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Critical state in disk-shaped superconductors

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We have calculated the magnetic fields and currents occurring in a disk-shaped superconductor (radius$\gg$thickness) in the critical state in a self-consistent way using finite-element analysis. We find that the field shielded (or trapped) in the center of the disk is roughly equal to $J_c d$, where d is the thickness of the disk. The shielding currents also create radial fields which are of order $J_c d/2$ on the disk surface. For low applied fields $H_{appl} < J_c d$ these self-field effects dominate, leading to a deviation of the local field direction from the applied field, which can exceed 90° towards the outside perimeter of the disk. If $J_c d$ is large, as is the case for $YBa_2Cu_3O_{7-\delta}$ single crystals at 4.2 K, self-field effects persist up to several tesla applied field. The field dependence of the calculated magnetic moment in the self-field dominated regime is independent of whether J_c is weakly or strongly ($\propto 1/H$) dependent on field. The calculations were validated by comparison to both magnetic and resistive measurements on a disk-shaped section of Nb_3Sn tape.

The critical current density J_c in type-II superconductors can be determined in various ways. For wires, J_c is traditionally measured by applying a transport current and measuring the voltage generated in a four-probe configuration. Contactless measurements rely on measuring the magnetic moment in increasing and decreasing magnetic fields, and ac methods determine the flux profile inside the superconductor.[1-4] All of the magnetic methods share a common basis, namely that the flux trapped in the conductor arranges itself into the critical state.[1] The J_c determined from resistive and inductive methods usually agrees well in metallic superconductors. Most measurements and calculations using the critical state are carried out in slab or cylindrical geometry where demagnetization corrections are negligible. Unfortunately almost all single crystals of the new oxide superconductors are in the form of thin plates with the c axis along the thin dimension (e.g., Kaiser *et al.*[5]) If the field is aligned parallel to the c axis, we can approximate this configuration by a thin superconducting disk in transverse field. The disk still has cylindrical symmetry, but we expect the critical state to be different from that of a long cylinder due to large demagnetization effects.

We have extended the magnetic field computations done by Frankel[6] for thin superconducting disks in a transverse field to disks with varying aspect ratio, and in addition we have included radial field effects. The calculation was carried out for a disk of height d and radius r_0 in cylindrical coordinates. The field H_{appl} was applied in the z direction. From the cylindrical symmetry it follows that currents only flow in the circumferential direction. The disk was divided into $n\times m$ ring segments (n along the radius and m along the thickness), yielding up to 10^5 segments. The radial and axial fields (h_r and h_z, respectively) at arbitrary coordinates r and z created by a current loop of radius a with the current I flowing are given by[7]

$$h_r(r,z)=\frac{I}{2\pi}\frac{z}{r[(a+r)^2+z^2]^{1/2}}\left[-K(k)+\frac{a^2+r^2+z^2}{(a-r)^2+z^2}E(k)\right], \quad (1)$$

$$h_z(r,z)=\frac{I}{2\pi}\frac{1}{[(a+r)^2+z^2]^{1/2}}\left[K(k)+\frac{a^2-r^2-z^2}{(a-r)^2+z^2}E(k)\right], \quad (2)$$

with $k^2=4ar[(a+r)^2+z^2]^{-1}$. The center of the loop is located at $r=z=0$. K and E are complete elliptic integrals of the first and second kind. The magnetic field at the coordinates of each segment is then the sum of the fields generated by all the other segments, plus the applied field. The calculation proceeds as follows: First a constant current density J is assumed to flow in each segment, the direction of which determines whether an increasing or decreasing field case is to be computed. Then the field generated by this current is determined and added to the applied field. In most cases the computations were stopped at this point, assuming J_c to be field independent across the disk.

