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Electrokinetic effects in fluid-saturated poroelastic media

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The Biot theory of fluid-saturated poroelastic media is extended to include the electrokinetic effects of streaming potential and electro-osmosis in the low-frequency or "resistive" domain. The equations presented are shown to reduce to the familiar equations under steady-state conditions, where they also satisfy the Onsager reciprocity conditions. Plane-wave solutions to the linearized equations show that significant electric potentials accompany the passage of compressional waves for a broad range of material properties and frequencies. Mechanical motion is shown to be essentially unaffected by the electromechanical coupling as it is in second order in the electrokinetic effects.

INTRODUCTION

Whenever a fluid flows along the surface of a solid, an electric field may develop in the direction of the flow, the strength of the field depending on the properties of the two materials and the rate of flow. This phenomenon and its complement, the production of relative fluid flow by an applied electric field, are referred to as electrokinetic effects. In different physical situations, this effect is referred to by different names such as streaming potential, electro-osmosis, electrophoresis, and the Dorn effect. Experimental observation of these effects was recorded as early as 1808 by Reuss and has been studied intensely both experimentally and theoretically ever since. Many good reviews exist.[1] In recent years investigation has been directed toward the phenomenon of streaming potential associated with the flow of fluid in porous media. At the same time, comprehensive treatment of the mechanical behavior of porous media have been developed, and new understanding of their elastodynamic properties has been achieved.[2,3] In the laboratory, for example, experiments demonstrated the effect for water flow in permeable rock samples,[4] while in the field the effect was associated with the seepage of water through earthen dams,[5] and with the diffusion of fluids into dilatant regions prior to earthquakes.[6,7]

In this paper we extend Biot's theory of fluid-saturated, poroelastic media[8] to include the effects of streaming potential and electro-osmotic pressure. Our equations reduce to the accepted form in the steady state, wherein they also satisfy the Onsager reciprocity conditions. In this regard our equations differ from other published ones.[9]

Finally we consider plane-wave solutions to linearized versions of our field equations. Here we show that there are three characteristic frequencies which play important roles in determining the properties of the system; one of the frequencies involves the viscosity of the fluid, another the electrical resistivity of the fluid, and the third the electrokinetic coefficient (the ζ potential). The amplitude of the electric potential is calculated relative to the mechanical displacement of the solid and is found to be significant over broad ranges of frequency and material properties.

On the other hand, we demonstrate that the feedback effect of this electrical potential on the mechanical behavior of the medium is negligible. This is not surprising since such an effect is second order in the electrokinetic coupling. The elastic behavior of the medium can, therefore, continue to be treated by Biot's theory, as long as no significant external electric field is present.

I. ELECTROKINETIC EFFECTS

The nature of the electrokinetic phenomena can be understood in terms of an electric double layer existing at the solid-fluid interface. A surface charge density develops on the solid due to the adsorption of ions from the fluid or other mechanism; by Coulombic attraction, ions of the opposite sign, called counterions, will concentrate near the solid surface thus forming a double layer. Thermal agitation within the fluid, however, prevents the counterions from forming a distinct, immobile layer on the surface, but rather produces a diffuse distribution of them near the surface. With flow of the fluid relative to the surface, at least part of the diffuse layer can be transported with the fluid, allowing for the possibility of charge separation.

When a constant electric field is applied to a fluid-saturated porous media, the mobile portion of the diffuse layer will flow because of the excess charge it carries. A steady flow rate will be achieved when the electrical force is balanced by the viscous force within the fluid. Without consideration of the structure of the diffuse layer, it has been shown that the average rate of flow (per unit area of composite) in the absence of applied fluid pressure p is

$$J_i^f=(\beta\epsilon z/\mu)\phi_{,i}, \quad p_{,i}=0 , \tag{1}$$

where ϵ is the electric permittivity, μ is the fluid viscosity, and z is a quantity proportional to the ζ potential, which is defined as the difference in potential between the effective shear plane of mobility near the surface of the solid and the body of the fluid. The subscript $_{,i}$ denotes a partial derivative with respect to the Cartesian coordinate $x_i, i=1,2,3$. The constant of proportionality is dependent on the pore geometry (for a capillary tube, the porosity $\beta=1$ and the electrokinetic coefficient of the composite $z=\zeta/2$). The electric potential ϕ must be understood as the mean potential at a point in the composite. Equation (1) describes electro-osmotic flow. (Please note that the Einstein summation convention is used throughout.)

