Sound attenuation and dispersion in a diluted Ising model

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The critical behavior of sound attenuation and dispersion in a diluted Ising system with a nonconserved order parameter is studied above T_c. The dynamical scaling functions are computed by the ϵ-expansion method up to the first order.

As it is evident from the heuristic argument of Harris[1] and from the renormalization-group (RG) approach,[2] the coupling of the order parameter (OP) with quenched nonordering impurities, which induce random variations in the local transition temperature leads to nontrivial consequences when the specific-heat exponent of the pure system is positive. In this paper we shall only consider weak disorder far from the percolation threshold. When we study the behavior of the Ising model (which is the only one from three-dimensional n-vector models with $\alpha > 0$) by means of the RG approach and ϵ $(=4-d)$ expansion, then the $O(\epsilon)$ Ising fixed point is driven by quenched disorder effects to the $O(\sqrt{\epsilon})$ Khmel'nitskii fixed point[3] with negative specific-heat exponent α. On the other hand if the Ising spins are coupled with isotropic elastic degrees of freedom then a first-order transition, usually proceeded by a pseudocritical region, is expected, under a constant pressure.[4] In the case of pinned boundary conditions a continuous transition with Fisher-renormalized exponents is predicted. In this work we are going to determine the influence of dilution on critical sound propagation in a solid under a constant pressure in the limit of small compressibility.[5] We shall only discuss the high-temperature disordered phase whose dynamics, unlike in the ordered phase, is not expected to be affected by long-lived droplet fluctuations.[6] The Hamiltonian of the elastically isotropic system may be specified as

$$H=\int d^dx\left[[r_0+\phi(x)-\nabla^2]S^2+\tfrac{1}{4}\tilde{u}_0S^4+\tfrac{1}{2}C_{12}^0\left(\sum_\alpha e_{\alpha\alpha}\right)^2 + C_{44}^0\sum_{\alpha,\beta}e_{\alpha\beta}^2+[p+\pi(x)]\sum_\alpha e_{\alpha\alpha}+g_0S^2\sum_\alpha e_{\alpha\alpha}\right], \tag{1}$$

consisting of (i) magnetic terms of the Ginzburg-Landau form, (ii) an elastic part where $e_{\alpha\beta}(x)$ denotes the strain tensor, p is an external pressure, and $C_{\alpha\beta}^0$ are the bare elastic constants, (iii) a term describing the interaction of the OP and the elastic degrees of freedom, where g_0 is the bare coupling constant. The random variables $\phi(x)$ and $\pi(x)$ are the local transition temperature fluctuations and induced random stress, respectively. We assume them to follow a Gaussian distribution with mean values of zero.

There are six fixed points[7] in the space of the parameters (r,u,v,Δ), where v represents a total effect from the homogeneous deformations (involving bulk modulus B) as well as from phonon modes (involving longitudinal modulus C_{11}) and can be regarded as the strength of a strain-mediated interaction in the effective OP Hamiltonian; Δ describes the amount of disorder. These parameters are defined as follows:

$$u=\tilde{u}-2g^2/C_{11},\quad v=2g^2(1/C_{11}-1/B),$$

$$\Delta=\Delta_{\phi\phi}+4g\Delta_{\phi\pi}/C_{11}+4g^2\Delta_{\pi\pi}/C_{11}^2,$$

where

$$B=C_{11}-\frac{2(d-1)}{d}C_{44}.$$

It was shown[7,8] that a random Ising (Khmel'nitskii) fixed point (RI) is stable with respect to the elastic perturbations and its asymptotic behavior is characterized by the exponents of the rigid random model. However, a first-order transition may appear if the quenched disorder is sufficiently weak and a constant pressure is applied.

The dynamics of the system are described by the stochastic Langevin equations for the OP field and the acoustic phonons[9]

$$\dot{S}_{\mathbf{k}}=-\Gamma_0\frac{\partial H}{\partial S_{-\mathbf{k}}}+\xi_{\mathbf{k}},$$
$$\ddot{Q}_{\mathbf{k},\lambda}=-\frac{\partial H}{\partial Q_{-\mathbf{k},\lambda}}-D_\lambda^0k^2\dot{Q}_{\mathbf{k},\lambda}+\eta_{\mathbf{k},\lambda}, \tag{2}$$

where $\xi_{\mathbf{k}}$ and $\eta_{\mathbf{k},\lambda}$ are Gaussian white noises. Using the functional representation of the above equations and having in mind that the transversal modes are not coupled to the OP in our model, the self-energy of the acoustic response function

$$\Sigma(k,\omega)=-2g^2k^2\frac{C(k,\omega)}{1+v^{\mathrm{ph}}C(k,\omega)}, \tag{3}$$

which is the quantity of interest, can be expressed in terms of the frequency-dependent specific heat

$$C(k,\omega)=[\langle\Gamma_0(\tilde{S}S)_{\mathbf{k},\omega}(S^2)_{-\mathbf{k},-\omega}\rangle_{L_{\mathrm{OP}}^{\mathrm{eff}}}]_{\mathrm{av}},$$

where unitary density was assumed and $\tilde{S}$ is an auxiliary response field.[10] The first average in the above expression is calculated for the effective OP Langrangian, $L_{\mathrm{OP}}^{\mathrm{eff}}$, containing the strain-mediated interactions. The specific heat of the quenched random system is then obtained by averaging this expression over static random variables $\phi(x)$ and $\pi(x)$.

