vector $\mathbf{q}$. We introduce the effect of the applied electric field to the electron distribution through a Fermi-Dirac distribution $f(E)$ with an electron temperature T_e larger than the lattice temperature T_l. In the analysis of the energy-loss rates, it is impossible to apply the elastic approximation to acoustic-phonon scattering. Thus $\partial N_q/\partial t$, the variation of number of phonons with wave vector $\mathbf{q}$ per unit time is given by[5]

$$\begin{aligned}\frac{\partial N_q}{\partial t}=2\frac{2\pi}{\hbar}\sum_{\mathbf{q}}|I(q_z)|^2\frac{1}{S^2(q_\parallel)}|M(q)|^2 \\ \times\delta(E_{\mathbf{k}}+\hbar\omega_q-E_{\mathbf{k}+\mathbf{q}_\parallel}) \\ \times\{(N_q+1)f(E_{\mathbf{k}+\mathbf{q}_\parallel})[1-f(E_{\mathbf{k}})] \\ -N_q f(E_{\mathbf{k}})[1-f(E_{\mathbf{k}+\mathbf{q}_\parallel})]\} .\end{aligned} \tag{29}$$

The factor 2 comes from the spin of the electron which absorbs or emits a phonon. $E_{\mathbf{k}}=\hbar^2k^2/2m^*$ and ω_q is related to q by $\omega_q=uq=u(q_\parallel^2+q_z^2)^{1/2}$, where u is the sound velocity and $q_\parallel$ is the wave vector parallel to the heterointerface. $|I(q_z)|^2$ in (29) is already given by (18).

For the deformation-potential coupling, the above equations can be reduced to

$$\begin{aligned}\left\langle\frac{\partial E}{\partial t}\right\rangle_{def}=-\frac{2D^2(2m^*)^{1/2}}{\rho E_F(2\pi)^2} \\ \times\int_0^\infty dE_{\mathbf{k}}\frac{f(E_{\mathbf{k}})}{\sqrt{4E_{\mathbf{k}}}}\int_0^\infty dq_\parallel\int_{(q_z)_{min}}^{(q_z)_{max}}dq_z\left\{\exp\left[\left[\frac{1}{k_BT_l}-\frac{1}{k_BT_e}\right]\hbar\omega_q\right]-1\right\}N_q|I(q_z)|^2 \\ \times\frac{1}{S^2(q_\parallel)}(q_\parallel^2+q_z^2)\left[1-\frac{(\hbar\omega_q-E_{\mathbf{q}_\parallel})^2}{4E_{\mathbf{k}}E_{\mathbf{q}_\parallel}}\right]^{-1/2}[1-f(E_{\mathbf{k}}+\hbar\omega_q)] ,\end{aligned} \tag{30}$$

where

$$(q_z)_{min}=[(E_{\mathbf{q}_\parallel}-2\sqrt{E_{\mathbf{k}}E_{\mathbf{q}_\parallel}})^2/\hbar^2u_l^2-q_\parallel^2]^{1/2} , \tag{31}$$

for $(E_{\mathbf{q}_\parallel}-2\sqrt{E_{\mathbf{k}}E_{\mathbf{q}_\parallel}})^2\geq(\hbar u_l q_\parallel)^2$ and $(q_z)_{min}=0$ otherwise, and $(q_z)_{max}$ is given by

$$(q_z)_{max}=[(E_{\mathbf{q}_\parallel}+2\sqrt{E_{\mathbf{k}}E_{\mathbf{q}_\parallel}})^2/\hbar^2u_l^2-q_\parallel^2]^{1/2} . \tag{32}$$

ρ is the mass density and u_l is the longitudinal sound velocity.

For the piezoelectric coupling,

$$\begin{aligned}\left\langle\frac{\partial E}{\partial t}\right\rangle_{piez}=-\frac{2e^2P^2(2m^*)^{1/2}u^{1/2}}{\pi\kappa_0E_F} \\ \times\int_0^\infty dE_{\mathbf{k}}\frac{f(E_{\mathbf{k}})}{\sqrt{4E_{\mathbf{k}}}}\int_0^\infty dq_\parallel\int_{(q_z)_{min}}^{(q_z)_{max}}dq_z\left\{\exp\left[\left[\frac{1}{k_BT_l}-\frac{1}{k_BT_e}\right]\hbar\omega_q\right]-1\right\}N_q|I(q_z)|^2 \\ \times\frac{1}{S^2(q_\parallel)}\left[1-\frac{(\hbar\omega_q-E_{\mathbf{q}_\parallel})^2}{4E_{\mathbf{k}}E_{\mathbf{q}_\parallel}}\right]^{-1/2}[1-f(E_{\mathbf{k}}+\hbar\omega_q)] ,\end{aligned} \tag{33}$$

where $(q_z)_{\min}$ and $(q_z)_{\max}$ are given by (31) and (32) in which u_l is replaced by u, the average of the longitudinal and transverse sound velocity. The total energy-loss rates are given by the sum of (30) and (33).

