Flux lattice melting in high-T_c superconductors

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We derive the wave-vector-dependent elastic moduli for a flux line lattice in compounds with underlying tetragonal crystalline symmetry. We find that it is essential to retain wave-vector dependence of the moduli when dealing with compounds where κ is large, as it is in the high-T_c materials. We use our results to establish a Lindemann criterion for flux lattice melting, which we then compare with experimental data on two materials, and find excellent agreement. The melting curves are suppressed well below the mean-field superconducting-normal transition line and are linear in temperature over a wide range of magnetic fields. The point $H=0$, $T=T_c$ is approached as $1-T/T_c \sim H^{1/2}$. The degree of suppression of the melting curves among the different compounds is accounted for in the main by differences in mass anisotropy.

I. INTRODUCTION

Many potential applications of the high-T_c superconductors demand high critical currents. This, in turn, makes detailed knowledge of the microscopics of the superconducting flux line lattice (FLL) of great importance. To obtain high critical currents the formation of a flux lattice is essential since collective pinning allows relatively few pinning centers to pin all of the flux lines (FL's). While there is now considerable evidence from flux decoration experiments[1] and mechanical measurements[2] that the vortex lattice melts well below T_c, which has several obvious implications for potential applications, there is at this time no theoretical agreement as to the melting mechanism. One possibility which is appealing because of the quasi-two-dimensional nature of these systems is the Kosterlitz-Thouless[3] vortex unbinding theory. However, as the observed melting in $Bi_{2.2}Sr_2Ca_{0.8}Cu_2O_8$ (Ref. 2) is essentially independent of whether the magnetic field is applied parallel or perpendicular to the c axis, this seems to be unlikely. Another possibility is simply that the small coherence lengths and high-transition temperatures make conventional thermal-fluctuation-induced melting more likely.[4–7] It is this scenario that we will investigate here.

We determine, within Ginzburg-Landau-Abrikosov-Gorkov (GLAG) (Ref. 8) theory, the wave-vector-dependent elastic moduli for a FLL in a compound with underlying tetragonal crystalline symmetry. This calculation generalizes Brandt's[9] earlier work on the elastic moduli of an isotropic superconductor. As in the latter case, we find that nonlocal effects (i.e., the finite wave-vector dependence of the moduli) are crucial when dealing with compounds where κ, the ratio of penetration depth to correlation length, is large, as it is in the high-T_c materials. Specifically, the moduli are significantly softer than they would be in a purely local theory. We then find the mean-square displacement of a FL due to thermal fluctuations and use the Lindemann[10] criterion to estimate the melting temperature of the FLL. For simplicity, we only consider the geometry in which the magnetic field is aligned parallel to the c axis of these highly anisotropic materials (i.e., perpendicular to the Cu-O planes). We show that the melting curves given by this criterion are essentially parallel to the mean-field superconducting-normal phase boundary $H_{c2}(T)$ over a wide range of field values, but the point $H=0$, $T=T_c$ is approached as $(1-T/T_c)\sim H^{1/2}$. The melting curves are suppressed well below $H_{c2}(T)$ due to the large value of κ, for κ values $\lesssim 10$ the melting curve is indistinguishable from the mean-field phase boundary. For compounds with κ values in the same range the degree of suppression of the melting curve below $H_{c2}(T)$ can be accounted for in terms of the difference in mass anisotropy (i.e., the spacing between Cu-O planes). It should be noted that the conclusions drawn from this analysis do not apply to the immediate vicinity of the lower critical field H_{c1} where the FL spacing is of the order of the penetration depth or more.[11] In this regime it has been argued that the ground state is a superfluid entangled vortex liquid.[7]

This paper is organized as follows. In the next section we derive the elastic moduli by generalizing the earlier work of Brandt[9] to the case of tetragonal symmetry (i.e., uniaxial anisotropy of the effective mass tensor). These results are then used in Sec. III to formulate a Lindemann criterion for melting which is then compared with experimental data. Technical details appear in the appendices.

II. THE ELASTIC MODULI OF AN ANISOTROPIC FLUX LATTICE

In this section we derive the nonlocal (i.e., wave-vector dependent) elastic moduli for a FLL in a compound with underlying tetragonal symmetry. The starting point of our analysis is the Ginzburg-Landau (GL) free energy of an anisotropic superconductor given by

$$F=\alpha|\psi|^2+\tfrac{1}{2}\beta|\psi|^4 + \frac{1}{2}\left[\left(\frac{\hbar\nabla}{i}-\frac{2e}{c}\mathbf{A}\right)\psi^*\overleftrightarrow{M}^{-1}\left(\frac{\hbar\nabla}{-i}-\frac{2e}{c}\mathbf{A}\right)\psi\right]+\frac{H^2}{8\pi}. \tag{2.1}$$

Here $\overleftrightarrow{M}^{-1}$ is an inverse mass tensor, $\nabla\times\mathbf{A}=\mathbf{H}$, e is the electronic charge, and F is the free-energy density of the superconductor relative to the normal state at $H=0$. To a very good approximation the mass tensor for the high-T_c oxides can be written as

$$\overleftrightarrow{M}=M\overleftrightarrow{T}^{-1}\ ,\qquad \overleftrightarrow{T}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&\dfrac{M}{M_z}\end{bmatrix}\ . \tag{2.2}$$

Here the $\hat{\mathbf{c}}$ axis has been chosen parallel to $\hat{\mathbf{z}}$ and the Cu-O planes lie in the perpendicular (x,y) plane; M_z is therefore a quasiparticle effective mass along the $\hat{\mathbf{c}}$ axis and M describes the Cu-O planes.