In order to discuss acoustic properties of the system, an additional parameter, $v^{\rm ph}=2g^2/C_{11}$, is required.[5] It fulfills the same RG equations as v but contrary to v_0 the bare value of $v^{\rm ph}$ is always positive. As the RG trajectory approach the random Ising fixed point, both v and $v^{\rm ph}$ tend to zero. Therefore the asymptotic behavior is only determined by the numerator of Eq. (3).

In the calculations the presence of impurities brings about additional diagrams which contribute to the value of the dynamic critical exponent z and strongly affect the OP shape function at small frequencies as $T\to T_c$.[11] For this reason some changes in the usual calculations used for the pure system are necessary.[5,12] Apart from "mass renormalization" it is convenient to perform exponentiations of the $O(\Delta)$ logarithmic singularities in the OP propagators[11] found in diagrams for the frequency-dependent specific heat. These singular terms come from the first-order graph in Fig. 1(a). After exponentiation we have

(a) (b)

FIG. 1. (a) Diagram contributing to the OP frequency-dependent self-energy calculated up to $O(\Delta)$. (b) The additional diagram contributing to the acoustic self-energy in first order in Δ. The lines and circled lines represent the OP response and correlation functions, respectively. The dashed line represents a factor of Δ.

$$G_{\tilde{S}S}^{-1}(k,\omega)=\chi_l^{-1}+k^2-i\omega F(\omega;l)\,, \qquad (4)$$

with

$$F(\omega;l)=\Gamma_0^{-1}\chi_l^{\Delta^*/2}(1-ix)^{-\Delta^*/2}\left\{1+\frac{\Delta^*}{2}\left[\frac{1}{ix}\ln(1-ix)+\ln(1+\chi_l^{-1}-i\omega/\Gamma_0)+\ln\left[\frac{1+\chi_l^{-1}}{1+\chi_l^{-1}-i\omega/\Gamma_0}\right]\right]\right\},$$

where $\chi_l=t^{-\gamma}e^{-2l}$ is the static susceptibility with t being the reduced temperature and $x=\omega\chi_l/\Gamma_0$. To the first order in Δ the diagram displayed in Fig. 1(b) should also be included in the frequency-dependent specific heat besides those for the pure case.[12]

Integration of the RG equations leads to the following expression for the acoustic response function:

$$G^{-1}(k,\omega)=c_l^2k^2-v_l^{\rm ph}c_l^2k^2C(\omega e^{zl})-\omega^2\,, \qquad (5)$$

with $c_l^2=c_0^2R$, $v_l^{\rm ph}=v_0^{\rm ph}Re^{(\alpha/\nu)l}$, $R=(1-a+ae^{(\alpha/\nu)l})^{-1}$, $a=v_0^{\rm ph}/v^*$, $v^*=\alpha/\nu+O(\epsilon)$, where less singular terms have been omitted in Eq. (5) and because the ultrasonic wavelength is much longer than the correlation length the limit $k=0$ in the frequency-dependent specific heat has been taken. In this expression c is the longitudinal sound velocity and the critical exponents correspond to the random Ising fixed point.

Finally the flow parameter l^*, for which the function $C(\omega e^{zl^*})$ can be evaluated by perturbation expansion, is determined by the condition

$$(\omega_{l^*}/2\Gamma_0)^{4/z}+\chi_{l^*}^{-2}=1\,, \qquad (6)$$

with the explicit solution

$$e^{l^*}=t^{-\nu}\Phi(y)\,, \qquad (7)$$

where

$$\Phi(y)=[1+(y/2)^{4/z}]^{-1/4}\,,$$

and $y=\omega t^{-z\nu}/\Gamma_0$. Inserting Eq. (7) into Eq. (5) we find that the sound attenuation coefficient and dispersion obey the asymptotic scaling relations

$$\alpha(\omega,t)\sim t^{-\rho}\omega^2 g(y)\,,$$

$$c^2(\omega,t)-c^2(0,t)\sim t^{-\alpha}f(y)\,,$$

where $\rho=z\nu+\alpha$,

$$g(y)=y^{-1}\Phi^{\alpha/\nu}(y)\,{\rm Im}C(y)+O(\epsilon)\,,$$

and $f(y)=\tilde f(y)-\tilde f(0)$ with

$$\tilde f(y)=\Phi^{\alpha/\nu}[1+v^*{\rm Re}C(y)]+O(\epsilon)\,.$$

To the first order in $\sqrt{\epsilon}$ the function C may be written as

$$C(y)=C^0(y)+\tfrac{3}{2}u^*\left[\frac{E^2}{F_2^2}+\frac{(1-iF_1)^2}{4F_1^2}A^2-\frac{(1-iF_1)}{F_1F_2}AE\right]$$
$$+\Delta^*\left\{\frac{(1-iF_1)}{F_2^2}\left[2DE-D^2+2{\rm Li}_2\left[\frac{1-iF_2}{2-iF_2}\right]+\frac{\pi^2}{6}\right]-\frac{E^2}{F_2^2}+\frac{i(1-iF_1)}{2F_1}A^2\right\},$$

