Electron-heating effects and the electron-phonon scattering time in thin Sb films

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A simple electron-heating experiment is described which demonstrates, in an especially striking manner, that the electron temperature can be quite different from the lattice temperature. The results are used to determine the magnitude of the electron-phonon scattering time, which is found to be in good agreement with the theory.

I. INTRODUCTION

During the course of an experimental study of weak localization in thin Sb films, we have come across some behavior which was, initially, very surprising. Figure 1 shows results for the resistance of an Sb film as a function of temperature. The important point to note here is that R displays a minimum at $T_{\min} \approx 2.8$ K. The physics responsible for this is well understood, and will be discussed in detail below; at this point it is only important that the minimum exists. In Fig. 2 we give results for R as a function of an applied ac electric field, E, at a fixed value of T. In these measurements the resistance was determined with a small ac measuring field at one frequency using standard ac bridge techniques (hence R might be termed the differential resistance), while the ac field, E, was at a different frequency, and was applied independently. From Fig. 2 we see that at temperatures below $T_{\min}$ the resistance decreases as E is increased. This is not surprising since we would expect the Joule heating due to E to cause the temperature of the electron

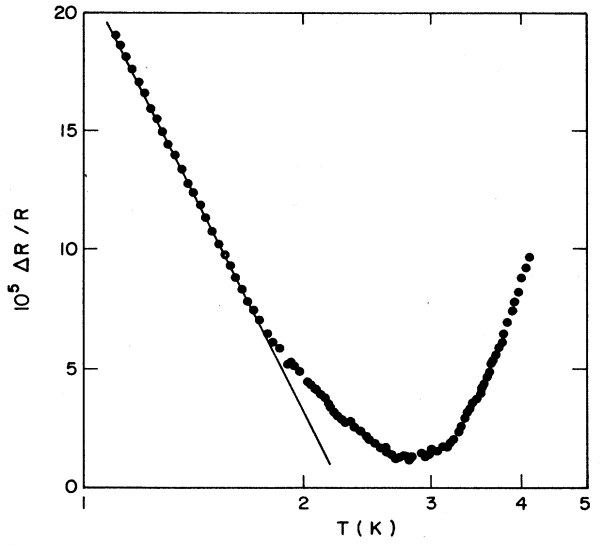
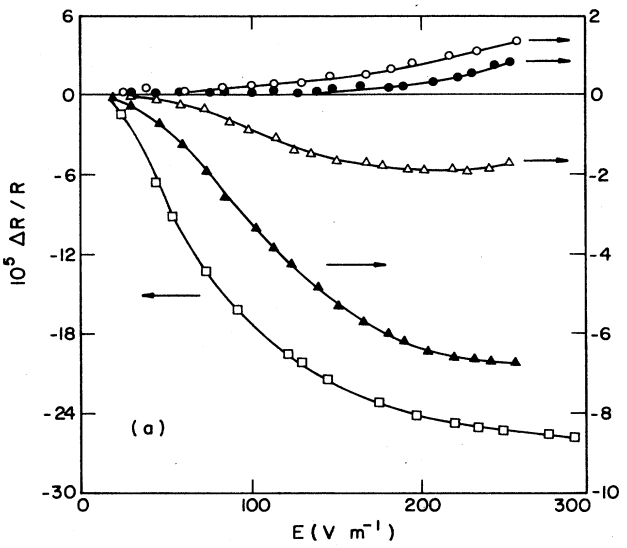


FIG. 1. Fractional resistance change, $\Delta R/R$, as a function of T, for an Sb film which had a sheet resistance $R_\square = 79\ \Omega$. The ac field, E, used to heat the electrons was zero for these measurements.