We consider the situation in which an external magnetic field is applied parallel to the $\hat{\mathbf{z}}$ axis. The solution of the linearized GL equations [derived from Eq. (2.1) on setting $\beta=0$] will therefore be the well-known Abrikosov triangular FLL with FL's parallel to the $\hat{\mathbf{z}}$ axis as in the isotropic case. Hence, to derive an elastic Hamiltonian from Eq. (2.1) we need only generalize Brandt's[9] work on the isotropic case where $M_z=M$. We imagine a general FLL defined by the locii of the zeros of the order parameter, equivalently the centers of the FL's,

$$\mathbf{r}_\nu(z)=\mathbf{R}_\nu+\mathbf{s}_\nu(z)\ , \tag{2.3}$$

where $\mathbf{s}_\nu=[s_\nu^x(z),s_\nu^y(z),0]$ is the displacement of the νth FL from the position $\mathbf{R}_\nu=(X_\nu,Y_\nu,z)$ it acquires in the Abrikosov solution. Then to study the fluctuations of the FLL in the continuum limit, we define an effective elastic Hamiltonian governing these fluctuations. For an ideal triangular FLL the elastic Hamiltonian in the harmonic approximation takes the simple form

$$H=\tfrac{1}{2}\sum_{\mathbf{k}}\mathbf{s}_i(-\mathbf{k})\{c_L(\mathbf{k})k_ik_j+\delta_{ij}[c_{66}(\mathbf{k})k_\perp^2+c_{44}(\mathbf{k})k_z^2]\}\mathbf{s}_j(\mathbf{k})\ ,\quad (i,j)=x,y\ , \tag{2.4}$$

where c_{66}, c_L, and c_{44} are the wave-vector-dependent shear, bulk, and tilt elastic moduli, respectively; to be determined. An obvious approach one might take to derive a Hamiltonian of this form from Eq. (2.1) would be to take the Abrikosov solution with modified argument Eq. (2.3), insert it into the free energy, and expand to second order in $\mathbf{s}$. However, not only does this *linearized* solution fail to reproduce results obtained from thermodynamics in the local limit ($\mathbf{k}\to 0$), but c_L and c_{44} even diverge in this limit. The shear modulus, c_{66}, however, depends only weakly on $\mathbf{k}$, and we will follow the usual procedure of assuming it to be dispersionless. We now outline a procedure, due to Brandt,[9] which removes these divergencies and moreover correctly reproduces the thermodynamic limit.

For convenience, we rewrite the free energy in dimensionless form

$$f=-\omega+\tfrac{1}{2}\omega^2+H^2+\left|\left(\frac{\nabla_\perp}{i\kappa}-A_\perp\right)\psi\right|^2+\frac{M}{M_z}\left|\left(\frac{\nabla_z}{i\kappa}-A_z\right)\psi\right|^2\ . \tag{2.5}$$

Here $\kappa=\lambda_\perp/\xi_\perp$, $\lambda_\perp^2=[Mc^2/4\pi(2e)^2(\beta/|\alpha|)]$, $\xi_\perp^2=(\hbar^2/2M|\alpha|)$, and $\omega=|\psi|^2$. The lengths $\lambda_\perp$ and $\xi_\perp$ are the penetration depth and correlation length in the Cu-O planes, respectively. The order parameter ψ is now measured in units of the London solution $|\psi_\infty|^2=|\alpha|/\beta$, the magnetic field is measured in units of $\sqrt{2}H_c$, where the critical field $H_c^2=4\pi(|\alpha|^2/\beta)$, and lengths are measured in units of $\lambda_\perp$. By ω_l we denote the two-dimensional (2D) linearized solution obtained by inserting Eq. (2.3) into the Abrikosov solution $\omega_0(x,y)$ and expanding the order parameter to linear order in $\mathbf{s}$. For the moment, if we neglect the shear deformation of the lattice, the result is

$$\omega_l(x,y)=|\psi_l(x,y)|^2=\omega_0(x,y)[1+\tfrac{1}{2}\eta(x,y)]^2+O(s^2)\ . \tag{2.6}$$

An explicit expression for ω_0 will not be needed and we do not specify it further. In the continuum limit, η is given by

$$\eta\to\bar{\eta}=2b\kappa^2(\nabla\cdot\mathbf{s})/k_\perp^2\ , \tag{2.7}$$

where $k_\perp^2=k_x^2+k_y^2$ and $b=\langle H\rangle_{\rm sp}/H_{c2}$, H_{c2} being the upper critical field, which is κ in these units. The brackets $\langle\ \rangle_{\rm sp}$ indicate a space average.

The strategy now is to make a variational ansatz for the solution of the full nonlinear GL equation by multiplying ψ_l by a slowly varying complex function modulating both the amplitude and the phase of the order parameter, but leaving the zeros unchanged,

$$\psi=\psi_l(1+\theta/2)^2e^{i\kappa\chi}\ ,\qquad \omega=|\psi|^2=\omega_l(1+\theta)\ . \tag{2.8}$$

The functions θ and χ are then variationally determined by minimizing the free energy, Eq. (2.5). In Appendix A we show that after minimizing with respect to the local field the free energy can be written

$$f=-\omega+\tfrac{1}{2}\omega^2+\omega Q_{B\perp}^2+(M/M_z)\omega Q_{Bz}^2-\mathbf{h}\cdot\mathbf{h}_B+B^2\ . \tag{2.9}$$

Here $\mathbf{B}=\langle H\rangle_{\rm sp}$, $\mathbf{H}=B\hat{\mathbf{z}}+\mathbf{h}=\nabla\times\mathbf{A}_B+\nabla\times\mathbf{A}_h$, $\nabla\times\mathbf{A}_B=\mathbf{B}$, and $\nabla\times\mathbf{A}_h=\mathbf{h}$. The fields $\mathbf{h}$ and $\mathbf{h}_B$ satisfy the second Gl equation which here takes the form

$$\nabla\times\mathbf{h}=-\omega\overleftrightarrow{T}\cdot\mathbf{Q}\ ,\qquad \nabla\times\mathbf{h}_B=-\omega\overleftrightarrow{T}\cdot\mathbf{Q}_B\ , \tag{2.10}$$

where $\overleftrightarrow{T}$ is defined by Eq. (2.2). The supervelocity $\mathbf{Q}$ is given by

$$\mathbf{Q}=\mathbf{Q}_B+\mathbf{A}_h\ , \tag{2.11}$$

where $\mathbf{Q}_B=\mathbf{Q}_{B,l}-\nabla\chi$ and $\mathbf{Q}_{B,l}$ is the supervelocity of the linearized theory

$$\mathbf{Q}_{B,l} = -\frac{\hat{\mathbf{z}}\times\nabla\omega_l}{2\kappa\omega_l} + \hat{\mathbf{z}}Q_{B,l}^z ,$$
$$Q_{B,l}^z \equiv \tilde{Q}_z = -\frac{B}{k_\perp^2}\frac{\partial}{\partial z}(\hat{\mathbf{z}}\cdot\nabla\times\mathbf{s}) . \qquad (2.12)$$