In order to investigate the case of large self-fields, some calculations were carried out with a field-dependent J_c. After the first step the local J values then were modified according to the critical state, fulfilling the condition $J=J_c(H)$ at all points in the disk where H is the total (radial plus axial) field at each point. Then the field was again calculated from the modified current distribution, thus changing the fields (and currents) again, and so on. A self-consistent state was usually reached within five iterations. The field dependence of the current used here was of the Kim type[8] with J_c proportional to $1/(H+H_0)$, where H_0 is a constant. Finally, the magnetic moment of the sample was computed by summing up the contributions from the individual current loops, averaging over the values found for increasing and decreasing fields. The magnetization was then obtained by dividing by the sample volume.

The field profiles generated here agree well with those measured (and calculated) by Frankel[6] for his geometry, using his value for the current density. However, we note that he used J_c and H_0 as fitting parameters and did not determine them independently (i.e., by a resistive J_c measurement). We have also computed the long-cylinder geometry ($d \gg r_0$) and obtain the standard results for the field distribution.[1]

Figure 1 shows h_r (at the surface) and h_z (in the center plane), normalized by $J_c d$ (in this case a field-independent J_c was used), as a function of normalized radius r/r_0 for two disks with different radii but identical thickness. h_z is almost identical for the two disks, although the radii are different by 2 orders of magnitude. h_z depends only weakly on z and is largest for $z=0$. The radial field is largest at the surface ($|z|=d/2$), zero in the specimen center (at $r=0$ for all z values) as well as the center plane at $z=0$, and changes sign at $z=0$.

There are two central results of the calculations. (i) The field h^* that is shielded (or trapped) in the center now is of order $J_c d$,[9] instead of $J_c r_0$. The critical state in the disk (where the field gradient is J_c) therefore actually occurs through the thickness, not the radius. (ii) The maximum radial field is roughly $h^*/2$. The exact location of the maximum in h_r depends on the field dependence of J_c. This field is absent in the slab and long cylindrical geometries.

The axial field generated by the disk reverses polarity at about $0.85r_0$. This leads to a demagnetization field of roughly 15% of $J_c d$ for moderate r_0/d ratios (higher for $r_0/d \gg 10$). This demagnetization field does not depend on the applied field (but on J_c), contrary to the diamagnetic case where the applied magnetic field at the edge of the disk is enhanced by a factor of $(4/\pi)(r_0/d)$.[10,11] If $h^* \ll H_{\text{appl}}$, then obviously a demagnetization correction (in a measurement of the magnetization) is not necessary.

Problems can occur in deriving the field dependence of J_c from the field dependence of the magnetic moment. In Fig. 2 the calculated magnetization and the current density (both normalized to their zero-field values) are plotted as a function of applied field for a disk with $r_0/d=5$. We see that the field dependence of the magnetization is significantly less steep than the field dependence of J_c for fields that are lower than the self-field h^*, which in this case is about 3 T with no external field applied. Therefore the field dependence of J_c can only be derived from magnetization measurements in fields which are much larger than h^*.

For $h^* \ll H_{\text{appl}}$ the variation of B and thus J_c across the disk is small and the field gradient across the disk can be approximated by a straight line, with deviations occurring in the center and at the outer edge. The local-field direction at all points in the disk is now very close to the direction of the applied field. In this case we obtain for the hysteresis of the magnetization (the difference between the up and down branch),

$$\Delta M = \tfrac{2}{3} J_c r_0 . \quad (3)$$

This expression is not dependent on the sample thickness and holds for all cases in which the current density is constant throughout the disk. It also is identical to the expression for a long cylinder.[1]

We have measured the flux gradient in thin disk specimens of Nb_3Sn in a transverse field, using the Campbell ac technique,[2] thus obtaining h^* directly. The J_c determined both resistively (at a voltage criterion of 1 μV/cm) and from the h^* measurement (using a frequency of 17 Hz) for the Nb_3Sn specimen are shown in Table I. The

