Theory of contacts in a two-dimensional electron gas at high magnetic fields

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General properties of a contact in a two-dimensional electron gas (2D EG), where more than one Landau level is occupied at high magnetic fields, and is quantitatively analyzed on the basis of a Landauer-Büttiker formalism. When acting as a current source (drain), a contact generally populates different Landau levels to different degrees. When acting as a voltage probe, a contact does not generally indicate a mean value of the chemical potentials of different Landau levels. A voltage-probe contact works also to partially equalize populations of different Landau levels. These properties, first pointed out qualitatively by Büttiker, are quantitatively analyzed in terms of n^2 parameters when n Landau levels are occupied in the 2D EG. The analysis is applied to the $n=2$ case to calculate characteristic resistances of multiprobe samples. Results of recent experiments are explained on the basis of the analysis. The properties of electrical transport as well as the properties of energy transmission and dissipation in multiprobe samples are quantitatively discussed.

I. INTRODUCTION

Recently approaches to the integral quantum Hall effect[1] (IQHE) based on a Landauer-type formalism for electron transport[2] have been presented by several authors.[3-8] The descriptions rest on the fact that the current ΔJ carried by each Landau level is determined by the difference $\mu'-\mu$ of the chemical potentials of the Landau level on opposite sides of the channel of a two-dimensional electron gas (2D EG); namely, $\Delta J=(e/h)(\mu'-\mu)$. The total current J carried by n Landau levels is given by the sum of the contribution of each Landau level, $J=(e/h)\sum_i(\mu_i'-\mu_i)$, where the ith Landau level has respective chemical potentials μ_i' and μ_i on the opposite sides of the channel. In the case when n Landau levels are equally occupied on respective sides of the 2D EG channel ($\mu_i'=\mu'$ and $\mu_i=\mu$), the Hall resistance $R_H=(\mu'-\mu)/eJ$ is quantized to $h/(ne^2)$ as usual. A Landau state at which the energy equals its chemical potential on one boundary of the 2D EG forms a one-dimensional (1D) channel located near the boundary of the 2D EG. This 1D channel is called an edge channel of the Landau level. A number of transport properties of the 2E EG system can thus be discussed only by specifying the energy of each edge channel.

Based on a Landauer-type treatment, Büttiker has provided a general and extremely instructive formalism of electrical contacts in 2D EG systems at high magnetic fields.[6] He argued that electrical contacts play crucial roles in a 2D EG: If a contact is "not ideal," it populates different Landau levels to different degrees when acting as a current source; a nonideal contact does not indicate a mean energy value of edge channels when it acts as a voltage probe. A contact is termed "not ideal" or "disordered" if an electron incident on the contact from edge channels is reflected back into the 2D EG with a finite probability. Hence, in the absence of an adequate equalization of the populations among different edge channels on sample boundaries, a deviation of the Hall resistance R_H from an exact quantization and an anomalous longitudinal resistance R_{xx} can be expected to appear if the contacts are disordered.[6] On the other hand, a number of theoretical treatments except those of Refs. 6–8 have not addressed possible roles of contacts.[3-5] Contacts would not influence the observations if interchannel scattering of electrons is significant enough to rapidly equalize the occupations of different Landau levels. Hence the problem of unequal occupation of edge channels as well as the influence of contacts have been assumed to be a specific issue to be discussed in small devices.[6-8]

Experimentally, however, the present authors have observed in samples with an interprobe distance of 100 μm that four-terminal measurements with disordered voltage-probe contacts exhibit a significant deviation of R_H and an occurrence of anomalous R_{xx} when unequal occupations of different edge channels are introduced by a gate-induced backscattering of edge currents.[9,10] Selective injection of electrons into different edge channels by disordered current contacts have also been observed subsequently.[11] These experiments have definitely indicated that an edge current comprising unequally occupied edge channels travels a distance in excess of 100 μm without exhibiting significant equilibration. van Wees *et al.* have also reported anomalous integer quantizations of two- and three-terminal resistances of 2D EG in a small device having quantum point contacts located at a distance of 1.5 μm from each other and whose property is controlled by split gates.[12] More recently, they have also pointed out that interchannel scattering of electrons is extremely weak,[13] similarly to our experiments. Hence the effect of a contact is not an issue to be argued only in small samples but is a more general problem to be considered in samples with ordinary sizes. In view of the fact that any real contacts may be disordered in a strict sense, however nearly they are ideal, it may also be of definite importance to figure out a possible deviation of R_H due to the influence of contacts.

This work is an extension of the pioneering work of Büttiker.[6,14] The aim is to bring the original discussion into quantitative arguments. For satisfactory treatments

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of a multichannel situation, one needs to explicitly deal with transmission and reflection probabilities of electrons among different edge channels and between a contact and the different edge channels separately, but this has not been worked out by Büttiker.[6] Here, we apply Büttiker's formalism of a contact and derive explicit equations satisfied by disordered contacts. It should be noted that "disordered contacts" studied here are not particular contacts with specific properties but can be any kind of contacts encountered in experiments. An ideal contact will be understood as a specific limiting example in the general family of disordered contacts. We will show that, whatever complexities disordered contacts may have in actual constructions and to whatever extent they are different from ideal contacts, they cannot exhibit arbitrary properties but obey quite systematic regulations including rigorous symmetry relations. Further, it would be worth mentioning that "disordered contacts" are useful contacts in the sense that they provide information about the occupations of different Landau levels in the 2D EG but "ideal contacts" are useless in this respect because they always completely mix up all the Landau levels.

Recently, Woltjer *et al.* discussed geometry effects on the magnetotransport of a 2D EG in which finite size of contacts is important.[15] The discussion is classical and based on the assumption that the current distribution within a 2D EG is determined by local resistivity tensors. The problem discussed there may be important in magnetic field ranges where the diagonal resistivity ρ_{xx} in the 2D EG cannot be neglected and the potential distribution within the 2D EG does influence the observation. In this paper we discuss more general and essential effects of contacts.

Section II starts from describing a Landauer-Büttiker formalism of contacts in an n-channel case and derives basic relations satisfied by contacts. In Sec. III several characteristic resistances of a multiprobe sample in the two-channel ($n=2$) case are calculated as a function of the parameters of disordered contacts. The magnetic-field–reverse reciprocity symmetry of the Hall resistance is derived to show that it is the manifestation of the symmetry properties of contacts in the case where the 2D EG are completely quantized while the contacts are disordered. A contact with a cross gate structure is shown to be a specific example of disordered contacts, and recent experimental results are explained. Section IV discusses quantitatively the origin of finite resistances in the regime of IQHE. In Sec. V incompatibility between the current conservation and the charge-density conservation in the presence of scattering is pointed out. The generality of the formalism of a contact given in this work is also discussed.