The central part of the minimization procedure is the determination, by solving Eq. (2.10), of the fields $\mathbf{h}$ and $\mathbf{h}_B$ in terms of the variational functions θ and χ. The solution is given in Appendix B and the results are

$$\mathbf{h} = \mathbf{h}^{\text{iso}} + \mathbf{h}_1 ,$$
$$\mathbf{h}_B = \mathbf{h}_B^{\text{iso}} + \mathbf{h}_{1B} . \qquad (2.13)$$

Here the isotropic component of the field $\mathbf{h}$ which was derived by Brandt[9] is given by

$$\mathbf{h}^{\text{iso}} = \hat{\mathbf{z}}\left[\frac{\langle\omega_0\rangle(1+\bar{\eta}+\theta)-\omega_0}{2\kappa}\right] + \frac{B\langle\omega_0\rangle}{k^2+\langle\omega_0\rangle}\left[-\hat{\mathbf{z}}\nabla\cdot\mathbf{s}+\frac{\partial\mathbf{s}}{\partial z}\right] , \qquad (2.14)$$

the spatial average of the Abrikosov order parameter

$$\langle\omega_0\rangle = (1-b)2\kappa^2/[(2\kappa^2-1)\beta+1)]$$

and $\beta = 1.16$ for the triangular ideal FLL. We find within the geometry considered here that

$$\mathbf{h}_1 = -\left[\frac{M}{M_z}-1\right] \times\langle\omega_0\rangle\frac{i(\mathbf{k}\times\hat{\mathbf{z}})C}{[k^2+\langle\omega_0\rangle+(M/M_z-1)\langle\omega_0\rangle k_\perp^2/k^2]} , \qquad (2.15)$$

where $C = \tilde{Q}_z - \partial\chi/\partial z + A_{h\,\text{iso}}^z$ and

$$A_{h\,\text{iso}}^z = \frac{B}{k^2}\frac{\langle\omega_0\rangle}{k^2+\langle\omega_0\rangle}\frac{\partial}{\partial z}(\hat{\mathbf{z}}\cdot\nabla\times\mathbf{s}) , \qquad (2.16)$$

$\mathbf{h}_B^{\text{iso}}$ is found by deleting $\langle\omega_0\rangle$ in the denominator of the last term of $\mathbf{h}^{\text{iso}}$ and

$$\mathbf{h}_{1B} = -\left[\frac{M}{M_z}-1\right]\langle\omega_0\rangle\frac{i(\mathbf{k}\times\hat{\mathbf{z}})C_B}{k^2} , \qquad (2.17)$$

where $C_B = \tilde{Q}_z - \partial\chi/\partial z$.

The unknown functions θ and χ are now determined by minimizing the free-energy function equation (2.9) with respect to θ and χ. The details of this calculation are given in Appendix C; the result is

$$f - f_0 = \tfrac{1}{2}[c_{11}(\nabla\cdot\mathbf{s})^2 + c_{44}(\partial_z\mathbf{s})^2] , \qquad (2.18)$$

where f_0 is the Abrikosov contribution to the free energy and the elastic moduli are given by

$$c_{44}(\mathbf{k}) = \frac{B^2}{4\pi}\left[\frac{M}{M_z}\right]\langle\omega_0\rangle\left[\frac{1}{k_\perp^2+(M/M_z)(k_z^2+\langle\omega_0\rangle)} + \frac{1}{2b\kappa^2}\right] , \qquad (2.19)$$

and

$$c_{11}(\mathbf{k}) = c_L(\mathbf{k}) + c_{66} = \frac{B^2}{4\pi}\left[\frac{[k^2+(M/M_z)\langle\omega_0\rangle]}{(k^2+\langle\omega_0\rangle)[k_\perp^2+(M/M_z)(k_z^2+\langle\omega_0\rangle)]} - \frac{1}{k_\perp^2+(M/M_z)k_z^2+k_\psi^2}\right] , \qquad (2.20)$$

where $k_\psi^2 = 2(1-b)/\xi_\perp^2$. Since the shear modulus is essentially dispersionless and the magnetic field is applied parallel to the $\hat{\mathbf{c}}$ axis, we may use the result obtained previously for the local isotropic case[9]

$$c_{66} \cong \frac{B_{c2}^2}{4\pi}\frac{b(1-b)^2}{8\kappa^2} . \qquad (2.21)$$

All of the results of this section were derived from the GL theory and are applicable when the fields of many FL's overlap, or if $\kappa \gg 1$ when $\frac{1}{2}\kappa^2 < b < 1$, i.e., at essentially all fields of interest in the oxides. In the isotropic case the moduli have been rederived from microscopic theory[12] and shown to apply even at low temperatures.

This completes the derivation of the effective Hamiltonian governing the fluctuations of the FLL. The elastic propagator therefore can be written

$$G_{ij}(\mathbf{k}) = k_B T\left[\frac{P_T}{c_{66}(\mathbf{k})k_\perp^2+c_{44}(\mathbf{k})k_z^2} + \frac{P_L}{c_{11}(\mathbf{k})k_\perp^2+c_{44}(\mathbf{k})k_z^2}\right] \qquad (2.22)$$

with elastic moduli given by Eqs. (2.19)–(2.21), $P_T = (\delta_{ij} - k_i k_j/k_\perp^2)$ is the transverse projection operator, and $P_L = k_i k_j/k_\perp^2$ $(i,j = x,y)$.

III. LINDEMANN CRITERION AND THE MELTING OF THE FLL

To examine the stability of the FLL against thermal fluctuations we need to determine the mean-square displacement of a FL from equilibrium

$$d^2(T) = \left\langle\sum_\nu s_\nu^2\right\rangle = \sum_k T_r G_{ij}(\mathbf{k}) . \qquad (3.1)$$

Here $\langle\ \rangle$ indicates a thermal average with respect to the elastic Hamiltonian Eq. (2.4) and $G_{ij}(\mathbf{k})$ is given by Eq. (2.22); hence

$$d^2(T) = k_B T\int_0^\infty\frac{dk_z}{2\pi}\int_0^{\Lambda^2}\frac{dk_\perp^2}{2\pi}\left[\frac{1}{c_{66}(k)k_\perp^2+c_{44}(k)k_z^2} + \frac{1}{[c_L(k)+c_{66}(k)]k_\perp^2+c_{44}(k)k_z^2}\right] = d_1^2(T) + d_2^2(T) . \qquad (3.2)$$

