PHYSICAL REVIEW B VOLUME 40, NUMBER 13 1 NOVEMBER 1989

Consequences of time-reversal-symmetry violation in models of high-T_c superconductors

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(Received 5 July 1989)

We consider some observable consequences of the possible breaking of the discrete space-time symmetries P and T in high-T_c superconducting materials, as occurs in anyon models. It is argued, within these models, that at least two species of anyons are expected to occur, as a result of a quasimagnetic modification of the algebra of translations. We find that there is an intrinsic orbital magnetic moment perpendicular to each anyon layer, whose sign depends on the sign of the broken symmetry in the layer. If the coupling between layers is ferromagnetic, there should be a number of observable bulk effects, including optical rotation and anomalous transport properties analogous to a Hall conductance, which would occur even in the absence of an external magnetic field. Depending on the sample geometry, there may be a magnetic domain structure and/or fringing magnetic fields, and there may be a difference in the value of H_{c1} for positive and negative magnetic fields. If the coupling between planes is antiferromagnetic, so that the sign of the broken P and T symmetry alternates between planes, the bulk effects are absent, but the broken symmetry may be detected in principle by surface-sensitive probes or by weak effects in neutron scattering. Measurements of muon spin relaxation provide a local probe that should be a sensitive detector of broken P and T symmetry in either the ferromagnetic or antiferromagnetic case. For the model of dilute, weakly interacting anyons, we show that the magnitude of the intrinsic orbital magnetic moment can be obtained exactly by a direct physical argument. Our analysis determines all of the coefficients in the effective London Lagrangian, including the Chern-Simons–type terms, if the value of the compressibility is known.

I. INTRODUCTION

Recently theoretical studies have indicated the existence of a new mechanism of superconductivity in two spatial dimensions,[1,2] this mechanism being abstracted from studies of the behavior of an ideal gas of fractional-statistics particles (anyons) at zero temperature. We would like to know if this new possibility applies to real materials. Particularly promising in this regard are the high-temperature superconducting materials based on copper oxide planes, which are, in a useful approximation, two dimensional, and which seem to become superconducting by a mechanism that is new, at least to the extent that it involves something other than the formation of Cooper pairs by phonon exchange.

One essential feature of the anyon models is a violation of the discrete symmetries of time reversal, T,[3,4] and the two-dimensional reflection symmetry, P, while the combined operation PT remains a valid symmetry in these models.[3] As was previously noted and remarked upon in Ref. 3, the violation of these discrete symmetries should have experimentally observable consequences, so that an experimental search for these properties could be very important in establishing whether or not the anyon models have relevance to actual superconducting materials. The major purpose of this paper will be to discuss some of the more promising experiments that might detect broken T or P symmetry, and, in as far as possible, to estimate the magnitudes of the effects using the simplest models of dilute, weakly interacting anyons with half-Fermi statistics, as follows from the microscopic spin wave function of Kalmeyer and Laughlin.[1] We note, however, that there exists in the literature on models of high-temperature superconductivity a number of papers with exotic ground states, including several with broken T and P symmetry, and a number which have varying degrees of similarity with the Laughlin-Kalmeyer ground state.[5,6] The qualitative discussions in the present paper should apply to any of the models with broken P and T symmetry.

The anyon mechanism for superconductivity is best understood in two stages. In the first stage, it is argued that certain plausible forms of the ground-state wave function for an insulating two-dimensional spin system have the property that electrons or holes doped into the system become dressed quasiparticles with strange quantum numbers.[7,8] For example, the quasiparticles with charge $\pm e$ carry no electron spin, and in the Laughlin-Kalmeyer model, at least, they obey fractional statistics.

In two spatial dimensions, fermions and bosons are no longer the only allowed possibilities—one can have multiparticle states whose wave functions are multiplied by a phase $e^{i\theta}$, not necessarily equal to ± 1, as two of the identical particles are slowly interchanged along a path that encloses no other particles.[9–13] In the general case the single-particle states whose statistics interpolate between bosons and fermions are said to have θ statistics. In the simplest versions of this idea, the charged quasiparticles are half fermions—that is, a trajectory whereby the quasiparticles are slowly interchanged in a counterclockwise fashion is weighted with phase $e^{i\pi/2}$, so that $\theta=\pi/2$.

The existence in actual condensed-matter systems of quasiparticles with fractional statistics was first noted in the case of the fractional quantized Hall effect (FQHE).[14] There it was pointed out[14–16] that fractional statistics could be deduced from the explicit form of the trial wave functions for charged excitations from Laughlin's ground state,[17] and moreover that fractional statistics provides a natural language with which to describe the observed hierarchy of fractional quantum Hall states.[14,18–20] Indeed, Kalmeyer and Laughlin first argued for the possibility of fractional-statistics quasiparticles in pure electron systems—that is, in the absence of an external magnetic field—by making an approximate mapping of the frustrated Heisenberg antiferromagnet on a triangular lattice onto a problem with an effective background field, and taking over the many-body variational wave functions used successfully to analyze the quantized Hall effect into this new context.

A more abstract and general definition of the type of ordering necessary to support fractional-statistics quasiparticles was given by Wen, Wilczek, and Zee,[5] who called the ordered states, chiral spin states. (See also Ref. 21.) The defining property of a chiral spin state, roughly speaking, is that the expectation value of the line integral for a "particle" of some type transported around a large loop should acquire a phase proportional to the area of the loop or, more accurately, to the number of particles inside. Thus one can assign a fictitious magnetic flux to each particle inside the loop, which in the simplest case is an integer multiple of 2π. As in the FQHE one is able to construct a quasiparticle or quasihole in such a way that there is a phase enhancement or deficit of precisely 2π for a particle transported around it on a counterclockwise path, so the quasiparticle is associated with a flux of 2π. The quasiparticles also acquire the associated particle number, which is generally fractional. If one quasiparticle is transported around another, it accumulates a phase which is the product of the particle number times flux and is therefore a fraction of 2π.

In the model considered by Kalmeyer and Laughlin,[1] the particles are bosons which represent the occurrence of an up spin on a given lattice site, and the flux associated with each boson is 4π. The quasiparticles in this case carry particle number $\frac{1}{2}$. When one quasiparticle winds around another the extra phase is π—for there is one-half the flux of the underlying particles, and one-half the particle number, and so altogether one-quarter the phase. The phase π for a full winding implies phase $\pm\pi/2$ for interchange, reflecting half-Fermi statistics for the quasiparticles.

The particle number in the Laughlin-Kalmeyer model is actually the number of up spins, and the quasiparticle referred to earlier is actually a spin excitation with no electric charge (a spinon).[1,7] If one assumes in addition that a free electron (or hole) added to the system will form a spin-singlet bound state with a spontaneously produced spinon, the resulting charged excitation will acquire the fractional statistics of the spinon.

In the second stage, it is argued that fractional statistics in themselves lead to superfluidity, at low temperatures, and at least in the case where $\theta=\pi(1-1/p)$, where p is an integer, which is the only case we consider here. The argument has been made quite forcefully and in great detail recently.[2,22,23] (See also Refs. 24–28.) Hence we shall not go into any detail here but merely recall the essential points, which are really quite simple and intuitive. The key step in the analysis is the following. Anyons may be represented as fermions with fictitious charge and flux attached. As a first approximation to the behavior of the anyon gas, one replaces the actual distribution of fictitious flux, tied to the particles, by a uniform background fictitious magnetic field with the same average strength.[2] It may be shown that this is a valid approximation for statistics near to fermions, more precisely when the statistical phase is $\theta=\pi(1-1/p)$ and $p\rightarrow\infty$. Then, for the indicated value of the statistics, one finds that the effective fermions exactly fill p Landau levels, and that there is a gap in the single-particle spectrum. On the other hand, the underlying anyon gas is clearly compressible, and there must be low-energy excitations corresponding to slow modulations in density (sound waves). Thus the first approximation is incomplete in one crucial respect—there must be gapless collective modes in addition to the single-particle excitations.[22,23] (In this compressibility argument, it is essential that the fictitious magnetic field is tied to the particle density. It is precisely in this point that the ideal anyon gas in itself differs from the anyon gas that arises in the fractional quantized Hall effect. While it has been convincingly demonstrated that the low-energy excitations in the fractional quantized Hall effect carry fractional statistics, and effective anyon field theories have been proposed to describe them, in that context the anyons are subject to a large external magnetic field. This leads to the anyon gas being incompressible—the charged anyons resist crossing real magnetic field lines—and there is no gapless mode in the quantized Hall system.)

The nature of the gapless excitations in the pure anyon gas is elucidated by a closer consideration of the symmetries of the problem. The underlying anyon gas contains a pair of commuting translation generators, but in the effective model of particles moving in a fictitious magnetic field, the translation generators, while they commute with the Hamiltonian, fail to commute with one another.[23] This mismatch involves surface terms, which can only be repaired by a massless field coupled in the appropriate way to the energy-momentum tensor. Since the anyon gas consisted of particles all having the same mass, the particle current is proportional to the momentum density. Thus the massless field also couples to the particle number current, leading to superfluidity—and, if the original anyons are electrically charged, to superconductivity. This qualitative picture is confirmed by direct calculation in a controlled approximation, viz., by perturbation theory in $1/p$.

In view of all this, it is important to determine whether the mechanism of anyon superconductivity does indeed apply to the copper-oxide superconductors. Fortunately, since P and T violation is inherent to the anyon mechanism, there are qualitative consequences of the anyon model which can be investigated experimentally by means of sensitive null experiments. In addition, the apparently unusual properties of the copper-oxide materials above T_c may well suggest that the transition to a chiral-spin-liquid-like state with broken P and T occurs at a higher temperature than the transition to the superconducting state. It is possible that many of the experimental consequences outlined later for the superconducting state also apply to the normal state. Effects of broken T and P symmetry do persist above the superconducting transition in the simplest model of weakly interacting anyons.

In Sec. II we briefly recall the definition of the anyon model. We also discuss some of the issues that arise when the derivation of this model from an underlying lattice theory is considered. Specifically, in the single quasiparticle subspace the lattice translations do not commute due to the chiral-spin ordering, which leads to unusual degeneracies among the single-particle states and, for the Laughlin-Kalmeyer model, forces the number of distinct anyon species to be even. This noncommutativity is related to, but not identical with, the noncommutativity in the effective macroscopic theory of anyon superconductors, emphasized in Ref. 23. The degeneracy significantly affects the predictions for several experimental situations.

Most of the experiments discussed in this paper will be *bulk* measurements that reflect the average properties of the system over a volume containing many copper-oxide layers. In order for there to be bulk manifestations of the breaking of P and T, it is necessary that the coupling between adjacent layers be such as to favor the same sign of spontaneous symmetry breaking in adjacent layers of the material. As far as we are aware, there has been no convincing argument advanced by proponents of anyon superconductivity as to whether the coupling between layers should favor like—"ferromagnetic"—or unlike—"antiferromagnetic"—signs of the symmetry breaking in adjacent layers. It is not even clear whether the sign of the coupling would necessarily be the same in the different crystal structures in which high-temperature superconductivity has been observed. However, for the majority of this paper we shall *assume* that at least in some cases the coupling between layers is favorable to having the same sign of symmetry breaking throughout the sample, and we shall explore the consequences of this assumption.

One of the most interesting consequences of the broken P and T symmetry is that there is predicted to be an intrinsic orbital magnetic moment perpendicular to the layers, whose sign depends on the the sign of the symmetry breaking parameter. According to the simple anyon model, the strength of the magnetic induction $\mathbf{B}_0$ that would be produced by this magnetic moment is of order 15 G. In the superconducting state, however, the magnetic induction is expelled from the interior of the sample and confined to a surface region the size of the London penetration depth λ_L.

In Sec. III we discuss the magnetic moment, and we investigate the possible magnetic domain structures that might be found in a layered superconductor with an intrinsic magnetic moment. We find that it should be possible under various circumstances to obtain a single-domain sample, which, of course, is extremely convenient for experiments that seek to measure bulk consequences of broken P and T symmetry.