II. FORMALISM AND ANALYSIS OF A CONTACT IN n-CHANNEL CASE

When the dispersion of a Landau level $\varepsilon(k)$ is given in terms of the wave number k along the direction of a 2D EG channel, the density of states is given by $\rho(k)=(1/\pi)|\partial\varepsilon/\partial k|^{-1}$ including spin degeneracy.[3–6] The extreme simplicity of the Landauer-type formalism results from the fact that the density of states given above is inversely proportional to the group velocity $v(k)=(\partial\varepsilon/\partial k)/\hbar$ of electrons. The current ΔJ carried by the electrons occupying a Landau level with the k vectors in the range $k_1\le k\le k_2$, where $\varepsilon(k_1)=\varepsilon_1$ and $\varepsilon(k_2)=\varepsilon_2$, is $\Delta J=e\int_{k_1}^{k_2}v\rho(\partial\varepsilon/\partial k)dk$. Here $v\rho$ is either $2/h$ or $-2/h$ depending on the sign of $\partial\varepsilon/\partial k$. Hence, $\Delta J=(2e/h)(\varepsilon_2-\varepsilon_1)$. When μ and μ' are the chemical potentials of the Landau level on opposite boundaries of a 2D EG channel, we carry out the integration over all the occupied states across the channel and obtain $J=(2e/h)(\mu'-\mu)$ for the current carried by all the electrons in the Landau level. Note that this result is independent of the detailed profile $\varepsilon(k)$ of the Landau level. In the presence of a random potential both along and across the 2D EG channel, the bulk Landau states are localized except at the band center. In this case also, the conclusion given above is unaltered so long as the bulk states at the Fermi level are localized.[6]

In the following, we will deal with a general case in which the distribution of electrons in a Landau level at a boundary of the 2D EG is not in thermal equilibrium. The distribution function $f(\varepsilon)$ generally depends on Landau levels (edge channels). In such a case, the "chemical potential" μ of a given edge channel has to be determined from the equation $(2e/h)\int_{\varepsilon_x}^{\infty}f\,d\varepsilon=(2e/h)(\mu-\varepsilon_x)$. Here, $\varepsilon_x=\varepsilon(k_x)$ is a reference energy well below μ such that the state k_x is occupied with probability 1 $[f(\varepsilon_k)=1]$, and the sign of $\partial\varepsilon/\partial k$ is the same over the k range from k_μ to k_x, where $\varepsilon(k_\mu)=\mu$. From the above equation, we have $\int_{\varepsilon_x}^{\infty}f\,d\varepsilon=\int_{\varepsilon_x}^{\mu}d\varepsilon$ or $\int_{\mu}^{\infty}f\,d\varepsilon=\int_{\varepsilon_x}^{\mu}(1-f)d\varepsilon$. Since ε_x can be replaced by $-\infty$, we define μ by

$$\int_{\mu}^{\infty}f(\varepsilon)d\varepsilon=\int_{-\infty}^{\mu}[1-f(\varepsilon)]d\varepsilon\ ,$$

where we limit the integration to one side of a 2D EG channel. The energy μ determined as above is reduced to the true chemical potential when electrons are in a thermal equilibrium state described by a Fermi distribution function. Note that our μ given above is generally different from the energy μ^* at which the number of electrons (occupied states) with $\varepsilon>\mu^*$ equals the number of holes (unoccupied states) with $\varepsilon<\mu^*$. The latter energy μ^*, instead of our μ, has been suggested in earlier work.[4,8] It follows from our definition of μ that the charge density of an edge channel cannot be conserved when intrachannel inelastic scattering and/or interchannel scattering take place, as will be discussed in Sec. V.

When an electrical contact is attached to a boundary of a 2D EG, two sets of edge channels are distinguished at the contact, along one of which electrons are incident on the contact and along the other of which electrons leave the contact as shown in Fig. 1(a) for the $n=2$ case in a magnetic field pointing out of the page. We will call briefly the former set of edge channels "incident channels" and the latter "outgoing channels." Following Büttiker,[6] we regard a contact as consisting of an electron reservoir and a disordered region. The electron reservoir

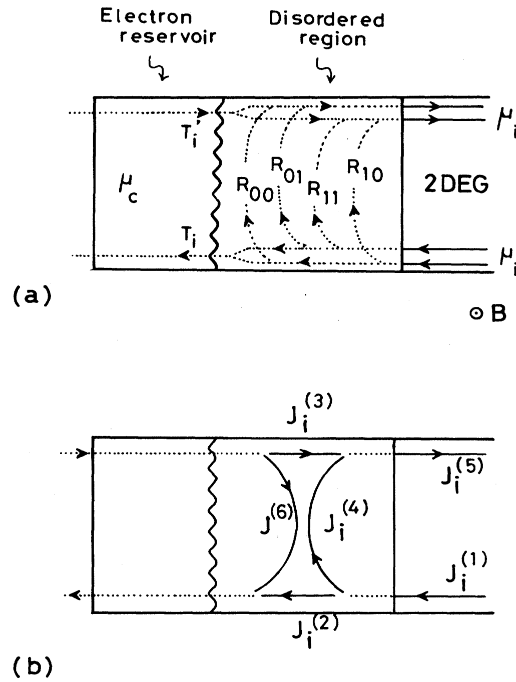

FIG. 1. A schematic representation of a disordered contact, consisting of an electron reservoir and a disordered region. (a) Transmission and reflection probabilities. (b) Characteristic currents relating to the ith Landau level.