Then according to the Lindemann criterion[10] the FL will melt when $d(T)\cong cl$; here l is the lattice parameter of the vortex lattice determined by the flux quantization condition, $\sqrt{3}l^2/2=\Phi_0/B$ for the triangular lattice considered here, Φ_0 is the flux quantum and c is a constant typically of order 0.1. In Eq. (3.2) the $k_\perp$ integral has been approximated by taking it over a circular Brillouin zone of radius Λ, $\Lambda^2=2b/\xi_\perp^2$. Henceforth, it will be convenient to introduce a dimensionless wave vector $q=k/\Lambda$, in which case the nonlocal elastic moduli, Eqs. (2.19) and (2.20), as functions of $\mathbf{q}$ take the form

$$c_{44}(\mathbf{q})=\frac{B^2}{4\pi}\frac{(1-b)}{2b\kappa^2}\left[\frac{M}{M_z}\right]\left[\frac{1}{q_\perp^2+(M/M_z)(q_z^2+m_\lambda^2)}+1\right] \quad (3.3)$$

and

$$c_{11}(\mathbf{q})=\frac{B^2}{4\pi}\frac{(1-b)}{2b\kappa^2}\left[\frac{(q^2+(M/M_z)m_\lambda^2)}{(q^2+m_\lambda^2)[q_\perp^2+(M/M_z)q_z^2+(M/M_z)m_\lambda^2]}-\frac{1}{q_\perp^2+(M/M_z)q_z^2+m_\xi^2}\right]. \quad (3.4)$$

Since c_{66} is dispersionless it is still given by Eq. (2.21). In Eqs. (3.3) and (3.4) we have defined $m_\xi^2=(1-b)/b$ and $m_\lambda^2=(1-b)/2b\kappa^2$.

The moduli, Eqs. (3.3) and (3.4), reduce to known expressions in the local limit but crossover to nonlocality is at $q^2\gtrsim m_\lambda^2$, which is extremely small in the high-T_c oxides, $\kappa\gtrsim 100$. In the local limit c_{66} vanishes at $b=1$, whereas c_{44} and c_{11} achieve their maximum values at the upper critical field. However, when $q>m_\lambda$, the lattice softens significantly; all moduli vanish at $b=1$. The anisotropy of the elastic moduli is manifest in the ratio (M/M_z); it should be noted that the effect of anisotropy is to modify the cutoff m_λ as well as to rescale q_z.

To determine $d^2(T)$ from Eq. (3.2) we must integrate over the full Brillouin zone $0<q_\perp<1$. Therefore, it is clear that a fully nonlocal theory is essential. For arbitrary field and temperature the integral over the two-dimensional Brillouin zone, Eq. (3.2), can only be evaluated numerically; however, with some restrictions, analytic results can be found. The contribution to $d^2(T)$ from the first term of Eq. (3.2) can be evaluated analytically for all $b>1/2\kappa^2$; however, useful analytic forms for the second term, which involves the bulk modulus c_L, can only be found subject to the further constraints that $m_\xi^2\ll 1$, i.e., $b\gg\frac{1}{2}$ or $m_\xi^2\gg 1$; $b\ll\frac{1}{2}$. We will discuss the latter in detail as this condition is generally well satisfied on the melting curve near T_c, and, as we shall see, in $Bi_{2.2}Sr_2Ca_{0.8}Cu_2O_8$ over the whole field and temperature range. We find that

$$d_1^2(T)=\left[\frac{k_BT}{4\pi}\left[\frac{\Lambda^2}{c_{44}^0c_{66}^0}\right]^{1/2}\right]\left[\frac{M_z}{M}\right]^{1/2}\times\kappa\left[\frac{2b}{1-b}\right]^{1/2}(\sqrt{2}-1) \quad (3.5)$$

for $b>1/2\kappa^2$ and

$$d_2^2(T)=\left[\frac{k_BT}{4\pi}\left[\frac{\Lambda^2}{c_{44}^0c_{11}^0}\right]^{1/2}\right]\left[\frac{M_z}{M}\right]^{1/2}\kappa^2\left[\frac{b}{1-b}\right] \quad (3.6)$$

for $b>1/2\kappa^2$ and $(1-b)/b\gg 1$. In Eqs. (3.5) and (3.6) the results of isotropic local elasticity theory are given by the terms in square brackets. The local elastic constants c_{44}^0, c_{66}^0, and c_{11}^0 can be obtained from Eqs. (3.3) and (3.4) in the limit $q\to 0$. In the local limit, $d_2^2(T)$ is negligible compared to $d_1^2(T)$ as c_{66}^0, which is always less than c_{11}^0, vanishes at $b=1$. The mean-square variance of a vortex line in an anisotropic superconductor within local elasticity theory has been inferred previously by Nelson and Seung.[7] Local elasticity theory, however, significantly underestimates the elastic response of the vortex lattice as can be seen from Eqs. (3.5) and (3.6); both $d_1^2(T)$ and $d_2^2(T)$ diverge as $b\to 1$. In addition, $d_1^2(T)$ is enhanced by a factor of κ, $d_2^2(T)$ by a factor of κ^2, a result of no import for conventional pure type-II superconductors, $\kappa\cong 1$, but a dramatic effect in the high-κ oxide superconductors. The variance d^2 is also enhanced as a result of anisotropy as conjectured in Ref. 7. However, the origin of the enhancement factor of $(M_z/M)^{1/2}$ in Eqs. (3.5) and (3.6) is in the anisotropy of the nonlocal elastic constants. In the local limit the elastic constants are independent of (M_z/M). As a result the in-plane elastic correlation length $\xi_\perp$, which can be estimated from local theory, is a factor of $(M/M_z)^{1/2}$ smaller than inferred in Ref. 7 and the aspect ratio of a correlation volume $(\xi_z/\xi_\perp)$ is correspondingly more anisotropic. In related work, within isotropic GL theory, Moore[6] has estimated $d^2(T)$ from an effective Hamiltonian deduced from the behavior of the shear modes of a *rigid* FLL. He found

$$d^2(T)\cong k_BT\Lambda\int_0^\infty\frac{dq_z}{2\pi}\int_0^1\frac{dq_\perp^2}{2\pi}\frac{q_\perp^2}{c_{66}(\mathbf{q})q_\perp^4+\rho_sq_z^2}, \quad (3.7)$$

where ρ_s is the superfluid density. This result is recovered from $d_1^2(T)$ in Eqs. (3.2) and (3.3) in the nonlocal limit $q>m_\lambda$ and as such gives a good estimate of this contribution to the variance of an FL. The effective Hamiltonian leading to Eq. (3.7), therefore, does not correctly describe the physics in the local limit $q\to 0$, the limit of interest for critical phenomena, when, as can be seen from Eq. (3.3), local elasticity theory is recovered. Therefore, we believe Moore's conclusion, that the lower critical dimension for superconductivity is $d=3$, to be incorrect.