III. ANALYSIS OF EXPERIMENTAL DATA AND DISCUSSIONS

The experimental data of the electron-energy-loss rates in $Al_xGa_{1-x}As/GaAs$ ($x=0.3$) heterojunctions have been reported by Hirakawa and Sakaki (HS)[9] and by Manion *et al.* (MAECH).[5] Both data are different from each other and it is not clear which data should be used to determine the deformation-potential constant D. We first fit the electron-energy-loss rates theoretically calculated to each data. The values of parameters used in our calculations are as follows. For the data of HS, the electron concentrations are $N_s=2.1\times10^{11}$ cm^{-2} (sample 1), 3.5×10^{11} cm^{-2} (sample 2), 4.6×10^{11} cm^{-2} (sample 3), 7.1×10^{11} cm^{-2} (sample 4), and 8.1×10^{11} cm^{-2} (sample 5). We assume $N_{\rm depl}=6.0\times10^{10}$ cm^{-2} and $W_{\rm sp}=20$ Å. N_I are given by the condition that the Fermi level of the 2D system is equal to the donor level in the $Al_xGa_{1-x}As$ layer and by the condition of charge neutrality of the whole system;[13] $N_I=3.4\times10^{16}$ cm^{-3} (sample 1), 1.6×10^{17} cm^{-3} (sample 3), and 5.2×10^{17} cm^{-3} (sample 4). For the data of MAECH, we use $N_I=1.3\times10^{18}$ cm^{-3} and $W_{\rm sp}=90$ Å given in their experiment and assume $N_{\rm depl}=6.0\times10^{10}$ cm^{-2}. N_s is determined to be 6.0×10^{11} cm^{-2} from the two conditions stated above. In real calculations we have neglected the term $\exp(-b'W_D)$ in Eq. (9) since it is much smaller than 1. We use the piezoelectric coupling constant $P=0.052$ (Ref. 20) and take into account the screening factor for piezoelectric scattering throughout the paper.

First we calculate the electron temperature dependence of energy-loss rates at $T_l=4.2$ K and fit it to experimental data. Figure 1 shows togther with experimental data of HS the result of fitting with $D=8$ eV and $N_s=4.6\times10^{11}$ cm^{-2}. Here the screening factor is not included for the deformation-potential coupling. On the other hand, if the screening factor is included, almost the same result of fitting is obtained with $D=11$ eV. Next we consider the data of MAECH. Figure 2 shows the result of fitting with $D=16.2$ eV for the data of MAECH and for a comparison the result of fitting with $D=11$ eV for the data of HS. Here the screening factor is included for both cases. If the screening factor is not included, the theoretical result with $D=11.8$ eV agrees quite well with the experiment of MAECH. Thus from the electron temperature dependence of energy-loss rates it is impossible to decide whether the screening factor should be included or not.

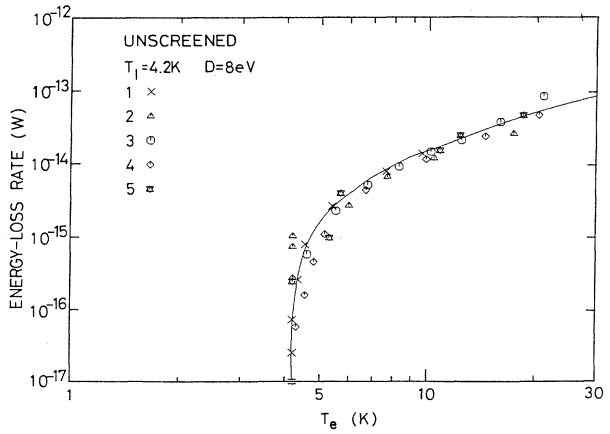

FIG. 1. Energy-loss rates at $T_l=4.2$ K vs the electron temperature. Experimental data are given by Hirakawa and Sakaki (HS). The solid line is the theoretical fitting with $D=8$ eV to the data of sample 3. The screening factor is not included for the deformation-potential coupling.

