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# Absence of effective electron-photon scattering in a tunneling chain of quantum dots

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We present a calculation of the dc electrical resistivity $\sigma^{-1}(0)$ of a tunneling chain of quantum dots (also called a quasi-zero-dimensional quantum-dot superlattice) due to the scattering by quasi-one-dimensional longitudinal-acoustic phonons. We find the vanishing of resistivity in such a system attributed to the restraint of electron-phonon scattering. This phenomenon is expected to be observable in the experiment.

## I. INTRODUCTION

Remarkable recent developments in semiconductor microfabrication technology have allowed the fabrication of structures with quantum confinement producing a one-dimensional quantum-well wire (QWW),[1-3] and have enriched the intriguing study of one-dimensional physics in real systems.[4-6] It is reasonably expected that the fabrication of a quasi-zero-dimensional quantum-dot superlattice will also produce equally intriguing phenomena. Recently, Reed *et al.*[7,8] have reported a successful fabrication of such a quasi-zero-dimensional system, in which electrons are confined in three spatial directions, and electronic states are discrete.[8] Energy-level splittings in such a quantum dot are generally of the order of 25 meV. If the thickness of the barrier is thin enough, electron tunneling between the adjacent quantum dots will occur. Physically, when no tunneling exists, the system cannot allow the presence of the plasmon waves. Huang *et al.*[9] calculated the tunneling intrasubband and intersubband plasmons in a quantum-dot superlattice. Leal *et al.*[10,11] reported the temperature dependence of the electrical conductivity in a two-dimensional electron gas (2D EG) and in quasi-one-dimensional conductor systems. Because a 1D EG can only be scattered by the short-range part of the scattering potential, in contrast to the cases of 2D and 3D EG which can be scattered not only by the short-range part, but also by the long-range part of the potential, the difference temperature behavior within the low-temperature region is expected.[1] Their calculations indicate that for longitudinal-acoustic (LA) interaction the temperature dependence of the electrical conductivity is uniquely determined by the dimensionality of the phonon system.[10,11]

In this paper, we present a calculation of the dc electrical resistivity of a quasi-zero-dimensional quantum-dot superlattice. The calculation indicates that when the period of the dot superlattice, $d$, is large enough, the energy restriction will dominate the maximum phonon energy absorbed, in contrast to the case of small $d$ where the momentum restriction $\hbar C_s q \le k_B \Theta_m$ dominates it. When $d$ further increases, the electrical resistivity is rapidly suppressed to zero due to the restraint of the electron-phonon scattering. Clearly, the larger the period of the dot superlattice is, the better the linear temperature dependence of the electrical resistivity is. The model system is only suitable to be applied to the one-dimensional phonon system[11] where the temperature is above the Peierls transition temperature. Besides, it allows one to include the presence of high magnetic fields without further formal difficulties. In Sec. II, we present the model Hamiltonian of the problem, as well as the memory-function formalism, to give an expression for dc electrical resistivity. The numerical calculation of the temperature behavior of the resistivity and some remarks are also given in this section.

## II. THE MODEL SYSTEM AND ITS HAMILTONIAN

The number of thermally excited optical phonons in the temperature range of interest is quite small, they will not play a fundamental role in the scattering process. Thus, we start with a Fröhlich Hamiltonian in tight-binding approximation (TBA)

$$H=\sum_k E(k)C_k^\dagger C_k+\sum_q \hbar\omega_q a_q^\dagger a_q+\sum_q D(q)\rho(q)A_q \;, \tag{1}$$

with an electron-phonon coupling constant $D(q)$, energy dispersion $E(k)$, and the symbols $\rho(q)$ and $A_q$ defined by

$$D(q)=[\hbar/(2m_i N'\omega_q)]^{1/2}q\xi \;, \tag{2}$$

$$E(k)=(W/2)[1-\cos(kd)] \;, \tag{3}$$

$$\rho(q)=\sum_k C_{k+q}^\dagger C_k \;, \tag{4}$$

and

$$A_q=(a_q+a_{-q}^\dagger) \;. \tag{5}$$

Here we have assumed in Eq. (3) that all the excited states are well above the ground level due to the very small lateral sizes $L_x, L_y$ and the width of quantum well $L_z$, and only the ground level is occupied. $C_k^\dagger$ ($C_k$) and $a_q^\dagger$ ($a_q$) are the electron and phonon creation (annihilation) operators with momentum $\mathbf{k}$ and $\mathbf{q}$, respectively. $\rho(q)$ is the Fourier transform of the electron-density operator, $\xi$ the deformation constant due to dilation, $m_i$ the ionic mass, and $N'$ the number of lattice cells.

