Quantized helicon absorption and nonlocal effects in a semiconductor superlattice

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The plateaus found previously in the imaginary part of the helicon-mode frequency are shown to be independent of the wave vector over a range of values of the latter. However, at still larger values of the wave vector, the plateaus become distorted due to the dominance of nonlocal effects, and finally disappear when the helicon modes themselves cease to exist. It is also shown that the plateaus in the imaginary part of the helicon frequency are primarily related to the plateaulike structure exhibited by the diagonal component of the conductivity tensor.

I. INTRODUCTION

The wave-vector and frequency dependence of the conductivity is of considerable interest in a system exhibiting the integral quantum Hall effect (IQHE), in view of the important role played by the localized states[1] in the occurrence of the IQHE. The frequency dependence of the conductivity in essentially two-dimensional electron-gas (2 DEG) systems has been studied both experimentally and theoretically by several groups.[2–12] However, the wave-vector dependence of the conductivity has not received the same attention.

Recently, we have shown theoretically that helicon-wave absorption[13] and helicon-wave damping[14] calculated on the basis of linear-response theory for a model superlattice exhibit plateaus when the applied magnetic field is varied. The model system represented a GaAs/(Al,Ga)As superlattice with a 3D electronic band structure and the values of the parameters of the Kronig-Penney model pertaining to it were taken from the work of Störmer *et al.*,[15] who had observed IQHE plateaus in that system. The plateaus in helicon absorption[13] and helicon damping[14] were found to occur at exactly the same locations as the plateaus in ρ_{xy} of the IQHE.[15]

The propagation of helicon waves, i.e., circularly polarized electromagnetic waves in a conducting medium propagating along the axis of an applied magnetic field is rendered possible because of the existence of the Hall effect and, in fact, helicon propagation is sometimes referred to as the dynamic or rf Hall effect.[16] Since they are used in contactless measurements[17] of the Hall effect, helicons offer an alternative technique with which to study the IQHE and are especially suited for studying the wave-vector-dependent effects.

It was originally thought that the plateaus in helicon absorption[13] also corresponded to the dynamic IQHE. However, this does not appear to be the case, as the dominant factor in determining the helicon absorption is the diagonal component of the conductivity tensor σ_{xx} and not the Hall conductivity. In order to understand better the physical origin of the plateau structure in helicon absorption, and the conditions under which the plateau structure manifests itself, it is necessary to study how factors such as the frequency, wave vector, and band structure of the superlattice influence the plateau structure in helicon absorption.

It is the purpose of this Brief Report to study the wave-vector dependence of the quantized helicon absorption in a GaAs/(Al,Ga)As superlattice on the basis of linear-response theory. The theoretical framework and the details of the model and the numerical method have been given before.[13,14,18,19] The results show that the plateaus in the imaginary part of the helicon-mode frequency remain well defined over the range of values of the wave vector when the local approximation is valid. However, when nonlocal effects do become significant, the plateaus become distorted and eventually disappear as the helicon modes themselves cease to exist. In addition, the real and imaginary parts of the wave-vector- and frequency-dependent conductivity-tensor components σ_{xx} and σ_{xy} have been computed for the same Kronig-Penney model. These numerical results and their relation to the plateau structure in helicon absorption will also be discussed.

II. OUTLINE OF THE THEORY OF HELICONS IN A SEMICONDUCTOR SUPERLATTICE

Consider a system with an electronic structure which is free-electron-like along the x and y directions, but is periodic along the z direction. The periodic potential in the z direction is represented by a Kronig-Penney model. Applied along the z direction is a static magnetic field $\mathbf{B}_0$, described by a vector potential $\mathbf{A}_0$, whose components in the Landau gauge are $(0,B_0X,0)$. There is also an electromagnetic disturbance that varies as $\exp(i\mathbf{q}\cdot\mathbf{r}-i\omega t)$. The wave vector $\mathbf{q}$ is also taken to be along the z direction. $\mathbf{A}_1(\mathbf{r},t)$ is taken to be the vector potential for the self-consistent field produced by the disturbance. SI units are used throughout.