Fluid flow is also produced by pressure gradients. If Darcy's law holds, the flow rate in the absence of electric fields is given by

$$J_i^f=-(\beta K/\mu)p_{,i}, \quad \phi_{,i}=0 , \tag{2}$$

where $\beta K=\kappa$, Darcy's coefficient of permeability. We write the permeability as βK to emphasize that, in the resistive regime, permeability is proportional to porosity, pore geometry being held constant. (For a capillary of radius $r, K=r^2/8$.)

In general, for a linear process, the flow rate should have the form

$$J_i^f=L_{11}p_{,i}+L_{12}\phi_{,i} , \tag{3a}$$

where L_{ab} are the kinetic coefficients. Thus, from Eqs. (1) and (2), we can write for the net flow,

$$J_i^f=-(\beta K/\mu)p_{,i}+(\beta\epsilon z/\mu)\phi_{,i} . \tag{3b}$$

Assuming that the fluid is electrically conductive (and that the solid is nonconducting), we can write Ohm's law for the current density (per unit area of composite) in the absence of fluid flow as

$$J_i^e=-\beta cg\phi_{,i}, \quad J_i^f=0 , \tag{4}$$

where g is the electrical conductivity of the fluid and c is a dimensionless coefficient that depends on pore geometry (in a capillary tube, $c=1$).

Electrical current also results from the transport of counterions. To see how to incorporate this contribution, we appeal to the fact that we are dealing with linear resistive processes; the current density then has the form,

$$J_i^e=L_{22}\phi_{,i}+L_{21}p_{,i} , \tag{5}$$

where, from the Onsager reciprocity principle,[10] $L_{21}=L_{12}=\beta\epsilon z/\mu$. To identify L_{22}, we note from (3a) that $p_{,i}=-(L_{12}/L_{11})\phi_{,i}$ when $J_i^f=0$; thus

$$J_i^e=-(L_{22}-L_{12}L_{21}/L_{11})\phi_{,i}, \quad J_i^f=0 , \tag{4$'$}$$

which, when compared with (4), requires that we set

$$L_{22}=-\beta[cg+(\epsilon z)^2/\mu K]=-\beta G . \tag{6}$$

The term βG now can be interpreted as the effective conductivity of the composite in the absence of pressure gradients, G being the effective electrical conductivity.

Finally, we write,

$$J_i^e=-(\beta G)\phi_{,i}+(\beta\epsilon z/\mu)p_{,i} . \tag{5$'$}$$

In the above analysis we have implicitly assumed that the absolute temperature T remained constant; under these conditions the entropy production rate is given by

$$d\tilde{S}/dt=-(J_i^f p_{,i}+J_i^e\phi_{,i})/T , \tag{7}$$

where $\tilde{S}$ is the entropy density and the quantity in the parentheses is the rate of energy production per unit volume. Substituting for the current densities from Eqs. (3b) and (5$'$), we see that $d\tilde{S}/dt$ is a quadratic function of the affinities $p_{,i}$ and $\phi_{,i}$; to be positive for all values of the affinities the determinant of the coefficients of the quadratic form must be positive, namely $L_{11}L_{22}-L_{12}^2>0$. This results in the condition that

$$\mu GK/(\epsilon z)^2>1 , \tag{8}$$

which will be useful to us later.

In the following, we shall assume that the kinetic equations (3) and (5) hold for slowly varying fields as well as for the steady state.

II. EQUATIONS OF MOTION

We propose that Biot's equations for fluid saturated poroelastic media [Eqs. (6) and (7) of Ref. 8] be extended to include the electrokinetic effects by simply adding to those equations terms accounting for the electric forces on the charge densities q_s and q_f bound to the solid and fluid phases, respectively. Thus,

$$\rho_{11}\ddot{u}_i+\rho_{12}\ddot{U}_i-Nu_{i,jj}-(A+N)u_{j,ji}-QU_{j,ji}+b(\dot{u}_i-\dot{U}_i)+q_s\phi_{,i}=0 , \tag{9}$$

$$\rho_{22}\ddot{U}_i+\rho_{12}\ddot{u}_i-(RU_{j,j}+Qu_{j,j})_{,i}-b(\dot{u}_i-\dot{U}_i)+q_f\phi_{,i}=0 , \tag{10}$$

where the overdot signifies a partial time derivative, u_i is the i component of the average solid displacement vector, ρ_{ab} is Biot's mass density coefficients, $b=\beta^2\mu/K$, Biot's dissipation coefficient, N and A are Biot's elastic constants (the second and first Lamé coefficient, respectively), Q is Biot's coupling constant, and R is Biot's elastic coefficient for a fluid.