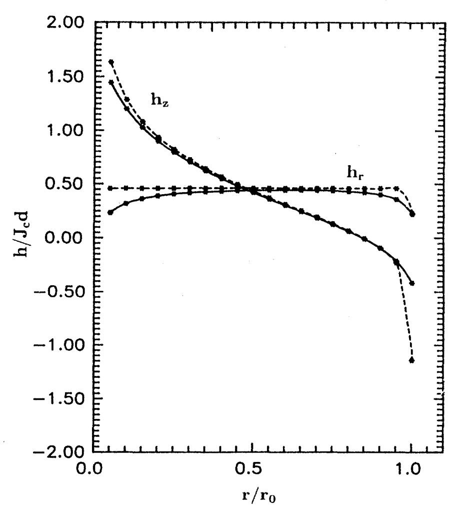

FIG. 1. Radial (h_r) and axial (h_z) self-field of the current carrying disk of thickness d normalized by $J_c d$ vs reduced radius for two different disks (——, $r_0/d=10$; ---, $r_0/d=10^3$). The lines shown represent a cubic spline fit through the calculated points.

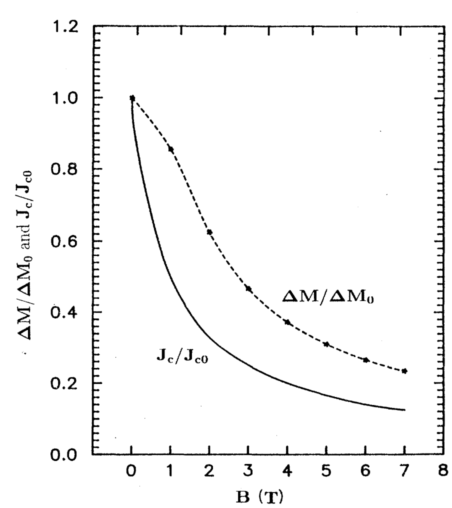

FIG. 2. Normalized magnetization $\Delta M/\Delta M_0$ (---) and critical current density J_c/J_{c0} (——) vs applied field. The lines are calculated using a spline fit.

TABLE I. Comparison of inductively and resistively (1 μV/cm) measured J_c for Nb_3Sn tape.

Field B (T)	J_c^{res} (A/mm^2)	J_c^{ind} (A/mm^2)
5	2170	2940
6	1630	2572
7	1390	2013

specimen always quenched in resistive measurements in fields below 5 T, and the highest field available for the inductive measurement was 7 T, thus only three values are given. The total Nb_3Sn layer thickness (5.4 μm in this case) was used as the scaling length. Since h^* in this case is several tens of mT, we can safely assume that J_c does not change appreciably across the specimen in our large applied fields. This configuration provides an explicit test of the model since r_0/d is ≈ 200 and there is no possibility of J_c being uncertain to more than a very small part of this factor. Indeed we found that $h^* = J_c d$, not $J_c r_0$.

All inductively measured J_c values tended to be roughly 40% larger than their resistive counterparts. We think that the discrepancy is mostly due to the different voltage criterion in the two cases. The resistive transition was found not to be very sharp. A 5 μV/cm criterion (the highest voltage measured resistively) already led to an increase in the resistive J_c of $\approx$15%–20%. With a frequency of 17 Hz, $h^* = 0.02$ T, and an average radius of 0.5 mm, we find an effective voltage criterion for the inductive measurement of 10 μV/cm. Thus, the real difference between the inductive and resistive measurements is less than 20%, which is really a very good agreement. Kroeger, Koch, and Charlesworth[4] also compared various methods of J_c determination for long cylindrical specimens. They found similar values for the discrepancy between J_c values determined by Campbell's method and the resistive method.

