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Limiting-path model of the critical current in a textured $YBa_2Cu_3O_{7-\delta}$ film

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A limiting-path model for the determination of the critical current in a polycrystalline superconductor is presented which allows for the exact calculation of the critical current given the individual intergrain couplings. The model is used to determine the a-axis texturing dependence of the critical current density in a c-axis–oriented polycrystalline $YBa_2Cu_3O_{7-\delta}$ film. The critical current density in untextured material is suppressed by a factor of $\simeq \frac{1}{30}$ as compared to the single-crystal value and can be raised considerably only by strong a-axis texturing (to within 5°). A nontrivial length dependence of the critical current density is found for long polycrystalline wires.

The problem of critical current flow in polycrystalline samples has obtained a new dimension with the advent of the oxide superconductors. In a recent Letter, Dimos *et al.*[1] reported on measurements of critical current densities across grain boundaries in (c axis oriented) bicrystals of $YBa_2Cu_3O_{7-\delta}$. Their data show a strong dependence of the critical current density on the misorientation angle in the basal plane a axis of the bicrystal, with a reduction in current density reaching a factor of $\frac{1}{50}$. These results indicate that the critical current in polycrystalline films depends strongly on texturing.

In this Rapid Communication we present a model for the determination of the critical current in a polycrystalline superconductor. Using the experimental data of Dimos *et al.*, we calculate the critical current and its texturing dependence in a c-axis–oriented thin film of $YBa_2Cu_3O_{7-\delta}$. In particular, we find that the current flow in an untextured film (i.e., with randomly directed a axis) is suppressed by a factor of $\frac{1}{30}$.

In a polycrystalline high-T_c superconductor the randomly oriented grains introduce random coupling strengths between the grains whose weak link behavior suppresses the intergrain current flow. Within a classical model we can determine the critical current of the sample by finding *that* interface which minimizes the sum of intergrain critical currents across the interface and which separates the current feeding contacts. This simple idea reduces the calculation of the critical current in a polycrystalline superconductor to a limiting interface problem. This class of interface problems has been studied in connection with several random systems, in particular domain-wall roughening in Ising models[2] and growth of directed polymers in random media.[3]

The problem of calculating the critical current of a granular superconductor starting from its individual intergrain couplings has been addressed previously: Huse and Guyer[4] treated the problem of superfluidity in a porous medium using ideas presented in this paper but had to introduce various approximations. Octavio *et al.*,[5] using the $Y \leftrightarrow \nabla$ transformation, were able to calculate the critical current exactly. This method, while being very efficient for large lattices, is a formal decimation technique, which yields no information on the actual current flow and on the interface across which the breakdown of superconductivity occurs. The present paper bridges between the two approaches by providing the exact calculation of the limiting interface.

In modeling the thin film we consider a single layer of grains and formulate the problem of current limitation as a linear optimization problem which we solve exactly by the simplex method. The result of this calculation will show how the problem can be reformulated as that of finding the best path in a two-dimensional random medium. We use a generalized version[6] of the transfer-matrix method of Derrida and Vannimenus,[7] and Kardar and Zhang,[3] which includes also the returning paths and thus provides an exact and numerically efficient solution of the problem. The algorithm is used to study the dependence of the critical current density on sample size and on texturing.

Consider a single layer of grains arranged on a $m \times n$ square lattice (lattice constant of 1); see Fig. 1. With their c axis orthogonal to the film, the individual grains are characterized by the direction ϕ of their a axis with respect to an arbitrary but fixed direction. The angles ϕ are chosen randomly between 0° and 90° with a Gaussian distribution $g_{\phi_0}(\phi)$ of width ϕ_0 $[g_{\phi_0}(\phi) \sim \exp(-\phi^2/\phi_0^2)]$. We assume a constant intragrain critical current density (normalized to 1) throughout the sample such that the intergrain critical current $i_c(\theta)$ is given solely by the misorientation angle $\theta = |\phi - \phi'|$ between the a axes of two neighboring grains.[1] The junctions are numbered consecutively as shown in Fig. 1, and periodic boundary conditions are applied along the current flow direction. The critical current I_c is found by solving the following linear problem for the current flow i_l, $l = 1, \ldots, 2mn$:

$$I_c = \max_{\{i_l\}} \sum_{l=1}^{n} i_{2l-1} , \tag{1}$$

$$-i_{cl} \le i_l \le i_{cl}, \quad l = 1, \ldots, 2mn , \tag{2}$$

$$0 = i_l + i_{l+1} - i_{l+2} - i_{2n+l+1}, \quad l = \nu + 2mn\delta_{\nu,0} , \tag{3}$$

where $\nu = [2(\mu+1)](\mathrm{mod}\,2mn)$ and $\mu = 0, \ldots, mn-1$.