We assume that the medium is nonmagnetic. The electric and magnetic fields associated with the wave are related through the Maxwell's equations. The theory for helicon propagation in a periodic structure based on the linear-response formalism, given by Narahari Achar,[18] can be applied to the present case. Using the notation of Ref. 18, it can be shown that the frequencies of the helicon waves are determined from

$$\epsilon_{\pm}(\mathbf{q},\omega)\omega_{\pm}^2=c^2q^2 . \tag{1}$$

Here, $\epsilon_{\pm}(\mathbf{q},\omega)$ are the wave-vector- and frequency-dependent components of the dielectric tensor in the polarization representation and are given in terms of the corresponding Cartesian components by

$$\epsilon_{\pm}(\mathbf{q},\omega)=\epsilon_{xx}(\mathbf{q},\omega)\pm i\epsilon_{xy}(\mathbf{q},\omega)\ , \tag{2}$$

and c is the speed of light.

The dielectric tensor is related to the conductivity tensor $\tilde{\sigma}(\mathbf{q},\omega)$ according to

$$\underline{\epsilon}(q,\omega)=\epsilon_1\mathbb{1}+\frac{i\tilde{\sigma}}{\omega\epsilon_0}\ . \tag{3}$$

Here, ϵ_1 is the dielectric constant of the lattice and $\mathbb{1}$ is the unit tensor. As has been described in Ref. 18, an expression for the conductivity tensor can be obtained on the basis of linear-response theory in terms of the energy eigenvalues,

$$E(n,k_z,k_y,l)=(n+\tfrac{1}{2})\hbar\omega_c+\epsilon_l(k_z)\ , \tag{4}$$

associated with the eigenstates

$$|nk_zk_yl\rangle=e^{ik_yY}U_n(x+l_H^2k_y)\sum_K b_l(k_z+K)e^{i(k_z+K)z} \tag{5}$$

of the unperturbed Hamiltonian

$$H_0=(\mathbf{P}-e\,\mathbf{A}_0)^2/2m+V(z)\ , \tag{6}$$

where $V(z)$ is the periodic potential along z. Here, U_n are the harmonic-oscillator wave functions and $l_H=(\hbar/m\omega_c)^{1/2}$ is the magnetic length. The $b_l(k_z+K)$ are the expansion coefficients associated with a set of plane-wave basis functions as described in Ref. 18.

Damping effects can be considered by treating the frequency ω to be a complex quantity $\omega-i/\tau$, as a result of which the dielectric tensor becomes a complex quantity:

$$\epsilon_{\pm}(q,\omega)=\epsilon'_{\pm}(q,\omega)+i\epsilon''_{\pm}(q,\omega)\ . \tag{7}$$

We choose to work with a real wave vector q and a complex frequency in the dispersion relation given by Eq. (1). Furthermore, we focus on the mode associated with the lower sign and henceforth drop the subscript and the arguments (q,ω). The real part ω' and the imaginary part ω'' of the helicon-mode frequency are then given by

$$\omega'^2-\omega''^2=c^2q^2\epsilon'/(\epsilon'^2+\epsilon''^2)\ , \tag{8}$$

$$2\omega'\omega''=-c^2q^2\epsilon''/(\epsilon'^2+\epsilon''^2)\ . \tag{9}$$

III. RESULTS AND DISCUSSION

The real and imaginary parts of the helicon-mode frequency in Eqs. (8) and (9) have been calculated as functions of the applied magnetic field using the same values of the model parameters as those given in Ref. 14. The calculations have been performed for several values of the wave vector ranging from 1.0×10^{-8} to 1.6×10^{-3} in units of $2\pi/d$, where d is the superlattice period. It has already been shown[14] that for a single value of the wave vector the real part ω' increases linearly with B and that the imaginary part ω'' exhibits plateaus. Although the same general behavior is exhibited over a range of values of the wave vector, significant deviations occur at larger values of the wave vector, as discussed below.