To see that these equations reduce correctly in the steady state, it is sufficient to consider the case where $\ddot{U}_i=0$ and $\ddot{u}_i=\dot{u}_i=0$, i.e., the fluid flow is steady and the solid is at rest. In this case, Eq. (9) can be written

$$Nu_{i,jj}+(A+N)u_{j,ji}=QU_{j,ji}-b\dot{U}_i+q_s\phi_{,i} , \tag{11}$$

where $j=1,2,3$, which is the static Navier equation with body forces due to fluid (pore) pressure gradients, fluid-flow resistance, and electric forces as given by the three terms on the right-hand side of (11), respectively.

Equation (10) reduces to

$$(RU_{j,j}+Qu_{j,j})_{,i}-q_f\phi_{,i}=b\dot{U}_i . \tag{12}$$

From Biot's theory,[8] the first term on the left is simply related to the fluid pressure,

$$-\beta p=RU_{j,j}+Qu_{j,j} . \tag{13}$$

Also, since flow is referred to a unit area of composite,

$$J_i^f = \beta \dot{U}_i \ . \tag{14}$$

Substituting the above results into (12), we can write that

$$-\beta p_{,i} - q_f \phi_{,i} = (b/\beta) J_i^f \ , \tag{15}$$

which is the same as (3b) provided that $b = \mu/K$ and

$$q_f = -\beta \epsilon z / K \ . \tag{16}$$

A seventh equation is needed in addition to the six represented by (9) and (10) to solve for the seven fields u_i, U_i, and ϕ. To this end we appeal to (a) the continuity of charge,

$$J_{i,i,}^e + \dot{q} = 0 \ , \tag{17}$$

where $q = q_f + q_s$ is the net charge density, and (b) Coulomb's law,

$$-\epsilon \phi_{,ii} = q \ . \tag{18}$$

Taking the divergence of the kinetic equation (5′) for current flow, we have that

$$J_{i,i}^e = -G \phi_{,ii} + (\beta \epsilon z / \mu p_{,ii}) \ . \tag{19}$$

The left-hand side of (19) can be replaced by $-q$ from (17) which, in turn, can be replaced by $\epsilon \phi_{,ii}$ from the time derivative of (18)—we are assuming that all time rates of change are sufficiently slow that electromagnetic effects are negligible. Finally, pressure can be eliminated in favor of the displacements by (13). Thus we have for our final field equation,

$$\{ z(R U_{j,j} + Q u_{j,j}) + \mu [\dot{\phi} + (\beta G / \epsilon) \phi] \}_{,ii} = 0 \ . \tag{20}$$

Note that the charge density on the solid is readily related to that on the fluid by (16) and Coulomb's law (18),

$$q_s = (\beta \epsilon z / K) - \epsilon \phi_{,ii} \ . \tag{21}$$

Thus, the equations of motion (9) and (10) are linear to the degree that $q_s \simeq -q_f$.

III. PLANE-WAVE SOLUTIONS

Our equations of motion are linear provided that all the material coefficients are constant (and that $q_s = -q_f$), conditions that might well be satisfied in homogeneous media under infinitesimal strain. For convenience, we rewrite our equations of motion (9), (10), and (20) in linearized form:

$$\rho_{11} \ddot{u}_i + \rho_{12} \ddot{U}_i - \mathrm{N}_{i,jj} - (A+N) u_{j,ji} - Q U_{j,ji} + b(\dot{u}_i - \dot{U}_i) - q_f \phi_{,i} = 0 \ , \tag{22}$$

$$\rho_{22} \ddot{U}_i + \rho_{12} \ddot{u}_i - (R U_{j,j} + Q u_{j,j})_{,i} - b(\dot{u}_i - \dot{U}_i) + q_f \phi_{,i} = 0 \ , \tag{23}$$

$$\{ q_f (R U_{j,j} + Q u_{j,j}) - \epsilon b [\dot{\phi} + (\beta G/\epsilon)] \phi \}_{,ii} = 0 \ . \tag{24}$$

