

Two-dimensional electron–acoustic-phonon interaction at high magnetic fields: Thermal conductance in GaAs/Al_xGa_{1-x}As heterostructures

S. Tamura and H. Kitagawa

Department of Engineering Science, Hokkaido University, Sapporo 060, Japan

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Magneto-oscillations of thermal conductance observed recently by Eisenstein, Gossard, and Narayanamurti in GaAs/Al_xGa_{1-x}As heterostructures at low temperatures are studied theoretically. We calculate the thermal-phonon lifetime limited by the scattering with Landau-quantized electrons at high magnetic fields. The calculation includes (i) both piezoelectric and deformation-potential couplings as the sources of electron-phonon interaction, (ii) Landau-level broadening with a Gaussian profile function, and (iii) magnetic-field-dependent screening effects. The results are compared favorably with the experimental data. In particular, the screening effects are crucial to explain both the magnetic field and temperature dependences of the observed thermal conductance.

Two-dimensional (2D) electrons formed at metal-oxide-semiconductor interfaces (e.g., inversion layers) and GaAs/Al_xGa_{1-x}As heterojunctions reveal a number of interesting physical properties. In particular, an external magnetic field applied perpendicular to the interfaces yields pronounced oscillatory behaviors in various quantities related to electrical, magnetic, and thermal properties.¹ In recent years 2D electron–acoustic-phonon interaction in these systems has been studied extensively by different techniques.^{2–6} Among these techniques, we are especially interested in the observation by Eisenstein, Gossard, and Narayanamurti of magneto-oscillations in the low-temperature thermal conductance of GaAs/Al_xGa_{1-x}As multilayers which contain 2D electrons.⁴ Because pure electronic contribution to the thermal conductance in these systems is estimated to be negligibly small, the oscillations are believed to arise from a modulation of thermal-phonon lifetime via coupling to the Landau-quantized electrons confined in the quantum wells of GaAs. In this regard, their measurements are expected to provide important information on the 2D density of states (DOS) of electrons as well as 2D electron-phonon interaction with magnetic-field-dependent screening effects in GaAs/Al_xGa_{1-x}As heterostructures.

In the present work we shall theoretically study both the magnetic field and temperature dependences of the thermal conductance observed experimentally. This will be carried out by calculating thermal-phonon lifetimes, limited by the interaction with 2D electrons under the strong magnetic fields. The calculation incorporates (i) both piezoelectric and deformation-potential couplings as the sources of electron-phonon interaction, (ii) Landau-level broadening with a Gaussian profile function, and (iii) magnetic-field-dependent screening effects in the long-wavelength approximation.

Now we shall consider the thermal conductance in a system with GaAs/Al_xGa_{1-x}As multilayers deposited on a GaAs substrate. Although their direct contribution is very small (typically 1 part in 10⁵ of the phonon contribution at 0.5 K,⁴ the 2D electrons in the heterolayers can influence the heat conduction through the electron-

phonon interaction. Let K and δK be the lattice thermal conductance of the system and its modulation due to the electron-phonon coupling, respectively. Then, we have

$$\frac{\delta(\Delta T)}{\Delta T} = -\frac{\delta K}{K}, \quad (1)$$

where $\delta(\Delta T)$ and ΔT are the temperature differences between thermometers induced by the electron-phonon coupling in the heterolayers and other scattering mechanisms present in the whole system, respectively. The experimental result shows that the magnitude of Eq. (1) is of the order of 10⁻² or less.⁴

The expression for the thermal conductance δK per unit volume is derived from the argument based on the Boltzmann equation as⁷

$$\delta K = -\frac{1}{V} \sum_{\lambda} (\bar{\tau}_{\lambda}^{e-ph})^{-1} \tau_{\lambda}^2 C_{ph} v_{\lambda}^2 \cos^2 \psi_{\lambda}, \quad (2)$$

where V is the normalization volume, $\lambda \equiv (\mathbf{q}, j)$ represents both wave vector \mathbf{q} and mode j of phonons, $\bar{\tau}^{e-ph}$ and τ are the lifetimes of phonons limited by the electron-phonon interaction and scattering mechanisms other than that, v is the sound velocity, and ψ is the angle between the temperature gradient and sound-velocity vector. In this equation the heat capacity C_{ph} per normal mode of phonons is defined by

$$C_{ph} = \frac{(\hbar\omega)^2}{k_B T^2} \frac{\exp(\hbar\omega/k_B T)}{[\exp(\hbar\omega/k_B T) - 1]^2}. \quad (3)$$