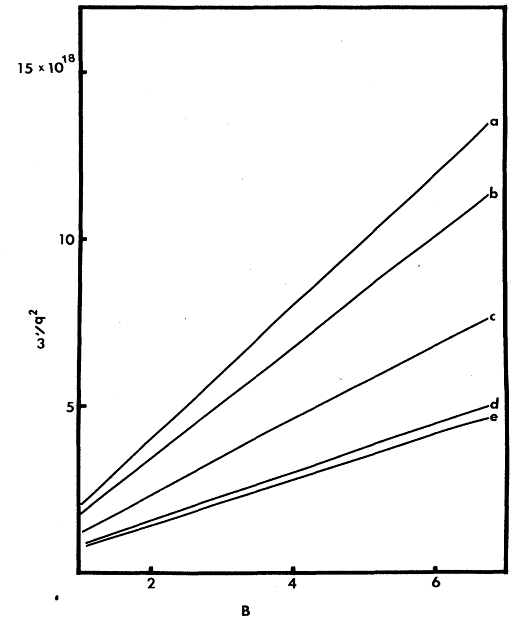

FIG. 1. Variation of the ratio ω'/q^2 with magnetic field for different values of the wave vector.

The linear variation of ω' with the magnetic field for helicons in a semiconductor superlattice is reminiscent of the behavior of helicons in a homogeneous medium according to the dispersion relation $\omega'=c^2q^2\omega_c/\omega_p^2$, in the usual notation. Therefore, a plot of ω'/q^2 versus B is expected to yield a straight line. Figure 1 shows a plot of ω'/q^2 versus B for several values of the wave vector, and Fig. 2 shows the variation of the imaginary part of the frequency ω'' with the increasing magnetic field for the values of q.

For all values of q ranging from 1×10^{-7} to 1×10^{-4},

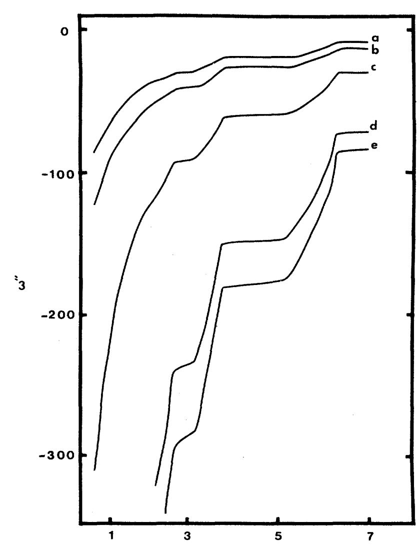

FIG. 2. Variation of ω'' with magnetic field for different values of the wave vector.

the plot of ω'/q^2 versus B results in a single straight line (labeled a in Fig. 1) with deviations of $\lesssim 1\%$. For values of q larger than 1×10^{-4}, the deviations become increasingly more significant, as shown by the four different straight lines (labeled $b-e$ in Fig. 1) for four different values of q, 5×10^{-4}, 1×10^{-3}, 1.5×10^{-3}, and 1.6×10^{-3}, respectively.

The plateaus in ω'' in Fig. 2 are sharp and practically flat, and, for a given value of the wave vector, occur at those values of ω'' which bear simple integer ratios to one another, and can be ranked according to the representative integers.

Just as in the case of ω', for values of q ranging from 1×10^{-7} to 1×10^{-4}, there is essentially a single curve of ω'' versus B, with deviations of $\lesssim 1\%$ (curve a in Fig. 2). For values of q larger than 1×10^{-4}, there are significant deviations. The curves labeled $b-e$ in Fig. 2 correspond to the same values of q, respectively, as those straight lines in Fig. 1 carrying the same labels.

For values of q larger than 1.6×10^{-3}, no helicon-mode solutions are found. For the lowest value of q, the wave vector used in this study, namely $q=1\times10^{-8}$, helicon-mode solutions are not obtained at low values of the magnetic field. At higher values of the magnetic field, solutions are found with the real part of the frequency ω' in the kHz range and with the imaginary part ω'' about 5% larger in magnitude than the values of ω'' in curve a, Fig. 2.