In the case of a multidomain sample there would be weak fringing magnetic fields exterior to the sample, in the vicinity of domain boundaries, which might be directly observable. In the case of a single-domain sample, there would be fringing fields near edges of the sample. In either case, the fringing field might be detectable.

In Sec. IV we go on to consider the general consequences of spontaneous P and T breaking for the structure of the kinetic coefficients. As was noted in Ref. 3, the standard Onsager reciprocity relations no longer apply, and, for instance, one might expect, at least for an isolated layer, an off-diagonal antisymmetry contribution $\sigma_{xy}=-\sigma_{yx}$ to the (electric or thermal) conductivity tensors, analogous to a Hall conductivity, which would be present even in the absence of an applied magnetic field. The Hall effect for a normal conductor in an external magnetic field is quite familiar, but this is just one of the many anomalous transport phenomena discussed, for example, in Ref. 29. Indeed, the anomalous properties of a two-dimensional system with spontaneously broken T and P symmetry are generally similar to the special properties of a normal material in an external magnetic field, of which many are well known.

Closely related to this is the suggestion of Wen and Zee,[30] that due to a possible antisymmetry contribution to the dielectric tensor at infrared or optical frequencies there would be a rotation of the plane of polarization of transmitted radiation (analogous to the Faraday effect in a magnetic field) or a difference in reflectivity for left and right circularly polarized light. We argue that these effects vanish in the simplest model, of a dilute anyon gas, but they should occur in more realistic models.

The most elegant way to discuss both the possibility of an antisymmetric term in the dielectric tensor, and some of the other consequences of P and T violation, is via an effective Lagrangian description of the low-energy excitations of the anyon gas. Indeed as discussed in greater detail in Ref. 23, it is possible to derive such an effective London Lagrangian by performing a matching operation to the results of the random-phase-approximation (RPA) calculation performed in Refs. 22 and 23. L_{eff} contains novel terms—specifically, terms of Chern-Simons type, of which there are two in a nonrelativistic theory. In Sec. V we show that all the coefficients in the effective Lagrangian, validly calculated to lowest order in p in the cited references, can be obtained exactly on the basis of physical arguments if the compressibility is known. The exact result differs significantly from the RPA result for small p.

If the sign of the T and P symmetry breaking alternates from layer to layer, so that there is a net cancellation of the anomalous contributions in the average bulk properties of the material, there remain several possibilities for detecting the broken symmetry. In Sec. VI we discuss some surface sensitive probes which, at least in principle, might reveal the broken T and P symmetry, assuming that the symmetry breaking persists in the layers closest to the surface. In Sec. VII we discuss two probes of the local magnetic order, neutron scattering and muon spin relaxation. The latter technique, which is very sensitive, should work equally well in the ferromagnetic and antiferromagnetic cases, and it may well be the most powerful method available for deciding whether models with broken T and P symmetry have any relevance to actual high-temperature superconductors.

The exact calculation of the intrinsic orbital magnetic moment of the dilute anyon superconductor, which is one of the parameters necessary for the specification of the Lagrangian of Sec. V, is discussed in some detail in Appendix A. This appendix also elucidates some of the subtleties connected with calculations of angular momentum in the anyon system. Two other appendixes (Appendixes B and C) discuss the antisymmetric part of the frequency-dependent conductivity tensor, in the presence of weak impurity scattering, and the field energy associated with a magnetic domain wall perpendicular to the plane of the superconducting layers.

It has been suggested that there should be no Josephson coupling between an ordinary BCS superconductor and a high-T_c superconductor, if the superconducting order parameter of the latter displays a broken time-reversal symmetry.[31] However, according to our analysis this is *not* necessarily the case if the high-T_c material is an anyon superconductor. Specifically, in the anyon model obtained by adding electrons or holes to the state of Kalmeyer and Laughlin, we have found that there exists a superconducting order parameter which is local in the electron coordinates and which has the quantum numbers appropriate to the creation operator for a pair of electrons in a spin singlet state. (Note again that the Kalmeyer-Laughlin model gives rise to an even number of anyon species.) More generally, depending on the microscopic details of the model, the electron pairs in an anyon superconductor may either condense in a state with total momentum zero and s-state rotational symmetry—in which case there should be normal Josephson coupling between the anyon system and an ordinary BCS superconductor—or the pairing might involve d-like rotational symmetry, or a nonzero total momentum. In the latter cases there could be Josephson coupling through a point contact but there would be a selection rule against Josephson coupling through a uniform junction to a BCS superconductor parallel to the layer.

It also appears that the Josephson coupling between two anyon layers should be independent of the signs of the broken T symmetry in the two layers. Therefore, the Josephson coupling by itself would not favor either a ferromagnetic or antiferromagnetic arrangement of alternate layers in a high-T_c superconductor. Since the Josephson coupling between layers need not, in the end, reflect the broken time-reversal symmetry between layers, we shall leave a detailed discussion of this subject for another paper.[32]

II. ANYON MODELS

Since we shall illustrate many of our points by reference to anyon models, we begin by defining these models.

A. Kinematics of fractional statistics

The most familiar form of anyon model contains a single species of particle described in "anyon gauge" by a multivalued wave function $\Psi^{(a)}(\mathbf{r}_1, \ldots, \mathbf{r}_N)$ defined on a $2N$-dimensional space, where N is the total number of particles. The wave function is required to satisfy the conditions $\Psi^{(a)} \rightarrow 0$ if any two particles come together, $\Psi^{(a)}$ is continuous elsewhere, and $\Psi^{(a)}$ is multiplied by a specified factor $e^{i\theta}$ if two particles are interchanged in a counterclockwise sense along a path which encloses no other particles. The Hamiltonian of the model in the anyon gauge may be written simply (in the absence of an external magnetic field) as,

$$H^{(a)} \equiv \sum_{j=1}^{N} \frac{\mathbf{p}_j^2}{2m^*} + V \ , \tag{2.1}$$

where V depends only on the positions of the particles, $\mathbf{p}_j = -i\hbar\nabla_j$, and points where $\mathbf{r}_j = \mathbf{r}_{j'}$, are excluded from the $2N$-dimensional space. Commonly we shall only discuss the free-anyon system, where $V=0$. It should be noted that in applications to solid-state physics where there is an underlying crystal lattice, the use of a continuum model such as (2.1) can only be strictly justified in the dilute limit, where the distance between anyons is large compared to the lattice constant. Further, the mass m^* is properly interpreted as an effective mass, which we assume isotropic, and the momentum $\mathbf{p}$ is actually the distance in momentum space from some point in the Brillouin zone where the energy of the anyon has a minimum.

The definition in terms of multivalued wave functions can be made more mathematically respectable by reference to the universal covering space. The configuration space for N identical particles is obtained from $(\mathbf{R}^2)^N$ by identifying points that differ by any permutation of coordinates, and excluding the singular points where two or more coordinates coincide. The wave function then lives on the universal covering space of this configuration space. On the universal covering space the relative angle coordinate ϕ between two nearby particles runs from $-\infty$ to $+\infty$ rather than from $-\pi$ to $+\pi$ (with endpoints identified). The anyon boundary condition for θ statistics is

$$\Psi(\phi+\pi) = e^{i\theta}\Psi(\phi) \ . \tag{2.2}$$

This boundary condition can be extended in a unique fashion over the whole covering space. For special values of the statistics, the wave function may be well defined on a simpler projection of the covering space. It is not necessary to let the relative angle run over the whole real line, if there is periodicity after a finite interval. Thus, of course, fermion and boson wave functions can be defined on the original configuration space, while to accommodate half fermions ($\theta=\pi/2$) it suffices to let the relative angles run from -2π to $+2\pi$. Several nonidentical types of anyons may be incorporated by imposing conditions similar to (2.2) but with 2π replacing π and 2θ replacing θ. This lesser periodicity is appropriate because, for distinguishable particles, winding one particle completely around the other leads to an indistinguishable configuration but simple interchange does not. We shall confine ourselves to models where the different species of anyons have identical effective masses and interactions, and we shall introduce an isospin index τ to distinguish the anyon species.

For computational purposes it is far more convenient to work in a fermion gauge, where the wave function is single valued, and obeys the restriction that it is multiplied by a factor of -1 on the interchange of the positions and isospins of a pair of anyons. The fractional statistics are incorporated by an appropriate effective interaction. Following the sign conventions used in Fetter *et al.*, the appropriate Hamiltonian is given by

$$H=\sum_{j=1}^{N}\frac{|\mathbf{p}_j-\mathbf{a}_j|^2}{2m^*}+V\ , \tag{2.3}$$

where

$$\mathbf{a}_j=\frac{\hbar}{p}\sum_{k\neq j}\frac{\hat{\mathbf{z}}\times\mathbf{r}_{jk}}{|\mathbf{r}_{jk}|^2}\ , \tag{2.4}$$

and $\mathbf{r}_{jk}\equiv\mathbf{r}_j-\mathbf{r}_k$. If $\Psi\{\mathbf{r}_j\}$ is an eigenstate of H with energy E, then we obtain an eigenstate $\Psi^{(a)}$ of $H^{(a)}$, with the same eigenvalue E, by setting

$$\Psi^{(a)}\{\mathbf{r}_j\}=\Psi\{\mathbf{r}_j\}\prod_{k<j}\left[\frac{z_k-z_j}{|z_k-z_j|}\right]^{-1/p}\ . \tag{2.5}$$

where $z_j=x_j+iy_j$. The difference between the two formulations is a singular gauge transformation.

B. Multispecies Models

It appears that the energy spectrum for an anyon on a lattice is always degenerate in the simplest cases, with several minima at different points of the Brillouin zone. For the spin model of Kalmeyer and Laughlin, for example, one finds that there is automatically twofold degeneracy for every state.[33] The origin of this degeneracy arises from the fact that the operators T_1 and T_2, which translate the anyon by a single lattice constant in the two independent lattice directions anticommute with each other. Roughly speaking, this comes about because the anyon quasiparticle, when transported around a loop by means of the product $T_1T_2T_1^{-1}T_2^{-1}$, acquires a phase 2π times the number of up spins inside, which is (on the average) $\frac{1}{2}$. One consequence is that T_1 and T_2 cannot both be used simultaneously to label quasiparticle states of definite energy, even though each separately commutes with the Hamiltonian. Instead one may use, for instance, T_1 and T_2^2 to label energy eigenstates, but then the representations of the full symmetry algebra based on T_1 and T_2 will necessarily contain several of these states. A closer analysis shows that in the case at hand the irreducible representations of the translation algebra are two dimensional. They contain states with T_1 labels related by $k_1'=k_1+\pi/l_1$, where l_1 is the lattice spacing in the 1 direction.

Of course, if the minimum of the energy band does not occur at a position which is invariant under the point symmetry group of the lattice, there may be additional degeneracies imposed by this point group. However, the total number of independent minima will always be even. Although the degeneracy of the spin quasiparticle spectrum was found by Kalmeyer and Laughlin in their analysis of the Heisenberg model on a triangular lattice, this feature has not been incorporated in previous calculations concerning the associated superconducting state.

We note that the degeneracy of minima of the anyon spectrum presumably could be lifted, if one were to generalize the Laughlin-Kalmeyer model, by adding a perturbation with a period of twice the lattice constant of the original lattice. Then one of the translation operators, say T_1, would no longer commute with the Hamiltonian, and all the remaining elements of the translation group would commute with each other. If the degeneracy were lifted in this way, and if the density of anyons were sufficiently low, only one minima would be occupied. At the same time, if the perturbation were sufficiently weak, the fractional statistics would not be affected by it.

