

New superconducting state of the Anderson-lattice model

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We discuss the possible existence of a new superconducting state, the two-component superconducting (TCSC) state, of the Anderson-lattice model, when the antiferromagnetic Heisenberg exchange energy J_H generated by the hybridization between the conduction band and localized orbitals exceeds a constant multiple of the Kondo energy $k_B T_K$. In this new state, holes in both the conduction band and localized orbitals exhibit two-particle off-diagonal long-range order, and the condensate wave function is a coherent mixture of both types of singlet pairs. We propose that the TCSC phase is a possible candidate for the superconducting phase in the high- T_c copper oxides.

The Anderson-lattice model¹ (which consists of localized orbitals with strong on-site Coulomb repulsion, a free conduction band, and hybridization between the two) captures the essential physics of a class of strongly correlated electron systems. For example, its solution obtained from slave-boson techniques² successfully describes a large class of low-temperature experiments on the heavy-fermion materials. The arising physical picture can be summarized as follows: Below the invariant Haldane level,³ charge fluctuations associated with the localized orbitals are strongly suppressed and local moments are formed. A single energy scale T_K marks the crossover to the low-temperature regime, where hybridization between the conduction electrons and the localized ones quenches the magnetic moments through the formation of singlets. The resulting low-temperature phase is a Fermi liquid with enhanced effective mass, spin susceptibility, etc., and a Fermi volume which encloses both the localized and the conduction electrons. The same low-temperature physics is present in the Kondo or Coqblin-Schrieffer⁴ models, which can be obtained from the Anderson-lattice model by eliminating the charge fluctuations via the canonical Schrieffer-Wolff⁵ transformation, and stopping at the second order in the dimensionless hybridization parameter.

The recent discovery of high- T_c copper-oxide superconductors and the consequent identification of strong Coulomb correlations as the primary feature of these materials⁶ renewed the interest in the searching of superconducting phases of the Hubbard and the Anderson-lattice models. A key consequence of the strong correlation is the appearance of localized antiferromagnetism in the insulating copper oxides. The studies of this limit and the metallic phase created by doping lead one school of thought to emphasize the differences,⁷ and another the similarities⁸ between the two models. The latter, in particular Zhang and Rice,⁸ argue that the Anderson-lattice model can in fact be reduced to the Hubbard model if the binding energy of the Kondo singlet far exceeds the typical kinetic energy of the oxygen holes and the antiferromagnetic ex-

change energy between the copper moments. Their argument becomes invalid if the oxygen bandwidth is appreciable. In that case, the singlet binding energy is greatly reduced and for a large parameter range the Heisenberg exchange energy J_H can actually exceed the Kondo binding energy. In this regime, the reduction of the Anderson-lattice model to the one-band Hubbard model does not occur and one must retain both types of carriers. We propose⁹ that in this case the ground state is superconducting, induced by the "Josephson hybridization" between the copper and oxygen pair states. At finite temperatures three distinct regions can be identified. In all of them the dynamic charge fluctuations on the localized orbitals are strongly suppressed. For $T > J_H/k_B$, like the $T > T_K$ regime of the heavy-fermion systems, copper local moments fluctuate paramagnetically. A crossover occurs at $T \approx J_H/k_B$. Below this temperature the local copper moments form a singlet spin liquid¹⁰ and the conduction band remains a normal Fermi liquid (which is decoupled from the spin liquid on the mean-field level) and the Fermi surface encloses *only* the oxygen holes. This coexisting spin and Fermi liquid (CSFL) is a new phase on the mean-field level, but becomes analytically connected to the paramagnetic phase in the high-temperature regime once fluctuations are included. These fluctuations frustrate the antiferromagnetic correlation via the Ruderman-Kittel-Kasuya-Yosida interaction and give a finite lifetime to the conduction carriers. The novel superconducting phase appears at a *real* phase transition at T_c . For $T < T_c$, the one-particle hybridization remains suppressed and only singlet pairs hybridize between the localized orbitals and the conduction band. This superconducting phase exhibits two-particle off-diagonal long-range order in both bands, and is characterized by two coupled $U(1)$ order parameters.¹¹ This two-fluid nature of the superconductivity is reflected in the difference between the excitation spectra of the quasiparticles as well. We propose that this two-component superconducting (TCSC) phase is a possible candidate for the superconducting phase in the high- T_c copper oxides.