is characterized by a large number of states at the Fermi energy and by frequent enough inelastic scattering to establish a chemical potential μ_c. Connected to the reservoir is the disordered region, in which elastic scattering of electrons takes place and through which electrons are exchanged between the reservoir and the 2D EG. As argued by Büttiker,[6] the property of the contact is determined by the scattering matrix of the disordered region, which is characterized by the quantities T_{mi}, T'_{im}, and R_{ji}, where i or j refers to the ith or jth edge channel and m refers to a state in the reservoir. Here, T_{mi} is the probability of an electron in the ith incident channel to be transmitted to a state m in the reservoir and leave the sample, T'_{im} is the transmission probability of an electron in the state m of the reservoir to the ith outgoing channel, and R_{ji} is the reflection probability of an electron in the ith incident channel into the jth outgoing channel. Here we assume that the sample is large enough that the contact does not interact with the edge channels located on the opposite side of the sample. The problem of a contact interacting with the edge channels on the opposite side of a sample has been addressed by the work of Peeters[7] and Akera and Ando.[8] What influences the property of a contact is not the individual quantities T_{mi} or T'_{im} but the sums of these quantities over all states in the reservoir, $T_i \equiv \sum_m T_{mi}$ and $T'_i \equiv \sum_m T'_{im}$, as schematically shown in Fig. 1(a). We assume that energy dependence of T_i, T'_i, and R_{ij} is negligible in a small energy interval near the Fermi energy. The property of a contact is then characterized by $2n+n^2$ parameters consisting of T_i, T'_i, and R_{ji}. Each of these parameters takes a value from zero to one. They may depend on magnetic field, but the symmetry relations, $T_{mi}(\mathbf{B})=T'_{im}(-\mathbf{B})$ and $R_{ji}(\mathbf{B})=R_{ij}(-\mathbf{B})$, have to be satisfied because of microreversibility; hence,[6,14]

$$T_i(\mathbf{B})=T'_i(-\mathbf{B}) \quad \text{and} \quad R_{ji}(\mathbf{B})=R_{ij}(-\mathbf{B}) \; . \tag{1}$$

Since an electron incident from the ith edge channel on the contact is either transmitted to the reservoir to leave the 2D EG or reappear in the 2D EG being reflected to one of the outgoing channels,

$$T_i+\sum_j R_{ji}=1 \tag{2}$$

has to be satisfied for each channel i. Here the sum is made for all the Landau levels $(j=0,1,\ldots,n-1)$. Since the same must hold in the reversed magnetic field, we have $T_i(-\mathbf{B})+\sum_j R_{ji}(-\mathbf{B})=1$, which is translated through the relations in (1) into

$$T'_i+\sum_j R_{ij}=1 \; . \tag{3}$$

By adding the difference of Eqs. (2) and (3) over all i, we have[6]

$$T=T' \; , \tag{4}$$

where $T\equiv\sum_i T_i$ and $T'\equiv\sum_i T'_i$ are the total transmission probabilities. Out of the former $2n+n^2$ parameters, n^2 variables suffice to fully characterize a contact because of Eqs. (2) and (3). A contact is called ideal if $T=n$ ($T_i=T'_i=1$ and $R_{ij}=0$), and is called disordered if $T<n$.

Let us consider a situation where the ith incident channel is occupied up to an energy μ_i, where μ_i is defined by the relation given at the beginning of this section. Figure 1(b) may be useful for the argument given below. The current carried by the ith incident channel is given by $J_i^{(1)}=(2e/h)(\mu_i-\mu_x)$ considering spin degeneracy.[6,14] Here μ_x is an arbitrary energy assumed to be smaller or equal to the lowest of all the chemical potentials in any electron reservoirs attached to the 2D EG. The fraction of this current which is transmitted to the reservoir and leaves the sample is $J_i^{(2)}=(2e/h)T_i(\mu_i-\mu_x)$. On the other hand, when the chemical potential of the reservoir is μ_c, the current injected from the reservoir into the ith outgoing channel is $J_i^{(3)}=(2e/h)T'_i(\mu_c-\mu_x)$. If the total net current fed from the reservoir to the 2D EG is J, the chemical potential μ_c of the reservoir adjusts itself so that $J=\sum_i (J_i^{(3)}-J_i^{(2)})$. Solving this equation by use of Eq. (4), we have

$$\mu_c=\sum_i (T_i/T)\mu_i+(h/2Te)J \; . \tag{5}$$

The current emitted from the contact into the ith outgoing channel comprises $J_i^{(3)}$ given above and the current $J_i^{(4)}=(2e/h)\sum_j R_{ij}(\mu_j-\mu_x)$ reflected from incident channels. On the other hand, if the ith outgoing channel is occupied up to energy μ'_i, the current carried by this channel must equal $J_i^{(5)}=(2e/h)(\mu'_i-\mu_x)$. Here, the energy μ'_i is defined in the same way as μ_i. The relation $J_i^{(3)}+J_i^{(4)}=J_i^{(5)}$ yields

$$\mu_i' = \sum_j [(T_i' T_j / T) + R_{ij}]\mu_j + (T_i'/T)(h/2e)J \qquad (6)$$

with the help of Eqs. (3)–(5). Equations (5) and (6) are the central results of this work, from which complete properties of a contact can be derived.

As shown in Fig. 1(b), there is yet another current $J^{(6)}$. This current enters the disordered region from the outside of the contact passing through the reservoir and is reflected back to the reservoir to leave the contact again. The presence of such current due to "external reflection" is necessary to assure the self-consistency of the present formalism of a contact.[6] However, we have not mentioned this current in the above because it does not affect net current J and does not influence our discussion given above.

From Eq. (6) we have $\bar{\mu}' - \bar{\mu} = (h/2ne)J$ by using Eqs. (2) and (4), where

$$\bar{\mu} \equiv \sum_{i=0}^{n-1} \mu_i / n \quad \text{and} \quad \bar{\mu}' \equiv \sum_{i=0}^{n-1} \mu_i' / n$$

are the mean values of μ_i and μ_i', respectively. Thus the difference of the mean energies of the edge channels on both sides of a contact is exactly quantized irrespectively of a fashion in which the incident and outgoing channels are occupied.[6] Particularly, the difference is zero when a contact acts as a voltage probe ($J=0$). Equation (5) indicates that when a contact acts as a voltage probe ($J=0$) the contact senses the ith incident channel selectively with the weight T_i/T. If incident channels are unequally occupied, it is only when T_i is the same for all i that the voltage-probe contact indicates a potential ($eV_c = \mu_c$) equal to $\bar{\mu}$. On the other hand, voltage-probe contacts by no means have μ_c outside the energy range of the incident edge channels; namely, $\min\{\mu_i\} \le \mu_c \le \max\{\mu_i\}$. It follows that, if all incident channels are equally occupied with $\mu_i = \mu$, any voltage-probe contact indicates correctly the energy μ of incident channels.[6] Generally, a voltage-probe contact works to partially equalize the populations of electrons among different edge channels as qualitatively pointed out by Büttiker.[6] This is explicitly described by Eq. (6) with $J=0$. When a contact serves as a current source (drain) under the condition of an equal occupation of incident channels ($\mu_i = \mu$), we have, from Eqs. (6) and (3),

$$\mu_i' = \mu + (T_i'/T)(h/2e)J \ . \qquad (7)$$

This equation indicates that a current-source contact populates the ith outgoing channel selectively to the degree T_i'/T. The equal population of outgoing channels is achieved only when T_i' is the same for all i. Especially, Eq. (6) shows that an ideal contact populates outgoing channels equally up to the energy $\bar{\mu} + (h/2ne)J$, which is equal to μ_c, irrespectively of the fashion in which the incident channels are occupied.