To describe the continuum limit of an anyon system with n_a degenerate minima in the Brillouin zone, we must consider that there are n_a flavors of anyons, distinguished by an isospin index τ. We assume as before that if two identical anyons are interchanged in a counterclockwise manner, around a path that contains no other anyon, the wave function is multiplied by a phase factor $e^{i\theta}$. For anyons with different values of τ, there is only the weaker restriction that the wave function is multiplied by $e^{2i\theta}$ when one particle is moved completely around the other, on a clockwise path that contains no other anyon. We continue to use the Hamiltonian (2.1), which is invariant under the group SU(n_a) of global rotations in isospin space. Once again, we can transform to a fermion representation, where the Hamiltonian is given by Eqs. (2.3) and (2.4), and the wave function Ψ, which obeys (2.5) is required to be antisymmetric under the interchange of $\mathbf{r}$ for two particles with the same τ.

Consider a situation where the potential energy V is circularly symmetric about the origin and chosen so as to give a ground state with a uniform particle density (except for edge effects) inside a circle of radius R. We will only consider cases where $(1-\theta/\pi)^{-1}$ is a positive integer p. Following,[2,22,23] we construct an approximate ground state by working with an unperturbed Hamiltonian

$$H_0=\sum_{j=1}^{N}\frac{|\mathbf{p}_j-\bar{\mathbf{a}}(\mathbf{r}_j)|^2}{2m^*}\ , \tag{2.6}$$

where

$$\bar{\mathbf{a}}(\mathbf{r})=\frac{N\hbar}{pR^2}(\hat{\mathbf{z}}\times\mathbf{r})\equiv\tfrac{1}{2}\bar{b}(\hat{\mathbf{z}}\times\mathbf{r})\ . \tag{2.7}$$

Eventually one will treat $(H-H_0)$ as a perturbation to generate a diagrammatic series for the energy and other properties of the system. The perturbation commutes with the total angular-momentum operator L_z.

The one-particle eigenstates of H_0 may be characterized by a Landau level index l, and an orbital quantum number m, with l and m being arbitrary nonnegative integers. For large values of m, and fixed l, the eigenfunction is localized near a ring of radius $r_m=(2\pi m)^{1/2}l_0$, where l_0 is the "magnetic length," $l_0=\bar{b}^{-1/2}$. The state with quantum numbers (l,m) is an eigenstate of L_z with eigenvalue $\hbar(m-l)$.

In the case of a single anyon species in the thermodynamic limit, where $N\rightarrow\infty$, with N/R^2 held constant, the ground state of H_0 is the state where the Landau levels are completely occupied for $0\leq l\leq(p-1)$ and empty for $l\geq p$. The density of particles in each filled Landau level is then $\bar{b}/2\pi$.

For the case of several anyon species, when p is a multiple of n_a, the ground state of H_0 has (p/n_a) filled Landau levels. The density of particles in each Landau level is $n_a\bar{b}/2\pi$ due to the isospin degeneracy. In the case $n_a=p=2$, the ground state has all particles in the lowest Landau level, with both isospin states full.

If p is not a multiple of n_a, then the highest Landau level is partially occupied, and the ground state of H_0 is highly degenerate. We expect that the actual interactions between the fermions will then lead to a spontaneous breaking of the isospin symmetry, so that in the highest Landau level we have one or more isospin states that are completely filled, while the remaining isospin states are empty. This is analogous to the exchange-induced spin splitting of Landau levels in the quantized Hall effect, at odd integer values of $2\pi\rho/B$.

Strictly speaking, the approximate ground state constructed from H_0 is justified as an approximation to the anyon system only in the limit of $p\rightarrow\infty$. Laughlin has argued, however, that one obtains reasonable results from this approximation even for the boson case $(p=1)$ and more certainly for the case of half fermions $(p=2)$. The limit $p\rightarrow\infty$ was explored in some detail in (Ref. 23).

In the present paper we will simply *assume* that the wave function for a group of noninteracting electrons in a set of filled Landau levels is a correct starting point for a description of the ground state of the anyon system, both in the case of a single anyon species, and in the more general case where p is a multiple of n_a. Specifically, we shall assume that (1) the trial wave function is a good approximation to the ground state and (2) that the exact ground state has the same quantum numbers as the approximate state, and can be derived from it by perturbation theory. This will be made more precise in Appendix A.

C. Order parameters

Finally, we state without derivation a superconducting order parameter we believe applicable to an anyon superconductor in the simplest case which is $n_a=p$. Let us define an operator

$$\Phi^\dagger(\mathbf{r})=\int d^2r_1\cdots d^2r_{p-1}f(\mathbf{r}_1,..,\mathbf{r}_p)\times\eta_1^\dagger(\mathbf{r}_1)\cdots\eta_p^\dagger(\mathbf{r}_p)U(\mathbf{r})\ , \qquad (2.8)$$

where $\eta_s^\dagger(\mathbf{r}_s)$ is the fermion creation operator for an anyon at a point $\mathbf{r}_s$ in isospin state s. The point $\mathbf{r}_p$ is chosen so that the center of mass of the points $\{\mathbf{r}_s\}$ is at the location $\mathbf{r}$. In addition the function $f(\mathbf{r}_1,\ldots,\mathbf{r}_p)$ is required to be totally symmetric in its arguments and is peaked when all points are close to $\mathbf{r}$, and $U(\mathbf{r})$ is an operator which multiplies the wave function by a phase factor depending on the positions of all the other anyons in the system. Specifically we choose

$$U(\mathbf{r})=\prod_{j=1}^{N}\frac{(z-z_j)}{|z-z_j|}=\exp\left[2\pi i\int d^2r'\arg(z-z')\rho(\mathbf{r}')\right]\ , \qquad (2.9)$$

where $\rho(\mathbf{r}')$ is the density operator at the point $\mathbf{r}'$. For the case of a single species of boson, with $n_a=p=1$, $\Phi^\dagger(\mathbf{r})$ is precisely the boson creation operator for a particle at point $\mathbf{r}$. More generally, we believe that $\Phi(\mathbf{r})$ has long-range order in the ground state of the system, in the sense that the correlation function $\langle\Phi^\dagger(\mathbf{r})\Phi(\mathbf{r}')\rangle$ approaches a nonzero constant for $|\mathbf{r}-\mathbf{r}|\rightarrow\infty$. Thus we can interpret $\langle\Phi(\mathbf{r})\rangle$ as an order parameter for the system. At finite temperature, for an isolated layer, up to the superconducting transition temperature T_c, the two-point correlation function should fall off as $|\mathbf{r}-\mathbf{r}'|^{-\eta}$, with $0<\eta<\frac{1}{4}$, because of the usual divergence of phase fluctuations associated with the long-wavelength phonon mode. The operator $\Phi^\dagger(\mathbf{r})$ may be generally interpreted as the creation operator for a p tuplet of anyons in an isospin singlet state.

In the case of a single species of anyon $(n_a=1)$ with $p\neq1$, a plausible generalization of this order parameter is obtained if we simply reinterpret the operators $\eta_s^\dagger(\mathbf{r}_s)$ as the fermion creation operator for a quasiparticle in Landau level s, in the state which is centered at $\mathbf{r}_s$. The order parameter we have chosen is closely related to the nonlocal order parameters introduced by various authors to describe the quantized Hall effect for Fermi systems.[34]

D. Characteristic magnitudes

At this point is seems appropriate to discuss two of the characteristic magnitudes that arise in anyon models. Of course it is only the product of fictitious field and fictitious charge that has physical meaning, but to relate the effect of the fictitious fields to our intuitions in dealing with real fields it is useful to ascribe the same unit e of charge. The magnitude of the average fictitious magnetic field is then related to the density according to

$$\bar{b}=\frac{\pi\rho}{2e}\ . \qquad (2.10)$$

If we take a typical value $\rho=10^{14}$ cm^{-2} then we find that $\bar{b}$ is about 10 MG. For the cyclotron frequency, which is also the gap in the charged particle spectrum, we have

$$\omega_c=e\bar{b}/m^*\ . \qquad (2.11)$$

If we take the effective mass to be just the bare electron mass, and the density as before, then we find that ω_c is about 100 meV, or in terms of an equivalent temperature about 1000 K.

The anyon models cannot be used to make refined numerical estimates, for several reasons—notably that in the real materials the geometric size of the quasiparticle clouds is likely to be comparable to or greater than the spacing between them, whereas the models treat them as structureless points and that the effective mass is not known. Nevertheless we take it as encouraging that the above estimate of the (zero-temperature) gap is of a reasonable order of magnitude for a high-temperature superconductor.

III. MAGNETIC MOMENT AND DOMAIN STRUCTURE

Since in the many-body ground state of the anyon gas the individual anyons are subject to an effective (fictitious) magnetic field, it seems very plausible that they carry an intrinsic orbital angular momentum. It is slightly tricky to relate this intrinsic moment to the macroscopic properties of the gas, however. For example the total angular momentum within, say, a large radius R around the origin, can receive large contributions from the particles at the boundary. The existence of currents at the boundary, or the occupation of "skipping" orbits, can drastically affect the total angular momentum. This sensitivity arises essentially because the angular-momentum operator contains a factor of the radius vector, and so the integral

$$L_z = m^* \int d^2r(\mathbf{r}\times\mathbf{j})\cdot\hat{\mathbf{z}} \tag{3.1}$$

receives a contribution proportional to the *area* from a *surface* current. The total angular momentum is therefore awkward to study, as it depends on the precise prescription for boundary conditions. A very instructive discussion of related issues has been given in Ref. 35.

We can finesse these difficulties to a great extent by considering the differential contribution to the angular momentum due to gradients in the density. With some benefit from hindsight, let us assume that in the interior of the sample the current density, in an equilibrium state, can be written in the form

$$j_k = \frac{\rho_s}{pm^*}(\hbar\nabla_k\Phi - pA_k) + \frac{1}{2m^*}\epsilon_{kl}\nabla_l(\gamma\rho) , \tag{3.2}$$

where $\Phi(\mathbf{r})$ is a function of position that we identify as the phase of the superconducting order parameter, $\mathbf{A}$ is the electromagnetic vector potential, $\rho(\mathbf{r})$ is the anyon density at point $\mathbf{r}$, ρ_s is a coefficient that we may identify as the local superfluid density and γ is a coefficient to be described later. We have set $e=c=1$ for this discussion.

The first term on the right-hand side of (3.2) is the usual supercurrent density, while the second term is a result of the broken P and T symmetry. Note that the coefficient γ is odd under P and T but even under PT, so it has the right quantum numbers to be related to an intrinsic moment. To cement this interpretation, simply consider the contribution of the additional current to the angular momentum:

$$\begin{aligned} m^* \int \epsilon_{kl} r_k j_l &= \tfrac{1}{2}\int \epsilon_{kl} r_k \epsilon_{ln} \nabla_n(\gamma\rho) = -\tfrac{1}{2}\int r_k\nabla_k(\gamma\rho) \\ &= \int (\gamma\rho) + \cdots \end{aligned} \tag{3.3}$$

where the ellipsis represents boundary terms. Evidently then, γ deserves to be called the intrinsic orbital angular momentum per particle. Equation (3.2) allows us to identify it from bulk properties.

We shall actually find that (at zero temperature) the coefficient γ is independent of density, so that it may be taken outside the derivatives and integrals in (3.2) and (3.3). In any case, we note that the contribution to the current from the second term in (3.2) is necessarily divergence free.

In Appendix A we shall argue that for a system with one type of anyon, with statistical parameter $\theta=\pi(1-1/p)$, the value of γ is given by

$$\gamma = -\frac{\hbar}{2}(p - p^{-1}) . \tag{3.4}$$

In contrast, the analysis of the electromagnetic response function, in the limit $p\rightarrow\infty$, which was carried out in Ref. 23, is equivalent to setting $\gamma=-\hbar p/2$.[36] For $p=2$, the large p formula gives $\gamma=-\hbar$, whereas the correct answer given by (3.4) is slightly smaller, $\gamma=-3\hbar/4$. For the case of bosons, with $p=1$, the large p formula gives $\gamma=-\hbar/2$, whereas the answer given by (3.4) is $\gamma=0$. This result is what we expect, of course, since there is no broken T symmetry in the boson case. For the case of two kinds of anyons, with $p=n_a=2$, we find instead

$$\gamma = -\hbar/4 . \tag{3.5}$$

Now we must consider what happens near a sharp boundary of the system. A direct application of (3.2) would imply an edge current $\mathbf{I}$ parallel to the boundary that is equal in magnitude to the value of $\gamma\rho/2m^*$ at a point slightly inside the boundary. However, we wish to allow for the possibility of an additional "anomalous" edge current $\mathbf{I}'$ that may depend on the details of the boundary conditions, so we write instead

$$\mathbf{I} = \left[\frac{\gamma\rho}{2m^*} + I'\right](\hat{\mathbf{z}}\times\hat{\mathbf{n}}) , \tag{3.6}$$

where $\hat{\mathbf{n}}$ is the unit vector directed outward from the boundary.