The thermal conductivity of pure dielectric solids at low temperatures is determined by boundary scattering. This leads to⁷

$$K = \frac{2\pi^2}{15} k_B \Lambda \langle v^{-2} \rangle \left[\frac{k_B T}{\hbar} \right]^3, \quad (4)$$

where Λ is the phonon mean free path due to the boundary scattering [we assume that Λ is the same for both the longitudinal (L) and transverse (T) phonons, i.e., $\Lambda = v_{\lambda} \tau_{\lambda}$] and $\langle v^{-2} \rangle \equiv \sum_j v_j^{-2} / 3 = (v_L^{-2} + 2v_T^{-2}) / 3$ in

the isotropic approximation we employ. The measured thermal conductivity in GaAs/Al_xGa_{1-x}As systems usually exhibits well the predicted T^3 dependence at temperatures below 3 K.^{8,9} According to this formula the experimental thermal conductance (in the sample 2 of Ref. 4) gives $\Lambda=1.2$ mm, and $\tau_L=2.3\times 10^{-7}$ s and $\tau_T=4.0\times 10^{-7}$ s for L and T phonons, respectively. Thus, our main subject is to calculate the phonon lifetime τ^{e-ph} against scattering by 2D electrons. Here it should be remarked that the relative magnitude of $(\bar{\tau}^{e-ph})^{-1}$ to τ^{-1} is proportional to $\delta d/d$, where d and δd are the thicknesses of the sample and heterolayers, respectively. Hence, it is convenient to introduce τ^{e-ph} defined by

$$(\bar{\tau}^{e-ph})^{-1} = \frac{\delta d}{d} (\tau^{e-ph})^{-1}, \quad (5)$$

where $(\tau^{e-ph})^{-1}$ represents the scattering rate of phonons by 2D electrons in a unit period of the multilayer GaAs/Al_xGa_{1-x}As system.

In a magnetic field B applied normal to the interfaces of heterolayers (along the z axis), the transverse 2D motion (in the x - y plane) of electrons is quantized and the energy spectrum is split into discrete Landau levels with energies

$$E_{n,\sigma} = \hbar\omega_c(n + \frac{1}{2}) + g^* \sigma \mu_B B / 2 + E_0, \quad (6)$$

where $\omega_c = eB/m^*$ is the cyclotron frequency (m^* is the effective mass), n is the Landau index, g^* is the effective g factor, $\sigma (= \pm 1)$ denotes the orientation of electron spin, μ_B is the Bohr magneton, and E_0 is the energy of the ground subband associated with the motion along the z direction in the GaAs quantum well. The energy of the first-excited state, E_1 , of the confining potential is typically 40 meV above E_0 (for the quantum-well thickness $d_w = 150$ Å), and for a 2D electron concentration of about 5×10^{11} cm⁻² the Fermi energy ($E_F \approx 20$ meV) locates well below E_1 .¹

Using the Landau gauge with vector potential $\mathbf{A} = (0, Bx, 0)$, the wave function associated with the energy (6) is

$$\Psi_\kappa = u_n(x - X) e^{iky} \phi_0(z) / \sqrt{L_y}, \quad (7)$$

where $\kappa \equiv (n, \sigma, k)$ is a set of quantum numbers of electrons, u is a wave function of a simple harmonic oscillator centered at the coordinate $X = -l^2 k$ with magnetic length $l = (\hbar/eB)^{1/2}$, ϕ_0 is the subband wave function, and L_y is the normalization length in the y direction. [The energy (6) is degenerate with respect to the wave number k .] The spin part of the wave function is suppressed in Eq. (7). Here we note that for electrons in the lowest subband the wave functions are well localized in each quantum well. However, the situation is somewhat different for phonons because they are extended in the whole system.

In the systems consisting of alternating GaAs and Al_xGa_{1-x}As multilayers, or superlattices, mini-Brillouin-zones are formed due to the introduction of artificial periodicity much longer than lattice spacings in the direction normal to the interfaces (i.e., zone-folding effects).¹⁰ The phonon dispersion relations are modified

by the appearance of frequency gaps at the center and edges of minizones. However, in the samples with periodicity of about 500 Å used in the experiment, the lowest-frequency gap (corresponding to the first-order phonon Bragg reflection) appears at a frequency around 50 GHz for normal propagation. On the other hand, in the experimental temperature range (0.1–1 K) the dominant frequencies of thermal phonons are 6–60 GHz. Hence, the modification of phonons by the superlattice periodicity should have only small effects on the electron-phonon interaction in the present situation. In addition, the elastic properties of Al_xGa_{1-x}As alloys are close to those of GaAs. Accordingly, we can approximate thermal phonons in this system by bulk acoustic phonons in GaAs.