When incident channels are equally occupied ($\mu_i = \mu$), it is convenient to define a characteristic resistance of a contact by $R_c = (\mu_c - \bar{\mu}')/eJ$ and to represent R_c by the dimensionless quantity $\delta \equiv R_c/(h/2ne^2)$. Using Eq. (7) and noting that $\mu_c = \mu + (h/2Te)J$ from Eq. (5), R_c or δ is written as

$$\delta \equiv R_c(h/2ne^2) = (n/T) - 1 \ . \qquad (8)$$

We note that $R_c(\mathbf{B}) = R_c(-\mathbf{B})$ from Eqs. (1) and (4). The resistance R_c is finite unless the contact is ideal. Let us denote a four-terminal resistance in a multiprobe sample by $R_{ij,kl} = (V_i - V_j)/J_{kl}$, where $V_i - V_j$ represents a voltage difference between contacts i and j and current J_{kl} is transmitted from contact k to contact l. Let contacts 2 and 3 be ideal in Fig. 2(a). The resistance R_c of a contact 1 is given by the three-terminal resistance $R_{12,13}$ in Fig. 2(a), where the magnetic field points out of the plane of the figure. We can easily show that the two-terminal resistance $R_{12,12}$ or $R_{13,13}$ deviates from the quantized value $h/(2ne^2)$ by the factor δ; namely, $R_{12,12} = R_{13,13} = (1+\delta)(h/2ne^2)$.

Equation (6) does not imply that the distribution function of the ith outgoing channel, $f_i'(\varepsilon)$, is one for $\varepsilon < \mu_i'$ and zero for $\varepsilon > \mu_i'$. Generally, $f_i'(\varepsilon)$ is finite at least in the energy range between the highest and lowest energies of incident channels, $\min\{\mu_i\} < \varepsilon < \max\{\mu_i\}$. This is because there are finite probabilities, R_{ji} and R_{ii}, in which electrons are elastically scattered from incident channels to the outgoing channels. Further, $f_i'(\varepsilon)$ is finite in the energy range up to the chemical potential μ_c of the electron reservoir. For example, $f_i'(\varepsilon)$ is T_i' in the range $\mu \le \varepsilon \le \mu_c$, if the distribution function of each incident edge channel $f_i(\varepsilon)$ is one in the range $\varepsilon \le \mu_i = \mu$.

As mentioned above, the distribution function $f_i'(\varepsilon)$ of

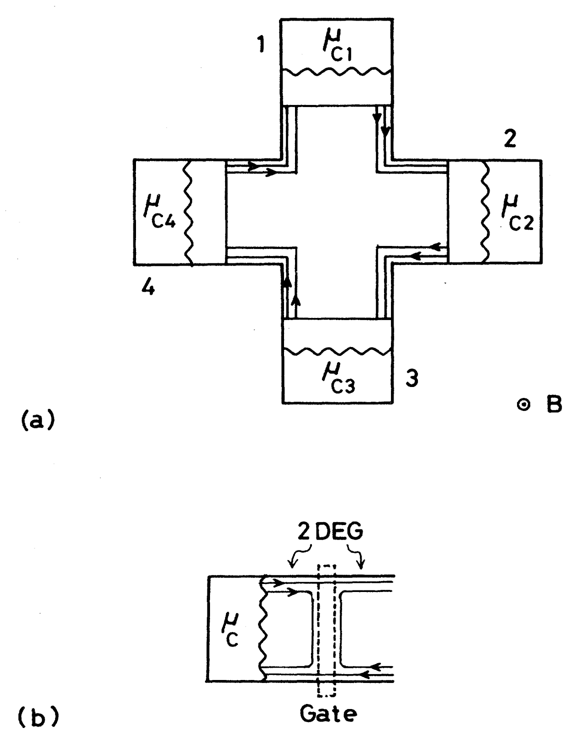

FIG. 2. (a) A sample with Hall bar geometry. (b) A specific example of a disordered contact, in which a gate produces a controllable reflection of different Landau levels.

each edge channel is not an equilibrium Fermi distribution function. The intrachannel nonequilibrium distribution will be relaxed through intrachannel inelastic scattering. Secondly, $f_i'(\varepsilon)$ can differ for different channels. This interchannel nonequilibrium distribution will be relaxed through interchannel elastic and/or inelastic scatterings. Experiments on $GaAs$-$Al_xGa_{1-x}As$ heterostructure devices show that the interchannel relaxation is unexpectedly weak, such that nonequilibrium populations of different edge channels do not equilibrate over a distance of order 100 μm.[10,11,13] However, these experiments do not provide evidence that the phase-coherence length is of comparable size. Intrachannel inelastic processes, probably due to electron-electron interaction, may be efficient to randomize the phase of electrons. The intrachannel scattering, however, does not cause observable effects in the experiments so long as the scattering matrices describing the contacts are independent of energy.