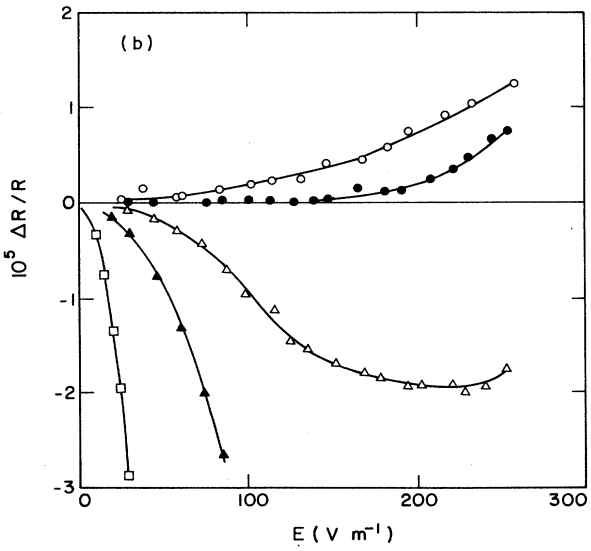


FIG. 2. $\Delta R/R$ as a function of E at several temperatures, for the same sample as considered in Fig. 1: ○, $T=4.20$ K; ●, $T=3.71$ K; △, $T=3.03$ K; ▲, $T=2.30$ K; □, $T=1.10$ K. (a) All of the data; note the different vertical scales for the different data sets. (b) Expanded view of the data for small $\Delta R/R$.

gas, T_e, to increase, and hence effectively move the resistance down the curve in Fig. 1. However, Fig. 2 also shows that for temperatures at and slightly above $T_{\min}$, the measured resistance *still* decreases with increasing E, and in fact the value measured in Fig. 2 can fall significantly *below* the value of R at $T_{\min}$. This clearly shows that one cannot understand the effect of Joule heating simply in terms of an increased effective temperature for the entire system. Rather, one must consider in detail how energy flows out of the electron gas, along with the possibility of having different parts of the system at different temperatures. While the need to allow for different electron and lattice temperatures has been noted by many previous workers, our results demonstrate this fact in a particularly striking manner. In the next section we will show how the behavior seen in Fig. 2 can be understood quantitatively, and use these results to determine the electron-phonon scattering time.

II. EXPERIMENT

The samples were produced by thermal evaporation of Sb onto room temperature glass substrates. They were typically 100–200 Å thick, and had sheet resistances, $R_\square$, in the range 10–200 Ω, corresponding to resistivities $\rho \approx 100\ \mu\Omega$ cm. The results given in this paper were obtained with a sample which had $R_\square = 79\ \Omega$ at 4 K; similar results were found for samples with other values of $R_\square$. The resistance was measured with a standard low frequency (200 Hz) bridge. This ac measuring field was always sufficiently small that it caused a negligible amount of Joule heating. An ac electric field at a different frequency, 3 Hz, was produced with a second voltage source. This electric field, which we denote as E, was used to heat the electrons. The sample was immersed directly in a bath of liquid helium; the temperature was controlled via the pressure of the helium, and was measured with a calibrated Ge thermometer.

Some typical results for the resistance as a function of temperature are shown in Fig. 1. These results were obtained with $E = 0$, i.e., the heating field was turned off. Figure 2 shows R as a function of E, for several different temperatures. At temperatures below $T_{\min}$ we find that R becomes smaller as E is increased. As noted in the Introduction, this is easily understandable in terms of simple Joule heating. As E is increased, the electron temperature increases, and one effectively moves to higher temperatures in Fig. 1. However, the magnitude of the change of R is much *larger* than one would expect from this simple argument. For example, at $T = 1.10$ K in Fig. 2, the relative (i.e., fractional) change, $\Delta R/R$, is more than 2.5×10^{-4} at the highest values of E, and comparing with Fig. 1 we see that this value of $\Delta R/R$ corresponds to a point *below* the value at the minimum. In addition, the results at $T = 3.03$ K in Fig. 2 show that for small E, R decreases with E, even though this temperature is well *above* the temperature at which R displays a minimum for $E = 0$, Fig. 1. The simplest heating model would predict that in this case R should increase with E, but this is definitely not found. Note, however, that at the highest E the resistance does increase slightly, and that at the highest temperatures in Fig. 2, R does increase monotonically with E. It seems clear from these results that an applied electric field does *not* simply lead to a higher overall effective temperature.