The dc electrical resistivity $\sigma^{-1}(0)=m^*/(Ne^2\tau)$, $m^*$ being the effective electronic mass and $N$ the carrier density, can be obtained by using the memory function $\tau^{-1}=\mathrm{Im}M_{xx}(0)$. Making use of the force-force correlation function $\Pi_{xx}^R(\omega)$, the zeroth-order approximation gives

$$M_{xx}^{(0)}(0)=(-1/Nm^*)\lim_{\omega\to 0}\Pi_{xx}^R(\omega)\omega^{-1}\ , \tag{6}$$

where

$$\Pi_{\alpha\beta}^R(\omega)=-i\int\Theta(t)\langle[U_\alpha(t),U_\beta(t)]\rangle\exp(i\omega t)dt\ . \tag{7}$$

$\Theta(t)$ is the unit step function and $U_\alpha$ is the generalized force acting on the center of mass given by

$$U_\alpha=i\sum_q q_\alpha D(q)\rho(q)A_q\ . \tag{8}$$

After a lengthy algebraic calculation, we finally express $\mathrm{Im}\Pi_{xx}^R(\omega)$ with the aid of the imaginary part of the dielectric function $\epsilon_2(q,\hbar\omega)$ in the Bohm-Pines random-phase approximation (RPA)

$$\mathrm{Im}\Pi_{xx}^R(\omega)=\sum_q q^2|D(q)|^2\frac{\exp(\beta\hbar\omega_q)[\exp(\beta\hbar\omega)-1]}{[\exp(\beta\hbar\omega_q)-1]\{\exp[\beta\hbar(\omega_q+\omega)]-1\}}\frac{\epsilon_2(q,\hbar(\omega_q+\omega))}{v_q}-\mathrm{term}(\omega\to-\omega)\ . \tag{9}$$

Using the Lindhard equation, the imaginary part of the dielectric function can be written as

$$\epsilon_2(q,\hbar\omega_q)/v_q=2[\Theta(E_F-E_-)-\Theta(E_F-E_+)]/(dW\sin(qd/2)\{1-[\hbar\omega/W\sin(qd/2)]^2\}^{1/2})\ , \tag{10}$$

with

$$E_\pm=(W\pm\hbar\omega-W\cos(qd/2)\{1-[\hbar\omega/W\sin(qd/2)]^2\}^{1/2})/2\ , \tag{11}$$

where we have assumed in Eq. (10) that the range of temperature of interest satisfies $k_BT\ll E_F$. Substituting Eq. (9) into Eq. (6) gives

$$\mathrm{Im}M_{xx}^{(0)}(0)=(2\hbar\beta/Nm^*)\sum_q q^2|D(q)|^2\{\exp(\beta\hbar\omega_q)/[\exp(\beta\hbar\omega_q)-1]^2\}\epsilon_2(q,\hbar\omega_q)/v_q\ . \tag{12}$$

Employing the Debye approximation, i.e., $\omega_q=C_sq$, and introducing the parameters defined as

$$c=k_Fd,\quad \alpha=k_B\Theta_m/W,\quad t=T/\Theta_m,\quad k_B\Theta_m=2\hbar C_sk_F\ , \tag{13}$$

we get

$$\begin{aligned}\sigma^{-1}(0)/A&=(m^*/Ne^2)\mathrm{Im}M_{xx}^{(0)}/A\\&=(d_0/d)^3\int dx\{\exp(x/2ct)/[\exp(x/2ct)-1]^2\}\{x^3/[\sin^2(x/2)-(\alpha x/2c)^2]^{1/2}\}\ ,\end{aligned} \tag{14}$$

with a constant $A$ as a unit for electrical resistivity,

$$A=[\xi^2h^2/(k_B\Theta_m)N^2N'e^2m_i\pi Wd^2C_sd_0^3]\ . \tag{15}$$

$d_0$ is a unit of length introduced for dimensional reasons.

It should be noted that the electron-phonon scattering will be dominated by the following four restrictions.

(i) For definite momentum transfer $\hbar q$, the momentum conservation law requires

$$q\le 2k_F\ . \tag{16}$$

(ii) For definite momentum transfer $\hbar q$ in the elastic electron-phonon scattering process, the energy conservation law gives

$$q\le(W/\hbar C_s)|\sin(qd/2)|\ . \tag{17}$$

(iii) For phonon absorption, the energy level related to the initial electronic state should be occupied, which means that

$$E_- \leq E_F . \tag{18}$$

(iv) For phonon emission, the energy level related to the final electronic state must lie below the Fermi surface, that is,

$$E_+ \leq E_F . \tag{19}$$

Let $x=qd$; combining Eqs. (16)–(19) will give the phonon energy ranges permitted, $X_-$ and $X_+$, for phonon absorption and phonon emission, respectively, as shown in Fig. 1.