We assume plane-wave solutions of the form:

$$u_i = u_i^0 \exp[i(k_i x_i - \omega t)] \ , \tag{25}$$

$$U_i = U_i^0 \exp[i(k_i x_i - \omega t)] \ , \tag{26}$$

$$\phi = \phi^0 \exp[i(k_i x_i - \omega t)] \ , \tag{27}$$

where the components of the propagation vector k_i and the plane-wave amplitudes u_i^0, U_i^0, and ϕ^0 (for the solid, fluid, and electric potential, respectively) can be complex. Substituting the solutions into the equations of motion and factoring out the common exponential term, we obtain a set of seven linear, homogeneous equations in the seven amplitudes:

$$[(A+N) k_i k_j - (\omega^2 \rho_{11} + i\omega b - N k^2) \delta_{ij}] u_j^0 + [Q k_i k_j - (\omega^2 \rho_{12} - i\omega b) \delta_{ij}] U_j^0 + (-i q_f Q k_i) \phi^0 = 0 \ , \tag{28}$$

$$[Q k_i k_j - (\omega^2 \rho_{12} - i\omega b) \delta_{ij}] u_j^0 + [R k_i k_j - (\omega^2 \rho_{22} + i\omega b) \delta_{ij}] U_j^0 + (i q_f R k_i) \phi^0 = 0 \ , \tag{29}$$

$$(-i q_f Q k_j) u_j^0 + (-i q_f R k_j) U_j^0 + [\epsilon b (\beta G/\epsilon - i\omega)] \phi^0 = 0 \ . \tag{30}$$

where ω is the angular frequency. For nontrivial solutions (nonzero amplitudes) the determinant of the coefficient of the amplitudes must vanish. This condition enables us to compute the wave number k (the eigenvalue) given the direction of propagation, i.e., the direction cosines k_i/k, $i=1,2,3$. Knowing k, we can determine the ratios of the amplitudes (the eigenvectors).

Since the physical system is isotropic, we can simplify our analysis in two ways. First, we can choose the x_1 axis to define the direction of propagation, then $k_1 = k$ and $k_2 = k_3 = 0$; second, we can treat longitudinal and transverse waves separately.

First, we consider transverse waves. With x_1 the direction of propagation, let us choose x_2 as the direction of polarization, i.e., the direction of particle motion for both solid and fluid phases. Equations (28)–(30) reduce to

$$[N k_2 - (\omega^2 \rho_{11} + i\omega b)] u_2^0 - (\omega^2 \rho_{12} - i\omega b) U_2^0 = 0 \ , \tag{31}$$

$$(\omega^2 \rho_{12} - i\omega b) u_2^0 + (\omega^2 \rho_{22} + i\omega b) U_2^0 = 0 \ , \tag{32}$$

$$[\epsilon b (\beta G/\epsilon - i\omega)] \phi^0 = 0 \ . \tag{33}$$

The first two equations are exactly Biot's equations for transverse waves. The third requires that $\phi^0 = 0$, which suggests that there is no charge separation under shear. Thus, there is nothing new for us to consider.

The case of longitudinal waves is of greater interest. Here, the polarization will be parallel to the x_1 axis, the direction of propagation. Equations (28)–(30) reduce to

$$[P k^2 - (\omega^2 \rho_{11} + i\omega b)] u_1^0 + [Q k^2 - (\omega^2 \rho_{12} - i\omega b)] U_1^0 - (i q_f k) \phi^0 = 0 \ , \tag{34}$$

$$[Q k^2 - (\omega^2 \rho_{12} - i\omega b)] u_1^0 + [R k^2 - (\omega^2 \rho_{22} + i\omega b)] U_1^0 + (i q_f k) \phi^0 = 0 \ , \tag{35}$$

$$(iq_f Qk)u_1^0+(iq_f Rk)U_1^0-[\epsilon b(\beta G/\epsilon-i\omega)]\phi^0=0\ , \qquad (36)$$

where $P=A+2N$. To expedite the analysis, let us cast the above equations in nondimensional form. To this end, we first divide the equations by k^2 to introduce the plane-wave velocity v $(=\omega/k)$; then we divide by $\rho v_c^2(=H)$ where v_c is Biot's characteristic velocity; finally we introduce the velocity parameter $V=(v/v_c)$ and the viscous, electric, and resistive characteristic frequencies ω_b, ω_e, and ω_g are equal to, respectively, b/ρ, $(\beta z/K)(\epsilon/\rho)^{1/2}$, and $\beta G/\epsilon$. Thus Eqs. (34)–(36) become

$$[\sigma_{11}-(\gamma_{11}+i\omega_b/\omega)V^2]u_1^0 +[\sigma_{12}-(\gamma_{12}-i\omega_b/\omega)V^2]U_1^0+(iV\omega_e/\omega)\tilde{u}^0=0\ , \qquad (37)$$