III. TWO-CHANNEL CASE

Let us consider a few important examples in some detail under the two-channel condition ($n=2$). The two-channel case is simplest to treat theoretically and has so far been the situation most extensively studied experimentally.[9–12] Equations (5) and (6) are rewritten into convenient forms as

$$\mu_c=\bar{\mu}+(\alpha/2)(\mu_0-\mu_1)+(1+\delta)(h/4e)J \tag{9}$$

and

$$\mu_i'=\bar{\mu}\pm(\beta/2)(\mu_0-\mu_1)+(1\pm\gamma)(h/4e)J\ , \tag{10}$$

where the plus and minus signs in Eq. (10) are for $i=0$ and $i=1$, respectively. Here,

$$\alpha\equiv(T_0-T_1)/T,\quad \gamma\equiv(T_0'-T_1')/T\ ,$$

$$\beta\equiv1-2(T_0T_1+R_{10}T_1+R_{01}T_0)/T\ ,$$

and

$$\delta\equiv(2/T)-1\ .$$

The property of any contacts in the two-channel condition can be conveniently characterized by parameters α, β, γ, and δ. Parameters α and β vary in the ranges $-1\le\alpha\le1$ and $0\le\beta\le1$ and describe properties when the contact serves as a voltage probe ($J=0$); α indicates the extent to which a contact selectively probes the energies of the two edge channels and it is the extent of deviation of μ_c from $\bar{\mu}$; β represents the degree of the contact equalizing the populations of edge channels. Parameters γ and δ vary in the ranges $-1\le\gamma\le1$ and $0\le\delta$ and describe the properties when a contact acts as a current source (drain); $\delta\equiv R_c/(h/4e^2)$ is the excess contact resistance; γ represents the degree to which the contact selectively populates the two edge channels. Note that an ideal contact is characterized by $\alpha=\beta=\gamma=\delta=0$. The symmetry relations

$$\alpha(\mathbf{B})=\gamma(-\mathbf{B})\ ,$$

$$\beta(\mathbf{B})=\beta(-\mathbf{B})\ ,$$

and

$$\delta(\mathbf{B})=\delta(-\mathbf{B})$$

are derived from Eqs. (1) and (4). These parameters are not completely independent of each other. For instance, we can easily show that the amplitudes of α and γ are limited to δ. When α and δ are given, the maximum value of β is limited to $(\delta+\alpha^2)/(\delta+1)$ corresponding to the case when $R_{01}=R_{10}=0$. On the other hand, $\beta=0$ (perfect equalization) is always possible even when the rest parameters α, γ, and δ are finite, if $R_{10}=R_{00}$ and $R_{01}=R_{11}$.

Let us consider a sample shown in Fig. 2(a), for which we assume only contact 2 to be ideal. We specify the energy of the ith channel incident on contact k by μ_{ik}, the energy of the ith channel leaving contact k by μ_{ik}', and the chemical potential of the reservoir of contact k by μ_{ck}. Let us denote the parameters α, β, γ, and δ for contact k by α_k, β_k, γ_k, and δ_k, respectively. The energies of edge channels will partially equalize during the travel of an edge current from one contact to another contact due to interchannel mixing processes caused by elastic and/or inelastic scattering. The effect is equivalent to the case in which an imaginary voltage-probe contact is attached to the sample boundary.[6,16] Let us denote the parameter β of such virtual contact on the sample boundary between contacts k and l by β_{kl} or β_{lk}. We transmit current J_{24} from contact 4 to contact 2. Since contact 2 is ideal ($\mu_{c2}=\mu_{c3}=\mu_{04}=\mu_{14}$), we have

$$R_{34,24}=R_{24,24}=(\mu_{c3}-\mu_{c4})/eJ_{24}=(1+\delta_4)(h/4e^2) \tag{11}$$

using $\mu_{c4}=\mu_{c3}-(1+\delta_4)(h/4e)J_{24}$ from Eq. (9). Equation (10) says that $\mu_{i4}'=\mu_{c3}-(1\pm\gamma_4)(h/4e)J_{24}$ for $i=0$ and 1 edge channels leaving contact 4. Equation (10) indicates that these energies change to $\mu_{i1}=\mu_{c3}-(1\pm\beta_{14}\gamma_4)(h/4e)J_{24}$ when the electrons reach contact 1. Hence, the Hall resistance $R_{31,24}=(\mu_{c3}-\mu_{c1})/eJ_{24}$ is given by

$$R_{31,24}=(1+\alpha_1\beta_{14}\gamma_4)(h/4e^2) \tag{12}$$

from Eq. (9). There is a deviation from the quantization by a factor $\alpha_1\beta_{14}\gamma_4$. A consideration similar to the above for opposite magnetic field leads to the resistance

$$R_{31,24}(-\mathbf{B})=-(1+\alpha_3\beta_{34}\gamma_4)(h/4e^2)\ ,$$

which demonstrates that the Hall resistance is not symmetric about $\mathbf{B}$.[6,14] Equation (12) shows that the exact quantization can be obtained irrespectively of the contacts if the interchannel scattering is strong enough to establish $\beta_{14}=0$. Similarly, it is trivial that a more accurate quantization is generally obtained if one or more contacts are indeed attached to the sample edge between contacts 1 and 4. This has been confirmed experimentally in Ref. 11. Equation (12) indicates that the maximum amplitude of possible deviation is limited to $\delta_1\delta_4(h/4e^2)$, since $|\alpha_1\gamma_4|$ is smaller than $\delta_1\delta_4$ as mentioned earlier. It follows from Eqs. (11) and (12) that

$$R_{14,24}=(\delta_4-\alpha_1\beta_{14}\gamma_4)(h/4e^2)\ . \qquad (13)$$

When the current contacts and the voltage-probe contacts are interchanged at the same time of the field reversal, the following relations are derived:

$$\mu'_{i1}=\mu_{c2}-(1\pm\gamma_1)(h/4e)J_{31}$$

from Eq. (10) and

$$\mu_{c4}=(\mu'_{01}+\mu'_{11})/2+(\alpha_4\beta_{14}/2)(\mu'_{01}-\mu'_{11})$$

from Eq. (9). Hence we have the resistance

$$R_{24,31}(-\mathbf{B})=(1+\alpha_4\beta_{14}\gamma_1)(h/4e^2)\ , \qquad (14)$$

which shows that $R_{31,24}(\mathbf{B})=R_{24,31}(-\mathbf{B})$ since $\alpha(\mathbf{B})=\gamma(-\mathbf{B})$ and $\beta(\mathbf{B})=\beta(-\mathbf{B})$ in any contact. The field-reverse reciprocity symmetry of a four-terminal resistance, $R_{kl,mn}(\mathbf{B})=R_{mn,kl}(-\mathbf{B})$, has been derived from general arguments without assuming particular properties of conductors or contacts.[14,16,17] The symmetry has been experimentally observed in different systems.[17–19] We have shown in the above that the reciprocity symmetry $R_{31,24}(\mathbf{B})=R_{24,31}(-\mathbf{B})$ in the present example is guaranteed by the symmetry in the properties of contacts. This symmetry of contacts has recently been observed experimentally.[20]