From Fig. 1 we know that when $d$ is less than about 200 Å, the phonon energy ranges permitted for phonon absorption and emission are almost independent of $d$ (as seen in Fig. 1, $X_-$ and $X_+$ are nearly proportional to $d$ when $d<200$ Å). However, when $d$ exceeds $d_1$ [the intersection of the curves defined by Eqs. (16) and (17), respectively], as shown in Fig. 1, the restriction for phonon energy absorbed Eq. (17) (long-dashed line) begins to

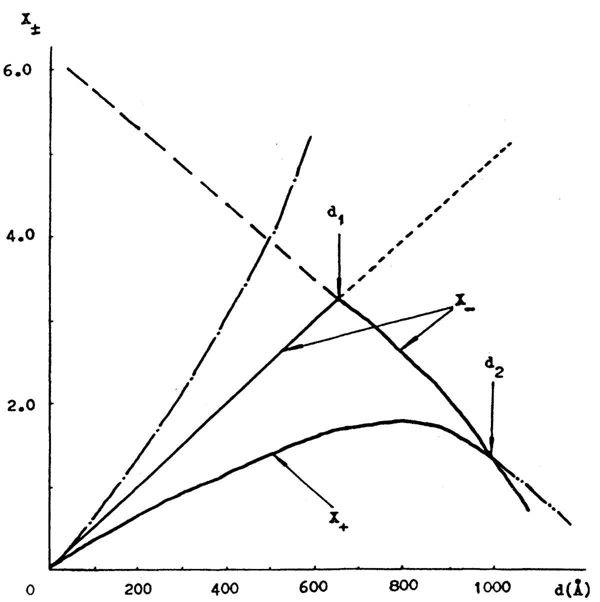


FIG. 1. The period of the dot superlattice $d$ dependence of the phonon energy ranges permitted, $X_-$ and $X_+$, for phonon absorption and phonon emission respectively. The parameters are as follows: $n_{1D}=1.58\times10^5$ cm$^{-1}$, $k_B\Theta_m=0.8631$ meV, and $m^*=0.041m_e$. The long-dashed line is related to the restriction for energy conservation, and the short-dashed line the restriction for momentum conservation. The dotted-dashed line and the doubly dotted-dashed line refer to $E_-\leq E_F$ and $E_+\leq E_F$, respectively.

dominate $X_-$ instead of the momentum restriction Eq. (16) (short-dashed line), then the range permitted for phonon energy absorbed is greatly reduced. When $d$ further approaches to $d_2$ [the intersection of the curves defined by Eqs. (17) and (19), respectively], the phonon energy ranges for phonon absorption as well as phonon emission are coincident. This gives the vanishing of the resistivity in such a system. To some extent, we can say that there seems to exist a "transparency window" for propagation of Bloch waves in this system when $d$ approaches $d_2$. The temperature dependence of the electrical resistivity is shown in Fig. 2.

From Fig. 2 we can see the clear increase of the electrical resistivity in the region $(0\leq d\leq d_1)$ and the decrease of the electrical resistivity in the region $(d \lesssim d_2)$. The inset of Fig. 2 shows the low-temperature behavior of the electrical resistivity $(T/\Theta_m \sim 0.0-0.5)$. It is evident that the larger the period of the dot superlattice is, the better the linear temperature dependence of the electrical resistivity is. In the more interesting regime $\Theta_m \ll T \ll E_F/k_B$, the screening effects can be disregarded, and the linear temperature dependence for a one-dimensional phonon system (1D PS), 2D PS, and 3D PS has been obtained. Hence, we can suppose that in this regime the LA-phonon system loses the information about its dimensionality in the interaction with the electron gas.

Furthermore, it is worthwhile to mention that our theory can be easily generalized to include the magnetic field effects in this quasi-zero-dimensional dot-superlattice systems. Moreover, the many-body effects can also be calculated in our formalism using the complete electron density correlation $S(\mathbf{q},ip_n)$ instead of the "simple bubble" noninteracting electron density correlation $S^0(\mathbf{q},ip_n)$ which implies neglecting "quantum corrections" to the electrical resistivity. However, these effects do not seem to play an important role for the LA-phonon scattering in the temperature range we are interested in.