Introducing the explicit values of the elastic constants we find

$$d^2(T)=\frac{1}{2\pi}\left[\frac{\epsilon M_z}{M}\right]^{1/2}\frac{t}{(1-t)^{1/2}}\frac{b^{1/2}}{(1-b)}\left[\frac{4(\sqrt{2}-1)}{(1-b)^{1/2}}+1\right]l^2 \tag{3.8}$$

and the melting criterion is given by

$$[t/(1-t)^{1/2}][b^{1/2}/(1-b)][4(\sqrt{2}-1)/(1-b)^{1/2}+1]\gtrsim 2\pi\left[\frac{\epsilon M_z}{M}\right]^{-1/2}c^2\equiv\alpha\ . \tag{3.9}$$

Here $t=T/T_c$, and ϵ is the Ginzburg criterion parameter

$$\epsilon=16\pi^3\kappa^4(k_BT_c)^2/\Phi_0^3H_{c2}^0\ .$$

In arriving at Eqs. (3.8) and (3.9), we have used the result $H_{c2}(T)=H_{c2}^0(1-t)$. substituting the values of the universal constants we find

$$\alpha\cong 2\times 10^5c^2(H_{c2}^0/T_c^2)^{1/2}(M/M_z)^{1/2}/\kappa^2\ .$$

Typical parameters for a conventional superconductor are $\kappa\cong 1$, $H_{c2}^0\cong 10^4$ G, and $T_c\cong 10$ K which, when setting the mass ratio equal to unity and taking $c=0.1$, gives $\alpha\cong 10^4$ which, in turn, means that the effect of thermal fluctuations in these materials is negligible unless $H\cong H_{c2}$. However, in the high-T_c superconductors $\kappa\cong 100$, $H_{c2}^0\cong 10^6$ G, $T_c\cong 100$ K, and the mass ratio $(M_z/M)\cong 10^2$ which gives $\alpha\cong 10^{-1}$ a reduction of 5 orders of magnitude and hence the flux lattice should be melted over a wide range of field $H<H_{c2}^0$. Most of this effect is due to the increase in κ, i.e., $\alpha\sim 1/\kappa^2$. However, for fixed κ the effect is more pronounced as the mass ratio (M_z/M) becomes larger, i.e., as the spacing between the Cu-O planes increases. For $T\cong T_c$ it is easily seen from Eq. (3.9) that the shape of the melting curve near T_c is given by $H^{1/2}\propto(1-T/T_c)$.

Finally we compare the melting curve deduced from the Lindemann criterion with the experimental results of Gammel *et al.*[2] We have integrated Eq. (3.2) numerically over the two-dimensional Brillouin zone with elastic moduli given by Eqs. (2.21), (3.3), and (3.4). To compare with the data for $YBa_2Cu_3O_7$ we took $\kappa=50$, the mass ratio $(M_z/M)^{1/2}=5$ (deduced from the ratio of the parallel and perpendicular critical field slopes given by Iye *et al.*[13]), $H_{c2}^0=440$ kG, and $T_c=87$ K, the latter two values were given in Ref. 2. The Lindemann parameter c was then varied to give the best fit to the data. The result with $c=0.4$ is shown in Fig. 1. Fixing c and using parameters for $Bi_{2.2}Sr_2Ca_{0.8}Cu_2O_8$ obtained from Palstra *et al.*,[14] $(M_z/M)^{1/2}=60$, $H_{c2}^0=440$ kG, $T_c=87$ K (these values were found using the midpoint of the resistivity curve), the data was best fit with $\kappa=95$, which is within the range quoted by Batlogg *et al.*[15] (Fig. 2).

IV. CONCLUSIONS

In conclusion, we have derived a nonlocal elastic theory for a FLL in a superconductor with underlying tetragonal symmetry. In the high-T_c materials where κ and M_z/M are large, the elastic moduli are found to be significantly softer than those predicted by an isotropic, local elasticity theory. Accordingly, a Lindemann criterion based on the nonlocal theory predicts melting curves suppressed well below the mean field $H_{c2}(T)$ due to the large value of κ, with features in general agreement with experimental data on a variety of compounds. The

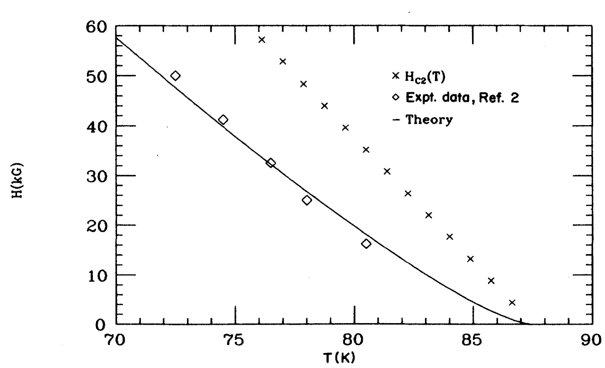

FIG. 1. The phase diagram for flux lattice melting in $YBa_2Cu_3O_7$. The solid line is given by Eq. (3.9) with $c=0.4$, $\kappa=50$, $(M_z/M)^{1/2}=5$, and $T_c=87$ K. The experimental data from Ref. 2 is denoted by the squares. The dashed line indicates $H_{c2}(T)=440$ kG $(1-t)$. For values of κ less than about 10, the melting line given by Eq. (3.9) would be indistinguishable from the straight dashed line.