$$[\sigma_{12}-(\gamma_{12}-i\omega_b/\omega)V^2]u_1^0 +[\sigma_{22}-(\gamma_{22}+i\omega_b/\omega)V^2]U_1^0-(iV\omega_e/\omega)\tilde{u}^0=0\ , \qquad (38)$$

$$(iV\gamma_{12})u_1^0+(iV\gamma_{22})U_1^0-[(\omega_b/\omega_e)(i-\omega_g/\omega)V^2]\tilde{u}^0=0\ , \qquad (39)$$

where $\tilde{u}^0=\phi^0\sqrt{\epsilon/H}$ has the dimensions of length, like u_1^0 and U_1^0 and where $\sigma_{11}=P/H$, $\sigma_{22}=R/H$, and $\sigma_{12}=Q/H$; $\gamma_{ab}=\rho_{ab}/\rho$. For nontrivial solutions, the determinant of the coefficient matrix must vanish; this leads to the characteristic polynomial in V, which we write in condensed form,

$$[(\tilde{A}-W_bW_g/W^2)+(\tilde{A}W_b+W_g)/W]V^6 -[(\tilde{B}-W_bW_g/W^2)+(\tilde{B}+W_g-\tilde{D}/W_g)/W]V^4 +\tilde{C}[1+(W_bW_g-1)/WW_g]V^2=0\ . \qquad (40)$$

The parameters $\tilde{A}(\gamma_{11}\gamma_{22}-\gamma_{12}^2)$, $\tilde{B}(\gamma_{11}\sigma_{22}+\gamma_{22}\sigma_{11}-2\gamma_{12}\sigma_{12})$, $\tilde{C}(\sigma_{11}\sigma_{22}-\sigma_{12}^2)$, and $\tilde{D}[\sigma_{22}(\gamma_{11}+\gamma_{12})-\sigma_{12}(\gamma_{12}+\gamma_{22})]$ are dimensionless coefficients which depend only on the mechanical properties of the system. Second, since we are interested in the electrokinetic effects, it is convenient to normalize all frequencies to ω_e; thus $W_b=\omega_b/\omega_e$, $W_g=\omega_g/\omega_e$, and $W=\omega/\omega_e$. Third, for a given porous solid, and fluids of a given density, the parameters W_b and W_g determine the electrokinetic behavior. Finally, the thermodynamic stability condition given in Eq. (8) can be written

$$W_bW_g>1\ , \qquad (41)$$

which enters directly in the last term of (40) and will play an important role later.

Returning to Eq. (40), we see that one root is simply $V=0$. Although trivial, it tells us that there is no "third" type of wave (in addition to Biot's type one and type two) involved in the motion. Solving for the other two roots by means of the quadratic formula is straightforward, but the analytical expressions we obtain are complicated and not particularly revealing. Numerical calculations are readily carried out giving V for any particular case; results of such a study are discussed below.

We can learn some things about the nature of our solutions by examining the limiting forms of the equation with frequency. As $\omega\to\infty$, Eq. (40) assumes the form

$$\tilde{A}V[\infty]^4-\tilde{B}V[\infty]^2+\tilde{C}=0\ , \qquad (42)$$

the roots of which give the velocities of Biot's types-1 and -2 waves, i.e., $v_{\text{Biot}}=v_cV[\infty]$. As $\omega\to 0$, the equation assumes the simple form,

$$V[0]^4-V[0]^2=0\ . \qquad (43)$$

the roots of which are $V^2=1,0$; thus $v=v_c,0$ for the types-1 and -2 waves, respectively. A more careful analysis of the limits show that, in the first case, $V^2=V[\infty]^2+O(1/\omega)$ as $\omega\to\infty$; in the second case, $V^2=V[0]^2+O(\omega)$ as $\omega\to 0$.

Figure 1 shows how normalized plane-wave velocity $\text{Re}(V)$ and normalized attenuation $L=\lambda V_c/\omega_e$ [the at-

