PHYSICAL REVIEW B VOLUME 40, NUMBER 10 1 OCTOBER 1989

Electromagnetic absorption in anisotropic superconductors

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(Received 23 March 1988; revised manuscript received 27 March 1989)

We calculate the absorptive part of the frequency-dependent conductivity for a large class of anisotropic superconductors. The effect of nonmagnetic impurity scattering is included at the level of a self-consistent t-matrix approximation. It is found that low-frequency absorption $(0<\Omega<2\Delta)$ takes place for any superconducting state with nodes of the energy gap on the Fermi surface. We demonstrate that the limiting low-frequency dependence of $\sigma(\Omega)$ on impurity concentrations can help to determine the structure of the order parameter in candidate systems such as heavy-electron compounds or high-T_c superconductors. We also discuss how strong scattering, similar to that observed in ordinary superconductors doped with Kondo impurities, may lead to absorption with a threshold of $\Omega \simeq \Delta$ rather than 2Δ as in quasiisotropic systems. Finally, we consider the contribution of order-parameter collective modes to the electromagnetic absorption, and show that this absorption may be substantial and depends sensitively on the impurity scattering rate.

I. INTRODUCTION

The observed temperature dependence of thermodynamic and transport properties of heavy-fermion superconductors has led to suggestions that the order parameter is highly anisotropic, perhaps displaying points or lines of nodes on the Fermi surface.[1-5] Several current model calculations predict anisotropic, in particular, "d-wave"-like pairing.[6-8] With less conclusive experimental evidence, some theoretical model calculations of high-T_c superconductors also propose "extended s-wave"- or "d-wave"-like pairing for these systems.[9-12]

One probe which holds out considerable promise of shedding further light on the nature of a possible anisotropic superconducting state is electromagnetic absorption. As first shown by Mattis and Bardeen,[13] the absorptive part of the frequency-dependent conductivity $\sigma(\Omega)$ at zero temperature T vanishes for $\Omega<2\Delta$, except for a δ-function peak at $\Omega=0$ reflecting the ideal response of the superconducing condensate (here Δ is the isotropic energy gap). At finite temperatures the thermal excitations give rise to an additional Drude-type contribution of width of order $1/\tau$. The excitation threshold at 2Δ is a consequence of the existence of Cooper pairs: excitations at $T=0$ involve the breaking of a Cooper pair requiring at least the binding energy 2Δ. By contrast, in an anisotropic superconductor with gap nodes, excitation of Cooper pairs involving electrons near the location of the nodes on the Fermi surface requires an arbitrarily small energy. Consequently, the conductivity $\sigma(\Omega)$ should be finite down to $\Omega=0$, often obeying a power-law dependence on Ω. Moreover, in the presence of sufficiently strong impurity scattering such that the density of states $N(\omega)$ at $\omega=0$ is nonvanishing (true gapless superconductivity), $\sigma(\Omega)$ may be expected to tend to a nonvanishing limit as $\Omega \rightarrow 0$ as well.

In this paper we examine the frequency-dependent conductivity for a large class of superconducting states both analytically and numerically. Our work thus generalizes the calculation of $\sigma(\Omega)$ for isotropic superconductors by Mattis and Bardeen, Abrikosov *et al.*,[13] Skalski *et al.*,[14] and others, although employing a substantially different formulation. A simple adaptation of Mattis and Bardeen's treatment, however, assuming their result to be true for each direction in $\hat{\mathbf{k}}$ space and averaging over the Fermi surface, as attempted recently by Maekawa *et al.*,[15] is found to be in error even for the polycrystalline systems they consider.