At these temperatures, three different effects contribute to the temperature dependent part of the resistance.[1–5] First, electron-electron interaction effects make a contribution

$$\frac{\Delta R}{R} = -R_\square \frac{e^2}{2\pi^2\hbar}\left(1-\tfrac{3}{4}F\right)\ln(T_e)\ , \tag{1}$$

where F is a screening factor whose value is predicted to be approximately in the range 0–1. In (1) we write the electron temperature, T_e, since it is the relevant temperature for this process. Second, there is a contribution from weak localization

$$\frac{\Delta R}{R} = -R_\square \frac{e^2}{2\pi^2\hbar}\ln(L_\phi)\ , \tag{2}$$

where L_ϕ is the phase breaking length. This length is related to the phase breaking time, τ_ϕ, by $L_\phi = \sqrt{D\tau_\phi}$, where D is the electron diffusion constant. In (2) we have assumed that the spin orbit scattering is strong, as is appropriate for Sb. Third, there is a contribution to the resistance from electron-phonon scattering through the Drude relation for the bulk resistivity, $\rho = ne^2\tau/m$. At temperatures above a few K, the temperature dependent part of τ in this expression leads to a significant temperature dependence of the resistance which yields[6]

$$\Delta R \sim T_{ep}^{-q}\ . \tag{3}$$

Here T_{ep} is the temperature associated with the electron-phonon scattering time, τ_{ep}, and we expect[7–10] T_{ep} to be near the lattice temperature T_{lattice}. The parameter q characterizes the temperature dependence[11] of τ_{ep}.

The contribution (1) decreases as the temperature increases, while (3) increases with temperature. The effect of localization, (2), also increases with temperature, since L_ϕ becomes smaller as T increases. At low temperatures electron-electron interactions, (1), dominate, while at high temperatures the direct contribution of electron-phonon scattering, (3), dominates. With this in mind, we can now qualitatively understand the behavior seen in Fig. 2. When E heats the electrons, T_e increases, and the interaction contribution (1) decreases. However, the temperature of the lattice, T_{lattice}, will be largely unaffected by E, so the direct contribution of electron-phonon scattering, (3), will change relatively little.[7] If, for the moment, we assume that the phase breaking is due to electron-phonon scattering, then τ_ϕ and L_ϕ, and hence the localization contribution (2), will also be little changed. The net result is that R will decrease, even at temperatures above $T_{\min}$ in Fig. 1. This also explains why R can assume values below the value at $T_{\min}$. This argument will break down at high temperatures, and at large values of E, as in these cases even a relatively small change in T_{lattice} will, because of the rapid temperature dependence of the direct contribution of electron-phonon scattering,[12] (3), dominate the behavior of R. If the phase

breaking, i.e., L_ϕ, is determined by electron-electron scattering, then the localization contribution, (2), will change with E. However, since the electron-phonon scattering time, and hence (3), will still be largely independent of E, the same qualitative behavior will be found, and it will still be possible for R to take on values below the minimum in Fig. 1.

To analyze the behavior quantitatively, we first consider the situation at the lowest temperatures in Fig. 1, for which R varies logarithmically with T. Here, the temperature dependence of the direct electron-phonon contribution, (3), is negligible, and only interactions and localization are important. The logarithmic slope observed in Fig. 1 can then be compared with the sum of (1) and (2). Since E was zero for the measurements in Fig. 1, all of the relevant temperatures, T_e, T_{ep}, etc., are the same, and are equal to the bath temperature, which we will denote simply as T. The temperature dependence of τ_ϕ is generally given by a power law, $\tau_\phi \sim T^{-p}$, where the parameter p depends on the inelastic process which is involved. As a result, the sum of (1) and (2) can be written in the well-known form

$$\frac{\Delta R}{R} = -R_\square \frac{e^2}{2\pi^2\hbar}\left[\frac{-p}{2}+(1-\tfrac{3}{4}F)\right]\ln(T) \ . \tag{4}$$