Our considerations are simplified if we consider a sample with circular symmetry, where the density $\rho(\mathbf{r})$ is assumed to be a slowly varying function of the radius r, up to a "sharp" boundary at radius $r=R$, and also let us take $\mathbf{A}=0$. If we further assume that there are no vortices present in the interior of the sample, then the superfluid phase $\Phi(\mathbf{r})$, as well as $\rho(\mathbf{r})$, can only depend on the radius r. Then there is no contribution from the first term in (3.2) to the azimuthal current, J_θ, and we may write

$$J_\theta(r) = -\frac{\gamma}{2m^*}\frac{\partial\rho}{\partial r} + I'\delta(r-R) . \tag{3.7}$$

The radial current must be zero in equilibrium since $\nabla\cdot\mathbf{J}=0$. Now if we compute the orbital angular momentum M_z we find

$$M_z=2\pi m^*\int_0^\infty r^2 J_\theta(r)dr$$
$$=\gamma N+2m^*\pi R^2 I' , \tag{3.8}$$

where N is the total number of particles in the system, and I' is determined by the specific boundary conditions and by the density of particles in the region just inside the boundary.

If we can neglect the contribution of the possible boundary current I' (as is certainly justified if the density of the anyons tapers slowly to zero near the boundary of the two-dimensional system), then the layer will have an orbital magnetic moment arising from the anyon current which is given by

$$\mu=\frac{\gamma}{\hbar}\mu_B^* N , \tag{3.9}$$

where μ_B^* is the anyon Bohr magneton

$$\mu_B^*=\frac{\hbar e}{2m^*c} . \tag{3.10}$$

The direction of μ depends on the sign of the statistics parameter θ, and thus depends on the sign of the broken symmetry in the layer.

So far we have only discussed the orbital contribution to the magnetic moment—but there is also the intrinsic magnetic moment of the individual anyons to consider. It is known for anyons in the continuum limit, an intrinsic angular momentum ("spin") is required by the spin-statistics theorem.[10,37] For the case of half-Fermi statistics, this intrinsic angular momentum must be equal to $\hbar(k\pm\frac{1}{4})$, where k is an integer. However, the g factor associated with this intrinsic angular momentum depends on the details of the short distance cutoff. There is no reason to expect any simple relation to the effective mass m^* of the anyons, and it is unlikely that the intrinsic magnetic moment would cancel the moment arising from the orbital motion of the anyons. Note that for a condensed-matter system on a square lattice, the angular momentum operator L_z is strictly defined only modulo $4\hbar$. The magnetic moment remains defined, however, and the continuum analysis remains useful in discussing its magnitude. In order to estimate the effects of the intrinsic magnetic moment, we ignore any contribution from the anyon spin, and assume $I'=0$.

The magnitude of the three-dimensional magnetic moment density is

$$M_0=\rho\gamma\mu_B^*/s\hbar , \tag{3.11}$$

where s is the interlayer spacing, and ρ the anyon density. Using our previous value $\rho=10^{14}$ cm^{-2}, $|\gamma|=\hbar/4$, and taking the effective mass equal to twice the electron mass, which is appropriate for low-frequency phenomena, and $s=7$ Å, we find $4\pi M_0\approx 15$ G.

Let us now consider the consequences of an intrinsic magnetic moment for the bulk properties of the system, first assuming coupling between layers favors the same sign of broken time-reversal in every layer. Then the intrinsic magnetic moment of the layers generates a macroscopic magnetic induction $\mathbf{B}_0$, which depends on the shape of the sample, but is different from zero except in the case of an infinite slab or film oriented parallel to the layers of the material. If, as seems likely, B_0 is below the critical field H_{c1} of the superconductor, this field will be expelled from the sample due to the Meissner effect, and there is only a nonzero value of $\mathbf{B}$ in a region near the surface whose depth is essentially the London penetration depth λ_L.

A simple case to consider is a cylindrical, single-domain sample, illustrated in Fig. 1. We assume that the axis of the cylinder is in the z direction, which is perpendicular to plane of the layers, and we assume that the intrinsic magnetization $\mathbf{m}$ is parallel to the z axis, as indicated by the bold arrow. A surface current $\mathbf{I}$ then appears on the lateral surface of the cylinder as indicated. The supercurrent $\mathbf{j}(\mathbf{r})$ in the surface region is in the opposite direction to $\mathbf{I}$, and cancels the surface current over a region of depth λ_L. In the limit where the sample size becomes infinite, the magnetic induction $\mathbf{B}$ is vanishingly small in the region exterior to the lateral surface. The induction jumps to the value $B_0=4\pi M_0$ just inside the sample, and falls off exponentially with the skin depth λ_L, away from the sample boundary.

In a finite sample, there will be a small, nonzero magnetic field $\mathbf{B}$ outside the sample, which may be thought of as a fringing field that emanates from the ends of the

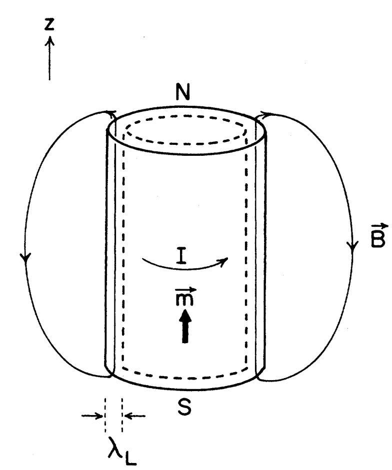

FIG. 1. Cylindrical sample with a single magnetic domain. The axis of the cylinder is taken perpendicular to the copper-oxygen planes; the sign of the hypothesized broken time-reversal symmetry is assumed to be the same in every layer, so that there is a uniform intrinsic orbital magnetic moment in the direction of the heavy arrow, with an associated surface current I on the side of the cylinder. The resulting magnetic induction is expelled from the sample except for a surface region the thickness of the London penetration depth λ_L. The curves show typical lines of magnetic induction $\mathbf{B}$, and the symbols N and S denote the north and south poles of the magnetization.

sample in a region of width λ_L near the perimeter. The magnetic field outside the cylinder looks something like the field of an annular bar magnet, and the total magnetic dipole moment is seen to be proportional to the lateral surface area rather than the volume of the cylinder.

Now let us consider the lowest-energy configuration of an oriented polycrystalline sample, where the z axes of all crystallites are assumed to lie in the same direction, but the magnetization of each crystallite is free to point in either the $+z$ or $-z$ direction. If two identical cylindrical crystallites are brought together end to end, it is clear that the energy is minimized when their magnetizations point in the same direction. If they are brought together side by side, however, the energy is minimized when their magnetizations point in opposite directions. Thus we would expect that in an oriented polycrystalline film, with the planes of the layers parallel to the plane of the film, the optimum configuration would be a domain structure, indicated schematically in Fig. 2, which is qualitatively similar to the domain structure of an ordinary ferromagnetic film.

In an ordinary ferromagnet, the energy of the magnetic interactions becomes increasingly important as the size of the sample becomes large, so that for a macroscopic sample, even if it is a single crystal, it is always favorable to introduce domain walls and create a sample with no net magnetization. This is not the case for our ferromagnetic superconductor, however, because the total magnetic moment of a domain does not increase proportional to the volume of the domain. If the film indicated in Fig. 2 were a single crystal of high-T_c superconductor, we would actually expect to find a single domain, regardless of the size of the sample. In particular we expect that there should be an energy cost to create a domain structure which would be of the order of an in-plane spin exchange energy J ($\approx 0.01-0.1$ eV) for every copper atom adjacent to the vertical domain wall, while the total gain in magnetic energy is limited to $E \approx B_0^2 S \lambda_L / 4\pi$, where S is the

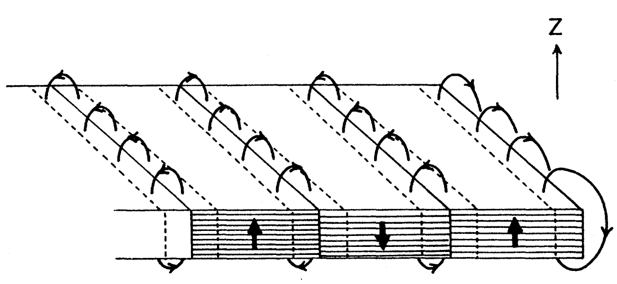

FIG. 2. Possible domain structure to minimize the energy in an oriented polycrystalline film. The copper-oxide planes are assumed to be parallel to the film plane as indicated by the parallel sets of lines on the front face of the sample, and the directions of intrinsic magnetization in different domains are indicated by the heavy arrows. Heavy solid lines indicate the boundaries of the domains, dashed lines suggest the surface region which extends a distance λ_L away from the domain walls, and the curves indicate typical lines of magnetic induction outside the sample.

area of the domain wall (see Appendix C). The ratio of the magnetic field energy gain to the exchange energy loss for a domain wall, at low temperatures, is thereby estimated to be

$$\frac{E_B}{E_J} = \frac{\pi\gamma^2}{\hbar^2}(\rho^{1/2}a)\left[\frac{\hbar^2\rho}{m^*J}\right]\left[\frac{e^2}{sm^*c^2}\right]^{1/2}, \tag{3.12}$$

where a is the in-plane lattice constant. This is a number of order 10^{-2} or smaller.

We note that the fringing field in the vicinity of a domain boundary in a polycrystalline sample, or at the corner of a single-crystal sample, is estimated to be of order $B_0 \approx 15$ G in a region of linear dimension $\lambda_L \approx 1000$ Å. This might be detectable with an appropriate experiment.

The state of lowest absolute energy is not necessarily the structure that would be observed in an actual experiment. In particular the difference between magnetic energies associated with different structures is very small, and the exchange energy for spin coupling between planes may also be very small, especially in materials where there is a large separation between cooper-oxide planes. In these circumstances, it may be possible to prepare a single domain, ferromagnetically aligned, sample regardless of the sign of the coupling between layers. To do this, one should cool the sample in the presence of a large external magnetic field in the z direction, through the temperature T_{TP}, where the broken time-reversal symmetry sets in. The sign of the broken symmetry is then the same in each layer, and is such as to align the orbital magnetic moment with the applied field. When the external field is turned off, it is likely that the sign of the symmetry breaking will remain fixed in each layer, because the energy barrier to nucleate a region with reversed magnetization is likely to be insurmountably large at low temperatures.

Finally, we note that the intrinsic moment should manifest itself through an asymmetry in the response of the superconductor to an applied magnetic field; and specifically through a dependence of the critical field H_{c1} on the orientation of the applied field relative to the fictitious field. For a single domain our estimates indicate that the fractional asymmetry should be quite substantial.

IV. ASYMMETRIC TRANSPORT AND RESPONSE

Consider a single copper-oxide layer of the new superconducting materials. Within this layer we can define both the operations of time-reversal T and two-dimensional parity P. Here P should be distinguished from the usual three-dimensional parity operation in that it is just a reflection in one axis (within the layer) rather than a full inversion in all three spatial axes, and as such, for example, two-dimensional parity P would reverse the orientation of an external magnetic field in the z direction, $B_z \to -B_z$. For a system that is rotationally invariant within a layer, it clearly does not matter along which axis we choose to reflect, but in the nonisotropic case we will always choose the reflection axis to be directed along a crystal axis.