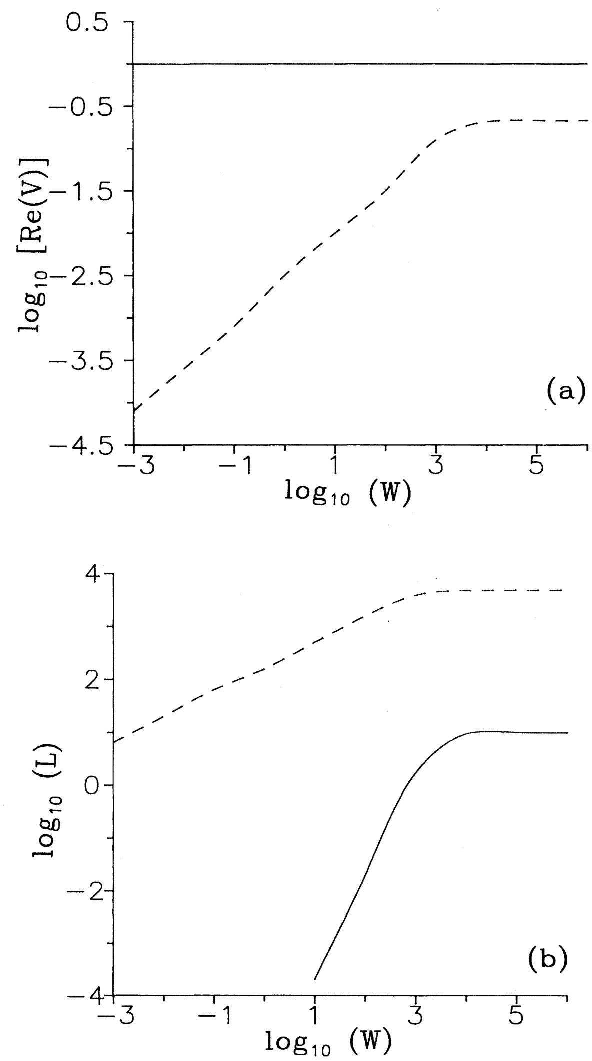

FIG. 1. (a) Normalized plane-wave velocities ($V=v/v_c$) and (b) normalized attenuation coefficients ($L=\lambda v_c/\omega_e$) vs normalized frequency ($W=\omega/\omega_e$). The solid curve is for type-1 waves; the dashed, type 2.

TABLE I. Values of the physical properties used in the water-saturated quartz-sandstone example.

Properties of the solid (quartz):	
density 2.65 g/cm^3	incompressibility 37.9×10^{10} dyn/cm^2
Properties of the fluid (slightly saline water):	
density 1.00 g/cm^3	incompressibility 2.25×10^{10} dyn/cm^2
$\mu=1.0\times10^{-2}$ g/cm s	$g=1.0\times10^{-3}$ $(\Omega\mathrm{m})^{-1}$
$\epsilon=80\times\epsilon_0=7.1\times10^{-10}$ F/m	
Properties of unsaturated skeleton:	
rigidity 9.0×10^{10} dyn/cm^2	incompressibility 12.0×10^{10} dyn/cm^2
$=0.10$	$\kappa=2.0\times10^{-9}$ cm^2
Properties of saturated skeleton (composite):	
$\rho_{11}=2.6$ g/cm^3	$\rho_{12}=-0.2$ g/cm^3
$\rho_{22}=0.3$ g/cm^3	$\rho=2.5$ g/cm^3
$P=4.10\times10^{10}$ dyn/cm^2	$Q=0.29\times10^{10}$ dyn/cm^2
$R=0.05\times10^{10}$ dyn/cm^2	$H=4.75\times10^{10}$ dyn/cm^2
$z=3.0\times10^{-3}$ V	$G=1.0\times10^{-3}$ $(\Omega\mathrm{m})^{-1}$
Properties associated with plane-wave motion:	
$v_c=1.4\times10^5$ cm/s	$(H/\epsilon)^{1/2}=8.2\times10^7$ V/cm
$\omega_b=2.0\times10^4$ s	$\omega_c=5.0\times10^5$ s
$\omega_e=80$ s	$\omega_g=1.4\times10^5$ s
$W_b=250$	$W_g=1750$
$Z_1[\infty]=3.0\times10^{-4}$	$Z_2[\infty]=13.4\times10^{-4}$
$W_1^*=1800$	$W_2^*=800$

tenuation coefficient $\lambda=-\omega\,\mathrm{Im}(v)/|v|^2$] vary with frequency for a medium-grained quartz sandstone saturated with slightly saline water. Numerical values of the material properties used are given in Table I; limiting values of v, λ, and other derived quantities are given in Table II. Values for the various mechanical properties of the skeleton and fluid-saturated composite were calculated from the basic properties of the solid and fluid components using the formulas of Biot and Willis.[11] A value for the electrical parameter z was estimated from Chandler's paper (Ref. 4) wherein a fluid pressure gradient of 1 atom per 10 cm produced a streaming potential of roughly 20 mV. From Eq. (5′) with $J_i^e=0$ we find that $z=3$ mV. For this model, the electrokinetic effect has little effect on the velocities and minor effect on the attenuation.