To understand why a naive generalization of the Mattis-Bardeen (MB) result is not appropriate, we consider a plane electromagnetic wave incident on a superconductor occupying the half-space $z>0$. In the clean limit, only two length scales are relevant to the electromagnetic response: the penetration depth $\Lambda(\Omega)$ over which the field amplitude decays, and the coherence length ξ_0, which determines the order-parameter spatial variation near the surface. The original Mattis-Bardeen calculation[13] was carried through for a conventional s-wave type-I superconductor, for which $\xi_0 \gg \Lambda$ (cf. Appendix B). In addition, it was shown that an identical result for the normalized conductivity σ/σ_N, where σ_N is the normal-state conductivity, obtains in the dirty limit, $\xi_0 \gg l$, independent of Λ. Both situations correspond to the extreme nonlocal electrodynamics of Pippard, where the current at a point **r** is determined by contributions from the vector potential in a neighborhood around **r** of size ξ_0. Field-theoretical treatments by Abrikosov *et al.*[13] and Skalski *et al.*[14] reproduced these results. In particular, the latter work examined the London limit $\Lambda \gg \xi_0$ appropriate to type-II superconductors, but again recovered the classic MB result in the dirty limit. Extremely important in this regard, however, is the large separation between the energy scales determining transport (e.g., normal impurities) and pair breaking (e.g., magnetic impurities).

As the heavy-fermion (and high-T_c) compounds are strong type-II superconductors, the Pippard limit calculations will become relevant only if experiments are eventually performed on films of thickness $d \ll \xi_0$. Nevertheless, we give a brief discussion of this case in Appendix B. Our primary focus here will be the London limit, appropriate for electromagnetic absorption measurements in *bulk* type-II systems. In the *collisionless* limit $\Omega\tau \gg 1$, we find a universal limiting form for the normalized conductivity $\sigma(\Omega)/\sigma_N(\Omega)$ of an unconventional superconductor which bears a surprising resemblance to the MB result even in the anisotropic case. By contrast, the simple universal MB form is appropriate neither for clean ordinary type-II superconductors nor for anisotropic type-I systems. Furthermore, we do *not* recover the MB result for an unconventional superconductor in the dirty London limit. This is because for both the anisotropic states considered here and in Ref. 15, as well as the *isotropic* p-wave state considered below, normal impurities are pair-breaking and there is only a single relevant energy scale. Any finite concentration of impurities also leads to depression of T_c and smearing of the gap "edge" in a manner similar to that discussed by Skalski *et al.*[14] in the case of magnetic impurities in a conventional superconductor.

In Sec. II, we present the general formalism from which we derive $\sigma(\Omega)$, and discuss the collisionless limit. A number of conceptual differences from the Bardeen-Cooper-Schrieffer-Mattis-Bardeen result are then illustrated in Sec. III, where we discuss the isotropic p-wave state. One interesting new effect which arises in this case is the important influence of the phase shift of the scattering centers on absorption in the superconducting state, leading in the extreme resonant limit of scattering phase shifts near $\pi/2$ (Kondo-type impurities) to an absorption threshold at $\omega \simeq \Delta$ rather than 2Δ. Order-parameter collective modes may lead to absorption near $\Omega \sim \Delta$ as well.

In Sec. IV, the results of Sec. III are generalized to anisotropic superconductors, and analytical and numerical results for model states with points and lines of nodes on the Fermi surface are presented. It is then possible to derive power laws in frequency for the various tensor components of $\sigma(\Omega)$ in the collisionless limit, as well as gapless behavior at $\omega = 0$.

In Sec. V we consider the effect of order-parameter collective modes on the electromagnetic absorption in the simplest case, the pseudoisotropic Balian-Werthamer p-wave pairing state in the clean limit. The effect of impurities and anisotropy is qualitatively discussed.

In Sec. VI, finally, we present our conclusions, reserving some technical remarks regarding impurity vertex corrections and the Pippard limit for the Appendixes.

II. GENERAL FORMALISM

We begin by considering the usual expression for the current response to an electromagnetic vector potential **A**:

$$\begin{aligned}\mathbf{j}(q,\Omega) &= \overleftrightarrow{\mathbf{K}}(q,\Omega)\,\mathbf{A}(\mathbf{q},\Omega) \\ &= [\overleftrightarrow{\mathbf{K}}_p(q,\Omega) - Ne^2/mc]\,\mathbf{A}(\mathbf{q},\Omega)\ ,\end{aligned} \quad (1)$$