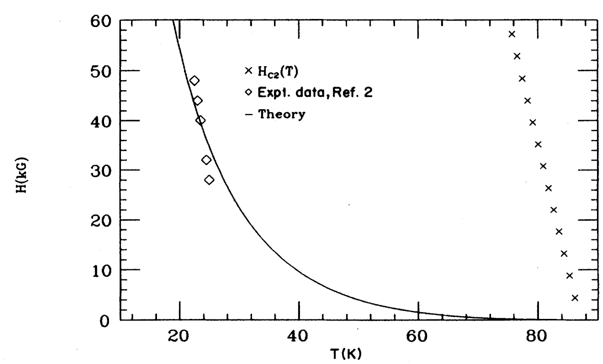

FIG. 2. The phase diagram for flux lattice melting in $Bi_{2.2}Sr_2Ca_{0.8}Cu_2O_8$. The solid line is given by Eq. (3.9) with $c=0.4$, $\kappa=95$, $(M_z/M)^{1/2}=60$, and $T_c=87$ K. The squares denote the data from Ref. 2 for this material, and the dashed line indicates $H_{c2}(T)=440(1-t)$. Note that the data points would extrapolate linearly to a point on the T axis, well below, while the theoretical curve predicts a sharp bend in the melting line at low fields towards the point $H=0$, $T=T_c$, which is approached with zero slope.

predicted melting curves are linear over a wide range of fields but the point $H=0$, $T=T_c$ is approached as $1-T/T_c \sim H^{1/2}$. It is remarkable that despite this curvature, the data of Ref. 2 can be fit with parameters which are in the general range of those quoted for $YBa_2Cu_3O_7$ and $Bi_{2.2}Si_2Ca_{0.3}Cu_2O_8$. Measurements on $TlBa_2CaCuO$ (Ref. 16) and on $Bi_{2.2}Sr_2Ca_{0.8}Cu_2O_8$ and $TlBa_2CaCuO$ (Ref. 17) in weak fields exhibit positive curvature consistent with the picture given here and in contrast to the melting curve deduced from 2D melting theory which has negative curvature.[15] The degree of suppression of the melting curves below $H_{c2}(T)$ among the different compounds is mostly accounted for in terms of the difference in mass anisotropy (i.e., the spacing between Cu-O planes). It should be noted that the conclusions drawn from this analysis do not apply to the immediate vicinity of the lower critical field H_{c1} where the FL spacing is of order of the penetration depth or more.[11]

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APPENDIX A: THE FIELD-DEPENDENT TERM IN THE FREE ENERGY

We consider the field-dependent terms, denoted by f^H in the free energy, Eq. (2.5), with general anisotropy

$$f^H = \langle \omega Q_i T_{ij} Q_j + H^2 \rangle \ . \tag{A1}$$

Here $\mathbf{Q}=\mathbf{Q}_B+\mathbf{A}_h$, $\mathbf{H}=B\hat{\mathbf{z}}+\hat{\mathbf{h}}$, $\nabla\times\mathbf{A}_h=\mathbf{h}$, and $\mathbf{Q}_B$ is defined in the text. Noting that, given the symmetry of the mass tensor,

$$Q_i T_{ij} Q_j = Q_{Bi} T_{ij} Q_{Bj} + A_{hi} T_{ij} A_{hj} + 2 A_{hi} T_{ij} Q_{Bj} \ . \tag{A2}$$

and that $H^2=B^2+h^2$, since the cross term represents a total derivative, we minimize (A1) with respect to $\mathbf{A}_h$ and find

$$\omega T_{ij} A_{hj} + 2\omega T_{ij} Q_{Bj} + 2(\nabla\times\mathbf{h})_i = 0 \ , \tag{A3}$$

or equivalently

$$(\nabla\times\mathbf{h})_i = -\omega T_{ij} Q_j \ . \tag{A4}$$

Here we have used the identity

$$\frac{\delta}{\delta \mathbf{A}_h}(\nabla\times\mathbf{A}_h)^2 = 2(\nabla\times\mathbf{h}) \tag{A5}$$

valid in the gauge $\nabla\cdot\mathbf{A}_h=0$. Furthermore, as

$$(\nabla\times\mathbf{A}_h)^2 = \mathbf{A}_h\cdot(\nabla\times\mathbf{h}) + \cdots \ , \tag{A6}$$

where the ellipsis represents surface terms; using this result together with Eq. (A5) we find that f^H at its minimum is given by

$$f^H = \langle \omega Q_{Bi} T_{ij} Q_{Bj} + \omega A_{hi} T_{ij} Q_{Bj} \rangle + B^2 \ . \tag{A7}$$

Now, after discarding total derivatives, we *define* a field $\mathbf{h}_B$ via

$$(\nabla\times\mathbf{h}_B)_i = -\omega T_{ij} Q_{Bj} \tag{A8}$$

and then the second term in Eq. (A7) can be written

$$-\mathbf{A}_h\cdot(\nabla\times\mathbf{h}_B) = \nabla\cdot(\mathbf{A}_h\times\mathbf{h}_B) - \mathbf{h}_B\cdot(\nabla\times\mathbf{A}_h) \ . \tag{A9}$$

Again, after discarding the total derivatives, we find

$$\begin{aligned} f^H &= \langle \omega Q_i T_{ij} Q_j + H^2 \rangle_{\min} \\ &= \langle \omega Q_{Bi} T_{ij} Q_{Bj} - \mathbf{h}_B\cdot\mathbf{h} \rangle + B^2 \ . \end{aligned} \tag{A10}$$

Consequently, when the specific form of T_{ij} [Eq. (2.2)] is used, the free energy is given by

$$f = \langle -\omega + \tfrac{1}{2}\omega^2 + \omega Q_{B\perp}^2 + (M/M_z)\omega Q_{Bz}^2 - \mathbf{h}\cdot\mathbf{h}_B \rangle + B^2 \ , \tag{A11}$$

where the fields $\mathbf{h}$ and $\mathbf{h}_B$ satisfy Eqs. (A4) and (A8), respectively.

APPENDIX B: SOLUTION TO EQ. (2.9)

We now consider in some detail the solution to Eq. (2.10) of the text

$$(\nabla\times\mathbf{h})_i = -\omega T_{ij} Q_j \ . \tag{B1}$$

In vector form, with T_{ij} given by Eq. (2.2), (B1) can be written

$$\nabla\times\mathbf{h} = -\omega\left[\mathbf{Q} + \left[\frac{M}{M_z}-1\right]\hat{\mathbf{z}}Q_z\right] \ . \tag{B2}$$

Here $\mathbf{Q}=\mathbf{Q}^{\text{iso}}+\mathbf{A}_{h1}$, $\mathbf{Q}^{\text{iso}}=\mathbf{Q}_B+\mathbf{A}_h^{\text{iso}}$, $\mathbf{A}_h=\mathbf{A}_h^{\text{iso}}+\mathbf{A}_{h1}$,

$$\nabla\times\mathbf{A}_h = \mathbf{h} = \nabla\times\mathbf{A}_h^{\text{iso}} + \nabla\times\mathbf{A}_{h1} = \mathbf{h}^{\text{iso}} + \mathbf{h}_1 \ ,$$

where $\mathbf{h}^{\text{iso}}$ satisfies

$$\nabla\times\mathbf{h}^{\text{iso}} = -\omega\mathbf{Q}^{\text{iso}} \ . \tag{B3}$$