Now taking these considerations into account, the scattering rate of phonons by the interaction with 2D electrons at low temperatures becomes

$$(\tau_\lambda^{e-ph})^{-1} = 2\pi\omega_\lambda \sum_{\kappa, \kappa'} |M_{\kappa\kappa'}^\lambda|^2 D_\kappa(E_F) D_{\kappa'}(E_F - \hbar\omega_\lambda), \quad (8)$$

where M is the matrix element for the electron-phonon interaction and D_κ denotes the DOS of electrons in the states specified by κ . First, we shall consider the matrix element M . Explicitly, it takes the form

$$M_{\kappa\kappa'}^\lambda = \left[\frac{\hbar}{2\rho\omega_\lambda V} \right]^{1/2} \Xi_\lambda \langle \kappa | e^{iq \cdot r} | \kappa' \rangle, \quad (9)$$

where ρ is the mass density and Ξ stands for the momentum representation of the sum of the deformation and piezoelectric potentials, i.e.,

$$\Xi_\lambda = iC\mathbf{q} \cdot \mathbf{e}_\lambda + \frac{2h_{14}e}{q^2} (q_1 q_2 e_{3,\lambda} + q_2 q_3 e_{1,\lambda} + q_3 q_1 e_{2,\lambda}), \quad (10)$$

In this equation, C ($= 7$ eV) is the deformation potential, \mathbf{e} is the phonon polarization vector, and h_{14} ($= 1.45 \times 10^9$ V/m) is the piezoelectric constant.¹¹ At temperatures below 1 K, the piezoelectric coupling dominates the deformation-potential coupling as we shall see below.

The wave function associated with the state $|\kappa\rangle$ in Eq. (9) is given by Eq. (7). Thus the evaluation of the matrix element yields, for the intra-Landau-level transition ($n \rightarrow n$) (besides an unimportant phase factor),¹

$$\langle \kappa | e^{iq \cdot r} | \kappa' \rangle = \exp(-q_\parallel^2 l^2 / 4) L_n(q_\parallel^2 l^2 / 2) \times F(q_z) \delta_{\sigma, \sigma'} \delta_{k, k' + q_y}, \quad (11)$$

where $q_\parallel^2 = q_x^2 + q_y^2$, L_n is the Laguerre polynomial, and $F(q_z)$ is the form factor defined by

$$F(q_z) = \int |\phi_0(z)|^2 \exp(iq_z z) dz, \quad (12)$$

which acts to cut off the electron-phonon interaction for $q_z d_w > 1$, i.e., for the z projection of thermal-phonon wavelength much shorter than the z extent of the lowest subband wave function. This implies that for $d_w \approx 100$ Å the form factor becomes important for phonon frequencies higher than 50 GHz or the corresponding tempera-

tures $T > 0.8$ K, while at low temperatures below 0.8 K we can set $F = 1$.

It is important to note, here, that in the experimental temperature range 0.1–1 K no phonons which can contribute to inter-Landau-level transitions are excited since the typical energies of phonons and inter-Landau-levels are 0.3 meV (at $T = 1$ K) and 2 meV (at $B = 1$ T), respectively. Hence, only intra-Landau-level transitions are responsible for the phonon scattering by 2D electrons. These transitions are indeed possible owing to the broadening of Landau levels in the presence of scatters; otherwise the sharp Landau levels prohibit the possibility of such transition.

The characteristics of Landau-level broadening directly reflect in the DOS and through Eq. (8) they play substantial roles in determining the scattering rate $(\tau^{e-ph})^{-1}$. Thus, the level broadening manifests itself in various observed effects. More specifically, in the presence of the scatterers of electrons the DOS of 2D electrons $\bar{D} = \sum_{\kappa} D_{\kappa}$ is not simply a series of well-separated δ functions. The finite width of a Landau level is determined by the strength and the range of the scattering potential in the system. Many different approximations have so far been used to calculate the physically plausible level broadening.^{12–16} Current theoretical^{1,13} and experimental^{17,18} investigations suggest that in the presence of strong magnetic fields the DOS is a sum of Gaussian functions with a level width proportional to $B^{1/2}$. Thus, we employ

$$D_{\kappa}(E) = \frac{1}{(2\pi)^{1/2}\Gamma} \exp\left[-\frac{(E - E_{n,\sigma})^2}{2\Gamma^2}\right], \quad (13)$$

where $\Gamma = \gamma B^{1/2}$, and γ is a coefficient independent of magnetic field. The degeneracy of energy (6) with respect to the wave number k of electrons assigns an orbital degeneracy $(2\pi l^2)^{-1}$ to each Landau spin sublevel indexed by (n, σ) .