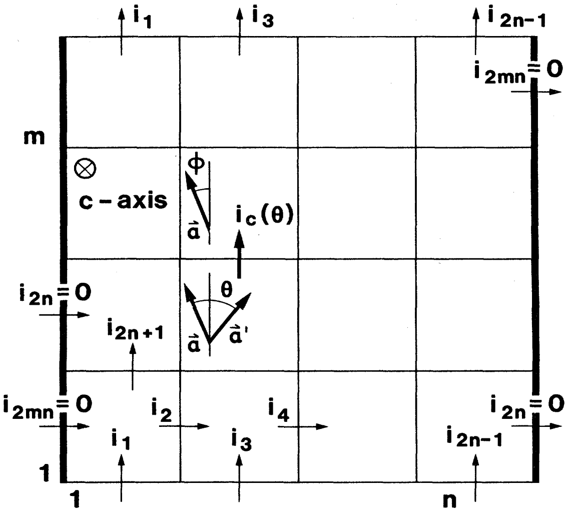

FIG. 1. $m\times n$ lattice modeling a single-layer c-axis–oriented polycrystalline film. The directions ϕ of the a axis are chosen randomly with a Gaussian distribution of width ϕ_0. The intergrain critical currents i_c depend on the misorientation θ between neighboring grains. Periodic boundary conditions are applied in both directions with a cut $i_{c,2nk}=0$ $(k=1,\ldots,m)$ at the left and right boundaries, forcing the global current flow into the vertical direction.

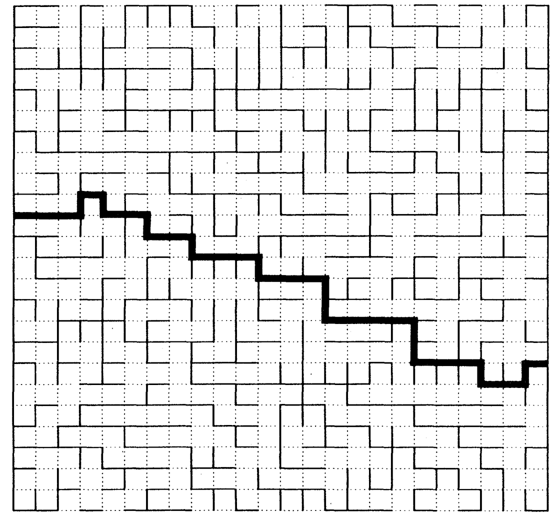

FIG. 2. Critical state of a 24×24 lattice determined with the simplex method. The directions ϕ of the a axis are distributed uniformly, $\phi_0=\infty$. Dotted lines are undercritical junctions, continuous lines denote critical junctions, and the thick line is the critical path γ_c. The critical current is determined by summing up the couplings i_{cl} along the critical path, $I_c=0.813$. Whereas the critical path is uniquely determined, other critical junctions can be made undercritical by a small rearrangement of the current flow. For a textured sample the number of critical vertical junctions decreases, indicating an increasingly laminar flow.

Equations (1)–(3) constitute a linear optimization problem which can be solved exactly using the simplex method.[8] In Fig. 2 we show the result of such a calculation ($m=n=24$) for a uniform distribution of angles ϕ and the following parametrization of $i_c(\theta)$ based on experimental data:[1]

$$i_c(\theta)=i_{c\infty}+i_{c0}\left[\frac{1}{1+\theta/\theta_0}+\frac{1}{1+(90^\circ-\theta)/\theta_0}\right], \qquad (4)$$

with the parameters $i_{c\infty}=-0.08$, $i_{c0}=0.6$, and $\theta_0=4.0^\circ$. The main result is the following. As the critical current is reached there is one and only one path γ_c crossing the sample in the direction orthogonal to the current flow with all junctions lying on this path carrying the critical current $|i_l|=i_{cl}$, $l\in\gamma_c$. This path minimizes the sum $\sum_{l\in\gamma} i_{cl}$ taken over all possible paths γ crossing the sample from left to right and therefore $I_c=\sum_{l\in\gamma_c} i_{cl}$. We call γ_c the *critical* or *limiting path*. The critical path always exists, since otherwise we can find at least one percolating chain of grains along the direction of current flow on which all the junctions are undercritical. In this case the current through the sample can still be further increased, contradicting the assumption that the critical current has been reached.