Although it is still a controversial subject, we shall base our discussion on the assumption that the holes created by doping in the cuprate superconductors go to the oxygen $2p$ orbitals⁷ that point toward the copper $d_{x^2-y^2}$ orbitals. Moreover, although the purpose of this Rapid Communication is more general, we shall restrict ourselves to the bonding topology of the CuO_2 planes in the following discussions. The picture we arrive at is a square lattice of copper sites, each capable of accommodating only one hole due to the large U_{dd} . Associated with each copper pair is an oxygen in between. The direct oxygen-oxygen hopping t_{pp} (≈ 0.5 eV) broadens the relevant oxygen levels into a band of width W (≈ 4 eV). This band hybridizes with the localized copper orbitals via t_{pd} (≈ 1.5 eV).

The basis of our calculation is the infinite- U_{dd} Anderson model:

$$\mathcal{H} = E_p \sum_{\mu} n_{\mu} + E_d \sum_i n_i - t_{pp} \sum_{\langle \mu, \nu \rangle} \sum_{\sigma} (p_{\mu\sigma}^{\dagger} p_{\nu\sigma} + \text{H.c.}) - t_{pd} \sum_{i, \mu(i)} \sum_{\sigma} (p_{\mu(i)\sigma}^{\dagger} b_i^{\dagger} d_{i\sigma} + \text{H.c.}), \quad (1a)$$

where $p_{\mu\sigma}$ and $d_{i\sigma}$ are the hole annihilation operators of the oxygens and coppers, respectively, and b_i^{\dagger} is the slave-boson creation operator;¹² E_p and E_d are measured relative to the chemical potential; μ, ν label the oxygen sites and i, j label the copper sites; $\langle \mu, \nu \rangle$ represent pairs of nearest-oxygen neighbors; $\mu(i)$ denotes the four neighboring oxygen sites μ of the copper sites i ; $n_{\mu} = \sum_{\sigma} d_{\mu\sigma}^{\dagger} d_{\mu\sigma}$, $n_i = \sum_{\sigma} p_{i\sigma}^{\dagger} p_{i\sigma}$. Because of the infinite U_{dd} associated with the copper d orbitals, Eq. (1a) is supplemented with the constraint

$$b_i^{\dagger} b_i + n_i = 1, \quad \text{all } i. \quad (1b)$$

As usual, we write down the coherent-state path-integral representation for (1a) and enforce the constraints (1b) via Lagrange multipliers λ_i . We perform the following transformations: (i) Replace the local Lagrange multipliers λ_i by a global one, λ ; (ii) anticipating $\lambda \geq |E_d|$, integrate out the slave bosons to generate an effective action to order t_{pd}^4/λ^3 . This leads to the following Hamiltonian:

$$\mathcal{H} = E_p \sum_{\mu} n_{\mu} + \varepsilon_d \sum_i n_i - t_{pp} \sum_{\langle \mu, \nu \rangle} \sum_{\sigma} (p_{\mu\sigma}^{\dagger} p_{\nu\sigma} + \text{H.c.}) - J_K \sum_{i, \mu(i), \nu(i)} d_{i\sigma}^{\dagger} p_{\mu(i)\sigma} p_{\nu(i)\sigma}^{\dagger} d_{i\sigma} - J_H \sum_{\langle ij \rangle} B_{ij}^{\dagger} B_{ij} - J_H \sum_{\langle ij \rangle} \sum_{\mu_1(i), \mu_2(i)} \sum_{\nu_1(j), \nu_2(j)} B_{ij}^{\dagger} A_{\mu_1(i)\nu_1(j)} A_{\mu_2(i)\nu_2(j)} B_{ij}. \quad (2)$$

Here, $B_{ij} = (d_{i\downarrow} d_{j\uparrow} - d_{i\uparrow} d_{j\downarrow})/\sqrt{2}$, $A_{\mu\nu} = (p_{\mu\downarrow} p_{\nu\uparrow} - p_{\mu\uparrow} p_{\nu\downarrow})/\sqrt{2}$, $\varepsilon_d = E_d - \lambda$, $J_K = t_{pd}^2/(\varepsilon_d - E_d)$, $J_H = J_K^2/(\varepsilon_d - E_d)$. The same low-energy effective Hamiltonian can be reached by performing a Schrieffer-Wolff transformation up to fourth order in $t_{pd}/(\varepsilon_d - E_d)$. The commonly neglected last term, however, describes Josephson tunneling and gives rise to the TCSC state.