The case of interest to us is that of a system in which the Hamiltonian, H, is invariant under both T and P, but these discrete symmetries are spontaneously broken in the ground state. In particular we can suppose that there are two inequivalent ground states, $|\mu\rangle$ and $|-\mu\rangle$, which are interchanged under the actions of P and T, similar to the interchange of the two degenerate ground states in the ferromagnetic two-dimensional Ising model under the action of these two operations. The existence of microscopic time-reversal and parity invariance for a system implies strong constraints on the allowed transport phenomena, which appear formally as the Onsager relations among the kinetic coefficients.[29] We thus need to consider the weaker relations that hold among the kinetic coefficients when some, or all, of these symmetries are spontaneously broken.

The kinetic coefficients in question, $C_{ij}^{\alpha\beta}$, are defined by the relation between the generalized currents J_i^α and the generalized forces X_j^β in a system, namely,

$$J_i^\alpha=\sum_{j\beta} C_{ij}^{\alpha\beta} X_j^\beta \ , \tag{4.1}$$

where the upper indices label the type of current, and the lower indices label the direction. The currents J_i^α are themselves the currents of various conserved densities q^α of the system. Typical such densities might be charge and energy density. In this simple case the generalized forces are then proportional to gradients of the temperature and chemical potential, and the kinetic coefficients are essentially the usual thermoelectric coefficients.

In terms of these $C_{ij}^{\alpha\beta}$'s, what are the relations that hold as a consequence of the individual discrete symmetries P, T, and PT? Under a parity reflection through the x axis (which we take to be a crystal axis) the components of the currents and forces transform as

$$\begin{aligned} &J_x^\alpha \rightarrow -\varepsilon^\alpha J_x^\alpha \ , \\ &J_y^\alpha \rightarrow \varepsilon^\alpha J_y^\alpha \ , \\ &X_x^\alpha \rightarrow -\varepsilon^\alpha X_x^\alpha \ , \\ &X_y^\alpha \rightarrow \varepsilon^\alpha X_y^\alpha \ , \end{aligned} \tag{4.2}$$

where $\varepsilon^\alpha=\pm$ depending on the intrinsic parity transformation properties of the density q^α. Demanding microscopic P invariance therefore tells us that

$$\begin{aligned} &C_{ij}^{\alpha\beta}=-\varepsilon^\alpha\varepsilon^\beta C_{ij}^{\alpha\beta}, \quad i\neq j \ , \\ &C_{ii}^{\alpha\beta}=\varepsilon^\alpha\varepsilon^\beta C_{ii}^{\alpha\beta} \ , \end{aligned} \tag{4.3}$$

regardless of whether T is violated or not. In the common case where we have $\varepsilon^\alpha\varepsilon^\beta=1$ for all α and β this forces the off-diagonal elements to vanish $C_{xy}^{\alpha\beta}=-C_{yx}^{\alpha\beta}=0$.

If in addition we have rotational invariance in the plane, or a smaller Z_4 or Z_6 discrete rotational symmetry (square or hexagonal symmetry), then $C_{xx}=C_{yy}$ as well. In general, isotropy or square or hexagonal symmetry within the plane forces the kinetic coefficients $C_{ij}^{\alpha\beta}$ to be proportional to a linear combination of δ_{ij} and ε_{ij}.

If the Hamiltonian H commutes with T and the ground state is invariant under T, then the standard reciprocity relation applies and we have

$$C_{ij}^{\alpha\beta}=\delta^\alpha\delta^\beta C_{ji}^{\beta\alpha} \ , \tag{4.4}$$

where δ^α is the intrinsic time-reversal transformation property of the density q^α.[38] On the other hand, combined PT invariance requires,

$$\begin{aligned} &C_{ij}^{\alpha\beta}=-\varepsilon^\alpha\varepsilon^\beta\delta^\alpha\delta^\beta C_{ji}^{\beta\alpha}, \quad i\neq j \ , \\ &C_{ii}^{\alpha\beta}=\varepsilon^\alpha\varepsilon^\beta\delta^\alpha\delta^\beta C_{ii}^{\beta\alpha} \ . \end{aligned} \tag{4.5}$$

These are the general relations that hold among the kinetic coefficients, but let us specialize, for instance, to the case of the thermal conductivity tensor,

$$\kappa=-[C^{TT}-C^{TE}(C^{EE})^{-1}C^{ET}] \ ,$$

and consider under what circumstances anomalous effects are allowed, in particular the possible appearance of a nonzero antisymmetric contribution to κ—a thermal equivalent of the Hall effect. (For the quantities involved in the thermoelectric processes, all the ε's and δ's are $+1$). The occurrence of an antisymmetric contribution to κ has been discussed in the literature for the case of a normal metal in an external magnetic field, where it is known as the Righi-Leduc effect.[29] We shall refer to it here by the more graphic name of a "thermal Hall effect." The results may be summarized as follows.

(1) If the Hamiltonian H is T conserving, and the ground state is T invariant, then $\kappa_{xy}=\kappa_{yx}$ regardless of P invariance. Isotropy would in addition force $\kappa_{xy}=-\kappa_{yx}=0$ and $\kappa_{xx}=\kappa_{yy}$. In any case T invariance does not allow a thermal Hall effect.

(2) If H is P conserving and the ground state P invariant, then $\kappa_{xy}=-\kappa_{xy}=0$ regardless of whether T is violated or not. Again there is no thermal Hall effect, with or without isotropy.

(3) If P and T are both violated in the ground state, then we can have a nonzero antisymmetric contribution to κ. The existence of PT as a good symmetry requires $\kappa_{xy}=-\kappa_{yx}$ but does not force the off-diagonal part to vanish. Rotational invariance does not change this situation. Therefore as long as both P and T are broken we *are* allowed a thermal Hall effect.

In principle, a nonzero thermal Hall effect can be detected by an experiment in which heat is forced to flow in the x direction through a rectangular sample, and a temperature difference is observed between a pair of thermometers whose separation is purely in the y direction. To check against spurious effects arising from misalignment of the sample or due to unknown inhomogeneities or anisotropies of the sample, one should interchange the positions of the thermometers with the source and sink of heat current. When the heat current flows in the y direction, there should be a temperature difference (with reversed sign) for thermometers separated in the x direction.

If broken time-reversal and parity invariance sets in, for a cooper-oxide material at a temperature T_{TP}, which is higher than the onset temperature T_c for superconductivity, then there should be in addition to the thermal

Hall effect, a spontaneous Hall effect of the ordinary type, as well as nonzero antisymmetric contributions to the cross coefficients in the thermoelectric tensor. Since electrical measurements are generally much simpler to perform than thermal measurements, it would be easiest, in this case, to look for a nonzero result in a standard Hall geometry, in the absence of an applied magnetic field. The spontaneous Hall effect should in fact be large, since the fictitious internal magnetic field b is predicted to be very large.

In the case where $T_{TP}=T_c$, asymmetry in the transport coefficients is harder to detect. In the superconducting state, the results of a standard Hall measurement are necessarily zero, because the electrochemical potential, which is measured by a standard voltage probe, is always constant in a superconductor in a steady state at constant temperature. This problem is reflected in the relations (4.1) by the divergence of the diagonal transport coefficients for the electric charge. However, the thermal-conductivity tensor κ_{ij} remains well defined in the superconducting state, and the thermal Hall effect should be observable if P and T are broken.

There exists another version of the Hall effect which can be observed in a superconductor.[39] Here one uses a contactless capacitive pickup to measure changes in the electrostatic potential produced by current flows in the sample. Such experiments are generally much more difficult than the conventional Hall-effect measurement, but a nonzero result is predicted, for a superconductor with broken P and T symmetry, in the absence of an external magnetic field. (This was discussed in Ref. 23 for a model with one species of anyon.)

The size of the thermal Hall effect that one might see in a high-T_c superconductor can only be estimated very crudely. The effect should vanish at low temperatures, where the thermal conduction takes place primarily via long-wavelength phonons, which are expected to have a large mean free path in the superconducting state. The electronic contribution to the thermal-conductivity tensor at low temperatures, within the anyon model, should come primarily from excitations of low-energy electron fluctuations, which have a small wave vector $k_{\parallel}$ in the x-y plane, and a nonzero wave vector in the direction perpendicular to the layers. As in the case of phonons, these excitations give no contribution to the thermal Hall effect when $k_{\parallel}$ is small.

The best hope to observe a thermal Hall effect, if $T_{TP}=T_c$, is to work at a temperature which is somewhat below the transition temperature, say $T=0.9T_c$. At this temperature we may hope that there are reasonably large numbers of short-wavelength electronic excitations, which we may think of as loosely bound pairs of anyons or pairs of vortices in a copper-oxygen plane. Energy and momentum is exchanged as a vortex from one pair is scattered by a vortex from another, and the effects of broken P and T symmetry are manifest as an asymmetry in the scattering cross section for the vortices. If one neglects the correlations between vortices, one is led by this reasoning to an off-diagonal contribution to the thermal-conductivity tensor, which could be a significant fraction, say 10%, of the electronic contribution to κ_{xx}. Heremans *et al.*[40] have estimated, assuming the approximate validity of the Wiedemann-Franz law, that in $YBa_2Cu_3O_7$ high-temperature superconductors the electronic contribution to the thermal conductivity is about 10% of the phonon contribution. Thus a crude estimate of the ratio κ_{xy}/κ_{xx} is

$$\frac{\kappa_{xy}}{\kappa_{xx}}\approx 0.1\alpha\ , \tag{4.6}$$

with α a numerical constant that might be as large as 0.1. However, recent measurements of the resistivity of $YBa_2Cu_3O_7$ epitaxial films show a substantially higher electrical conductivity, at temperatures just above T_c, than that found by Heremans *et al.* It is therefore likely that the electronic contribution to the thermal conductivity could be greater than 10%, leading to an increase in our estimate of the ratio κ_{xy}/κ_{xx}.

Our symmetry analysis can of course be extended to the complex finite-frequency electrical conductivity tensor $\sigma_{ij}(\omega)=C_{ij}^{EE}(\omega)$. At optical frequencies it is often more convenient to discuss the frequency-dependent dielectric tensor,

$$\varepsilon_{ij}(\omega)=\delta_{ij}+\frac{4\pi i\sigma_{ij}(\omega)}{\omega s}\ , \tag{4.7}$$

where s is the interplane spacing. When both P and T are violated, antisymmetric contributions to $\sigma_{ij}(\omega)$ and $\varepsilon_{ij}(\omega)$ are allowed.

As was noted by Wen and Zee,[30] the P and T violating term in $\varepsilon_{ij}(\omega)$, for $\omega>0$, would lead to rotation of the plane of polarization of light that is transmitted in the direction normal to the superconducting planes, and a difference in the reflectivity for the two types of circularly polarized light. However, as discussed in Ref. 23 and in Sec. V, these optical rotation effects, for normally incident light, are absent in the simplest model, where the anyons are described in an effective mass approximation, and impurity scattering is ignored. For more realistic systems where T and P violation occurs we do expect to find an antisymmetric contribution to $\sigma_{ij}(\omega)$, and hence optical rotation should be observable.

V. REMARKS ON THE EFFECTIVE LONDON LAGRANGIAN

The most transparent way to discuss many of the P and T violating features of the anyon gas is via an effective Lagrangian description of its low-lying collective excitations. At small $\mathbf{q}$ and ω, the response to weak external electromagnetic fields is dominated by the contribution of the collective sound or Nambu-Goldstone mode. This mode can be represented by a massless scalar field, and its interactions with the electromagnetic field expanded in gradients. The lowest terms yield an effective Lagrangian density

$$\begin{aligned}L_{\text{eff}}=&\tfrac{1}{2}(\partial_0\phi-CA_0)^2-\frac{v^2}{2}(\partial_i\phi-CA_i)^2\\&+a\varepsilon_{ij}\partial_i A_j(\partial_0\phi-CA_0)\\&+b\varepsilon_{ij}(\partial_0 A_i-\partial_i A_0)(\partial_j\phi-CA_j)\ ,\end{aligned} \tag{5.1}$$

where A_μ is the external electromagnetic field and ϕ represents the collective mode. Notice that in the absence of electromagnetic coupling the effective Lagrangian reduces to that of a free scalar, with speed of propagation v.