Figure 2(b) shows a system comprising an ideal contact and a cross gate spanning the 2D EG channel connected to the contact. A gate electrode is insulated from the 2D EG and produces a potential barrier for electrons when it is negatively biased with respect to the 2D EG. Samples with a cross gate have been studied experimentally in several groups.[9–11,20–22] Contacts equivalent to the structure shown in Fig. 2(b) have been used by van Wees *et al.*[12,13] In order to demonstrate the usefulness of the analysis presented in this work, we will show that the variety of experimental results[9–13,20–22] can be understood on a common basis. Since the potential barrier underneath the gate causes a reflection of edge currents, the system as a whole can be regarded as a specific example of disordered contacts. If the potential induced by the gate slowly varies everywhere in the 2D EG in such a way that the potential difference ΔU at a distance of magnetic length $l=(\hbar/eB)^{1/2}$ is much smaller than the energy separation between Landau levels $\hbar\omega_c$, mixing of different Landau levels due to scattering at the barrier may be ignored. This may be probable in the experimental situation, where a gate electrode is placed at a distance much larger than the magnetic length l from a 2D EG. This makes it reasonable to assume the parameters R_{01} and R_{10} of the complex contact to be zero and hence $T_i=T'_i$: The property of the contact can be completely determined merely by T_0 and T_1, which are identical to the transmission probabilities of an electron across the barrier underneath the gate. As shown in the top of Fig. 3, the parameters T_0 and T_1 are both unity when the potential barrier height W is zero. When the gate is negatively biased, the barrier causes a reflection first in the first Landau level to reduce T_1, and the perfect reflection of the first Landau level ($T_1=0$) is achieved when $W=\hbar\omega_c$ is approached. The barrier does not cause a

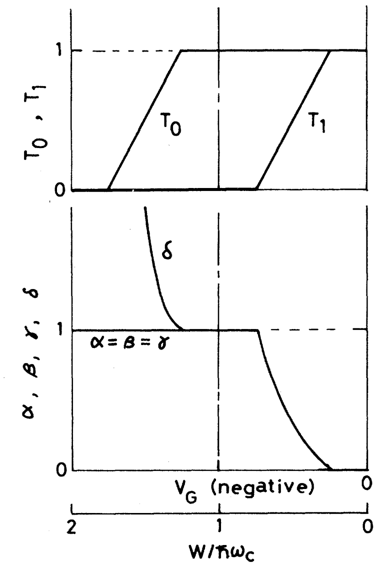

FIG. 3. Characteristics of the disordered contact shown in Fig. 2(b) in the two-channel case. Transmission probabilities T_0 and T_1, and the parameters α, β, γ, and δ are schematically shown as a function of negative gate-bias voltage, V_G, with respect to the 2D EG.

reflection in the zeroth Landau level until the gate is more strongly biased such that W exceeds $\hbar\omega_c$ to a certain extent. It follows that $T_0=1$ and $T_1=0$ in a finite range of V_G. The zeroth Landau level causes perfect reflection ($T_0=0$) when $W=2\hbar\omega_c$ is approached. It follows that the parameters, $\alpha=\gamma=(T_0-T_1)/(T_0+T_1)$, $\beta=1\ -2T_0T_1/(T_0+T_1)$, and $\delta=2/(T_0+T_1)-1$, vary with V_G as shown in the lower half of Fig. 3: α, β, and γ are equal to each other in the entire range of V_G; they change their values from zero to one as V_G decreases in a vicinity of $W=\hbar\omega_c/2$ and remain one with further decreasing V_G; δ is equal to α, β, and γ until it departs from one to increase indefinitely as V_G decreases in the vicinity of $W=3\hbar\omega_c/2$. The essential features of the contact described above are unaffected by the detailed construction of the contact. The size and the geometrical shape of the gate, the mobility of the 2D EG, and the temperature at which the contact is operated, do not influence the essential features: They will affect only detailed features such as the plateau width of δ in the vicinity of $W=\hbar\omega_c$ and the detailed line shape of $\alpha=\beta=\gamma=\delta$ versus V_G in the vicinity of $W=\hbar\omega_c/2$.

Suppose that contact 4 of the sample shown in Fig. 2(a) is replaced by the gated contact discussed above. Equations (11)–(13) indicate that, when $R_{31,24}$, $R_{34,24}=R_{24,24}$, and $R_{14,24}$ are measured as a function of V_G, they start from the quantized values $R_{31,24}=R_{34,24}=R_{24,24}=h/4e^2$, and $R_{14,24}=0$, and show plateaus at the values $R_{31,24}=(1+\alpha_1\beta_{14})(h/4e^2)$, $R_{34,24}=R_{24,24}=h/2e^2$, and $R_{14,24}=(1-\alpha_1\beta_{14})(h/4e^2)$, respectively, in the vicinity of V_G giving $W\sim\hbar\omega_c$. The resistance $R_{34,24}$ or $R_{24,24}$ in the plateau range of V_g is quantized to $h/2e^2$ irrespectively of the property of contact 1 but the other resistances deviate from $h/4e^2$ by a factor $\alpha_1\beta_{14}$. It is also predicted that $R_{31,24}$ shows an anomalously wide plateau developing over the entire range of V_G corresponding to $1 \lesssim W/(\hbar\omega_c) \lesssim 2$ while the plateaus of $R_{32,24}=R_{24,24}$ and $R_{14,24}$ are narrower. These features have been clearly observed in experiments.[10,20,23,24] Results reported in Refs. 21 and 22 correspond to the specific case where either α_1 or β_{14} is nearly zero.[23] When contacts 1 and 4 are both replaced by the gated contacts, and if $\beta_{14}=1$, the resistances $R_{31,24}, R_{34,24}=R_{24,24}$, and $R_{14,24}$ are quantized to $h/2e^2$, $h/2e^2$, and 0 for the gate voltages of the contacts both corresponding to $W \sim \hbar\omega_c$. This configuration has been studied in Ref. 12.