The critical path is in fact a long (length l_c) inhomogeneous Josephson junction crossing the whole sample. As the current is driven beyond the critical value a finite voltage will appear across the critical path. The characteristic length describing the junction on a microscopic scale is the transverse penetration depth[9] $\lambda_t=(c\Phi_0 l_c d/16\pi^2\lambda I_c)^{1/2}$, where $\Phi_0=hc/2e$, λ is the grain penetration depth, and d is the film thickness. For our classical model to be applicable we have to exclude the existence of *intergrain* vortices which would have to be described by a model taking into account the phase of the coherent superconducting state, e.g., by the XY model. For strongly coupled grains, i.e., $\lambda_t<L$, where L denotes the grain size, no intergrain vortices can exist.[10] In the oxide superconductors the intergrain current densities are of the order of 10^4 A cm^{-2} and the grain penetration depth is $\lambda\simeq 0.2$ μm. The resulting transverse penetration depth if $\lambda_t\simeq 2.5$ μm, which is smaller than the typical grain size $L\simeq 10$ μm and we conclude that our classical model can be applied.[11]

The reformulation of the maximal current problem (1)–(3) as that of finding the limiting path is the following one: Find the path γ_c from the left to the right edge which minimizes the functional $\sum_{l\in\gamma} i_{cl}$. This best path problem can be solved, within the restricted set of *nonreturning* paths, by the transfer matrix method used in Refs. 3 and 7. We have generalized this method to include returning paths, irrespective of how complicated their structure is. The basic idea is to include both forward *and* backward transfer steps.[6] The importance of returning paths depends on the details of the intergrain coupling distribution $w_{\phi_0}(i)$ which can easily be obtained from $g_{\phi_0}(\phi)$ and Eq. (4). Whereas the omission of returning paths changes the results for the critical current density $\langle J_c\rangle$ by less than 0.1% for a distribution $w_{\phi_0}(i)$ based on Eq. (4), their inclusion turns out to be important in the percolation problem[4] with only two values for i_{cl}, one of which is zero.[6]

We first address the problem of the existence of a welldefined critical current density characterizing the film. We expect the critical current I_c of a film to be proportional to the length l_c of the critical path. In Fig. 3 we show the scaling behavior of $\langle l_c \rangle$ with sample size n for square-shaped samples, $m=n$. The critical path length depends linearly[12] on the sample dimensions for $n \gtrsim 50$, $\langle l_c \rangle \simeq [1+\kappa(\phi_0)]n$, with the coefficient $\kappa(\phi_0)$ determined by the coupling distribution. As a consequence the critical current density $\langle J_c \rangle = \langle I_c/n \rangle$ saturates and thus is a meaningful quantity characterizing the film. In addition, we have found that $\langle J_c \rangle$ is independent of the sample form $q=m/n$ in the limit $m=qn \to \infty$.[13] This result is due to the rapid narrowing of the distribution function for J_c with increasing size n (its width scales as $n^{-0.75\pm 0.02}$) showing that self-averaging takes place in a single large sample.

An interesting length dependence of the critical current is found for a long wire, $m \to \infty$, $n=$const. The critical current density *always drops to the minimal coupling* i_a since the probability to find a straight critical path ($l_c = n$) with a current density in the interval $[i_a, i_a+\delta i]$ approaches unity for arbitrarily small positive δi. The asymptotic form of $\langle J_c(m) \rangle$ depends on the properties of $w(i)$ near i_a. We have calculated the asymptotic length dependence for two classes of coupling distributions.

(i) *Two-junction model:* $w(i) = p_a \delta(i-i_a) + (1-p_a) \times \delta(i-(i_a+\Delta))$, $p_a < 1$. In this case the critical current density approaches i_a exponentially:

$$\langle J_c(m) \rangle = i_a + i_0 e^{-m/L} + \cdots ,$$

$$i_0 = \min[i_a + \Delta/n, (1+1/n)i_a] ,$$

$$L^{-1} = -\ln(1-p_a^n) \simeq p_a^n .$$

The critical flow model of Huse and Guyer[4] corresponds to such a two-junction model with $i_a=0$. In this limit the critical current density can be found by counting the number of percolating paths connecting the contacts.

(ii) *Continuous coupling distribution*: $w(i) = A(i-i_a)^\alpha + O((i-i_a)^{\alpha+1})$, $\alpha > -1$. The decay of the current density is only algebraic with a strong dependence on the wire thickness n:

$$\langle J_c(m) \rangle = i_a + i_0(n,A,\alpha) m^{-\lambda} + O(m^{-(\lambda+1)}) ,$$

$$i_0(n,A,\alpha) = \frac{[An^{1+\alpha}\Gamma(1+\alpha)]^n}{\Gamma(1+n(1+\alpha))}\Gamma(1+\lambda) ,$$

with $\lambda = [n(\alpha+1)]^{-1}$. For $i_c(\theta)$ given by Eq. (4) the decay of the critical current density is algebraic with an exponent $\lambda = 2/n$ [class (ii), $\alpha = -\frac{1}{2}$]. The exponential decay of $\langle J_c(m) \rangle$ for the two-junction model is due to the gap in the coupling distribution $[w(i)=0]$ which separates i_a from the larger couplings. The above results illustrate that the length dependence of the critical current in a long wire is drastically weakened by increasing its width n.