Equation (B3) has been analyzed in detail in Ref. 9 and has the solution

$$\begin{aligned} \mathbf{h}^{\text{iso}} = {} & \hat{\mathbf{z}}\frac{[\langle\omega_0\rangle(1+\bar{\eta}+\theta)-\omega_0]}{2\kappa} \\ & + \frac{B\langle\omega_0\rangle}{k^2+\langle\omega_0\rangle}\left[-\hat{\mathbf{z}}\nabla\cdot\mathbf{s} + \frac{\partial\mathbf{s}}{\partial z}\right] \ , \end{aligned} \tag{B4}$$

where $\bar{\eta}=2b\kappa^2(\nabla\cdot\mathbf{s})/k_\perp^2$. We find that $\mathbf{h}_1$ satisfies

$$\begin{aligned} -\nabla^2 h_{1x} &= -\langle\omega_0\rangle h_{1z} - \left[\frac{M}{M_z}-1\right]\langle\omega_0\rangle\partial_y Q_z \ , \\ -\nabla^2 h_{1y} &= -\langle\omega_0\rangle h_{1y} + \left[\frac{M}{M_z}-1\right]\langle\omega_0\rangle\partial_x Q_z \ , \\ h_{1z} &= 0 \ , \end{aligned} \tag{B5}$$

where

$$Q_z = Q_z^{\rm iso} + A_{h1z} \ , \qquad \text{(B6)}$$

$$C \equiv Q_z^{\rm iso} = \tilde{Q}_z - \partial\chi/\partial z + A_{hz}^{\rm iso} \ , \qquad \text{(B7)}$$

$$A_{hz}^{\rm iso} = (B/k^2)[\langle \omega_0 \rangle (k^2 + \langle \omega_0 \rangle)][(\partial/\partial z)(\hat{\mathbf{z}}\cdot\nabla\times\mathbf{s})] \ ,$$

and

$$\tilde{Q}_z = -(B/k_\perp^2)[(\partial/\partial z)(\hat{\mathbf{z}}\cdot\nabla\times\mathbf{s})] \ .$$

Then, on using $\nabla\times\mathbf{A}_{h1} = \mathbf{h}_1$ and $\nabla\cdot\mathbf{A}_{h1} = 0$, it can be shown that

$$A_{h1z} = -\frac{i}{k^2} X \ , \qquad \text{(B8)}$$

where $X \equiv k_y h_{1x} - k_x h_{1y}$; we can now solve for X. After Fourier transforming, Eq. (B10) takes the form

$$(k^2 + \langle \omega_0 \rangle) h_{1x} = -\langle \omega_0 \rangle \left[\frac{M}{M_z} - 1 \right] i k_y \left[C - \frac{i}{k^2} X \right] \ ,$$
$$(k^2 + \langle \omega_0 \rangle) h_{1y} = \langle \omega_0 \rangle \left[\frac{M}{M_z} - 1 \right] i k_x \left[C - \frac{i}{k^2} X \right] \ , \qquad \text{(B9)}$$

and therefore,

$$X = -\langle \omega_0 \rangle \left[\frac{M}{M_z} - 1 \right] \frac{i k_\perp^2}{(k^2 + \langle \omega_0 \rangle)} \left[C - \frac{iX}{k^2} \right] \qquad \text{(B10)}$$

and

$$A_{h1z} = -\frac{\langle \omega_0 \rangle}{(k^2 + \langle \omega_0 \rangle)} \left[\frac{M}{M_z} - 1 \right] \frac{k_\perp^2}{k^2} \frac{C}{[k^2 + \langle \omega_0 \rangle + (M/M_z - 1)\langle \omega_0 \rangle k_\perp^2/k^2]} \ . \qquad \text{(B11)}$$

Using this result together with Eqs. (B5) and (B6), we find h_{1x} and h_{1y} are given by

$$h_{1x} = -\langle \omega_0 \rangle \left[\frac{M}{M_z} - 1 \right] i k_y \frac{C}{[k^2 + \langle \omega_0 \rangle + (M/M_z - 1)\langle \omega_0 \rangle k_\perp^2/k^2]} \ ,$$
$$h_{1y} = \langle \omega_0 \rangle \left[\frac{M}{M_z} - 1 \right] i k_x \frac{C}{[k^2 + \langle \omega_0 \rangle + (M/M_z - 1)\langle \omega_0 \rangle k_\perp^2/k^2]} \ . \qquad \text{(B12)}$$

By a similar method we obtain

$$h_{1Bx} = -\langle \omega_0 \rangle \left[\frac{M}{M_z} - 1 \right] \frac{i k_y}{k^2} C_B \ ,$$
$$h_{1By} = \langle \omega_0 \rangle \left[\frac{M}{M_z} - 1 \right] \frac{i k_x}{k^2} C_B \ , \qquad \text{(B13)}$$
$$C_B = \tilde{Q}_z - \frac{\partial \chi}{\partial z} \ .$$

Equations (B12) and (B13) can be written in vector form as follows:

$$\mathbf{h}_1 = -\left[\frac{M}{M_z} - 1 \right] \langle \omega_0 \rangle (i\mathbf{k}\times\hat{\mathbf{z}}) \frac{C}{[k^2 + \langle \omega_0 \rangle + (M/M_z - 1)\langle \omega_0 \rangle k_\perp^2/k^2]} \ , \qquad \text{(B14)}$$

$$\mathbf{h}_{1B} = -\left[\frac{M}{M_z} - 1 \right] \langle \omega_0 \rangle (i\mathbf{k}\times\hat{\mathbf{z}}) \frac{C_B}{k^2} \ . \qquad \text{(B15)}$$

Equation (B14) together with Eq. (B4) then solves Eq. (B1).