The logarithmic slope observed in Fig. 1 can thus be used to estimate the quantity $-p/2+(1-3F/4)$ in (4). From Fig. 1 we find that this quantity is $\approx 0.27\pm 0.01$. Much work on weak localization in recent years[1-5] has shown that just two inelastic processes need to be considered in our case, electron-electron and electron-phonon scattering. For two dimensional electron-electron scattering we expect[2-4] $p=1$, while for electron-phonon scattering one generally finds[5,13-17] a value which lies somewhere between 1.5 and 3. If both processes are important, τ_ϕ^{-1} will be given by the sum of the electron-electron and electron-phonon scattering rates, and the effective value of p is (usually weakly) temperature dependent. To determine p we have conducted magnetoresistance measurements. Of the three contributions to R, (1)–(3), only weak localization has an appreciable field dependence at low fields, and it has been well established[1,18] that this dependence can be used to accurately measure L_ϕ. Results for L_ϕ for a sample which had nearly the same $R_\square$ as the one considered in Figs. 1 and 2, are shown in Fig. 3, where it can be seen that L_ϕ follows a power law $L_\phi \sim T^{-p/2}$ with $p=1.3\pm 0.1$. Combining this with the result for the logarithmic slope in Fig. 1, we find $(1-3F/4)\approx 0.90$, hence $F\approx 0.12$, which is consistent with theoretical expectations.

We next consider[19-22] how the change of R due to heating is related to τ_{ep} and E. The energy generated by the electric field is removed from the electron gas only via electron-phonon scattering. The rate at which this process brings the electron system into equilibrium with the lattice is

$$\frac{dT_e}{dt} = -\frac{T_e - T_{\text{lattice}}}{\tau_{ep}^E} \ . \tag{5}$$

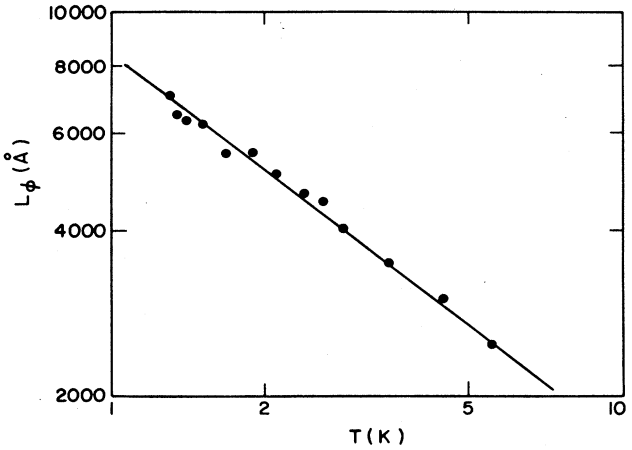


FIG. 3. Phase breaking length, L_ϕ, as a function of T, obtained from magnetoresistance measurements for an Sb film with $R_\square = 77\ \Omega$. The solid line is a fit to a power law which yielded $p=1.3\pm 0.1$.

The relaxation time is written here as τ_{ep}^E to distinguish the energy relaxation time from the scattering time, τ_{ep}, which is relevant for phase breaking. These two quantities are, of course, closely related, and theoretical work[9,10] has shown that they differ by a constant factor which is of order unity. We will return to this point below. Joule heating deposits energy, U, at the rate

$$\frac{dU}{dt} = I^2 R \ , \tag{6}$$

where R is the electrical resistance and I is the current. Since

$$\frac{dT_e}{dt} = \frac{1}{C_v V}\frac{dU}{dt} \ , \tag{7}$$

where C_v is the electronic heat capacity per unit volume, V, equating the energy input with the energy loss we find

$$T_e - T_{\text{lattice}} = \tau_{ep}^E \frac{I^2 R}{C_v V} = \tau_{ep}^E \frac{E^2}{\rho C_v} \ . \tag{8}$$

If T_e is known for a given value of E, then (8) can be used to determine τ_{ep}^E. We should note that while the approach used to derive (8) seems quite plausible, and has been used successfully in interpreting previous experiments, it is conceivable that the assumption of well defined electron and lattice temperatures may not be correct. For example, at high temperatures where τ_{ep}^E is shorter than the electron-electron phase breaking time, τ_{ee}, one might well imagine that the electron system would not be in internal equilibrium, since there is no scattering process faster than τ_{ep}^E by which the electrons can exchange energy. This is an interesting problem which will hopefully attract more attention in the future. In any case, we now proceed to use (8) to analyze our results.