Now we perform a mean-field decoupling by introducing

$$\phi_1 \equiv \frac{1}{4N} \sum_{i, \mu(i)} \frac{J_K}{t_{pd}} \langle p_{\mu(i)\sigma}^{\dagger} d_{i\sigma} \rangle,$$

and

$$\phi_2 \equiv \frac{1}{16N} \sum_{\langle ij \rangle} \sum_{\mu(i), \nu(j)} \frac{J_H}{J_K} \langle A_{\mu(i)\nu(j)}^{\dagger} B_{ij} \rangle,$$

and factorize the Hamiltonian to obtain

$$\mathcal{H} = E_p \sum_{\mu} n_{\mu} + \varepsilon_d \sum_i n_i - t_{pp} \sum_{\langle \mu, \nu \rangle} \sum_{\sigma} (p_{\mu\sigma}^{\dagger} p_{\nu\sigma} + \text{H.c.}) - t_{pd} \phi_1 \sum_{i, \mu(i)} \sum_{\sigma} (p_{\mu(i)\sigma}^{\dagger} d_{i\sigma} + \text{H.c.}) - J_K \sum_{\langle ij \rangle} \sum_{\mu(i)\nu(j)} (\phi_2 A_{\mu(i)\nu(j)}^{\dagger} B_{ij} + \text{H.c.}) - J_H \sum_{\langle ij \rangle} B_{ij}^{\dagger} B_{ij} + (\varepsilon_d - E_d) \sum_i (\phi_1^2 + 2|\phi_2|^2 - 1). \quad (3)$$

Here ϕ_1 , ϕ_2 , and ε_d are variational parameters to be determined by a free-energy minimization. (See below regarding the other possible decouplings.)

Ignoring the term proportional to J_H , the Heisenberg Fermi liquid (HFL) phase is obtained as the $\phi_1 \neq 0$, $\phi_2 = \phi_1^{\dagger}$ solution. As shown by Rice and Ueda,¹³ within the Gutzwiller approximation¹⁴ the ground-state wave function corresponding to this phase is given by

$$|\Psi_{\text{HFL}}\rangle = P_{n_d} P_d |\Psi_0\rangle, \quad (4)$$

where P_d removes configurations involving doubly occupied d orbitals, and P_{n_d} is the projection operator that fixes the average occupation number of d orbitals at $n_d = 1 - \alpha\phi_1^2$ [$\alpha = O(1)$]. $|\Psi_0\rangle$ is the free-fermion state described by a Slater determinant made of one-electron hybridized-band wave functions. At zero temperature, the mean-field solutions for ε_d and ϕ_1 are given by

$$\varepsilon_d = E_c e^{-1/\rho J_K}, \quad \phi_1^2 = \frac{1}{1 + \rho t_{pd}^2/\varepsilon_d}, \quad (5)$$

where ρ is the total conduction-band density of states at the Fermi level, and E_c is an energy cutoff $\approx \delta/\rho$ for small doping $\delta = \langle n_{\mu} \rangle$. The Kondo temperature T_K at which the crossover from the high-temperature paramagnetic solution ($\phi_1 \neq 0$) to the low-temperature HFL solution ($\phi_1 = 0$) takes place is given by $T_K = \varepsilon_d/k_B$.

The Hamiltonian given by (2) supports qualitatively different phases when $J_H/k_B T_K > C$ [$C = O(1)$]. One can understand this by following Doniach¹⁵ in observing that for $J_H > k_B T_K$ the system can lower its energy relative to the heavy Fermi liquid by forming singlets from pairs of copper spins. This qualitative argument was recently confirmed by detailed studies in several special limits. For example, in the double-impurity problem it was found that when J_H exceeds $2.2k_B T_K$ the Kondo-singlet formation is preempted.¹⁶ An analogous problem for the lattice was studied by Castellani and Kotliar¹⁷ assuming a zero t_{pp} . Contrary to the double impurity result, they find that in order to prevent the Kondo-singlet formation, J_H has to exceed J_K —a condition which cannot be satisfied if $|t_{pd}| < |\varepsilon_d - E_d|$. We have studied the lattice problem allowing $t_{pp} > 0$. The Doniach criterion is manifested in the competition between ϕ_1 and the mean-field parameter introduced to decouple the Heisenberg term (see below). Solving the eigenvalue equation and minimizing the free energy determines the mean-field parameters. We find that $\phi_1 \rightarrow 0$, if $J_H > Ck_B T_K$, even if the copper spins have a gapless excitation spectrum (here C is a constant of the order of unity). A remark is in order here about the other possible mean-field decouplings. The Doniach argument in general suppresses every expectation value containing