APPENDIX C: MINIMIZATION OF THE FREE ENERGY

Using Eq. (2.9) in conjunction with the results of Appendices A and B, the free energy can be written

$$\begin{aligned} f - f_0 = {} & \frac{\langle \omega_0 \rangle}{4\kappa^2} \left\langle k_\psi^2 (\bar{\eta} + \theta)^2 + k_\perp^2 \theta^2 + \frac{M}{M_z} \left[\frac{\partial(\bar{\eta} + \theta)}{\partial z} \right]^2 \right\rangle + \langle \omega_0 \rangle \left\langle k_\perp^2 \chi^2 + \frac{M}{M_z} \left[\tilde{Q}_z - \frac{\partial \chi}{\partial z} \right]^2 \right\rangle \\ & - B^2 \langle \omega_0 \rangle^2 \frac{\langle s^2 k_z^2 + (\mathbf{s}\cdot\mathbf{k})^2 \rangle}{k^2 (k^2 + \langle \omega_0 \rangle)} + \frac{M}{M_z} \frac{B^2}{2b\kappa^2} s^2 k_z^2 \langle \omega_0 \rangle - \langle \omega_0 \rangle^2 \left[\frac{M}{M_z} - 1 \right]^2 \frac{k_\perp^2}{k^2} \langle C C_B \rangle \\ & + B \frac{\langle \omega_0 \rangle^2}{k^2 + \langle \omega_0 \rangle} \left[\frac{M}{M_z} - 1 \right] \left\langle \frac{C_B}{k^2} \frac{\partial}{\partial z} (\hat{\mathbf{z}}\cdot\nabla\times\mathbf{s}) \right\rangle + B \frac{\langle \omega_0 \rangle^2}{k^2} \left[\frac{M}{M_z} - 1 \right] \frac{\langle C (\partial/\partial z)(\hat{\mathbf{z}}\cdot\nabla\times\mathbf{s}) \rangle}{[k^2 + \langle \omega_0 \rangle + (M/M_z - 1)\langle \omega_0 \rangle k_\perp^2/k^2]} \ , \end{aligned} \qquad \text{(C1)}$$

where f_0 is the free energy of the ideal, nondeformable FLL (the Abrikosov solution) given by

$$f_0 = -\langle \omega_0 \rangle (1-b) + \frac{1}{4\kappa^2} \langle \omega_0 \rangle^2 [(2\kappa^2-1)\beta+1] + B^2$$

which, when minimized with respect to $\langle \omega_0 \rangle$, gives

$$\langle \omega_0 \rangle = \frac{(1-b)2\kappa^2}{(2\kappa^2-1)\beta+1} .$$

The variational functions θ and χ are now determined by minimizing the free energy, Eq. (C1), with respect to θ and χ. Varying with respect to θ gives

$$\theta = -\frac{[k_\psi^2 + (M/M_z)k_z^2]}{[k_\psi^2 + k_\perp^2 + (M/M_z)k_z^2]} \overline{\eta} \tag{C2}$$

and

$$\overline{\eta} + \theta = 2b\kappa^2 \frac{\nabla \cdot \mathbf{s}}{[k_\psi^2 + k_\perp^2 + (M/M_z)k_z^2]} , \tag{C3}$$

and varying with respect to χ gives

$$\chi = -\frac{B}{k^2} \frac{k_z^2}{k_\perp^2} \left[1 + \left[\frac{M}{M_z} - 1 \right] \frac{k_\perp^2}{k_\perp^2 + (M/M_z)(k_z^2 + \langle \omega_0 \rangle)} \right] (\hat{\mathbf{z}} \cdot \nabla \times \mathbf{s}) \tag{C4}$$

and

$$\tilde{Q}_z - \frac{\partial \chi}{\partial z} = -\frac{B}{k^2} \left[1 - \left[\frac{M}{M_z} - 1 \right] \frac{k_z^2}{k_\perp^2 + (M/M_z)(k_z^2 + \langle \omega_0 \rangle)} \right] \frac{\partial}{\partial z} (\hat{\mathbf{z}} \cdot \nabla \times \mathbf{s}) . \tag{C5}$$

Substituting these expressions involving the variational functions θ and χ into the free energy, Eq. (C1), and using $\overline{\eta} = 2b\kappa^2(\nabla \cdot \mathbf{s})/k_\perp^2$ together with

$$(\mathbf{k} \cdot \mathbf{s})^2 + (\mathbf{k}_\perp \times \mathbf{s})^2 = k_\perp^2 s^2 , \tag{C6}$$

the free energy reduces to

$$f - f_0 = \tfrac{1}{2}[c_{11}(\nabla \cdot \mathbf{s})^2 + c_{44}(\partial \mathbf{s}/\partial z)^2] , \tag{C7}$$

where c_{44} and c_{11} are given in Eqs. (2.19) and (2.20), respectively.

[1] P. L. Gammel, D. J. Bishop, G. J. Dolan, J. R. Kwo, C. A. Murray, L. F. Schneemeyer, and J. V. Waszczak, Phys. Rev. Lett. **59**, 2592 (1987).

[2] P. L. Gammel, L. F. Schneemeyer, J. V. Waszczak, and D. J. Bishop, Phys. Rev. Lett. **61**, 1666 (1988).

[3] J. M. Kosterlitz and D. J. Thouless, in *Progress in Low Temperature Physics,* edited by D. F. Brewer (North-Holland, Amsterdam, 1978), Vol. 7-B.

[4] A. Houghton and M. A. Moore, Phys. Rev. Lett. **60**, 1207 (1988).

[5] D. R. Nelson, Phys. Rev. Lett. **60**, 1973 (1988).

[6] M. A. Moore, Phys. Rev. B **39**, 136 (1989).

[7] D. R. Nelson and H. S. Seung, Phys. Rev. B **39**, 9153 (1989).

[8] See, e.g., A. L. Fetter and P. C. Hohenberg, in *Superconductivity,* edited by R. D. Parks (Marcel Dekker, New York, 1969), Vol. II.

[9] E. H. Brandt, J. Low Temp. Phys. **26**, 709 (1977); **26**, 735 (1977); **28**, 263 (1977); **28**, 291 (1977).

[10] F. Lindemann, Phys. Z **11**, 69 (1910).

[11] D. R. Nelson, J. Stat. Phys. (to be published).

[12] A. I. Larkin and Yu. N. Ovchinnikov, J. Low Temp. Phys. **34**, 409 (1979).

[13] Y. Iye, T. Tanegai, H. Takeya, and H. Takei, Physica **148B**, 224 (1987).

[14] T. T. M. Palstra, B. Batlogg, L. F. Schneemeyer, R. B. van Dover, and J. V. Waszczak, Phys. Rev. B **28**, 5102 (1988).

[15] B. Batlogg, T. T. M. Palstra, L. F. Schneemeyer, R. B. van Dover, and R. J. Cava, Physica C **153-155**, 1062 (1988).

[16] R. S. Markiewicsz, J. Phys. C **21**, L1173 (1988).

[17] P. L. Gammel (private communication).