The results presented in Fig. 1 (and subsequent figures) are plotted over a broad frequency range to show the possibilities. It is not presumed that these results are valid beyond Biot's "characteristic frequency" for low-frequency behavior $\omega_c=b/\rho(\gamma_{12}+\gamma_{22})$ (where $\rho=\rho_{11}+\rho_{22}+2\rho_{12}$, the mass density of the composite), which for our model sandstone, has the value of 5.0×10^5/s. For other cases, with higher values of ω_c, the latter parts of the curves would be of interest.

It is not only the wave velocities which interest us, but the amplitudes of the waves. Because only two of the three amplitudes are independent, we consider ratios of the fluid-displacement amplitude U_1^0 and the electrical amplitude ϕ^0 to the solid displacement amplitude u_1^0. Being complex, these ratios give us both the relative magnitudes and the phase differences relative to the solid displacement. Subtracting Eq. (35) from (34) to eliminate ϕ^0, then dividing by u_1^0 we find that

$$Y\equiv\frac{U_1^0}{u_1^0}=\frac{-(\sigma_{11}+\sigma_{12})+(\gamma_{11}+\gamma_{12})V^2}{(\sigma_{12}+\sigma_{22})-(\gamma_{12}+\gamma_{22})V^2}\ . \tag{44}$$

Then, from (36),

$$Z\equiv\frac{\epsilon}{H}\frac{\phi^0}{u_1^0}=\frac{(\sigma_{12}+\sigma_{22}Y)}{(1+iW_g/W)W_bV}\ , \tag{45}$$

where $H=P+R+2Q$. As with the velocities, expanding out the above expressions provides us with little but complication; we rely on numerical solutions and limiting cases for analysis.

Consider Y first. In both the high- and low-frequency limits, V^2 approaches non-negative real values; the quantity Y, in turn, approaches positive real values for type-1

TABLE II. Limiting values of some important quantities associated with plane-wave motion, evaluated from the values of Table I.

	Type-1 waves		Type-2 waves	
Quantity	$\omega\to0$	$\omega\to\infty$	$\omega\to0$	$\omega\to\infty$
v ($\times10^5$ cm/s)	1.4	1.4	0	0.3
λ (cm^{-1})	0	0.006	0	2.8
U_1^0/u_1^0	1	1.29	-12.8	-12.6
ϕ^0/u_1^0 (V/cm)	$0.17(i^{-1}\omega)$	2.5×10^4	$-434(i^{-1}\omega)^{1/2}$	-1.1×10^5

waves and negative real values for type 2. The fluid is thus in phase with the solid for type-1 waves and 180° out of phase with the solid in type 2. For our model water-saturated sandstone we find that Y varies little with frequency, thus it is in approximate agreement with Biot's theory in the absence of electrokinetic effects.

Consider next the electrical amplitude Z. In the high-frequency limit, for both types-1 and -2 waves,

$$Z_{1,2}[\infty]=\frac{(\sigma_{12}+\sigma_{22}Y_{1,2}[\infty])}{W_b V_{1,2}[\infty]} . \tag{46}$$

As all terms are real and positive except for $Y_2[\infty]$ which is real and negative; the electrical signal is in phase with solid displacement for type-1 waves and 180° out of phase for type 2.

In the low-frequency limit, we must consider the types-1 and -2 waves separately. First, noting that $Y_1[0]=1$,

$$Z_1[0]=-iW(\sigma_{12}+\sigma_{22})/W_b W_g , \tag{47}$$

which shows that in the low-frequency regime, Z_1 is proportional to frequency ω. The type-2 wave, on the other hand, looks like

$$Z_2[0]=\frac{(i^{-1}W)^{1/2}(\sigma_{12}+\sigma_{22}Y_2[0])}{[2\tilde{C}W_g(W_b W_g-1)]^{1/2}} , \tag{48}$$

which shows that $Z_2[0]$ is proportional to $\omega^{1/2}$ in the low-frequency regime. Notice also that the stability condition $W_b W_g > 1$ guarantees that the denominator does not vanish, producing singular results. Figure 2 shows the variation of Z with frequency for our model sandstone; limiting values are given in Table II.