IV. TRANSMISSION AND DISSIPATION OF ENERGY

When current J is passed through a conductor from a reservoir of chemical potential μ_c to another reservoir of chemical potential μ_c', energy has to be fed into the system with the rate $P=(\mu_c-\mu_{c'})J/e$. In our situation where the 2D EG is in a dissipationless state, it is interesting to ask where and how the energy can be dissipated. Büttiker has argued that the quantized Hall resistance $R_H=h/(2ne^2)$ is a "contact resistance" in the sense that the energy dissipation occurs totally in the contacts.[6] He further argued that the dissipation totally occurs in the current sink but does not occur at all in the current source if the contacts are ideal.[6] When there are unequal occupations in edge channels, energy dissipation occurs also in voltage-probe contacts.[6] These problems need a quantitative examination. Let us take the sample shown in Fig. 2(a), where we assume n Landau levels are occupied. Let contacts 4 and 2 be a current source and a current sink, respectively ($\mu_{c4}>\mu_{c2}$). Suppose contact 4 is disordered and the other contacts are ideal. Let contact 4 be characterized by δ (R_c), T_i', and $T\equiv\sum_i T_i'$, etc. We need a total power

$$P_t=(\mu_{c4}-\mu_{c2})J/e=(h/2Te^2)J^2$$

to transmit current J from contact 4 to contact 2 since $\mu_{c4}=\mu_{c2}+(h/2Te)J$ from Eq. (5). We will consider the problem at absolute zero temperature. The results are the same also when considered at finite temperatures. The Landau states with energies $\varepsilon \le \mu_{c2}$ are completely occupied, and the energy fluxes carried by those electrons with $\varepsilon \le \mu_{c2}$ on the lower and the upper boundaries of the sample cancel each other. The net energy flux emitted from contact 4 into the ith edge channel is given by

$$P_i=\int_{\mu_{c2}}^{\infty}(\varepsilon-\mu_{c2})v_i\rho_i f_i\,d\varepsilon\ .$$

Here, $v_i(\varepsilon)$, $\rho_i(\varepsilon)$, and $f_i(\varepsilon)$ are, respectively, the velocity of electrons, the density of states, and the distribution function of the ith edge channel on the upper boundary of the sample. Using the relation $v_i\rho_i=2/h$, we have

$$P_i=(2/h)\int_{\mu_{c2}}^{\infty}(\varepsilon-\mu_{c2})f_i\,d\varepsilon\ . \qquad (15)$$

Here, $f_i(\varepsilon)=T_i'$ for the energy interval of $\mu_{c2}\le\varepsilon\le\mu_{c4}$ and $f_i(\varepsilon)=0$ for $\mu_{c4}<\varepsilon$. Hence,

$$P_i=T_i'(\mu_{c4}-\mu_{c2})^2/h=(T_i'/T^2)(h/4e^2)J^2\ .$$

The total net flux emitted from contact 4 is $P^{(4)}=\sum_i P_i=(h/4Te^2)J^2$. This flux is exactly half of the total power necessary to drive the system, $P_t=(h/2Te^2)J^2$. The other half of the total power, $P_t-P^{(4)}=(h/4Te^2)J^2$, has been dissipated in the electron reservoir of current-source contact 4. This is because electrons with energies $\varepsilon<\mu_{c4}$ are extracted from the reservoir of contact 4, where the chemical potential is μ_{c4}. The holes so created in the reservoir have to be refilled with electrons through inelastic scattering to maintain equilibrium. When the energy flux emitted from contact 4 reaches voltage-probe contact 1, complete equilibration of electrons takes place so that the distribution function changes to $f_i(\varepsilon)=1$ for $\varepsilon<\mu_{c1}$ and $f_i(\varepsilon)=0$ for $\varepsilon>\mu_{c1}$, where $\mu_{c1}=\mu_{c2}+(h/2ne)J$. Thus, from Eq. (15), contact 1 remits energy flux $P^{(1)}=(h/4ne^2)J^2$. This amount of energy flux $P^{(1)}$ is fed to the electron reservoir of contact 2 and dissipated there through inelastic scattering. The difference between $P^{(4)}$ and $P^{(1)}$, which is equal to

$$\Delta P=(T^{-1}-n^{-1})(h/4e^2)J^2=\delta(h/4ne^2)J^2=R_cJ^2/2\ ,$$

is dissipated partially in the electron reservoir of voltage-probe contact 1 and partially on the sample boundary between contacts 4 and 1, depending on the extent of the sample edge equilibrating the energies of edge channels. Thus, when there is nonequilibrium distribution of electrons in edge channels, a net energy flux flows into the voltage-probe contact to be dissipated there. This is because, whereas net current flow into the contact is zero, electrons with energies $\varepsilon>\mu_{c1}$ are fed into the reservoir and electrons with $\varepsilon<\mu_{c1}$ are extracted from the reservoir. It is only when incident edge channels are completely occupied up to μ_{c1} that the net energy flow into the voltage-probe contact is absent.

We have needed additional power

$$P_t-(h/2ne^2)J^2=\delta(h/2ne^2)J^2=R_cJ^2$$

to drive the disordered contact. Half of it, $\delta(h/4ne^2)J^2$, has been dissipated within the reservoir of contact 4 to create partial distributions in edge channels, and the other half has been transferred to the 2D EG in the form of the partial distributions.

V. DISCUSSION

It is sometimes stated in literature based on a Landauer-type approach that edge Landau states carry the current but bulk Landau states in the interior region of the 2D EG do not contribute to the current.[6,12,22] However, as we noted at the beginning of Sec. II, the basic equation $J=(2e/h)\sum_i(\mu_i'-\mu_i)$ used in the approach is independent of a detailed profile of the dispersion $\varepsilon(k)$ of a Landau level. In other words the validity of the treatment is independent of whether or not the group velocity of electrons in the interior region of the 2D EG channel is vanishing. In the regime of IQHE, the bulk Landau states at the Fermi level are localized and do not contribute to the net current. However, the complete bulk states below the Fermi level, as a whole, can carry a net current. The problem of how the local current, or the local electric field, is distributed in a 2D EG channel under a given condition of a total current being transmitted has been studied theoretically from several different approaches by assuming a system at absolute zero temperature, and a variety of predictions have been derived.[25–29] Roughly described, the current is strongly confined to the vicinity of the boundaries of a 2D EG in the approaches applied by Heinonen and Taylor[26] and Johnston and Schweitzer,[27] while the current is carried essentially by bulk Landau states in the approaches of MacDonald *et al.*,[25] Ono and Ohtsuki,[28] and Gudmundsson *et al.*[29] Another important aspect is that experiments are made at low, but not absolute zero, temperatures. This implies that energy loss of real 2D EG systems can never be zero; however, it is small. A finite loss works to uniformly distribute the current across the sample, as pointed out by Thouless.[30] The possibility of an extremely small (but finite) loss of a real system being important in the determination of a real current distribution has been completely disregarded in the existing theoretical studies. Experimentally, an accurate quantization of the Hall resistance is established with a total current magnitude corresponding to a chemical potential difference between the sample boundaries much larger than the Landau level spacing; i.e., $\mu'-\mu \gg \hbar\omega_c$.[31,32] It appears to the present authors to be quite reasonable to expect that a major portion of the current is being carried by bulk Landau states, at least, in such a condition. We emphasize again that all the discussion given in the present work is completely independent of the unsettled problem of how local current is distributed in the 2D EG.