Now, the thermal conductance δK due to 2D electron-phonon interaction in the heterostructure is basically calculated from Eqs. (8)–(13). However, we further have to consider the screening effects of electron-phonon interaction because the wavelength of the relevant phonons is very long. An important aspect of the screening is the fact that the screening length is determined by the DOS at the Fermi level and hence it oscillates with magnetic field.^{14,15,19} More explicitly, the screened potential is given by $\Xi_{\lambda}/\epsilon(q_{\parallel})$, where the dielectric function $\epsilon(q_{\parallel})$ takes the form

$$\epsilon(q_{\parallel}) = 1 + \frac{e^2}{2\epsilon_0 \bar{\kappa} q_{\parallel}} \Pi(q_{\parallel}) f(q_{\parallel}). \quad (14)$$

In this equation, $\bar{\kappa} = 12.9$ is the static dielectric constant, Π is the static polarizability, and $f(q_{\parallel})$ is defined by

$$f(q_{\parallel}) = \int dz \int dz' \exp(-q_{\parallel}|z - z'|) \times |\phi_0(z)|^2 |\phi_0(z')|^2. \quad (15)$$

In order to take account of the magnetic field dependence of the screening semiquantitatively, we assume the long-wavelength approximation for the polarizability, i.e.,

$\Pi(q_{\parallel} = 0) = \bar{D}(E_F)$.^{14,19} Thus the screening is very effective if a Landau level is nearly half-filled, and it breaks down if the Landau level is either completely filled or empty. We shall find that this property of the screening of electron-phonon interaction is crucial in understanding experimental findings.

Figure 1 displays the calculated magnetic field dependence of $\delta(\Delta T)/\Delta T$ at 0.35 K. The numerical calculation was performed with the same parameters given by Eisenstein *et al.* for their sample 2, e.g., $d = 48 \mu\text{m}$, $\delta d = 2.7 \mu\text{m}$, the number of periods is 50 and 2D carrier density $N_s = 5.3 \times 10^{11} \text{cm}^{-2}$.^{4,20} In our calculation the unknown prefactor γ of the Landau-level width was varied as a parameter and spin gaps were neglected. The oscillations of thermal conductance are well reproduced. Local maxima (minima) of $\delta(\Delta T)/\Delta T$ occur at the magnetic field for which the filling factor $\eta = 2\pi l^2 N_s$ becomes an odd (even) integer value, or when the Fermi level E_F lies in a spin gap within a given Landau level (E_F lies in a gap between neighboring Landau levels). A large level width Γ of the order of 1 meV yields a nonvanishing DOS at energies between Landau levels, producing nonoscillatory background for $\delta(\Delta T)/\Delta T$.

The trace of Fig. 1(a) is a result obtained without screening effects, whereas the traces of Fig. 1(b) for three different values of the coefficient γ of the level width include screening effects. The profiles of the oscillation in the latter are similar to the experimental result, but the former profile exhibits a rather rapid decrease at high

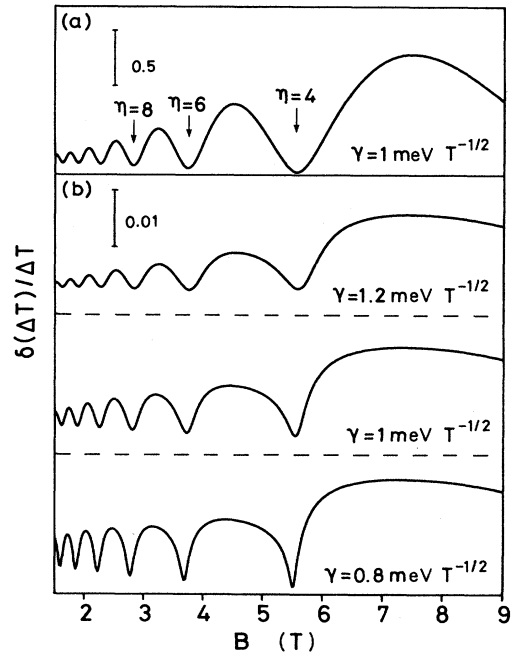


FIG. 1. Magnetic field dependence of temperature difference caused by electron-phonon interaction, (a) without screening effect and (b) with screening effect. Arrows indicate the magnetic fields at which the filling factor η takes integer values. Assumed Landau-level widths γ are also indicated. Note the difference of vertical scales between (a) and (b).