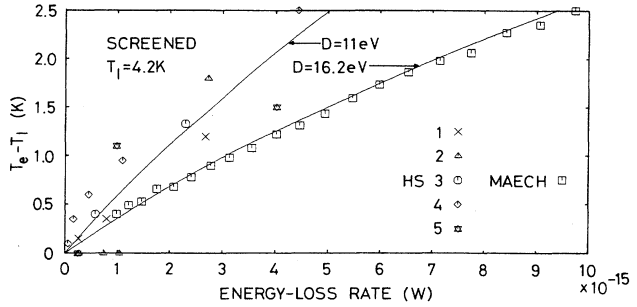

FIG. 2. Energy-loss rates at $T_l=4.2$ K vs the electron temperature. Experimental data are given by Manion *et al.* (MAECH) and by Hirakawa and Sakaki (HS). The solid lines are the theoretical fitting with $D=16$ and 11 eV. The screening factor is included for the deformation-potential coupling.

Now it should be noticed that N_s dependence of energy-loss rates seems to exist from Fig. 2. This dependence is not clearly shown in Fig. 1, since it is given in log scale. We calculate the electron temperature dependence of energy-loss rates varying the electron concentration as a parameter. In order to clarify N_s dependence of energy-loss rates we show in Figs. 3(a) and 3(b) the results obtained in terms of $D=11$ eV with screening and 8 eV without screening, respectively, for three different values of N_s, 2.1×10^{11}, 4.6×10^{11}, and 7.1×10^{11} cm^{-2}. From a comparison with the experimental data of HS it seems that the choice of $D=8$ eV without screening is preferable. Now we examine which value of D obtained above is compatible with the experimental data on electron mobilities in $Al_xGa_{1-x}As/GaAs$ heterojunctions. We consider the temperature coefficient α of inverse mobility given in (27), for which many experimental data have been accumulated.[1,6,8,21–23] In our calculation we take the following values: $N_{\rm depl}=6.0\times10^{10}$ cm^{-2}, $N_I=1.0\times10^{18}$ cm^{-3}, and $W_{\rm sp}=50$ Å. Figure 4 shows α as a function of N_s for three different values of D: $D=8$ eV without screening, 11.8 eV without screening, and 16.2 eV with screening. Experiments prefer a choice of $D=8$ eV without screening. The increase of α with N_s is due to suppression of the contribution from piezoelectric scattering as a result of screening.

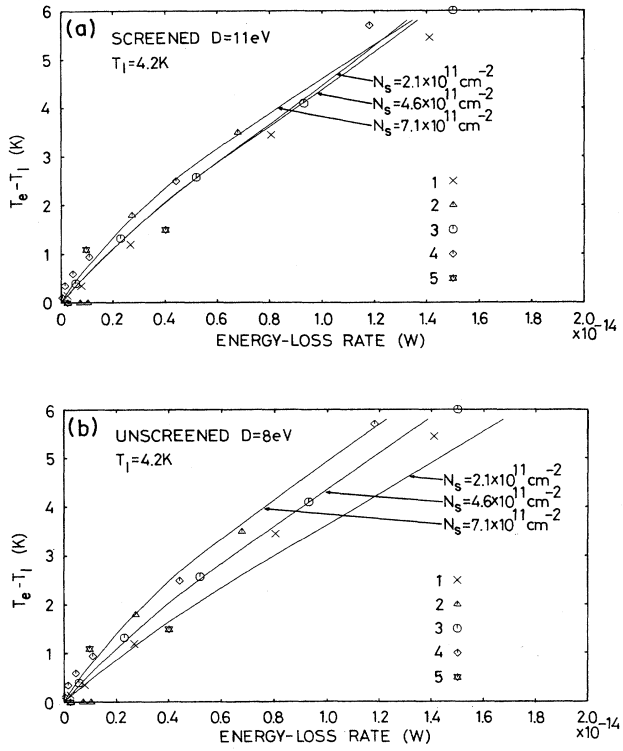

FIG. 3. (a) Energy-loss rates at $T_l=4.2$ K vs the electron temperature for three different electron concentrations. Experimental data are given by HS. The theoretical curves are obtained in terms of $D=11$ eV with screening. (b) Same as (a). The theoretical curves are obtained in terms of $D=8$ eV without screening.

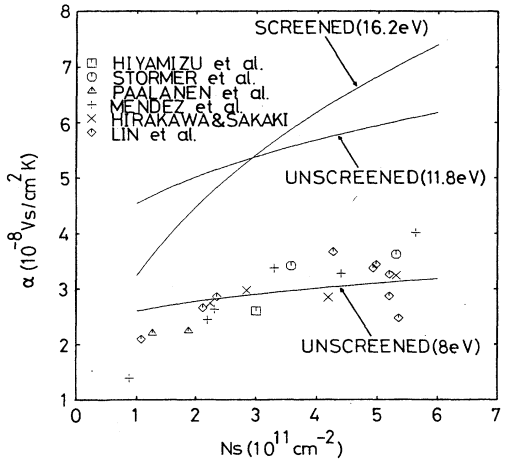

FIG. 4. Temperature coefficient α of inverse electron mobility vs the electron concentration. The theoretical curves are obtained for $D=8$ eV without screening, 11.8 eV without screening, and 16.2 eV with screening. Experimental data are given in Refs. 1, 6, 8, and 21–23.