The characteristic features are the widths and slopes of the plateaus. The width of the plateaus for a given value of q increases with increasing magnetic field (i.e., the width of the plateaus depends on the order of the plateau). For a given order, the width of the plateau is practically independent of q. However, the slope of the plateau changes with q, initially slowly, but rapidly at larger values of q. This can be seen more clearly from Fig. 3, where the slope $\partial\omega''/\partial B$ is plotted as a function of $\log_{10}q$ for plateaus of different order. As q increases, especially near the maximum value of q, the slope changes so rapidly as to distort the plateaus.

This strong dependence of the real as well as imaginary parts of the helicon-mode frequency on the wave vector implies strong nonlocal effects arising from the q dependence of the conductivity tensor. It has been shown[20] that nonlocal effects strongly influence the nature of helicon dispersion in semiconductor superlattices. The helicon dispersion exhibits a maximum at a value of $q=q_{\max}$ and a sharp drop beyond the maximum. There are no solutions of the familiar type of helicon modes for values of q beyond $q_{\max}$. A few discrete solutions do exist, but their nature is not clear. In the present work, $q_{\max}$ is slightly larger than 1.6×10^{-3} and no helicon-mode solutions are obtained for $q>q_{\max}$. The imminent nature of the nonlocal effects on the plateaus in ω'' as q approaches $q_{\max}$ is clear from Fig. 3.

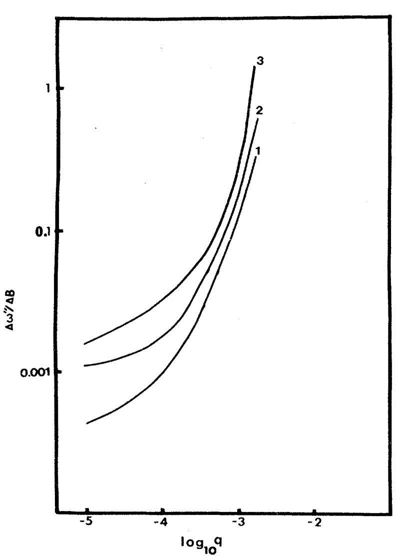

FIG. 3. Variation of the slope of the plateau with wave vector for different orders. 1, 2, and 3 refer to the plateaus at about 7, 5, and 3, respectively, in Fig. 2.

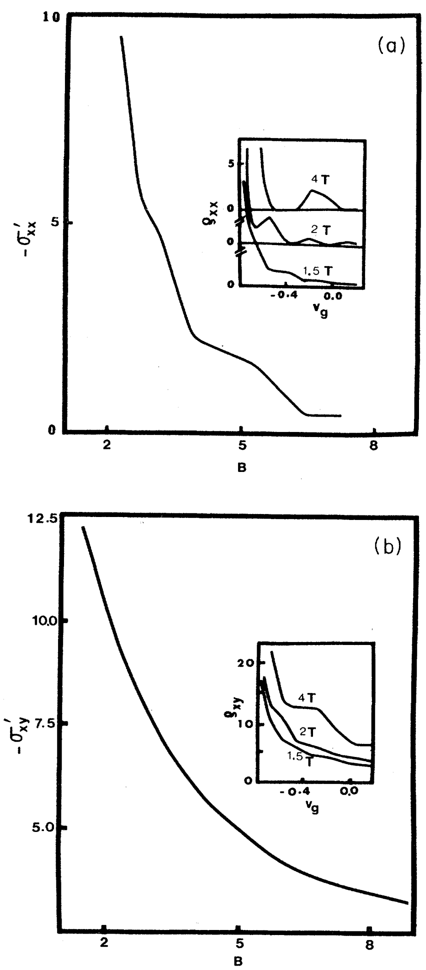

FIG. 4. (a) Variation of σ'_{xx} with B. Inset shows the variation of ρ_{xx} with gate voltage for different magnetic fields, observed in a GaAs-$Al_xGa_{1-x}As$ heterostructure (Ref. 21). (σ'_{xx} in units of $10^{-3}\Omega^{-1}\,m^{-1}$, B in T, ρ_{xx} in $k\Omega$, and V_g in V.) (b) Variation of σ'_{xy} with B. Inset shows variation of ρ_{xy} with gate voltage for different magnetic fields (Ref. 21). (σ'_{xy} in units of $10^3\ \Omega^{-1}\,m^{-1}$ and ρ_{xy} in kΩ.)