magnetic fields ($B > 7.5$ T), which is inconsistent with the experiment. More seriously, comparing with the experimental data, the amplitude of oscillation in Fig.1(a) is about 35 times larger than the experiment [from experimental data the amplitude of oscillation of $\delta(\Delta T)/\Delta T$ is 0.015 at $B = 4.5$ T],⁴ while in the results including the screening effects the magnitudes are in accord with the experiment within a factor of 2. The importance of the screening effects will also be recognized by plotting the modulated temperature difference $\delta(\Delta T)$ as a function of the ambient temperature of the sample.

Figure 2 displays the temperature dependence of the oscillatory amplitude, $\delta(\Delta T)_{\text{osc}}$, caused by the electron-phonon interaction, which can be directly compared with the experiment. The plotted $\delta(\Delta T)_{\text{osc}}$ corresponds to the difference between the local maximum of the temperature difference at $B = 4.5$ T and its neighboring local minima, or more accurately

$$\delta(\Delta T)_{\text{osc}} = \delta(\Delta T)_{B=4.5 \text{ T}} - [\delta(\Delta T)_{B=3.6 \text{ T}} + \delta(\Delta T)_{B=5.4 \text{ T}}] / 2 .$$

Here, we have chosen $\gamma = 1 \text{ meV T}^{-1/2}$ because the sample 2 used in the thermal-conductance measurement is practically identical to the sample A used in the magnetization measurement,¹⁸ and the latter result was fitted favorably with the same γ . The solid line at the top is the result obtained without screening, which exhibits the temperature dependence close to a T^{-4} power law. This temperature dependence is much stronger than T^{-3} be-

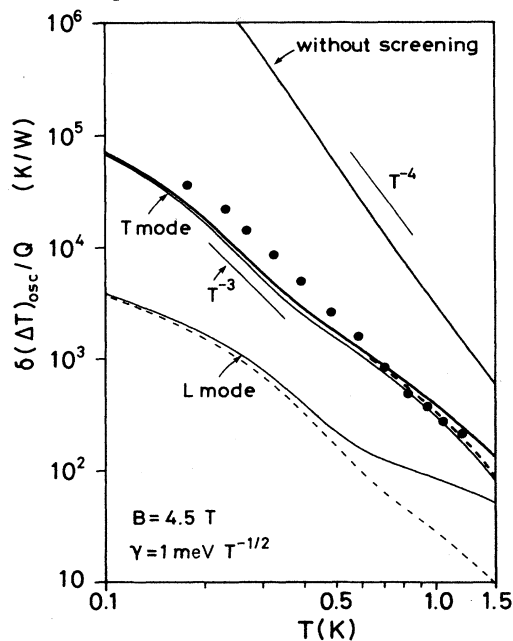


FIG. 2. Calculated temperature dependence of oscillatory amplitude $\delta(\Delta T)_{\text{osc}}$ at $B = 4.5$ T (bold line) together with experimental data (dots) from Ref. 4. (Q is the applied heat flux.) The Landau-level width $\Gamma = (1 \text{ meV/T}^{1/2})B^{1/2} = 2.12 \text{ meV}$ is assumed. The contributions of T and L modes are indicated by thin lines. Dashed lines are the results obtained without deformation-potential coupling. The temperature dependence without screening effect is also shown.

havior, which the experimental $\delta(\Delta T)_{\text{osc}}$ approximately obeys from 0.2 to 0.8 K. [The extra T^{-1} contribution multiplied by T^{-3} (due to the temperature dependence of the specific heat C_{ph}^{-1}) comes mainly from the Gaussian factor of Eq. (11).] More importantly, this calculation overestimates the strength of $\delta(\Delta T)_{\text{osc}}$ by more than 1 order of magnitude at temperatures below 1 K. In these respects, it is interesting to see whether the screening effects lead to a correct temperature dependence of $\delta(\Delta T)_{\text{osc}}$ in addition to the overall magnitude.

