Coherent and sequential tunneling in double barriers with transverse magnetic fields

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Coherent and sequential processes in resonant tunneling through a double barrier are theoretically analyzed by means of a magnetic field parallel to the interfaces. The use of a simple model for describing scattering effects and a generalized transfer Hamiltonian method allow us to conclude that presently available experimental information is restricted to the sequential regime. We show that, for systems with barriers narrower than the ones used in the published experiments, coherent tunneling becomes dominant on the sequential mechanism.

Tunneling of carriers through double-barrier potential involves two physically different phenomena: an elastic coherent part including virtual transitions to well states and a sequential process in which the elastic tunneling through the first barrier is followed by some inelastic event in the well before the elastic tunneling through the second barrier takes place. Some efforts have been devoted to clarify the role of these two mechanisms in resonant tunneling[1–4] but we presently lack a system in which their relative importance can be controlled. Since the characteristic length of coherent tunneling is the width of the two barriers plus the well, while that of the sequential process is the width of each barrier, the main requirement for a comparative analysis is the ability to modulate the spatial localization of the states so that differences are introduced between the two processes. The best way to change the carrier localization is to apply a magnetic field B parallel to the barriers allowing the control of the wave-function extension in the direction perpendicular to the barriers, in a given sample. Little light is thrown on this matter by the available experimental information,[5–7] where the general trend is that the peak resonance appearing in the current for $B=0$ shifts to higher energies and is monotonously quenched when the field is turned on and increased. Since no particular features appear, it is not straightforward to draw any information about the tunneling mechanism. Theoretical efforts on this problem are scarce and devoted to just a part of the phenomenon.[8,9] The aim of this paper is to perform a theoretical analysis of both coherent and sequential tunneling through a double barrier with an applied bias and a magnetic field perpendicular to the current (i.e., parallel to the barriers). The theoretical difficulties are different in each process. In the sequential case, tunneling through each barrier can be computed using previously developed transfer Hamiltonian methods[10] and the problem is to include the inelastic scattering in the well. For the coherent case virtual transitions are involved so that the problem is to calculate the transmission probability by means of a generalized transfer Hamiltonian (GTH) scheme.[11]

A system like this one of a double barrier in which both coherent and sequential ways are possible behaves like a circuit with two resistors in parallel. Each resistance is proportional to the inverse of its transmission probability, T_c and T_s for the coherent and sequential channels, respectively.[4] Then the total transmission probability is given by

$$T=T_c+T_s\ , \tag{1}$$

where all three magnitudes are taken at the same energy.[4] As will be discussed below, in our problem, a discrete set of tunneling channels exists and some care must be taken when mixing coherent and sequential processes.

Let us briefly sketch the way we use the GTH scheme for computing transmission probabilities. In order to describe the time evolution of wave packets the total Hamiltonian H is separated[11] into two spatial regions, left and right, in a way such that

$$H\equiv H_L+V_L=H_R+V_R\ , \tag{2}$$

where $H\equiv H_L$ (H_R) in the left (right) side. $|L\rangle$ and $|R\rangle$ are the eigenstates for the left and right Hamiltonians with energies E_L and E_R, respectively, and the wave functions $\Phi_L(\mathbf{r})=\langle\mathbf{r}|L\rangle$ and $\Phi_R(\mathbf{r})=\langle\mathbf{r}|R\rangle$ are the wave packets of the actual problem. We take the double barrier and the applied bias in the z direction, and the magnetic field $\mathbf{B}$ along the x direction. By considering the gauge $\mathbf{A}=(0,-Bz,0)$ the left and right electronic spectra are obtained by solving the Schrödinger equation for H_L and H_R, respectively, by means of a finite-element method.[10,12] Once the electronic spectrum is obtained, one calculates the coherent transmission probability between an initial state $\Phi_L(\mathbf{r})$ and a final one $\Phi_R(\mathbf{r})$ with

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the same energies $E_L=E_R$ by means of[11]

$$T_C=|t_{LR}|^2 \quad (3)$$

with

$$t_{LR}=\langle L|V_L+V_L(E_L-H+\mathbb{P}_R V_R)^{-1}\mathbb{P}V_R|R\rangle\ , \quad (4)$$

$\mathbb{P}_R$ being the projection operator on the state $|R\rangle$ and $\mathbb{P}=1-\mathbb{P}_R$. The first term in the matrix element gives the direct transition usually described by the transfer Hamiltonian method while the second one includes all the virtual transitions producing the resonance. In our case we have checked that a good approximation for the latter is to substitute $(E_L-H+\mathbb{P}_R V_R)^{-1}$ by the Green function G_c of a central Hamiltonian H_c (i.e., only the well) corrected by a self-energy[11] $\Sigma=V_P+V_P G_c V_P$, where V_P is $H-H_c-\mathbb{P}_R V_R$.

