Brief Reports

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Attenuation coefficient of first sound in liquid ^{4}He

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In consideration of a finite lifetime of thermal phonons and other factors which were neglected in our recent work, a new improved formula for the attenuation coefficient of first sound in liquid helium is derived.

The temperature dependence of the velocity of first sound in liquid helium is a basic subject which has been studied experimentally for many years,[1] resulting in significant information. For instance, a study of the ultrasonic attenuation coefficient led Maris and Massey[2] to the conclusion that the energy dispersion in the liquid is actually "anomalous" rather than "normal." Conclusive experimental evidence concerning the absence of a quadratic term in the dispersion relation was obtained by Roach, Abraham, Ketterson, and Kuchnir[3] through a direct measurement of the difference in velocity of simultaneous sound waves of different frequencies. The studies by Jackle and Kehr and Dynes and Narayanamurti[4] gave further information on the mechanism concerning sound propagation. On the other hand, in the pioneering theoretical work of Andreev and Khalatnikov,[5] and in a later article of Singh and Prakash,[6] the so-called normal energy dispersion was used. This and other aspects concerning the sound propagation in liquid helium have been studied.[7]

On the other hand, it has been known for many years that the attenuation coefficient of first sound shows a peak at around 1 K away from the transition point. Khalatnikov and Chernikova[8] noted that rotons play an important role for the temperature variation of first sound at low temperatures, but they did not use the anomalous dispersion. We have recently looked into the effect of the anomalous energy dispersion on the attenuation coefficient at low temperatures[9] and derived a formula:

$$\alpha(T)=\alpha_0(T)[1-(T/T^*)^2], \tag{1}$$

where

$$\alpha_0(T)=\frac{(u+1)^2\pi^3(k_BT)^4\omega}{30\rho_0\hbar^3c_0^6}, \tag{2}$$

$$T^*=\frac{c_0}{\pi k_B}\left(\frac{7}{60\gamma}\right)^{1/2}, \tag{3}$$

c_0 is the sound velocity, u is the Grüneisen's constant, ρ_0 is the mass density of liquid helium, and γ is the coefficient in the dispersion relation

$$\varepsilon=c_0p(1+\gamma p^2+\cdots). \tag{4}$$

γ is positive (anomalous dispersion) in liquid ^{4}He. Otherwise, T^* could not be introduced as in Eq. (3).

Equation (2) is of an interesting form, and yields $T^*=1.54$ K for $\gamma/\hbar=1.51$ Å^2. This temperature gives a peak of $\alpha(T)$ at $T_m=1.26$ K. However, experiments have shown that the peak can be below 1 K and depends on frequency. Therefore, improvement on the previous theory is necessary. The present Brief Report has been written for this reason.

At low temperatures, the limiting form of the attenuation coefficient is given by Eq. (2), although the numerical factor 30 may depend on specific theoretical conditions. For instance, Khalatnikov and Chernikova arrived at a formula with a factor 60. Such a difference can be understood when a finite lifetime of phonons is taken into consideration.

According to Kwok, Martin, and Miller, and Pethick and ter Haar,[10] the attenuation coefficient for such a case is given by

$$\alpha_0=\frac{\pi^2(u+1)^2(k_BT)^4}{30\rho_0\hbar^3c_0^6}\omega[\tan^{-1}(2\omega\tau)+\tan^{-1}(3\gamma p^2\omega\tau)], \tag{5}$$

where p is the average phonon momentum and the sign of the second term has been changed because the formula was derived originally for $\gamma<0$. Since $3p$ processes were used, it could be used for the anomalous case by this sign change. We note that in the limit $\omega\tau\to\infty$, it is reduced to Eq. (2), while if $\omega\tau\gg1\gg3\gamma p^2\omega\tau$, it agrees with Khalatnikov and Chernikova.

The finite lifetime of thermal phonons affects the tem-

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perature dependence of the attenuation coefficient given by Eq. (1) and is expected to cause α to depend on frequency. Moreover, the cross product of Grüneisen's constant u and γ which was ignored in our previous work[9] will be retained. The product is small, but when a small temperature range about 1 K is concerned, it must be retained. Such a product appears in the form $u\gamma p^2$. As a result, Eq. (2.24) of our previous article[9] is replaced by

$$\mathrm{Im}\left[\frac{\omega^2}{k^2}-c_0^2\right]=\frac{c_0^2}{\rho_0}\frac{\pi(u+1)^2}{h^3}\int\frac{\partial n}{\partial\varepsilon}\frac{p^3}{1+3\gamma p^2}\times\left[1+\frac{u-3}{1+u}\gamma p^2\right]^2 dp\,. \tag{6}$$

We note that frequency ω is coupled with a relaxation time τ. In a single-relaxation-time approximation, the frequency ω used in the previous work is replaced by $1/\omega+i/\tau$. This results in a factor $\omega\tau/[1+(\omega\tau)^2]$ in the expression of $\alpha(T)$. Moreover, for consistency we have introduced a cutoff momentum for momentum-space integrations. The reason is simply that the use of the dispersion relation up to the γ term in Eq. (4) is correct only for small momentum transfer. Therefore, momentum-space integrations become not integrable but require numerical calculations, and the results are expressed in terms of certain functions instead of constant factors.