The bold solid line in Fig. 2 shows the result including screening effects, which describes semiquantitatively the expected temperature dependence as well as the magnitude. The small oscillation of $\delta(\Delta T)_{\text{osc}}$ versus temperature arises from the Laguerre polynomial in Eq. (11). The corresponding oscillation is also observed experimentally, though it is much weaker than the theory. In obtaining Fig. 2, we used the expressions of the form factors $F(q_z)$ and $f(q_{\parallel})$ calculated from the simplest form $\phi_0(z) = (2/d_w)^{1/2} \sin(\pi z/d_w)$ for the subband wave function with $d_w = 140 \text{ \AA}$. (Comparing with the results with $F = f = 1$, we recognize that the effects of the form factors F and f become noticeable at temperatures higher than 0.5 K [F acts to decrease $\delta(\Delta T)$, whereas f acts to increase it].)

We have also plotted in Fig. 2 the contributions of both T (two branches are degenerate) and L modes to $\delta(\Delta T)_{\text{osc}}$. The predominance of T modes relative to L mode at low temperatures is attributed to their pronounced DOS. At high temperatures ($T > 1$ K), however, the L-mode contribution becomes important due to the increase of deformation-potential coupling. The dashed lines in Fig. 2 indicate $\delta(\Delta T)_{\text{osc}}$ obtained when the deformation-potential coupling is turned off. Note that, in the isotropic approximation, only L phonons interact with electrons via the deformation potential.

To summarize, we have calculated the thermal conductance, or, more accurately, the temperature difference in GaAs/Al_xGa_{1-x}As heterostructures caused by the scattering of thermal phonons by 2D electrons at high magnetic fields. Screening effects are essential to reproduce quantitatively both the magnetic field and temperature dependences of thermal conductance below 1 K. Our calculation is consistent with the Gaussian-broadened Landau levels with level width $\Gamma = (1 \text{ meV T}^{-1/2})B^{1/2}$. This is in accord with the result of magnetization measurement made with the sample practically identical to that used in the thermal conductance measurement.

More quantitative comparison of theory with experiment necessitates the inclusion of additional effects and/or an elaboration of the approximations we employed. One of them is related to spin gaps we neglected in the present calculation. Experimentally, a small dip in $\delta(\Delta T)$ is observed at $B = 7.3$ T or the corresponding filling factor $\eta = 3$. This is attributed to the decrease of the DOS between spin doublets in the corresponding Landau level ($n = 1$). Preliminary study suggests that in order to reproduce this small dip a large enhancement of the g factor, e.g., $g^* > 5$, is needed. It is now well established that g^* oscillates with magnetic field and its mag-

nitude should be determined self-consistently with the filling factors of spin sublevels.^{1,21,22}

In the present calculation we assumed the same energy width Γ for different Landau levels, though the level width generally depends on the Landau index n . The recent studies based on the self-consistent scheme show that the lower Landau levels are slightly broader than high levels.^{14,15} Such n -dependent level broadening should also affect the screening effects through the DOS at the Fermi level. Thus, the inclusion of the n dependence of level width will modify to some extent the relative magnitude of oscillatory amplitudes $\delta(\Delta T)_{\text{osc}}$ at different integer-filling factors.

The elastic anisotropy of constituent layers of the heterostructure is another important factor to be taken into account. The significant anisotropy of phonon energy flux called "ballistic phonon focusing" exists in elastically anisotropic media and it has been reported that this focusing effect has some influence on the magnitude of thermal conductance in the boundary-scattering re-

gime.²³ All these additional effects are now under consideration and will be published elsewhere.

Finally, in our numerical calculation we have used the values of the deformation-potential and piezoelectric coupling constants measured for bulk GaAs. An enhancement of these values, especially of the deformation potential up to two times in GaAs/ $\text{Al}_x\text{Ga}_{1-x}\text{As}$ heterostructures, has recently been claimed to explain electron mobility at low temperatures.^{24,25} Further experimental and theoretical studies on the thermal conductance in these systems will also shed a light on this problem.

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²⁰The carrier density $N_s = 5.1 \times 10^{11} \text{ cm}^{-2}$ was given by Eisenstein *et al.* (Ref. 4) for their sample 2, but we find $N_s = 5.3 \times 10^{11} \text{ cm}^{-2}$ fits better the location of the cusplike minima of the temperature difference $\delta(\Delta T)$ they measured.

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