The a and b terms are formally higher order in gradients, but are retained because they violate symmetries (P and T) that are "accidentally" respected by the leading terms. They bear a close resemblance to the famous Chern-Simons term of relativistic field theory. In the nonrelativistic theory, as we see, two such terms can in principle arise; furthermore, we have given a manifestly gauge-invariant form for these terms, by introducing the scalar field ϕ. The ordinary relativistic Chern-Simons term would arise if we put $a=b$ and $\phi=0$.

If we wish to treat the electromagnetic field as a dynamic variable, we must add to (5.1) a field term of the form $(E^2-B^2)/8\pi$. Now, however, we must distinguish carefully between three different situations: (1) We may wish to describe an isolated superconducting layer coupled to a three-dimensional electromagnetic field. Then the field term must be integrated over three-dimensional space, while (5.1) is confined to the plane of the sample. (2) We may consider a fictitious world of two space dimensions where the electromagnetic field, as well as the superconductor is restricted to the x-y plane. (3) We may consider a three-dimensional stack of superconducting layers, interacting with a three-dimensional electromagnetic field. Then the total Lagrangian is the sum of the three-dimensional field contribution and the sum of contributions of the form (5.1) for each superconducting layer.

The results in case (2) are the simplest. The model exhibits the Higgs mechanism in a form due to Stuckelberg: ϕ, which in the absence of electromagnetism represents a scalar degree of freedom—essentially describing sound waves with v^2 equal to the speed of sound squared—loses its independent significance, and becomes the longitudinal part of the effectively massive electromagnetic field. Indeed, ϕ can be set to zero by gauge transformation, since (5.1) is invariant against gauge transformations of the form

$$\begin{aligned}\phi &\to \phi + Cf \ ,\\ A_\mu &\to A_\mu + \partial_\mu f \ .\end{aligned} \tag{5.2}$$

When this choice is made we find the famous London proportionality between current and potential; more generally, variation of the first two terms of (5.1) yields a gauge-invariant version of the London equations.

The results in case (3), a stack of superconducting layers, are quite similar to the fictitious two-dimensional case, as long as one restricts oneself to situations where there is translational symmetry in the z direction. The propagating long-wavelength modes with $k_z=0$ are pushed up to the plasma frequency $\omega_p=(4\pi v^2C^2/s)^{1/2}$, where s is the interplane spacing, while static magnetic fields in the z direction are screened with the bulk London penetration length $\lambda_L=c/\omega_p$. In the first case of an isolated layer, the normal modes propagate with the speed of light c, at very small wave vectors k, and with a group velocity $v_g \propto vCk^{-1/2}$, for $k>v^2C^2/c^2$.

The response explicitly calculated in the random-phase approximation to the dynamics of the ideal anyon gas can indeed be expressed in the form (5.1), with specific coefficients. The values found by Chen *et al.*,[23] for the parameters v^2, C, and a, for the model with one species of anyon, are

$$\begin{aligned} v^2 &= \frac{2\pi p \hbar^2}{m^{*2}} \ ,\\ C &= e\sqrt{m^*/2\pi\hbar} \ ,\\ a &= -pe\sqrt{\hbar/32\pi m^*} \ ,\\ b &= 0 \ ,\end{aligned} \tag{5.3}$$

where p is defined as before by $\theta=\pi(1-1/p)$. The random-phase approximation in this problem can be recast in this context as a perturbative expansion in $1/p$; thus we expect that these calculated coefficients should approach the exact answer as $p\to\infty$.

Now we will argue that every parameter in this effective Lagrangian can be determined by direct physical arguments. First, the velocity of sound is determined by the compressibility, that is the second derivative of the energy with respect to density. It is easy to verify that the energy of a two-dimensional electron gas that *exactly fills* an integral number of Landau levels is exactly equal to that of the same gas in the absence of magnetic field. In our context, this means that the velocity of sound should be independent of p for large p, and equal to that formally derived from the compressibility of the free fermion gas. Since it is only at large p that the residual interactions among anyons can be neglected, we expect that corrections will arise for small p. Indeed, the free Bose gas can be regarded as having $v^2=0$.

The product v^2C^2 appears in the expression relating the supercurrent $\delta L/\delta A_i$ to the vector potential, and thus directly in the formula for the penetration depth. Since the anyon gas exhibits perfect diamagnetism, we expect that the penetration depth should be given by the classic London formula, and in particular that it should be independent of p when expressed in terms of density and effective mass. The explicit results for large p do bear this out.

The value of the coefficient a, or more precisely the product of aC, can be related to the coefficient γ discussed in Sec. III. Indeed, we have (setting, for reasons, that will soon become apparent, $b=0$):

$$j_k=\delta L/\delta A_k=v^2(\partial_k\phi-CA_k)+aC\varepsilon_{kl}\partial_l(\partial_0\phi-CA_0) \ , \tag{5.4}$$

and to lowest order

$$\rho=\delta L/\delta A_0=-C(\partial_0\phi-CA_0) \ . \tag{5.5}$$

We see that (5.4) is equivalent to (3.2), if we make the identifications

$$\frac{\rho_s}{p^2 m^*}\Phi = v^2\phi ,$$
$$\frac{\rho_s}{m^*} = Cv , \tag{5.6}$$
$$\frac{\gamma}{2m^*} = \frac{a}{C} .$$

The RPA results (5.3) imply that $\rho_s = \rho$ (at zero temperature), which is actually an exact result for this system.

In Appendix A we have calculated γ directly, with the result (for the single-anyon model)

$$\gamma = -\frac{\hbar}{2}\left[p - \frac{1}{p}\right] . \tag{5.7}$$

This result ought to agree with the RPA result for large p, and indeed it does. The arguments of Appendix A seem to be much more powerful, however, and it appears that they afford an exact evaluation of γ. In conjunction with the other results of this section, they allow us to evaluate the coefficients in the effective Lagrangian for multianyon models (and the associated prediction[23] for the zero-field Hall effect) without further ado.

Although it is somewhat outside the scope of this paper, we will now mention briefly some formal questions of interest in connection with the second term of (3.2). That contribution to the current is meaningful even for the pure scalar theory (with $C=0$). What is the Lagrangian which implements it? It is not hard to show that

$$\Delta L = -\frac{\gamma}{4m^*}\varepsilon_{lm}\partial_l\partial_0\phi\partial_m\phi = -\frac{\gamma}{4m^*}\partial_l(\varepsilon_{lm}\partial_0\phi\partial_m\phi) \tag{5.8}$$

does the job. Indeed,

$$\Delta j_k = \frac{\delta \Delta L}{\delta(\partial_k\phi)} = \frac{\gamma}{2m^*}\varepsilon_{kl}\partial_l\partial_0\phi \approx \frac{\gamma}{2m^*}\varepsilon_{kl}\partial_l\rho , \tag{5.9}$$

where the second expression is accurate to lowest order in gradients.

The additional term ΔL is a total divergence, and thus its inclusion does not affect the equations of motion. This feature alters when we include couplings to electromagnetism. This is achieved by substituting covariant for ordinary derivatives. For a Stuckelberg field the two possible orders of taking covariant derivatives give the alternatives

$$\partial_k\partial_0\phi \to \partial_k(\partial_0\phi - CA_0) ,$$

or (5.10)

$$\to \partial_0(\partial_k\phi - CA_k) .$$

Note that in either case the quantity in parentheses is gauge *invariant,* so the second covariant derivative reduces to the ordinary derivative. Thus starting from ΔL we arrive at the two possibilities,

$$\Delta L_{\rm I} = -\frac{\gamma}{4m^*}\varepsilon_{lm}\partial_l(\partial_0\phi - CA_0)(\partial_m\phi - CA_m) ,$$
$$\Delta L_{\rm II} = -\frac{\gamma}{4m^*}\varepsilon_{lm}\partial_0(\partial_l\phi - CA_l)(\partial_m\phi - CA_m) . \tag{5.11}$$

Now we can see that $\Delta L_{\rm I} - \Delta L_{\rm II}$ is proportional to our b term, while after integrating by parts, $\Delta L_{\rm I}$ becomes proportional to the a term. These gauge-invariant terms do, of course, affect the equations of motion. These results are quite reasonable from a physical point of view. In the pure scalar case an intrinsic angular momentum has limited dynamical significance, but it acquires direct physical meaning when a gauge field is introduced to which it couples.

Finally, we turn to the coefficient b. The vanishing of b found in the RPA calculation is no accident. Indeed, the effect of b is to induce a transverse current in response to an applied electric field. However, in a gas of nonrelativistic particles all of which have the same charge to mass ratio, a uniform applied electric field couples directly to the center-of-mass coordinate. The response to such a field is therefore, under very general assumptions, just the free-particle response—the motion of the center of mass is independent of the interparticle interactions. Of course the free-particle response does not include a transverse current; thus the coefficient b vanishes.

Essentially the same argument may be given in a more abstract form: From the microscopic universality of the charge-to-mass ratio we extract the relation

$$T_{0i} = \frac{m^*}{e} j_i \tag{5.12}$$

between the momentum density and the current density, that should also be implemented in any effective Lagrangian. There is a general way to implement such relations, as is explained in detail in Ref. 41. However, (5.12) is not respected by the b term, nor does there appear to be any way to repair the discrepancy.

Now let us consider the very interesting experiment proposed by Wen and Zee, namely, measurement of the rotation of the plane of polarization of linearly polarized normally incident light. The a term, despite its being P and T violating, does not lead to a rotation of the plane of polarization of the reflected light. Indeed this term couples the charge density to the perpendicular magnetic field, and does not affect the propagation of fields tangential to the plane. An effect of the type proposed by Wen and Zee would arise from the b term. Unfortunately, as we have seen, this term vanishes for the ideal anyon gas.

It is easy to see that off normal incidence, the a term *will* lead to an effect related to optical rotation, but in practice this is a more difficult experiment to perform. Since we fully expect, when the simple anyon gas is generalized to a more realistic model, to find an optical rotation at normal incidence we will not discuss the off-axis case here.

In order to obtain a nonzero antisymmetric contribution to $\varepsilon_{ij}(\omega)$, one must include in the Hamiltonian terms which violate the Galilean invariance of the dilute anyon model. We expect, for example, that by including terms in the kinetic energy which represent deviations from the effective mass approximation, or by employing a proper band structure for the anyons, we would find a nonzero optical rotation. Since the hole concentration in copper-oxide superconductors is typically of order 15%, the deviations from the effective mass approximation should be substantial.

Even within the effective mass approximation, Galilean invariance is broken if there is scattering from fixed impurities. We show in Appendix B that there is *no* antisymmetric contribution to $\epsilon_{ij}(\omega)$ when the calculation of the effects of impurities is truncated to second order in the strength of the impurity potential. However, we expect that there will be a nonzero value of $\varepsilon_{xy}(\omega)$, if terms of third order or higher are included.

Altogether, although the vanishing of b in the most idealized version of the anyon gas makes it difficult to form quantitative expectations, there is every reason to expect a nonzero effect in real materials. Given the sensitivity of optical techniques, the measurement proposed by Wen and Zee remains a very worthwhile exploratory experiment.

VI. SURFACE-SENSITIVE PROBES

As we mentioned in the Introduction, if it turns out that alternate layers of a high-temperature superconductor have opposite signs of the broken time-reversal symmetry, there would be no bulk asymmetry in the transport properties or in $\varepsilon_{xy}(\omega)$. Therefore it would be desirable to find a surface sensitive probe that could reveal a broken symmetry within the surface layer itself.