where the paramagnetic current response function is given by

$$K_p^{ij}(q,\Omega_m) = -\frac{e^2k_F^2}{m^2c}\sum_{\mathbf{k}}\hat{\mathbf{k}}_i\hat{\mathbf{k}}_j T \times \sum_n \mathrm{tr}[\underline{g}(k_+,\omega_n^+)\underline{g}(k_-,\omega_n^-)] \quad (2)$$

and $\mathbf{k}_\pm = \mathbf{k} \pm \mathbf{q}/2$, $\omega_n^+ = \omega_n + \Omega_m$, $\omega_n^- = \omega_n$. Vertex corrections to (2) will be discussed later. The single-particle matrix Green's function $\underline{g}(\mathbf{k},\omega_n)$ for an anisotropic superconducting state subject to nonmagnetic impurity scattering has been calculated in Refs. 16–18 to be

$$\underline{g}(\mathbf{k},\omega_n) = \frac{i\tilde{\omega}_n\underline{\tau}^0 + \xi_k\underline{\tau}^3 + \Delta_{\mathbf{k}}\underline{\tau}^1}{\tilde{\omega}_n^2 + \xi_k^2 + \Delta_k^2}\ , \quad (3)$$

where Δ_k is the anisotropic order parameter, $\xi_k = k^2/2m^* - \mu$ is the normal-state quasiparticle energy, and $\tilde{\omega}_n = \omega_n - \Sigma_0(\omega_n)$. The self-energy Σ_0 due to impurity scattering is given within a self-consistent T-matrix approximation by $\Sigma_0 = \Gamma G_0/(c^2 + G_0^2)$, where

$$G_0 = (1/\pi N_0)\sum_{\mathbf{k}} \tfrac{1}{2}\mathrm{tr}[\underline{\tau}^0\underline{g}(\mathbf{k},\omega_n)]\ ,$$

and N_0 is the density of states at the Fermi level. The scattering rate Γ is given in terms of the current relaxation time τ in the normal state and the (s-wave) scattering phase shift δ as $\Gamma = (2\tau\sin^2\delta)^{-1}$, and the constant $c = \cot\delta$. We have neglected an additional self-energy correction Σ_3 related to ξ_k, as it may be shown to be unimportant for this case.[18] The gap function Δ_k is unrenormalized in the case of s-wave impurity scattering and non-s-wave pairing, provided $\langle \Delta_k \rangle_{\hat{\mathbf{k}}}=0$ due to some parity or reflection symmetry over the Fermi surface. We will always make this assumption in what follows, but believe that the qualitative physics will remain the same if the restriction is relaxed to include gaps with general (large) anisotropy. This is a consequence of the breakdown of Anderson's theorem in the anisotropic state.

In evaluating (2) it is convenient to represent the Green's functions by their spectral representation and to do the frequency sum over ω_n, with the result

$$K_p^{ij}(\mathbf{q},\Omega)=\frac{e^2k_F^2}{m^2c}\int\int_{-\infty}^{\infty}d\omega\, d\omega'\frac{1}{(2\pi)^2}\sum_{\mathbf{k}}\hat{\mathbf{k}}_i\hat{\mathbf{k}}_j\mathrm{tr}[\underline{a}(\mathbf{k}_+,\omega)\underline{a}(\mathbf{k}_-,\omega')]\left[\frac{\tanh(\frac{1}{2}\beta\omega)-\tanh(\frac{1}{2}\beta\omega')}{\omega-\Omega-\omega'-i0}\right]. \quad (4)$$

Here $\underline{a}(\mathbf{k},\omega)$ is the single-particle matrix spectral function and the function $K_p^{ij}(q,\Omega_m)$ has been analytically continued in Ω from the points $\Omega_m=2\pi iTm$ to the real axis $\Omega+i0$.