Taking the strong-screening limit $(kd\ll1)$ in Eq. (3), we know that $E(k)$ will tend to the free-electron energy $\hbar^2k^2/2m^*$, thus we require that $W\sim 2\hbar^2/m^*d^2$. Considering a realistic model to account for the overlap of the wave functions in the calculation of $W$, we know that the empirical estimation of the bandwidth $W\sim 2\hbar^2\exp(-d^2/L_z^2)/m^*d^2$ will decrease more rapidly than $\sim 2\hbar^2/m^*d^2$ with the increase of the period $d$ of the dot superlattice. When $k_B\Theta_m/k_F=2\hbar C_s$ is larger than $Wd\sim 2\hbar^2\exp(-d^2/L_z^2)/m^*d$, the electrical resistivity will be suppressed to zero. In this case, if we take $L_z=d/2$, we estimate that $d_2$ is less than 100 Å, which implies that we can easily observe this tunneling phenomenon experimentally.[12] However, the qualitative features of the electrical resistivity from these two models remain the same.

Although we can adjust the one-dimensional carrier density $n_{1D}$ by changing the gate voltage, the adjustable range permitted is generally very small. In contrast, experimentally, we can easily change the period $d$ of the dot superlattice over a wide range. If $k_F$ exceeds $\pi/d$, the ex-

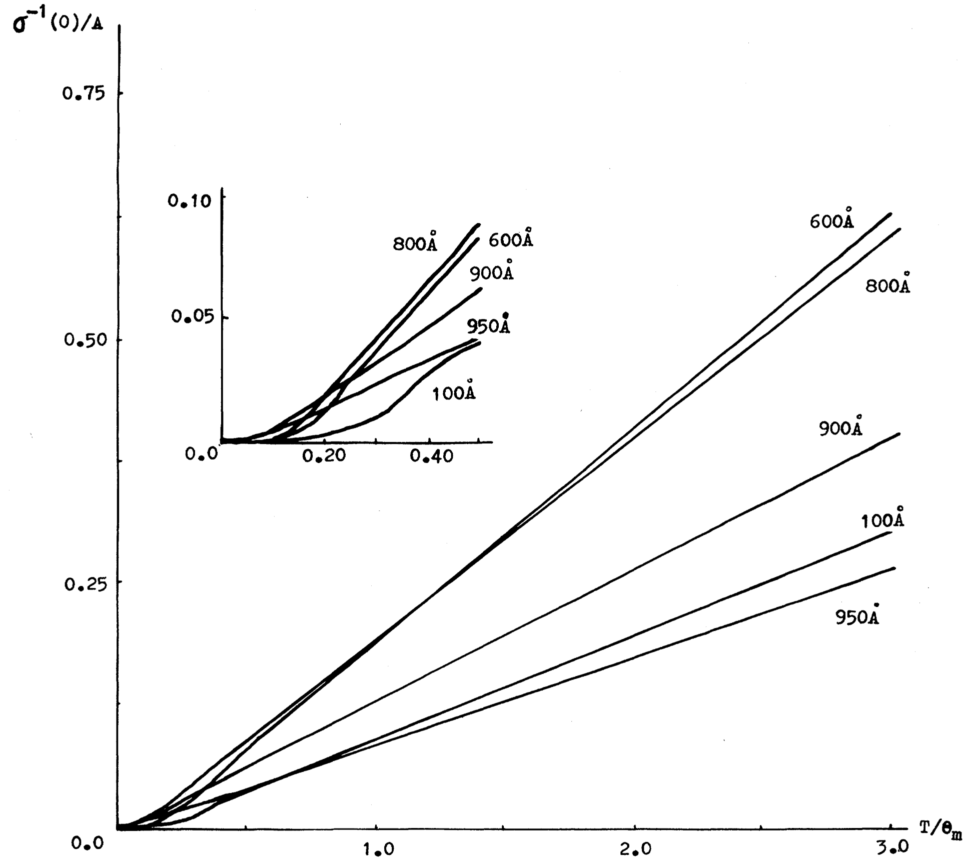


FIG. 2. The temperature behavior of the electrical resistivity for different periods of the dot superlattice (or different coupling strength). The constant $d_0$ is chosen to be 100 Å, and the other parameters are the same as those in Fig. 1. The inset shows the features of electrical resistivity in the low-temperature regime.

cited levels begin to be occupied, and the model system introduced above will fail to apply. In this case, the system has gone through a transition from a semiconductor to a metal, and the intrasubband acoustic-plasmon mode is completely suppressed, but the intersubband plasmon mode can still exist.

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