The effect of localization, (2), is determined by L_ϕ. Let us first assume that L_ϕ is dominated by electron-phonon scattering, and hence that $L_\phi = \sqrt{D\tau_{ep}}$; we will refer to

this as case (1) in the following. We will consider other possibilities below. With this assumption, T_{lattice} will, to a first approximation,[7,23] be the appropriate temperature for determining L_ϕ. This temperature should also be the one which determines the magnitude of the direct contribution of electron-phonon scattering to the resistivity, (3), and will be essentially independent of E. The quadratic behavior of R as a function of E seen in low fields in Fig. 2 is in accord with (8). From the data at 2.30 K in Fig. 2 we find $\Delta R/R = 1.0\times10^{-5}$ at $E = 50$ V/m. We next use this result for $\Delta R/R$ to obtain $T_e - T_{\text{lattice}}$. Since, by the above arguments, both the localization and direct electron-phonon contributions to R, (2) and (3), will be approximately independent of E, the variation seen in Fig. 2 must come solely from (1). We therefore use (1) together with the value of F estimated above to obtain the increase in T_e. Finally, using $\rho = 100\ \mu\Omega$ cm, and the known value for the electronic heat capacity,[24,25] in (8) we find $\tau_{ep}^E = 1.4\times10^{-10}$ s. If we had instead assumed that L_ϕ was dominated by electron-electron scattering, we would instead use (4) to convert the measured value of $\Delta R/R$ into an estimate of $T_e - T_{\text{lattice}}$; we will refer to this as case (2) in what follows. The end result in this case is a somewhat larger value of the electron-phonon scattering time, $\tau_{ep}^E = 4.8\times10^{-10}$ s. If we now use the prediction[26] that $\tau_{ep} \approx 1.2\tau_{ep}^E$, we find $\tau_{ep} = 1.7\times10^{-10}$ s and 5.8×10^{-10} s for cases (1) and (2), respectively.

Let us now compare these values of τ_{ep} with the phase breaking time obtained from the magnetoresistance measurements. Since these measurements yield L_ϕ, we must first estimate the diffusion constant D. Using free-electron theory we have shown previously[27] that for our Sb films $D \approx 25$ cm^2/s. Combining this with the results in Fig. 3, we find $\tau_\phi = 1.0\times10^{-10}$ s at 2.30 K. This value is smaller than the estimates of τ_{ep} obtained with the two different assumptions discussed above. Since in general, $\tau_\phi^{-1} = \tau_{ee}^{-1} + \tau_{ep}^{-1}$, this indicates that electron-electron scattering makes the dominant contribution to the phase breaking, although electron-phonon scattering cannot be completely neglected. Hence, case (2) considered above is most appropriate. Our results thus indicate that for Sb at 2.3 K, $\tau_{ep} \approx (5\pm2)\times10^{-10}$ s and $\tau_{ee} \approx (1.5\pm0.5)\times10^{-10}$ s. The value of p which was found can also be understood, since this value (1.3 ± 0.1) lies between the values expected for electron-electron and electron-phonon scattering.

We next consider how these results compare with the theory. The electron-electron phase breaking time predicted[4,28] for this value of $R_\square$ is 2.1×10^{-10} s at 2.30 K. Given the experimental uncertainties, particularly in D, our result for τ_{ee} is in very good agreement with this prediction.