an odd number of both p and d operators, and among the remaining even-even decoupling alternatives we have used the most singular one. For $T_c < T < J_H/k_B$ the stable mean-field solution $\phi_1=0$, $\phi_2=0$, and $\varepsilon_d=\varepsilon_d^0$ (the CSFL phase) describes a metallic O2p band that is decoupled from the spin liquid on the copper sites. For this solution, the Fermi surface encloses the *oxygen holes only*. Fluctuations in ϕ_1 and ϕ_2 recouple the oxygen holes with the excitations of the copper antiferromagnet (AFM). This coupling can both destroy the long-range AFM order among the copper spins and scatter the oxygen holes. The wave function corresponding to the fluctuation-corrected mean-field solution has the form

$$|\Psi_{\text{CSFL}}\rangle = P_{n_d=1} |\Psi_p\rangle |\Psi_d\rangle, \quad (6)$$

where $|\Psi_p\rangle = \prod_{\varepsilon_{pk} \leq 0} p_{k\sigma}^\dagger |0\rangle$ ($p_{k\sigma}^\dagger$ are the creation operators for quasiholes in the lower oxygen band and ε_{pk} is the corresponding energy dispersion), and $P_{n_d=1} |\Psi_d\rangle$ de-

scribes the copper singlet spin-liquid state. As discussed by many authors,⁶ due to the projection operator $P_{n_d=1}$, there exists a family of $|\Psi_d\rangle$ which gives the same $|\Psi_{\text{CSFL}}\rangle$. The members in this family of $|\Psi_d\rangle$ are related by local SU_2 transformations.

For $T < T_c$, there exists another stable mean-field solution—the TCSC phase. This phase is characterized by $\phi_1=0$, $\phi_2 \neq 0$, and $\varepsilon_d = \varepsilon_d^0 + O(\phi_2^2)$. Consequently, singlet pairs of holes can hop in a correlated way between the oxygen band and the copper orbitals, but single holes cannot. This is because the singlet correlation between the copper spins suppresses the uncorrelated single-particle hopping but enhances the coherent singlet-pair hopping. Triggered by this pair hopping, the TCSC phase possesses superconducting order in $\Delta_d(k) = \langle d_{k_1}^\dagger d_{-k_1} \rangle$ and $\Delta_p(k) = \langle p_{k_1}^\dagger p_{-k_1} \rangle$.

The mean-field free energy expressed in terms of $n_d(k) \equiv \langle d_{k\sigma}^\dagger d_{k\sigma} \rangle$, ϕ_2 , ε_d , $\Delta_p(k)$, and $\Delta_d(k)$ is

$$F = \sum_k (\varepsilon_{pk} - E_{pk} + \varepsilon_{dk}' - E_{dk}) + \frac{1}{N} \sum_{kq} \{ \sqrt{2} J_K \phi_2 \gamma_q [\Delta_p^*(k) \Delta_d(q) + \text{c.c.}] + 4 J_H \Gamma_{kq} \Delta_d^*(k) \Delta_d(q) \} \\ + \frac{J_H}{2N} \sum_{kq} (1 + \gamma_{k-q}) n_{dk} n_{dq} - 2k_B T \sum_k [\ln(1 + e^{-\beta E_{pk}}) + \ln(1 + e^{-\beta E_{dk}})] + N(\varepsilon_d - E_d)(2\phi_2^2 - 1), \quad (7)$$

where N is the total number of copper sites,

$$\gamma_k \equiv \frac{1}{2} (\cos k_x a + \cos k_y a),$$

$$\Gamma_{kq} \equiv \frac{1}{2} (\cos k_x a \cos q_x a + \cos k_y a \cos q_y a),$$

$$E_{pk} = (\varepsilon_{pk}^2 + |G_p|^2)^{1/2},$$

$$E_{dk} = (\varepsilon_{dk}'^2 + |G_{dk}|^2)^{1/2},$$

and

$$\varepsilon_{dk}' = \varepsilon_d - \frac{J_H}{N} \sum_q (1 + \gamma_{k-q}) n_{dq},$$

$$G_p \equiv \frac{\sqrt{2} J_K \phi_2}{N} \sum_q \gamma_q \Delta_d(q),$$

$$G_{dk} \equiv \frac{\sqrt{2} J_K \phi_2 \gamma_k}{N} \sum_q \Delta_p(q) + \frac{4 J_H}{N} \sum_q \Gamma_{kq} \Delta_d(q).$$