It is not difficult to think of probes that would show such an effect in principle, but it seems much more difficult to find a convincing argument for a sizeable effect. For example, one would expect to find a difference in the photoemission rate for the two possible circular polarizations of the incident light. Similarly, there should be slight circular polarization of the emitted light in an inverse photoemission experiment. As in the case of optical rotation, however, the polarization sensitivity can only occur as a result of complicated high-order excitation processes, so that the final polarization dependence may be very small.

Because of the inherent noise in a photoemission experiment, and because the greatest sensitivity to incident polarization may occur for light of relatively low frequencies (below the photoemission threshold), it is possible that the most practical method of looking for a broken time-reversal invariance in the surface layer would be to look for a small residual asymmetry in the reflection of circularly polarized light, due to the unbalanced contribution of the surface layer. For light in a frequency range which is strongly absorbed in the material, the polarization dependence of the reflectivity for the case of layers of alternating sign would be reduced, relative to the case where all layers have the same sign of broken time-reversal invariance, by a factor of order s/λ. Here s is the interlayer spacing, λ the penetration depth of the light, and we have assumed reflection from a region with an atomically flat surface.

A surface sensitive probe which should be capable, in principle, of revealing broken time-reversal symmetry in the surface layer, even if the broken symmetry alternates in sign from one layer to the next, is low-energy electron diffraction (LEED). For example, if an electron is incident on the surface with its in-plane component of momentum parallel to the x direction (which we take to be one of the symmetry axes of the crystal), there may be an asymmetry in the intensity of electrons scattered in the y and $-y$ directions.

The scattering of electrons with in-plane momentum parallel to y would show a similar asymmetry, with the same chirality. The expected chirality does not show up in the lowest Born approximation, however, and we do not have a reliable estimate of its magnitude.

Finally, and perhaps most fundamentally, in anyon superconductivity there is a close correlation between charge density and magnetic field. The a term, which is a contribution to the Lagrangian essentially proportional to the product of charge density and perpendicular magnetic field, is one sign of this. More pronounced charge concentrations, and electric potential gradients, should occur in the neighborhood of vortices. Recent experiments with scanning tunneling microscopes have been able to study in considerable detail the tunneling characteristics, and indirectly the charge distribution, of individual vortex cores at the surface of the superconductor $NbSe_2$ (Ref. 42). If similar experiments can be performed on a superconductor with broken T and P symmetries, one should observe a significant difference in the tunneling characteristics for vortices of opposite sign, for a given sign of symmetry breaking in the surface layer. Alternatively, if the sign of the broken T and P symmetry in the surface layer is different in two regions of the sample, there would be different tunneling characteristics in the two regions for vortex cores of the same sign. If there is antiferromagnetic ordering of the layers in the bulk of the superconductor, then a change of sign of the symmetry breaking in the surface layer will be produced by a step on the surface of a single lattice constant.

VII. NEUTRON SCATTERING AND MUON SPIN RELAXATION

Magnetic moments in solids can scatter neutrons because of the spatially varying magnetic induction $\mathbf{B}(\mathbf{r})$ produced by the moments. A uniform orbital magnetic-moment density in a copper-oxide plane would actually produce no neutron scattering, however, because $\nabla\times\mathbf{m}=0$ in the interior of the sample, and except at the boundary of the sample there will be no currents to produce a spatially varying $\mathbf{B}(\mathbf{r})$. It makes no difference to this whether alternate planes have the same or opposite signs of the magnetic moment. The situation is different if one takes into account modulation of the magnetic-moment density, due to variations of the orbital current density associated with the periodic electronic charge density within a unit cell of the copper-oxide planes. It is possible, in fact, that the modulation of the magnetization density will be a significant fraction of the average magnetization in the plane. In the case of ferromagnetic alignment of the planes, the result is a magnetic contribution to neutron scattering at the same points in reciprocal space where there will be Bragg scattering in any case due to the scattering by the nuclei of the system, so the magnetic scattering would surely be impossible to detect.

In the case of antiferromagnetic ordering of the planes, however, there would be new Bragg peaks associated with the magnetic order. For example, if the copper-oxide planes are stacked directly above each other, the new Bragg peaks should appear at scattering vectors of the form

$$\mathbf{Q}=(\mathbf{G}_{\perp},\tfrac{1}{2}G_z)\ , \tag{7.1}$$

where $\mathbf{G}_{\perp}$ is a nonzero reciprocal lattice vector parallel to the copper-oxide planes, and G_z is an odd multiple of the elementary reciprocal lattice vector perpendicular to the planes.

Unfortunately, the strength of the magnetic Bragg peak is proportional to the square of the local magnetic moment. In the present case, the maximum modulated moment in a unit cell is of order $\rho a^2\gamma\mu_B^*/\hbar$, which is a small fraction of Bohr magneton, so the magnetic scattering would be very weak.

An experimental technique, which is apparently very sensitive to the small spatially varying magnetic induction $\mathbf{B}(\mathbf{r})$, that would be produced by modulations of the orbital magnetic-moment density at the period of the lattice, is *positive-muon spin relaxation* (μSR) studies.[43] (A modulation of the anyon density, with an associated local magnetic field should also be produced by the Coulomb potential of the muon itself.) This technique should be equally sensitive whether the broken time-reversal symmetry has the same sign or opposite sign in different layers of the superconductor. Thus far, no intrinsic moment has been reported in μSR studies of high-T_c superconductors. A quantitative analysis of both the experimental limits and the size of the periodic magnetic field expected in realistic theoretical models is clearly called for.

VIII. CONCLUSION

We have seen that there are a variety of possible effects whose existence in the superconducting state of the copper-oxide materials would imply the existence of a broken P and T symmetry within the copper-oxide layers, and would support the hypothesis that the superconductivity itself results from an unconventional chiral-liquid order among copper spins. Although a true calculation of the magnitude of these effects is possible only in the context of a detailed model, we can at least estimate the possible size in some cases by using the anyon model of Laughlin and co-workers.

If the coupling between layers is ferromagnetic, so that there exist domains in which all layers have a single sign of the broken time-reversal symmetry, then there are a variety of bulk experiments which, it seems, should be adequate to see the predicted effects. On the other hand, failure to see positive results in these experiments could either indicate the absence of broken T and P symmetry or could indicate that the layers are antiferromagnetically ordered. Experiments that could detect broken T and P invariance in the antiferromagnetic case seem to be much more difficult to perform, except for the case of muon spin relaxation.

ACKNOWLEDGMENTS

This work was partially supported by National Science Foundation Grant Nos. PHY-87-14654 and DMR-88-17291 and by U.S. Department of Energy (DOE) Contract No. DE-AC02-76ER02220. We would like to thank Y. H. Chen, M. Greiter, B. A. Jones, N. P. Ong, A. Shapere, and E. Witten for their help and assistance. Enlightening conversations with J. Golovchenko, A. I. Larkin, R. B. Laughlin, M. Tinkham, Y. J. Uemura, and P. Wiegmann are also gratefully acknowledged.

APPENDIX A: CALCULATION OF THE ANGULAR MOMENTUM

The orbital momentum is most conveniently defined in the "anyon gauge" where we write simply

$$L_z=\sum_{j=1}^{N}\mathbf{r}_j\times\mathbf{p}_j\ , \tag{A1}$$

where $\mathbf{p}_j=-i\hbar\nabla_j$. If $\Psi^{(a)}$ is a wave function in the anyon gauge,which is an eigenstate of L_z with eigenvalue M_z, and if Ψ is the corresponding wave function in the fermion gauge, as given by (2.5), then Ψ is also an eigenstate of L_z with an eigenvalue $\widetilde{M}_z$ related to M_z by

$$M_z=\widetilde{M}_z-\frac{\hbar}{p}\frac{N(N-1)}{2}\ . \tag{A2}$$

We shall identify M_z as the actual orbital angular momentum of the anyon system.

Let us now consider a system containing one species of anyon, in the circularly symmetric situation described in Sec. II, and let us compute the angular momentum for a reference state which is chosen to be an eigenstate of the approximate Hamiltonian H_0. For a large but finite system, the number of particles N_l in Landau level l may be chosen slightly differently in each of the occupied Landau levels, depending on the nature of the confining potential at the boundary radius R. However, it will be convenient to choose as our reference state the simplest case, where the occupation is precisely the same in each Landau level,

$$N_l=N/p\ , \tag{A3}$$

and we require that N be a multiple of the integer p.

In each of the occupied Landau levels, we occupy the states with $0\le m\le N_l-1$. Then the orbital angular momentum of the reference state, in the fermion gauge, is equal to

$$\widetilde{M}_z=\hbar\sum_{l=0}^{p-1}[\tfrac{1}{2}N_l(N_l-1)-lN_l]=\frac{N^2\hbar}{2p}-\frac{N\hbar}{2}p\ . \tag{A4}$$

Thus the orbital angular momentum of the reference state in the anyon gauge is

$$M_z=-\frac{N\hbar}{2}(p-p^{-1})\ . \tag{A5}$$

For the case of particles with half-Fermi statistics ($p=2$), this gives an orbital angular momentum $M_z=-3\hbar N/4$. For bosons, with $p=1$, we have $M_z=0$ as expected. On the other hand, for $p\rightarrow\infty$ we do not recover the result $M_z=0$, that one would expect for a set of noninteracting fermions. This suggests that the reference state is not a correct description of the actual ground state for sufficiently large p.

It is possible to obtain a state with $M_z=0$, for $p\neq 1$, by small rearrangements of the occupation numbers, which lead to additional currents at the boundary. For example, we may choose

$$N_l=\frac{N}{p}+\alpha N^{1/2}(l-p+\tfrac{1}{2}) , \tag{A6}$$

for $0\leq l<p$, where α is a constant of order unity. The total number of particles is unaffected, but the angular momentum is now given by

$$M_z=-\frac{N\hbar}{2}\left[(p-p^{-1})-\frac{\alpha^2p^2(p-1)}{24}\right] . \tag{A7}$$

Clearly one can obtain $M_z=0$ with an appropriate choice of α.

Let us now return to our reference state where M_z is given by (A5). If we begin with this reference state, and adiabatically turn on the perturbation $(H-H_0)$, we obtain an eigenstate of H whose angular momentum is the same as that of the reference state. If we now apply the definitions and analysis of Sec. III to the perturbed reference state, we see that

$$m_z+2m^*I'=-\frac{\hbar}{2}(p-p^{-1})\rho , \tag{A8}$$

where $\rho=N/(\pi R^2)$ is the particle density, and $m_z\equiv\gamma\rho$ is the angular-momentum density, which are both constant in the interior of the system, not too close to the boundary, and I' is the anomalous boundary current, if any.

The true ground state of $H^{(a)}$ may differ from the perturbed reference state because of a repopulation of the Landau levels in the vicinity of the boundary. This could lead to a different value of I', and a different value of the angular momentum per particle M_z/N.

We shall now argue that I' is actually zero for our perturbed reference state, and therefore

$$\gamma=-\frac{\hbar}{2}(p-p^{-1}) . \tag{A9}$$

To see this, let us adiabatically add to the Hamiltonian $H^{(a)}$ a weak potential $\delta V(\mathbf{r})$, which is a slowly varying function of radius r, and which leads in turn to a radial variation of ρ and m_z. The values of N and M_z will be unaffected by this process. The radius R will also be unaffected if we choose $\delta V(r)$ in such a manner that

$$2\pi\int_0^R\delta\rho(r)r\,dr=0 . \tag{A10}$$

According to (3.8), the variation in M_z is given by

$$\delta M_z=\left[\frac{dm_z}{d\rho}\right]\left[2\pi\int_0^R\delta\rho(r)r\,dr\right]+2m^*\pi R^2\delta I' , \tag{A11}$$

so that $\delta M_z=0$ implies $\delta I'=0$.