The condition $E_L=E_R$ reduces the possibility of tunneling to a discrete set of channels.[10,13] The physical meaning is very simple. In the dispersion relation of the total problem shown in Fig. 1, there is a set of anticrossings. One state at one anticrossing has weight in the two sides of the double barrier, while other states are localized in just one side. Therefore the anticrossings constitute the tunneling channels. In the GTH formalism these anticrossings become crossings of the dispersion relations of H_L and H_R as shown in the inset of Fig. 1. On top of the energy conservation, in Eq. (4) all the potentials are just functions of z so that the transmission coefficient is only nonzero for states $|L\rangle$ and $|R\rangle$ with the same k_x and k_y, implying that crossings (and, correspondingly, anticrossings in the dispersion relation of the total Hamiltonian) are the tunneling channels. When one state localized in the well (i.e., eigenstate of H_c) is close in energy to a crossing with the same k_y, the virtual transition is very intense and coherent resonance exists. Due to the complicated structure of channels shown in Fig. 1, it can be expected that by varying the bias, centerlike states (i.e., with wave functions in the well) scan many possible channels giving rise to a lot of narrow resonances. This is shown in Fig. 2(a) for a double well n^+ GaAs-100 Å $Ga_{0.6}Al_{0.4}$-70 Å GaAs-100 Å $Ga_{0.6}Al_{0.4}As$-n^+ GaAs. There, the current density is calculated from coherent transmission probabilities by means of[10]

$$j=\frac{2e}{h}\sum_{n,m}\int dk_x dk_y T_C \times[\Theta(E_L^F-E_{Ln}(k_y))-\Theta(E_R^F-E_{Rm}(k_y))] \times\delta(E_{Ln}(k_y)-E_{Rm}(k_y))\ , \quad (5)$$

where E_L^F and E_R^F are the Fermi levels of the left and right contacts, respectively, and n, m are indices running over magnetic levels of H_L and H_R, respectively. Since in the reported I-V experimental curves[5,7] no evidence of several peaks exists, one can conclude that coherent tunneling is not the dominant mechanism in such cases. Therefore, the next step in the analysis is the study of the sequential mechanism.

Sequential tunneling can be visualized as a three-step process. First, the carrier traverses the left barrier, second it spends some time in the well losing memory of

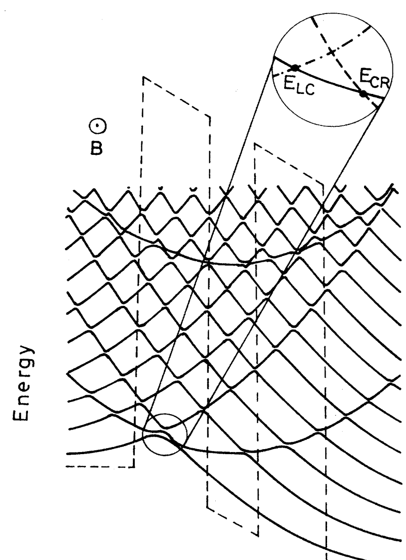

FIG. 1. Dispersion relation (solid line) of the total Hamiltonian describing a double barrier (dashed line) with a magnetic field B parallel to the barriers. The inset shows the dispersion relations of H_C (solid line), H_L (dashed-dotted line), and H_R (dashed line) for a tunneling channel (anticrossing of the total H).

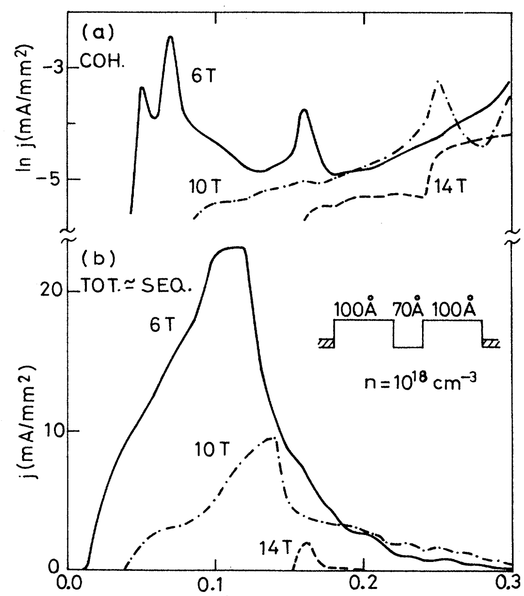

FIG. 2. (a) Logarithm of the coherent current density (in mA/mm^2) as a function of the bias (in V) for the double barrier shown in the inset for three different values of B. (b) Total (in this case almost equal to sequential) current density (in mA/mm^2) as a function of the bias (in V) for the double barrier shown in the inset for three different values of B.