The mean-field equations for $\Delta_p(k)$, $\Delta_d(k)$, and n_{dk} are given by the following:

$$\Delta_p(k) = \frac{G_p}{2E_{pk}} \tanh \left[\frac{\beta}{2} E_{pk} \right], \\ \Delta_d(k) = \frac{G_{dk}}{2E_{dk}} \tanh \left[\frac{\beta}{2} E_{dk} \right], \quad (8) \\ n_{dk} = 1 - \frac{\varepsilon_{dk}'}{E_{dk}} \tanh \left[\frac{\beta}{2} E_{dk} \right].$$

The free energy F expressed solely as a function of ϕ_2 and ε_d can be obtained by substituting (7) into (6). The mean-field equations for ϕ_2 and ε_d can be obtained by setting $\delta F / \delta \phi_2 = 0$ and $\delta F / \delta \varepsilon_d = 0$.

What we find is that as $T \rightarrow T_c^-$, $\phi_2 \rightarrow 0$, $\varepsilon_d = \varepsilon_d^0 + O(\phi_2^2)$, $\Delta_p(k) = O(\phi_2)$, $\Delta_d(k) = \Delta_d^0(k) + O(\phi_2^2)$, and $n_{dk} = n_{dk}^0 + O(\phi_2^2)$. Here $\Delta_d^0(k) = \langle \Psi_d | d_{k_1}^\dagger d_{-k_1} | \Psi_d \rangle$ with

$|\Psi_d\rangle$ defined by Eq. (6). Although, as discussed above, the $SU(2)$ symmetry of the CSFL phase makes the choice of $|\Psi_d\rangle$ ambiguous, the ambiguity is removed by demanding that $|\Psi_d\rangle_{\text{CSFL}} = \lim_{\phi_2 \rightarrow 0} |\Psi_d\rangle_{\text{TCSC}}$. The suitable $|\Psi_d\rangle_{\text{CSFL}}$ turns out to be the one with maximum s -wave component $\bar{\Delta}_d^0$ of $\Delta_d^0(k)$, where $\bar{\Delta}_d^0 = 1/N \sum_k \gamma_k \Delta_d^0(k)$. Expanding the mean-field equations to leading order in ϕ_2 , we find that the critical temperature T_c below which the $\phi_2 \neq 0$ solution exists satisfies

$$\frac{2(\varepsilon_d^0 - E_d)}{J_K^2 |\bar{\Delta}_d^0|^2} = \frac{1}{2N} \sum_k \tanh \left[\frac{1}{2k_B T_c} |\varepsilon_{pk}| \right] \frac{1}{|\varepsilon_{pk}|}. \quad (9)$$

In the weak-coupling limit, Eq. (8) yields the BCS result

$$k_B T_c = 1.14 E_c e^{-1/\rho V}, \quad (10)$$

where

$$V = |\bar{\Delta}_d^0|^2 J_K^2 |4(\varepsilon_d^0 - E_d) - O(J_H)|.$$

We note that Eq. (8) is the same as that for the BCS transition temperature obtained for oxygen holes with pairing strength V . Also, as in many other theories, T_c scales linearly with the doping concentration δ , because the cutoff energy E_c is determined by the Fermi energy $\varepsilon_F = \delta/\rho$ for small doping. Although the precise form of $\Delta_d^0(k)$ is unknown, from general arguments based on Marshall's sign rule for the AFM spin-singlet state $P_{n_d=1} |\Psi_d\rangle$, we expect $\Delta_d^0(k)$ to be a superposition of the order parameters with s and d symmetries.¹⁸ Therefore, $\bar{\Delta}_d^0$ (which is the s -wave component of the order parameter) is nonzero. For $T < T_c$, the s -wave component $\Delta_d(k)$ will increase by an amount proportional to ϕ_2^2 , which in turn opens up a spin gap for copper holes. This is necessary in view of the anomalously short quasiparticle lifetime for $T > T_c$ due to the low-lying spin excitations since long-range phase coherence can only be achieved by pair-