Finally we would like to make the following points. (i) If we adopt the Fang-Howard variational wave function as $\zeta(z)$, we get D by about 1.2 times as large. Our numerical results are reliable because almost the same value of D between our calculation and the self-consistent numerical calculation[5] is necessary to fit experimental data on energy-loss rates. (ii) Since the deformation-potential coupling is a short-range interaction, the screening effect is expected to be small. Our result agrees with this expectation. The screening factor used in our calculation does not correctly describe the screening effect for electron–acoustic-phonon interaction via the deformation-potential coupling. In most bulk GaAs cases,[10–12] the screening factor is not included for the deformation-potential coupling. (iii) In the analysis of N_s dependence of energy-loss rates in Fig. 3, we have taken only the data of three samples. We have not used the data of the other two samples, because they are too much deviated to be fitted with any value of D. (iv) We cannot explain the data of MAECH with the bulk value of $D=8$ eV. It is probable that their sample contains something to scatter electrons inelastically.

In summary, using the extended Fang-Howard variational wave function we have determined the deformation-potential constant D from a comparison with the experimental data of electron-energy-loss rates and electron mobilities in $Al_xGa_{1-x}As/GaAs$ heterojunctions. The determined value $D=8$ eV is almost the same as that in bulk GaAs. The screening factor is necessary for the piezoelectric coupling which is of long range, while it is not necessary for the deformation-potential coupling which is of short range. As a result we can conclude that there are no discrepancies between the electron-phonon interactions in the $Al_xGa_{1-x}As/GaAs$ heterojunctions and the bulk GaAs.

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[1]B. J. F. Lin, D. C. Tsui, and G. Weimann, Solid State Commun. **56**, 287 (1985).
[2]P. J. Price, Phys. Rev. B **32**, 2643 (1985).
[3]W. Walukiewicz, H. E. Ruda, J. Lagowski, and H. C. Gatos, Phys. Rev. B **32**, 2645 (1985).
[4]B. Vinter, Phys. Rev. B **33**, 5904 (1986).
[5]S. J. Manion, M. Artaki, M. A. Emanuel, J. J. Coleman, and K. Hess, Phys. Rev. B **35**, 9203 (1987).
[6]E. E. Mendez, P. J. Price, and M. Heiblum, Appl. Phys. Lett. **45**, 294 (1984).
[7]P. Pfeffer, I. Gorczyca, and W. Zawadzki, Solid State Commun. **51**, 179 (1984).
[8]K. Hirakawa and H. Sakaki, Phys. Rev. B **33**, 8291 (1986).
[9]K. Hirakawa and H. Sakaki, Appl. Phys. Lett. **49**, 889 (1986).

[10]D. L. Rode, Phys. Rev. B **2**, 1012 (1970).
[11]C. M. Wolfe, G. E. Stillman, and W. T. Lindley, J. Appl. Phys. **41**, 3088 (1970).
[12]D. L. Camphausen, G. A. N. Connel, and W. Paul, Phys. Rev. Lett. **26**, 184 (1971).
[13]T. Ando, J. Phys. Soc. Jpn. **51**, 3893 (1982); **51**, 3900 (1982).
[14]W. Walukiewicz, H. E. Ruda, J. Lagowski, and H. C. Gatos, Phys. Rev. B **30**, 4571 (1984).
[15]T. Ando, A. B. Fowler, and F. Stern, Rev. Mod. Phys. **54**, 437 (1982).
[16]F. F. Fang and W. E. Howard, Phys. Rev. Lett. **16**, 797 (1966).
[17]F. Stern, Phys. Rev. Lett. **18**, 546 (1967).
[18]P. F. Maldague, Surf. Sci. **73**, 296 (1978).
[19]P. J. Price, Ann. Phys. (N.Y.) **133**, 217 (1981); Surf. Sci. **113**, 199 (1982); **143**, 145 (1984).
[20]D. L. Rode, in *Semiconductors and Semimetals,* edited by R. K. Willardson and A. C. Beer (Academic, New York, 1975), Vol. 10.
[21]H. L. Störmer, A. Pinczuk, A. C. Gossard, and W. Wiegmann, Appl. Phys. Lett. **38**, 691 (1981).
[22]S. Hiyamizu, J. Saito, K. Nanbu, and T. Ishikawa, Jpn. J. Appl. Phys. **22**, L609 (1983).
[23]M. A. Paalanen, D. C. Tsui, A. C. Gossard, and J. C. M. Hwang, Phys. Rev. B **29**, 6003 (1984).