Normalized "corner" frequencies between high- and low-frequency behavior can be defined by the intersection of the asymptotic lines (on log-log graphs) as is evident upon inspection of Fig. 2. Analytically, these frequencies for type-1 and type-2 waves, respectively, are given by,

$$W_1^* = \frac{(\sigma_{12}+\sigma_{22}Y_1[\infty])W_q}{(\sigma_{12}+\sigma_{22}Y_1[0])V_1[\infty]} , \tag{49}$$

$$W_2^* = \frac{2\tilde{C}(\sigma_{12}+\sigma_{22}Y_2[\infty])^2 W_q(W_b W_q^{-1})}{(\sigma_{12}+\sigma_{22}Y_2[0])^2 W_b V_2[\infty]} + . \tag{50}$$

Numerical value of these frequencies for our model sandstone are given in Table I.

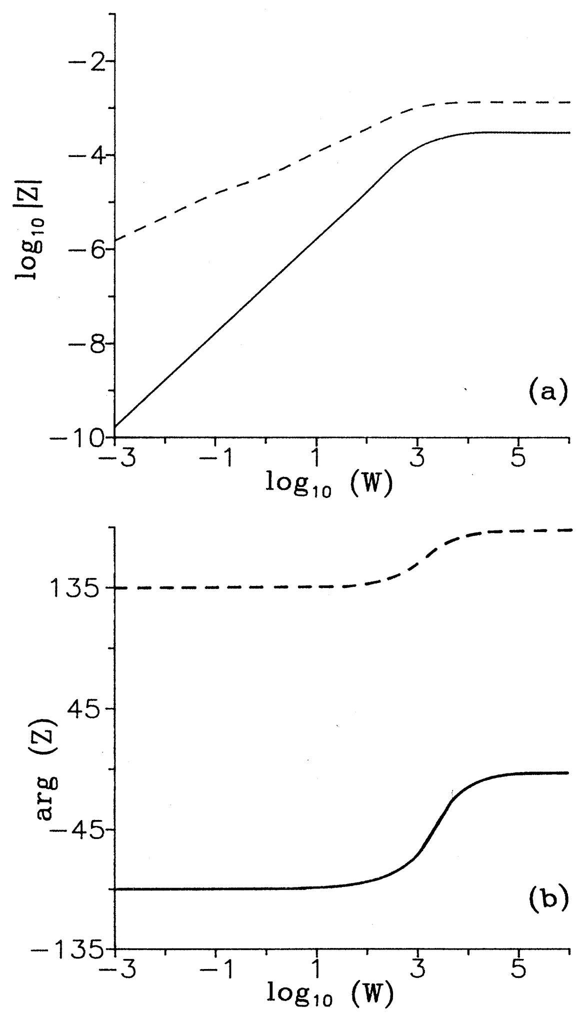

FIG. 2. (a) Amplitude and (b) phase (degree) of the normlized electrokinetic potential $[Z=\sqrt{\epsilon/H}(\phi^0/u_1^0)]$ vs normalized frequency ($W=\omega/\omega_e$). The solid curve is for type-1 waves; the dashed, type 2.

DISCUSSION AND CONCLUSION

The theory we have presented here is an attempt to extend Biot's theory to include the electrokinetic effects associated with the flow of fluid in poroelastic media. We have restricted the theory to the resistive (low-frequency) domain in both fluid flow and electrical conduction wherein the Onsager reciprocity conditions enable us to establish thermodynamic consistency. The extension of the theory into the dynamic (nonstationary) regime must be considered tentative, awaiting experimental test. Our "seventh" equation, in particular, is quasistationary and may need amending to include specific dynamic terms. Experiments in which fluid-saturated porous rods are driven by sinusoidal forces, for example, would provide the sorts of tests we need. Our study of the plane-wave solutions show that the electrokinetic interaction does not produce any third type of wave (in addition to Biot's types 1 and 2) but does allow for possibly significant effects on the motion of the solid and especially on the fluid. We have discovered that the nature of the electric field accompanying the (relative) motion of the solid and fluid is determined to a great extent by three characteristic frequencies which involve, separately, the viscosity of the fluid, its electrical conductivity, and its electrokinetic coefficient (the ζ potential).

Depending on the size of the ζ potential, our limited numerical study suggests that the streaming potential accompanying mechanical (seismic) vibrations in fluid-saturated porous media might well indeed be measurable and that the simultaneous measurement of a seismic wave signal and the accompanying streaming potential would provide important information about both the fluid and its reservoir. It is, of course, very important to discriminate between the streaming potential and other possible "seismoelectric" effects such as the J effect (the variation in resistance due to porosity changes) and piezoelectric effects. The necessary discrimination should be possible on the basis of the phase difference between the mechanical and electrical signals.

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