These results have important implications for the analysis of magnetization curves of typically aspected single crystals of high-T_c superconductors. In a typical experiment, the zero-field current density is determined by measuring the trapped flux generated in the specimen by ramping the applied field to a finite value and then back to zero. In this case the field present in the sample is generated exclusively by the shielding currents of the sample itself. The present analysis shows that the direction of the total magnetic field is not perpendicular to the surface of the disk, especially near the outside perimeter, where the field is actually parallel to the surface. For an anisotropic specimen such as a single crystal of $YBa_2Cu_3O_{7-\delta}$ this has important implications, since the direction of the magnetic field varies throughout the specimen. Values of the zero-field J_c anisotropy therefore cannot be reliably extracted from measurements of the trapped flux in the specimens of disk geometry. The overall current in the specimen is always flowing in the ab plane, supported by a fluxon density gradient. However, the local current flow around the center of each fluxon is more complicated, since the direction of the current field depends on the direction of the magnetic field and the anisotropy of the superconducting parameters. In isotropic superconductors the currents are always flowing perpendicular to the local-field direction. This is not the case for anisotropic materials.[12]

Problems occur also in determining the field dependence of J_c. At 4.2 K the self-field generated by a typical $YBa_2Cu_3O_{7-\delta}$ single crystal is several tesla. Therefore, the self-field dominated regime is quite large (0–10 T), and field dependencies of J_c determined from these measurements are not reliable. The anisotropy and field dependence of J_c derived from aligned powdered specimens[13,14] should therefore be more accurate, since they avoid the disk geometry and, due to the small size of the aligned particles, their self-field is lower.

Finally, we would like to make some remarks on the stability of the critical state. The adiabatic stability criterion[15] for a slab defines a stability parameter $\beta = \mu_0 J_c^2 a^2/[3\gamma C(T_c - T)]$, where $2a$ is the width of the slab, γ is the density, C is the specific heat per unit volume, and T (T_c) is the (critical) temperature. The slab is flux-jump unstable if $\beta > 1$. The calculation is based on the contribution of the shielding currents to the specific heat of the sample. At $\beta = 1$ the effective specific heat of the sample is zero. We have to modify this criterion for the disk geometry since the shielding fields are different for the two cases. The result is $\beta_{disk} = \mu_0 J_c^2 r_0 d/[5\gamma C(T_c - T)]$. Experimentally, we found that our Nb_3Sn specimen exhibited flux jumps at low applied fields, as soon as the Nb substrate tape became superconducting (below about 0.5 T). β_{disk} in this case was roughly 9. We calculate $\beta_{disk} = 10$ for Frankel's NbTi specimen, which, however, was stabilized by copper.

Others[16–18] have estimated the magnetic stability of $YBa_2Cu_3O_{7-\delta}$, however, only down to a temperature of 20 K, and in slab geometry. Taking values of J_c (Ref. 19) and C (Ref. 20) from the literature, we find $\beta_{disk} = 678$ for a crystal with $r = 0.5$ mm and $d = 50$ μm in transverse field at 4.2 K. Therefore, any disturbance should lead to the decay of the critical state. Actually, flux jumps should have occurred long before the sample reached this state (and they do, in Nb). To the knowledge of the authors only in one case have flux jumps been observed in $YBa_2Cu_3O_{7-\delta}$ single crystals.[21] A possible explanation could be found in the difference in T_c between the two materials. A flux jump in a 1:2:3 crystal would not be expected to run to completion since the material becomes stable in the 20-K range. However, we do not find this explanation satisfactory, since partial flux jumps should still take place. There is also the possibility that the specimen is subdivided internally, and therefore that only part of the current is flowing across the whole specimen.[22,23] This would certainly increase the stability of the system, while still keeping the magnetization of the specimen high.

In summary, we have calculated the magnetic fields generated by a superconducting disk in the critical state in a transverse magnetic field. We find that the field shielded at the center of the disk is $J_c d$, not $J_c r_0$. A radial field appears on the surfaces which is of order $J_c d/2$, a field which is absent in long cylindrical geometry. These calculations were compared to flux-penetration experiments done on Nb_3Sn tape and good agreement was obtained. The relevance of the field profiles to measurements of the remanence in high-T_c oxide superconductors was discussed. Finally, we discussed the flux-jump stability of the critical state in this geometry. We found that disks are more stable than cylinders (with identical J_c) and we noted that high-T_c crystals are much more flux-jump stable than expected.

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