ing quasiparticles with long lifetimes. The lack of k dependence of G_p indicates that $\Delta_p(k)$ has s -wave symmetry; hence a genuine gap will open up for the oxygen quasiparticles at the superconducting transition. Thus, above T_c we predict a Korringa-like relaxation rate for oxygen and a non-Fermi-liquid-like behavior for the copper nuclear spins. Below T_c however, due to the opening of the gaps, activated relaxation rates are obtained for both types of nuclear spins, in qualitative agreement with the recent NMR measurements.¹⁹

The ground-state wave function for the TCSC state has the form

$$\Psi_{\text{TCSC}} = P_{n_d} P_d |\Psi_p\rangle |\Psi_d\rangle,$$

with $n_d = 1 - 2\phi_2^2$, and $|\Psi_p\rangle$ and $|\Psi_d\rangle$ given by

$$\begin{aligned} |\Psi_p\rangle &= \prod_k \{u_{pk} + v_{pk} p_{k\uparrow}^\dagger p_{k\downarrow}^\dagger\} |0\rangle, \\ |\Psi_d\rangle &= \prod_k \{u_{dk} + v_{dk} d_{k\uparrow}^\dagger d_{k\downarrow}^\dagger\} |0\rangle, \end{aligned} \quad (11)$$

where

$$\begin{aligned} u_{pk} &= \frac{G_p}{[(E_{pk} - \varepsilon_{pk})^2 + |G_p|^2]^{1/2}}, \\ v_{pk} &= \frac{E_{pk} - \varepsilon_{pk}}{[(E_{pk} - \varepsilon_{pk})^2 + |G_p|^2]^{1/2}}, \\ u_{dk} &= \frac{G_{dk}^*}{[(E_{dk} - \varepsilon_{dk})^2 + |G_{dk}|^2]^{1/2}}, \\ v_{dk} &= \frac{E_{dk} - \varepsilon_{dk}}{[(E_{dk} - \varepsilon_{dk})^2 + |G_{dk}|^2]^{1/2}}. \end{aligned} \quad (12)$$

In general, disorder introduces single-particle hybridization between bands and hence tends to smear out the difference between the two components of the superconducting state. For the TCSC state however, due to the AFM Heisenberg exchange, the single-particle hybridiza-

tion term is suppressed ($\phi_1 = 0$); i.e., the two-fluid nature of the TCSC state will not be disrupted by weak disorder.

Whether or not the condition $k_B T_K < J_H$ holds in the cuprate materials is a controversial issue. If one takes the typical values $t_{pd} = 1.5$ eV, $t_{pp} = 0.5$ eV, $E_c \approx \varepsilon_F = 0.4$ eV, and $\varepsilon_d - E_d = 3$ eV, one finds $J_H = O(10^{-1})$ eV and $k_B T_K = O(10^{-2})$ eV, hence the condition for the suppression of the HFL formation is likely to be realized. The $T_K \equiv \varepsilon_d/k_B$ [with ε_d given by Eq. (4)] used here merely gives the temperature at which the crossover from the paramagnetic regime to the HFL regime occurs,²⁰ and it should not be confused with the self-consistent Fermi temperature obtained by solving Eq. (2) with ϕ_2 and the Heisenberg exchange term ignored.²¹

As for any slave-boson mean-field theory, it remains to be seen whether our results survive fluctuations. Moreover, the TCSC phase will not be stable if there is a strong repulsive interaction between the oxygen-band quasiparticles. The off-diagonal Coulomb matrix element between the two-particle states in the copper and oxygen orbitals, on the other hand, enhances the pair hopping and hence acts favorably.²²

Concerning numerical evidences, we refer to the work of Imada,²³ who finds an enhancement in pair susceptibility when $J_k \approx J_H$, consistent with our results.

Finally, we note that the stabilization of the TCSC fixed point due to the AFM Heisenberg exchange and the fact that the singlet pairs of copper holes behave as distinct entities at the TCSC fixed point are similar in spirit to Anderson's original resonating valence bond picture,¹⁰ and is rather different from the spin-fluctuation mechanism suggested previously²⁴ for heavy-fermion superconductivity.

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