The plateaus in ω'' seen in Fig. 2 have been obtained on the basis of linear-response theory for a model superlattice which has a three-dimensional electronic band structure. The model corresponds to the GaAs/(Al,Ga)As superlattice in which the IQHE was observed by Störmer *et al.*,[15] demonstrating that a strictly 2D EG is not absolutely necessary for the occurrence of the IQHE. Furthermore, the propagation of helicons is an essentially three-dimensional phenomenon and plateaus in ω'' would occur only if electron-scattering effects are taken into account. It is believed that IQHE-like behavior could occur in a three-dimensional structure provided the minibandwidth be less than the separation between Landau levels, so that there exists a gap, and that the Fermi levels lies in the gap. In order to check if there is any relationship between the plateaus in ω'' and the plateaus of the IQHE, we have calculated the real and imaginary parts of both the diagonal component $\sigma_{xx}(q,\omega)$ and the off-diagonal component $\sigma_{xy}(q,\omega)$ as functions of the magnetic field, using the formalism outlined in Sec. II, the model parameters of Ref. 13, and using the values $q=1\times10^{-6}(2\pi/d)$ and $\omega=1\times10^{8}$ rad/s. Figure 4(a) shows the variation of the real part $\sigma'_{xx}(q,\omega)$ with B and Fig. 4(b) shows the variation of the real part $\sigma'_{xy}(q,\omega)$ with B.

The dominant contribution to the real part of the helicon-mode frequency ω' arises from σ'_{xy}. As can be seen from Fig. 4(b), σ'_{xy} is a smooth function of B, and this fact explains the linear variation of ω' with B in Fig. 1.

The dominant contribution to ω'' arises from σ'_{xx}. The plateau structure in ω'' arises from the plateau structure in σ'_{xx}, as can be seen from Fig. 4(a).

Although the Kronig-Penney model used in the present work was proposed for the GaAs/(Al,Ga)As superlattice, in which the IQHE was found experimentally, it is clear that plateaus in σ_{xy} corresponding to the IQHE are not obtained in these calculations. This is not surprising, however, as none of the conditions appropriate for the occurrence of the IQHE have been assumed. Surprisingly, the calculations do yield plateaus in σ'_{xx}. The question naturally arises as to how plateaus in σ'_{xx} can occur while σ'_{xy} itself is a smooth function of B.

The insets of Figs. 4(a) and 4(b) show the experimental results of von Klitzing *et al.*,[21] for the magnetoresistivity ρ_{xx} and the Hall resistivity ρ_{xy} obtained for a GaAs-$Al_xGa_{1-x}As$ heterostructure as functions of the gate voltage at different magnetic fields. At $B=4$ T, ρ_{xy} exhibits well-defined plateaus and $\rho_{xx}=0$ in the plateau regions. At lower fields, the plateau structure in ρ_{xy} is not fully developed and ρ_{xx} exhibits oscillations, but may not reach the zero value. At $B=1.5$ T, ρ_{xy} has only kinks and ρ_{xx} exhibits almost a plateaulike structure. It is possible that, at still lower values of B, ρ_{xy} may be an almost smooth function of B and ρ_{xx} exhibits a steplike variation.

These results suggest that the plateaulike structure in σ'_{xx} may owe its origin to the same physical causes as the IQHE. Quantized Landau levels, and perhaps the pinning of the chemical potential to a Landau level, may cause plateaus to appear in σ_{xx} even though σ_{xy} may still be a smooth function of B, exhibiting no plateaulike structure corresponding to the IQHE. When the plateaus of the IQHE do develop in σ_{xy}, σ_{xx} goes to zero. Thus the plateaus in σ_{xx} may be the precursors of the plateaus in σ_{xy}. It would be interesting to study the effect of localization on the plateaus in σ_{xx}. Work is underway to study the effect of band structure and frequency on the plateau structure in σ_{xx} and of the quantized helicon damping, and the results are planned to be published separately.

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