Finally, we discuss the dependence of $\langle J_c \rangle$ and $\langle l_c \rangle$ on texturing; see Fig. 4. As the directions ϕ of the a axis in the grains are lined up, $\phi_0 \to 0$, the average coupling $\langle i_c \rangle$ shifts towards large values. The mean critical current density $\langle J_c \rangle$ increases with texturing but always remains well below the average coupling $\langle i_c \rangle$. This behavior illustrates that the critical current density cannot be determined from the average properties of the material. The suppression factor for the critical current density reaches $\frac{1}{30}$ in the case of an untextured material and we find that only very strong texturing ($\phi_0 \lesssim 5°$) will increase the current density close to the maximal coupling strength $i_b = 0.54$. The length of the critical path $\langle l_c \rangle$ changes with the degree of texturing and shows a maximum at $\phi_0 = 35°$ where the disorder in the couplings is largest. Increasing texturing further, the current flow pattern becomes laminar with fewer and fewer saturated longitudinal bonds and eventually the critical path approaches a straight line with $\langle l_c \rangle = n$.

In conclusion, we have reformulated the problem of

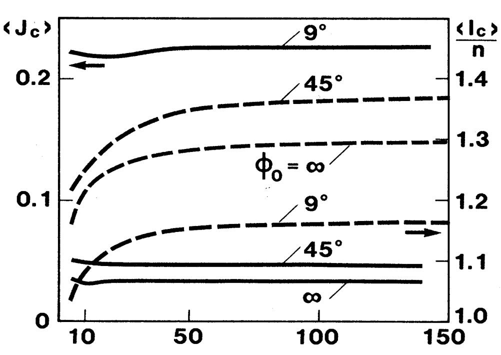

FIG. 3. Relative critical path length $\langle l_c \rangle/n$ and critical current density $\langle J_c \rangle$ vs sample size $m=n$. For $n > 50$ the critical path length increases linearly with the sample size n. This behavior implies the saturation of the critical current density $\langle J_c \rangle$, which therefore becomes a meaningful quantity characterizing the quality of a polycrystalline film.

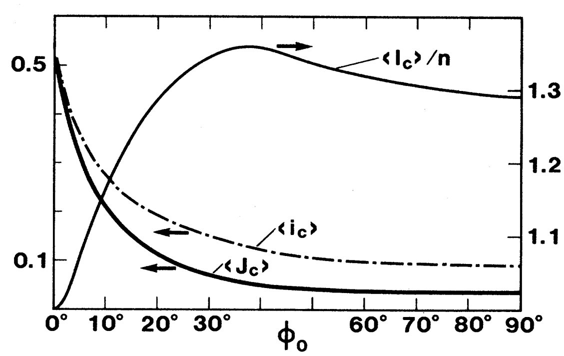

FIG. 4. Critical current density $\langle J_c \rangle$ and relative critical path length vs width ϕ_0 parametrizing texturing. $\langle J_c \rangle$ and $\langle l_c/n \rangle$ have been evaluated at 0.9°, 2.25°, 4.5°, 6.75°, and at all multiples of 9°. Average has been taken over 300 samples with $m=n=50$ (including returning paths); the error bars are smaller than the thickness of the lines. Note that the average coupling strength $\langle i_c \rangle$ is not a good approximation for $\langle J_c \rangle$. In untextured material the critical current density $\langle j_c \rangle$ is suppressed by a factor of $\frac{1}{30}$ as compared to the single-crystal value.

current limitation in a polycrystalline superconductor as a limiting path problem. We have applied the model to the determination of the critical current in a polycrystalline c-axis-oriented thin film. The results of our calculations show a very strong texturing dependence of the critical current in polycrystalline films. Only very strong a-axis texturing ($\phi_0 \leq 5°$) can increase i_c near the intragrain critical current density. This result provides a possible explanation of the difference between critical current densities found in single crystals and in polycrystalline c-axis-oriented thin films.[1,14]

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[11]For the Josephson coupling strengths and grain sizes typical for a polycrystalline $YBa_2Cu_3O_{7-\delta}$ film Coulomb coupling between the grains can be neglected, see, e.g., B. Abeles, Phys. Rev. B **15**, 2828 (1977).

[12]The critical path length $\langle l_c \rangle$ is bounded by $n i_{max}/i_{min}$, showing that $\langle l_c \rangle \sim n$ for $i_{max}/i_{min} < \infty$.

[13]The transverse fluctuations of the critical path grow only as $n^{2/3}$ (in accordance with Ref. 2), thus for $q < 1$ saturation of $\langle J_c \rangle$ is always found but shifted to larger values of n due to boundary effects.

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