its previous state, and finally it traverses the right barrier. Due to the memory-loss process, T_s becomes proportional to $(T_{LC}^{-1}+T_{CR}^{-1})^{-1}$, where T_{LC} and T_{CR} are the transmission probabilities through the left and right barriers, respectively.[4,14] The first and third steps (T_{LC} and T_{CR}) can be obtained as was done for the coherent tunneling, including only the first term in the matrix element of Eq. (4) because no resonance occurs. Since the argument for tunneling channels is applicable to these two processes, it is evident from Fig. 1 that they take place for different values of energy and k_y. Therefore, some inelastic process in the well is necessary to bring the carrier from the left channel (first step, described by the crossing of left and center states shown in the inset of Fig. 1) to the right channel (third step, described by the crossing of center and right states shown in the inset of Fig. 1). Such memory relaxation is difficult to calculate in detail so we opt for just using a weight factor to describe the process taking place in contact with a reservoir[4] and whose probability is very small when the variation of the quantum numbers of the channels is large. A good model for the weight factor can be borrowed from the use of path-integral methods to compute the impurity-induced transitions between edge states at the two surfaces of a narrow channel with a magnetic field.[15,16] Then, we take for the transmission probability of the sequential three-step process

$$T_S^{-1}=\exp[\alpha(l_m\Delta k_y)^2](T_{LC}^{-1}+T_{CR}^{-1}) , \qquad (6)$$

where l_m is the magnetic length and Δk_y is the difference in k_y between the left and right channels. The parameter α depends on the mechanism producing the memory loss and is typically of the order of the unity.[14,15] In our problem there are several channels for which Δk_y is much smaller than $1/l_m$ so that they have an exponential factor of almost 1 for any reasonable value of α and no special care need be taken for this parameter. It must be pointed out that in Eq. (6) T_{LC} and T_{CR} are transmission probabilities taken at different energies (E_{LC} and E_{CR} in Fig. 1). This implies, for the sequential transmission, an approximation which is good when $\Delta k_y \ll 1/l_m$, becoming worse in the converse limit case. This is not an objection because the latter limit has very little weight due to the exponential factor. In other words, the only processes playing an important role in sequential magnetotunneling are those corresponding to center states very close to a crossing of $E_{Ln}(k_y)$ and $E_{Rm}(k_y)$, i.e., to coherent channels. Moreover, only crossings close enough to the well give significant values of T_{LC} and T_{CR} because now there are no energy-difference denominators connected with resonances. This implies a rather featureless current, compared with the coherent case. Figure 2(b) shows the total density current calculated with an expression similar to (5) by replacing T_C by the total T computed using Eqs. (1) and (6). From the comparison with Fig. 2(a) it is easily seen that the main contribution is due to the sequential part of the tunneling while the coherent one is negligible in this case. The behavior of $j(V)$ for different values of the magnetic field is qualitatively that of the experiment so that one can conclude that sequential tunneling is that which is being experimentally observed.

From the results discussed above a question clearly arises. How can the relative intensity between sequential and coherent mechanisms be changed? An obvious answer is to vary the barrier widths. Figure 3 shows the total current density as well as that computed including only either the sequential or the coherent mechanism for a system n^+ GaAs-20 Å $Ga_{0.6}Al_{0.4}$-70 Å GaAs-20 Å $Ga_{0.6}Al_{0.4}As$-n^+ GaAs with $B=6$ T. Now the double barrier is so narrow that all the crossings between $E_{Ln}(k_y)$ and $E_{Rm}(k_y)$ take place close to the well, implying rather intense virtual transitions to well states. Then, the coherent part is strongly enhanced with respect to the sequential one and the two become comparable. The rich structure of the coherent part then becomes clear in the total current density. The comparison of Figs. 2 and 3 clarifies even more the relative importance of the two parts of the tunneling. We do not know of any experimental information comparing two samples with the same wells and different barriers, but we hope that the theoretical prediction presented here will soon be experimentally confirmed, throwing much light on the two tunneling mechanisms.

In summary, we have used a magnetic field B perpendicular to the current for studying theoretically the coherent and sequential contributions to resonant tunneling through a double barrer. The coherent part is described by a generalized transfer Hamiltonian method including all the virtual processes. For the sequential mechanism we propose a simple model to treat scattering

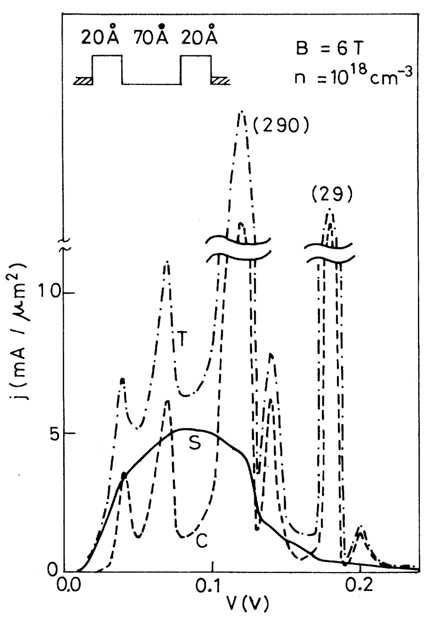

FIG. 3. Total (dashed-dotted line), sequential (solid line), and coherent (dashed line) current density (in mA/μm^2) as a function of the bias (in V) for the double barrier shown in the inset with $B=6$ T. Numbers in parentheses give the intensity of the two peaks which are out of scale.

effects. In the case of the samples where experimental information has been reported, sequential tunneling controls the process. We present a case where coherent processes should dominate so that a rich structure should be observed in the current-bias curves.

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