We have arrived at

$$\alpha(T)=\alpha_0(T)\frac{\omega\tau}{1+(\omega\tau)^2}\frac{G(x)}{4\pi^4/15}\left[1-\left(\frac{T}{T^*}\right)^2\right], \tag{7}$$

where

$$T^*=\frac{c\omega\tau}{k_B[3\gamma(1+\omega^2\tau^2)]^{1/2}}F(x)f(u)\,, \tag{8}$$

$$x=\frac{c\hbar}{k_BT(3\gamma)^{1/2}}\,. \tag{9}$$

The functions $F(x)$ and $G(x)$ are illustrated in Fig. 1. Their asymptotic values, which were used in the previous work, are $G(\infty)=4\pi^4/15$ and $F(\infty)=(\frac{7}{20})^{1/2}/\pi$. On the other hand, the function $f(u)$ in Eq. (8) is new, and is due to the cross products of γ and u. Its expression is

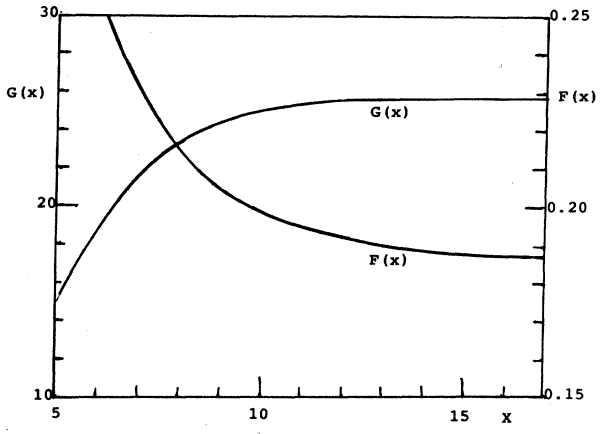

FIG. 1. Functions $G(x)$ (left ordinate) and $F(x)$ (right ordinate) in the range of relevant values of x.

$$f(u)=\left[1-\frac{2(u-3)}{3(1+u)}\right]^{-1/2}. \tag{10}$$

For $u=2.8$, $f(2.8)=0.98$ so that this correction is not very large. However, in general, it causes a nonnegligible effect on T^*.

Since $F(x)$ and $G(x)$ depend on T, the present attenuation coefficient is rather involved. Nevertheless, we note that both $G(x)$ and $F(x)$ approach their asymptotic value when x exceeds 10. This particular x value corresponds to a peak temperature $T_m=0.8$ K. Therefore, let us consider the neighborhood of $x=10$. For $\omega\tau\simeq1$ and $u=2.8$, we find from Eq. (8) that $T^*=1.17$ K. This yields a peak point $T_m=0.95$ K. This value is not very far from 0.8 K. Therefore, our approximate consideration in the vicinity of $x=10$ is self-consistent. We can conclude that with the present improvements the peak position has been reduced. Moreover, the new formula indicates that α increases when $\omega\tau$ increases. These are in accord with experiments.

[1]See, for instance, J. Wilks, *The Properties of Liquid and Solid Helium* (Clarendon, Oxford, 1967).

[2]H. J. Maris and W. E. Massey, Phys. Rev. Lett. **25**, 220 (1970); H. J. Maris, *ibid.* **28**, 277 (1972); Phys. Rev. A **8**, 2629 (1973).

[3]P. R. Roach, M. Abraham, J. B. Ketterson, and M. Kuchnir, in *Low Temperature Physics,* edited by K. D. Timmerhaus, W. J. O'Sulivan, and E. F. Hammel (Plenum, New York, 1974), Vol. 1, p. 493.

[4]J. Jackl and K. W. Kehr, Phys. Rev. Lett. **27**, 654 (1971); Phys. Rev. A **2**, 1757 (1974); R. Dynes and V. Narayanamurti, Phys. Rev. B **12**, 1720 (1975).

[5]A. Andreev and I. M. Khalatnikov, Zh. Eksp. Teor. Fiz. **44**, 2053 (1963) [Sov. Phys. JETP **17**, (1963)].

[6]K. K. Singh, and J. Prakash, Phys. Rev. B **17**, 1253 (1978).

[7]C. I. Um, H. K. Oh, W. H. Kahng, and A. Isihara, Phys. Rev. B **33**, 7550 (1986); C. I. Um, W. H. Kahng, K. H. Yeon, S. T. Choh, and A. Isihara, *ibid.* **29**, 5203 (1984); H. G. Oh, C. I. Um, W. H. Kahng, and A. Isihara, *ibid.* **34**, 6151 (1986).

[8]I. M. Khalatnikov and D. M. Chernikova, Zh. Eksp. Teor. Fiz. **50**, 411 (1966) [Sov. Phys. JETP **23**, 274 (1966)].

[9]A. Isihara, C. I. Um, C. W. Chun, W. H. Kahng, and S. T. Choh, Phys. Rev. B **37**, 7348 (1988).

[10]P. C. Kwok, P. C. Martin, and P. B. Miller, Solid State Commun. **3**, 181 (1965); C. J. Pethick and D. ter Haar, Physica **32**, 1905 (1966).