The theory for electron-phonon scattering in a disordered metal film has been discussed by a number of workers.[5,13–17] Unfortunately, a comparison with experiment is complicated by several factors, which are not encountered when one treats electron-electron scattering. First, it is not clear how to model the phonons. The wavelength of a thermal phonon is of the order of the film thickness, so one might expect the phonons to be two-dimensional. However, while coupling with the substrate could act to make them three-dimensional, the effect of boundary scattering at the film-substrate interface adds further uncertainties. In addition, it has been shown theoretically that the temperature dependence of τ_{ep} is not, in general, given by a simple power law, but rather the "effective" value of p is temperature dependent. The behavior of p, and also the magnitude of τ_{ep}, depend on material parameters such as the velocity of sound, which are again very difficult to estimate for a thin film. These issues have been discussed at length elsewhere, so we need not go into greater detail here. It seems sufficient to make use of the recent theoretical work of Belitz and Das Sarma,[17] in which a careful treatment of the temperature dependence and magnitude of τ_{ep} was given, along with a comparison with the magnitude of the electron-electron phase breaking time. They considered the situation for several different metals, although they did not treat Sb specifically, and found values of τ_{ep} which fall into the range $\approx(2-10)\times10^{-10}$ s at 2 K, which is in good agreement with the value we find for Sb. They also found that at this temperature, the electron-electron scattering time is generally somewhat smaller than τ_{ep}. Thus, their results are in good agreement with our findings.

III. CONCLUSIONS

We have conducted a simple electron-heating experiment with thin Sb films. The value of τ_{ep} found from an analysis of the heating measurements is in good agreement with the theory. It should be possible to design similar experiments to obtain much more detailed information concerning the electron-phonon scattering times in these systems.[29]

ACKNOWLEDGMENTS

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[6]See, for example, N. W. Ashcroft and N. D. Mermin, *Solid*

State Physics (Holt, Rinehart, and Winston, New York, 1976).

[7]Strictly speaking, the value of τ_{ep} will depend on both the lattice and electron temperatures. The behavior of τ_{ep} as a function of T_{lattice} and T_e has been discussed theoretically by several workers (Refs. 8–10). From those calculations one finds that the effective temperature which characterizes τ_{ep}, which we have denoted by T_{ep}, is closer to T_{lattice} than to T_e. For example, with $T_e = 2T_{\text{lattice}}$, the theory (Ref. 10) predicts $T_{ep} \sim 1.3T_{\text{lattice}}$. This justifies our assumption that T_{ep} changes much less than T_e.

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[11]For electron-phonon scattering in a clean system $\tau_{ep} \sim T^{-3}$, and together with a factor which takes into account the momentum transfer in the scattering process (Ref. 6), one finds $q = 5$. In a disordered system the temperature dependence of τ_{ep} is different and the momentum transfer factor will presumably also change, and hence the value of q will be different. For our purposes here the precise value of q is not important.

[12]The effect of T_e on τ_{ep} must also be taken into account in this case.

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[23]Strictly speaking, there will again be a correction for the effect of T_e on τ_{ep}, as discussed in Ref. 7, and this will affect the contributions (2) and (3). The size of this effect with regards to L_ϕ and (2) depends on the temperature dependence of τ_{ep} (see Ref. 10), but even in the worse case it changes the estimate of $T_e - T_{\text{lattice}}$ by only a small amount, so we have not included it here [for further discussion see J. Liu, Ph.D. thesis, Purdue University (unpublished)]. Note also that for the situation in which L_ϕ is determined by electron-electron scattering, which we will find below is appropriate for Sb, the determination of $T_e - T_{\text{lattice}}$ does not depend on these considerations. As for the contribution (3), since it is only weakly temperature dependent at temperatures much below T_{min}, the effect of T_e on τ_{ep} will be negligible at the temperatures of interest to us in the following.

[24]Using the handbook value for the electronic heat capacity of Sb [see, for example, C. Kittel, *Introduction to Solid State Physics,* 6th ed. (Wiley, New York, 1986)] gives $C_v = 6.0$ J/K m^3 at 1 K.

[25]The values of E in Fig. 2 are the amplitude of the ac field. These values were converted to rms before they were used in (8).

[26]The factor which relates τ_{ep}^E to τ_{ep} depends on the temperature dependence of τ_{ep}; i.e., the factor q in (3). The value we quote here is predicted for a disordered system (Ref. 10). For a clean system this factor is predicted to be 2.6 (Refs. 9 and 10).

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[28]This value of τ_{ee} was obtained from Eq. 4.47a of Ref. 4.

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