where

$$C^0(y)=i\left[\frac{E}{F_2}-\frac{(1-iF_1)}{2F_1}A\right],\;\; E=2\ln(\Phi),\;\; A=\ln(\Phi^2-iF_1\Phi^2),\;\; F_k=\frac{k}{2}\omega e^{zl^*}\chi_{l^*}F\left[\frac{k}{2}\omega e^{zl^*};l^*\right],$$

$D=\ln(2\Phi^2-iF_2\Phi^2)$, ${\rm Li}_2(x)=\sum_{k=1}(x^k/k^2)$, $u^*=\frac{2}{3}(6\epsilon/53)^{1/2}$, and $\Delta^*=(6\epsilon/53)^{1/2}$.

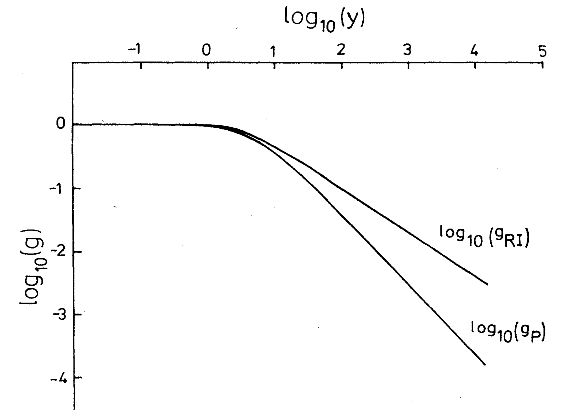

FIG. 2. Scaling functions for sound attenuation for $\epsilon = 1$ with normalization $g_{RI}(0) = g_p(0) = 1$. g_{RI} and g_p correspond to diluted and pure Ising behavior, respectively.

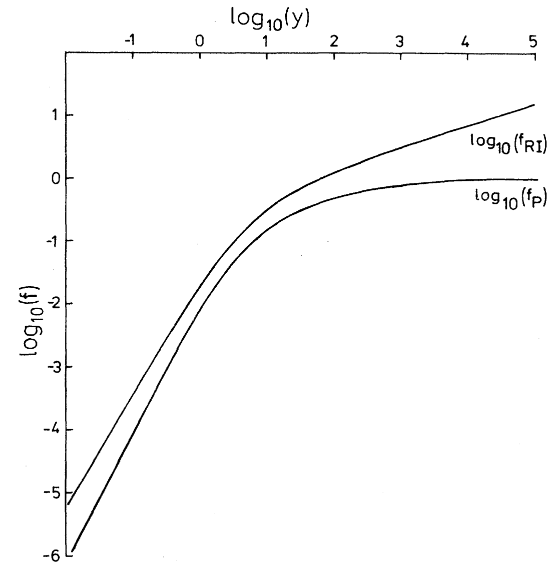

FIG. 3. Scaling functions of sound dispersion for pure (f_p) and random (f_{RI}) Ising model. The normalization $f_p(\infty) = 1$ and $f_{RI}(10) = 10^{-1/2}$ has been used.

In Figs. 2 and 3 the scaling functions for the diluted Ising model, calculated to the first order in $\sqrt{\epsilon}$, are compared with those corresponding to the pure Ising model, calculated to the first order in ϵ, in the limit of weak coupling g_0.[5,12] A rather great difference in the slope of the scaling functions for sound attenuation (Fig. 2) in the critical regime ($y \to \infty$) is to a considerable degree a result of an overestimation of critical exponents especially for the random Ising model. Although neutron scattering[13] and Mössbauer effect[14] experiments on $Fe_xZn_{1-x}F_2$ confirmed that the random exchange Ising system indeed had different critical exponents from those of the pure system and that the experimental values of static exponents agreed quite well with the theoretical predictions,[15] as far as the authors of this paper know, there is no satisfactory theoretical estimation as well as no reliable experimental data[16] on the dynamic critical exponent of the random Ising system. On the other hand,[14] it seems that dilution may produce more a dramatic effect on dynamic behavior than has been observed for the static critical exponents.

In the hydrodynamic regime ($y \to 0$) the scaling functions for sound dispersion (Fig. 3) behave like $y^{4/z}$ so in the first-order approximation the slope of the curve for the diluted model (f_{RI}) differs significantly from the conventional value of 2 obtained for the pure model. Thus, low-frequency measurements of sound dispersion might turn out to be very helpful in the determination of the value of the dynamic critical exponent in diluted Ising systems. Unfortunately, we have not come across such results.

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