We have noted the relation $\bar{\mu}=\bar{\mu}'$ from Eq. (6) for a contact acting as a voltage probe. This relation, which was first derived by Büttiker,[6] says that the mean energy value $\bar{\mu}$ of edge channels cannot change when the edge channels are subject to any kind of scattering (interchannel or intrachannel scattering, and elastic or inelastic scattering) except for the scattering into edge channels on the opposite side of the sample. This is a direct consequence of current conservation. On the other hand, we should note that the velocity $v_i(\varepsilon)$ of an electron (or the density of states) in an edge channel i with energy ε is generally dependent on channel i and varies with energy ε. On this basis we can consider a transient phenomenon in the following "gedanken" experiment. Imagine switching on a certain scattering mechanism at a certain moment at some location of a sample boundary. Whether the scattering is interchannel or intrachannel, it follows from the dependence of $v_i(\varepsilon)$ on i or on ε that the magnitude of the current leaving the scattering region is generally altered immediately after the onset of the scattering mechanism. The unbalance between the currents on the opposite sides of the scattering region results in a piling up of charge (whether positive or negative) on the side leaving the scattering region. The piling up lasts until the relation $\bar{\mu}=\bar{\mu}'$ is finally recovered so as to balance the currents on both sides of the scattering region. Thus what is conserved in a steady state is not the charge density but the current. (Note that "charge" must be conserved in any occasion but "charge density" can be altered.) Streda *et al.*[3] derived a different relation from Büttiker's and ours by assuming that the carrier density is unalterable despite the scattering, and their relation has been subsequently used by Sivan *et al.*[5] That the so-derived relation[3] does not agree with experimental results has been pointed out by Haug *et al.*[21]

It may be interesting to ask the question to which extent the present formalism of a contact is general. Büttiker's original formalism, adopted in this work, assumes that a contact is divided into two regions, one of which is characterized by strong enough inelastic scattering to establish a chemical potential (electron reservoir) and the other of which is characterized by the presence of elastic scattering and the absence of inelastic scattering (disordered region). This assumption appears to be a reasonable approximation of any realistic situation, but may not always be rigorously justified. There may be such contacts in which the regions of inelastic and elastic scattering cannot be clearly separated. To simplify the problem, we can imagine a case when a contact described by our formalism is connected to an external resistor R. If we regard the whole thing as a new contact, it will be most simply represented as a contact consisting of two disordered regions D and D', and two electron reservoirs of chemical potentials μ_c and μ_c' as shown in Fig. 4. Since the "external resistor" can be embedded in the contact, we may not be aware of whether or not a given contact is of such a complex structure. Suppose our contact is of the type shown in Fig. 4. We measure μ_c, analyze the property of the contact incorrectly assuming that the contact is of the simpler type shown in Fig. 1, and derive parameter values, $T_i^{(\mathrm{er})}$, $T_i'^{(\mathrm{er})}$, and $R_{ij}^{(\mathrm{er})}$. However, this error is not actually harmful. A contact described by $T_i^{(\mathrm{er})}$, $T_i'^{(\mathrm{er})}$, and $R_{ij}^{(\mathrm{er})}$ in the present formalism and the real complex contact are different only in that partial distributions of electrons in each of the Landau levels (introduced either by unequally occupied edge channels being incident on the contact or by a given current injection from the contacts) are different. Since the anomalous distribution within each of the Landau levels does not make a difference in the observations as noted earlier, the error does not cause observable effects in the transport properties unless the scattering matrices in the disordered regions depend on the energy. Hence no existing contacts may exhibit properties different from those described in this work, so long as the transport properties being studied are in the linear regime.

The disordered region of a contact may be characterized by an arbitrary combination of impurity potentials and tunneling barriers. Theoretically, different Landau

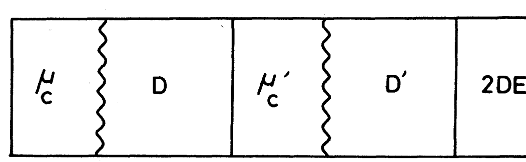

FIG. 4. An example of a generalized disordered contact consisting of two sets of electron reservoirs and disordered regions.

levels can have arbitrary values for the transmission probabilities T_i or T_i'. In realistic contacts, however, we suppose that T_i (T_i') should be larger for lower Landau levels. This is because the impurity potentials of a long force range tend to cause more reflection in higher Landau levels, similarly to the simplest case of the gate-bias-induced potential barrier discussed in this work and in Refs. 10, 12, 21, and 22. Tunneling barriers will also yield more reflection in higher Landau levels.[7,8] The impurity potentials of a short force range will work to cause mixing of Landau levels, diminishing the difference of T_i among different Landau levels. The character of the disordered contacts observed in the experiments is consistent with our conjecture here.[10,11]

VI. SUMMARY

Properties of a contact in a 2D EG at high magnetic fields have been analyzed in terms of transmission probabilities between the electron reservoir in the contact and the Landau levels in the 2D EG and reflection probabilities among different Landau levels, by utilizing a Landauer-type resistance formula originally applied by Büttiker to the analysis of contacts. General properties of a contact in the case when n Landau levels are occupied in the 2D EG (n-channel case) have been explicitly derived in terms of independent variables of number n^2 representing the transmission and reflection probabilities. The analysis has been applied to the two-channel case to calculate characteristic resistances of multiprobe samples. The magnetic-field–reverse reciprocity relation of the Hall resistance, $R_{kl,mn}(\mathbf{B})=R_{mn,kl}(-\mathbf{B})$, has been explicitly derived to show that the general validity of the relation in the particular system considered is guaranteed by the symmetry properties of contacts when they serve as voltage-probe contacts and when they serve as current-source contacts. Results of recent experiments made on samples with a cross gate or split gates have been explained on the basis of the present analysis of contacts. The formalism of a contact is supposed to be general and applicable to any existing systems.

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