A useful way of describing the nonuniform state is to generalize the "unperturbed" Hamiltonian H_0 of (2.6) to include a nonuniform effective magnetic field $\bar{b}(r)$, which satisfies $\bar{b}(r)=2\pi\hbar\rho(r)/p$ in the interior of the sample, and $\bar{b}(r)=2\pi\hbar\rho(R)/p$ for $r>R$. If we now put N/p fermions into each of the lowest p Landau levels, and then turn on the perturbation $(H-H_0)$, we should obtain precisely the same final state as before, when we started with the perturbed uniform reference state and then turned on $\delta V(r)$. We now see, however, that the occupation of the states at the edge of the inhomogeneous sample must be the same as for the uniform reference state with density equal to $\rho(R)$. It follows that

$$\delta I'=\delta\rho(R)\left[\frac{dI'(\rho)}{d\rho}\right] , \tag{A12}$$

where $I'(\rho)$ is the anomalous edge current for the perturbed uniform reference state at density ρ. Therefore $dI'/d\rho=0$, and from (A8),

$$\frac{dm_z}{d\rho}=-\frac{\hbar}{2}(p-p^{-1}) . \tag{A13}$$

Assuming that $m_z\to 0$ as $\rho\to 0$, we see that $m_z=\gamma\rho$, with γ given by (A9) and $I'=0$, as just asserted.

It is interesting to consider in more detail the behavior of a nonuniform system in the limit of large p. In particular, let us consider a system where the density $\rho(r)$ takes a constant value ρ_0 for $0\leq r<R_0$ and a different constant value ρ_1 in a region $R_1<r<R$, and where $\rho(r)$ varies smoothly between ρ_0 and ρ_1 in the intermediate region $R_0<r<R_1$. We may assume that ρ_0/ρ_1 is not very different from 1, and that R_0 and R_1 are both very large compared to the interparticle spacing ρ_0^{-1}, but R_1-R_0 is only a few times larger than ρ_0^{-1}. If the integer p is not very large, then the current density $J_\theta(r)$ given by (3.7) remains fairly small, and it is a reasonable proposition that the state we have obtained is the true ground state of the system. If we let p become large, however, while ρ_0/ρ_1 and R_1-R_0 are held fixed, the current density will grow, and eventually the energy cost will ensure that the system is no longer in its proper ground state. How can we lower the energy of the system? One possibility is to introduce a row of vortices of one sign at radius R_0, and a row of opposite sign at radius R_1, such that there is a supercurrent in the region $R_0<r<R_1$, which approximately cancels the current produced by ∇m_z. If we assume that the core energy of a vortex is given by the cyclotron energy $\hbar\bar{b}/m^*$, then the core energy vanishes in the limit $p\to\infty$, and it is plausible that the cancellation becomes more and more perfect as p increases. Thus for fixed values of ρ_0, ρ_1, and R_1-R_0, when we take the limit $p\to\infty$, we can recover the result of the noninteracting Fermi system, where there is no broken time-reversal symmetry, and $\langle\mathbf{J}(\mathbf{r})\rangle=0$, in an arbitrary potential $V(\mathbf{r})$. We may also deduce that $I'\neq 0$ in the correct ground state of a system of anyons with sharp boundary conditions and a large value of p.

Now let us consider a situation where there are two types of anyons $(n_a=2)$, with $p=2$. The reference ground state in this case consists of $N_0=N/2$ particles of each type in the lowest Landau level, filling the states out to the maximum radius $r_m=R$. In this case we find that the angular momentum M_z, in the anyon gauge, is given by $M_z=-\hbar N/4$, and the value of the coefficient γ is therefore $-\hbar/4$. For arbitrary integer n_a, with p a multiple of n_a, we find

$$\gamma=-\frac{\hbar}{2}\left[\frac{p}{n_a}-\frac{1}{p}\right] . \tag{A14}$$

It is interesting to compare the aforementioned results with the internal orbital angular momentum of a bound pair of anyons. Suppose that two identical anyons interact with a potential V, which is a function only of the distance r_{12} between the particles. We can choose as a trial wave function for the ground state of the Hamiltonian $H^{(a)}$ a function of the form

$$\Psi^{(a)}(\mathbf{r}_1,\mathbf{r}_2)=f(r_{12})e^{i\mu\phi_{12}} , \tag{A15}$$

where

$$\mathbf{r}_{12}=r_{12}(\cos\phi_{12},\sin\phi_{12})$$

and $\mu=\theta/\pi$ (mod 2). The energy of the trial wave function, for a fixed choice of f, can be written as,

$$\langle H^{(a)}\rangle=\varepsilon_0+\varepsilon_1\mu^2 , \tag{A16}$$

where

$$\varepsilon_1=\frac{\pi}{m^*}\int_0^\infty\frac{|f(r)|^2dr}{r} . \tag{A17}$$

Clearly the energy is minimized by choosing $|\mu|$ as small as possible, which means $\mu=\theta/\pi$, if $|\theta|\le\pi$. The orbital angular momentum of the pair is given by $M_z=\hbar\mu$, so that the angular momentum per particle is $\hbar\mu/2$.

In the case of $\theta=\pi/2$, a bound pair of anyons is a compound particle that obeys ordinary Bose statistics. For a suitable potential $V(r_{ij})$, which is attractive at short distances, we may expect that a pair of anyons may form a bound state, and that the effective interaction between pairs is repulsive. Then the ground state of the multi-anyon system at intermediate values of the density would presumably be equivalent to a Bose condensate of anyon pairs. The orbital angular momentum of this state would clearly be $M_z=\hbar N/4$, giving the value $\gamma=\hbar/4$ in this case. This is different from the result $\gamma=-3\hbar/4$ that we found above for the ground state of the *weakly interacting* anyon system, with $n_a=1$ and $p=2$, assuming that Laughlin's wave function is a correct starting point for the ground state.

For the case of two different species of anyon, the restriction on the trial wave function of a pair is now weakened to $\mu=\theta/\pi$ (mod 1). A choice $\mu=\theta/\pi$ (mod 2) implies that the two anyons occur in an isospin triplet state, while $\mu+1=\theta/\pi$ (mod 2) may be interpreted as an isospin singlet. For $p=2$, the lowest-energy state for an isospin singlet, has $\mu=-\frac{1}{2}$, which has angular momentum $M_z=-\hbar/2$. A Bose condensate of isospin-singlet pairs would lead to $M_z=-\hbar N/4$, or $\gamma=-\hbar/4$. It is interesting to note that this is identical to the result obtained earlier for the reference ground state of a noninteracting system of two species of anyons with $n_a=p=2$.

APPENDIX B: ABSENCE OF OPTICAL ROTATION TO SECOND ORDER IN THE IMPURITY POTENTIAL

We use here a well-known method which relates the $\mathbf{k}=0$ frequency-dependent conductivity tensor of a system of electrically charged particles with a fixed charge to mass ratio, in the presence of a collection of fixed impurities, to the longitudinal density response function of the interacting system with no impurities present, correct to second order in the impurity potential.[44]

The antisymmetric part of the two-dimensional conductivity tensor $\sigma_{ij}(\omega)$ is given by the difference $\sigma^+(\omega)-\sigma^-(\omega)$, where $\sigma^\pm(\omega)$ is the conductivity in response to an infinitesimal circularly polarized uniform electric field of the form

$$\mathbf{E}(t)=E_0e^{-i\omega t}(\hat{\mathbf{x}}\pm i\hat{\mathbf{y}}) .$$

It suffices to study the real parts of $\sigma^\pm(\omega)$, since the imaginary part can then be determined from the Kramers-Kronig relations.

We assume that the Hamiltonian in the absence of the electric field can be written in the form

$$H=H_{\text{pure}}+\int u(\mathbf{r})\rho(\mathbf{r})d^2r , \tag{B1}$$

where $u(\mathbf{r})$ is the "impurity potential" at point $\mathbf{r}$, $\rho(\mathbf{r})$ is the density of particles at point $\mathbf{r}$, and H_{pure} is the Hamiltonian of a collection of particles with mass m^*, which interact with each other via a velocity-independent potential that depends only on the separation of the particles. The particles may obey any statistics we choose.

Let us now make a Galilean transformation to an accelerating frame which is displaced from the laboratory frame by a vector

$$\mathbf{s}(t)=\text{Re}\left[-\frac{e\mathbf{E}(t)}{m^*\omega^2}\right] . \tag{B2}$$

In this frame the uniform electric field is cancelled by the "gravitational" field produced by the transformation, but the impurity potential acquires a time-dependent form

$$U(\mathbf{r},t)=u(\mathbf{r}+\mathbf{s}(t))=u(\mathbf{r})+\mathbf{s}(t)\cdot\nabla u(\mathbf{r}) . \tag{B3}$$

The spatial Fourier transform of U has a positive frequency part given by

$$\delta U(\mathbf{q},t)=\delta U(\mathbf{q})e^{-i\omega t} ,$$

where

$$\delta U(\mathbf{q})=-\frac{ieE_0}{m^*\omega^2}(q_x\pm iq_y)u(\mathbf{q}) . \tag{B4}$$

Let us assume that the system is initially in the ground state of H. The average power dissipated by this time-dependent potential $U(\mathbf{r},t)$ is given, correct to second order in u by the formula

$$\bar{P}=\frac{\omega}{2}\int\frac{d^2q}{(2\pi)^2}|\delta U(\mathbf{q})|^2\chi''_{\text{pure}}(\mathbf{q},\omega) , \tag{B5}$$

where $\chi''_{\text{pure}}(\mathbf{q},\omega)$ is the imaginary part of the density response function of the pure system at wave vector $\mathbf{q}$ and frequency ω. To obtain this formula, we have made use of the fact that, due to translational invariance, the response function $\chi''_{\rm pure}(\mathbf{q},\omega)$ is diagonal in $\mathbf{q}$. It is clear from (B4) and (B5), that $\bar{P}$ is independent of the sign of the circular polarization. Since $\bar{P}$ is related to $\sigma^{\pm}$ by

$$\bar{P}=\tfrac{1}{2}|E_0|^2\mathrm{Re}\sigma^{\pm}(\omega)\ ,$$

we find that $\sigma^{+}(\omega)=\sigma^{-}(\omega)$ to second order in u.

If we wish to calculate $\bar{P}$ correct to third order in u, the density response function on the right-hand side of (B5) must be calculated in the presence of the static potential $u(\mathbf{r})$. The response is no longer diagonal in the wave vector $\mathbf{q}$, and we can no longer argue that $\bar{P}$ is independent of the sign of the circular polarization.

APPENDIX C: MAGNETIC ENERGY OF A SINGLE DOMAIN WALL

We calculate here the magnetic energy of an infinite domain wall in the y-z plane, perpendicular to the planes of superconducting layers. Assume that the sign of the coefficient γ, introduced in Sec. III, is positive for $x<0$ and negative for $x>0$. The energy (or the free energy at finite temperatures), is written in the form

$$F=\int\frac{|\nabla\times\mathbf{A}|^2}{8\pi}d^3r+\int\frac{\rho_s}{2p^2m^*s}|\hbar\nabla\Phi-pe\,\mathbf{A}/c|^2d^3r-2\int\frac{\mathbf{I}\cdot\mathbf{A}}{sc}d^2r\ , \tag{C1}$$

where $2\mathbf{I}$ is the boundary current from the discontinuity of the magnetization in a single layer, s is the interlayer spacing, and the integral in the third term is over the area of the domain wall. The second integral, in principle, is confined to the volume of the superconductor, and only the x and y components $\mathbf{A}$ and $\nabla\Phi$ should be included since we neglect the Josephson coupling between layers. The current $2\mathbf{I}$ is in the y direction.

A solution which minimizes F, is obtained by choosing $\nabla\Phi=0$, and $\mathbf{A}\|y$, with

$$A_y=\lambda_L B_0 e^{-|x|/\lambda_L}\ , \tag{C2}$$

where

$$\lambda_L=(4\pi\rho_s e^2/m^*c^2)^{-1/2}$$

is the London penetration depth, and $B_0=4\pi I/c$ is the value of the magnetic induction for $x\to 0^-$. The energy per unit area of the interface is equal to $-\lambda_L B_0^2/4\pi$.

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