In type-II superconductors, to which we confine our discussion for the most part, the electromagnetic absorption is in good approximation given by the absorptive part of the local frequency-dependent conductivity

$$\sigma_{ij}(\Omega)\equiv\lim_{q\to 0}\sigma_{ij}(\mathbf{q},\Omega)=-(c/\Omega)\lim_{q\to 0}\mathrm{Im}K_p^{ij}(\mathbf{q},\Omega) .$$

This is a consequence of the fact that the magnetic penetration depth Λ is much larger than the coherence length ξ_0 (London limit), which governs the q dependence of $K_p(\mathbf{q},\omega)$ in (1). However, this standard argument does not take into account the existence of massive collective modes in anisotropic superconductors, which introduce additional length scales, i.e., the wavelengths of collective modes at the given frequency Ω. At the frequency Ω where the wavelength matches the magnetic penetration depth one expects maximal absorption into the collective mode. This will be discussed for the simplest case, the pseudoisotropic Balian-Werthamer state in the clean limit, in Sec. V. The opposite limit of Λ being much smaller than the coherence length (Pippard limit), appropriate for type-I superconductors, will be briefly discussed in Appendix B. In order to evaluate the frequency-dependent conductivity we now replace the momentum sum in (4) by an integral over energy ξ_k and an integral over all directions $\hat{\mathbf{k}}$ on the Fermi surface and perform the ξ_k integral by complex integration. The result is

$$\sigma_{ij}(\Omega)=\sigma_0\frac{1}{2\Omega}\int_{-\infty}^{\infty}d\omega\{\tanh(\tfrac{1}{2}\beta\omega)-\tanh[\tfrac{1}{2}\beta(\omega-\Omega)]\}\times S_{ij}(\omega,\Omega) , \quad (5a)$$

where

$$S_{ij}(\omega,\Omega)=\frac{3}{2\tau}\int\frac{d\Omega_k}{4\pi}\hat{\mathbf{k}}_i\hat{\mathbf{k}}_j\times\mathrm{Im}\left[\frac{\tilde{\omega}'_+}{\tilde{\omega}_+-\tilde{\omega}'_+}\left[\frac{1}{\xi'_{0+}}-\frac{1}{\xi_{0+}}\right]+\frac{\tilde{\omega}'_-}{\tilde{\omega}_+-\tilde{\omega}'_-}\left[\frac{1}{\xi_{0+}}+\frac{1}{\xi'_{0-}}\right]\right] . \quad (5b)$$

Here we have defined $\tilde{\omega}_\pm=\tilde{\omega}(\omega\pm i0)$ and $\tilde{\omega}'_\pm=\tilde{\omega}(\omega-\Omega\pm i0)$ as well as $\xi_{0\pm}=\mathrm{sgn}(\omega)(\tilde{\omega}_\pm^2-\Delta_{\mathbf{k}}^2)^{1/2}$ and $\xi'_{0\pm}=\mathrm{sgn}(\omega-\Omega)[(\tilde{\omega}'_\pm)^2-\Delta_{\mathbf{k}}^2]^{1/2}$ with branch cuts chosen such that $\mathrm{Im}\xi_{0+}>0$ and $\mathrm{Im}\xi'_{0+}>0$.

We first discuss a few limiting cases of (5). In the limit $T=0$ the ω integral in (5a) is restricted to the interval from 0 to Ω. For the normal state, putting $\Delta_k=0$ and replacing $\tilde{\omega}_\pm=\omega\pm i/2\tau$, etc., one recovers the Drude result,

$$\sigma_{ij}(\Omega)=\delta_{ij}\sigma_0/[1+(\Omega\tau)^2]\equiv\delta_{ij}\sigma_N(\Omega) .$$

In the limit of weak impurity scattering such that $c\gg 1$ and $1/\tau\ll\Delta$, we find a simple result in the collisionless limit, i.e., for $\Omega\tau\gg 1$ by expanding in $1/\Omega\tau$. For any anisotropic superconducting state with order parameter $\Delta_{\mathbf{k}}$ such that $\langle\Delta_{\hat{\mathbf{k}}}\rangle=0$, or, more precisely, with unrenormalized gap, i.e., $\tilde{\Delta}_{\mathbf{k}}=\Delta_{\mathbf{k}}$, we find

$$\frac{\sigma_{ij}(\Omega)}{\sigma_N(\Omega)}=\frac{1}{\Omega}\int_{-\infty}^{+\infty}d\omega\,\tilde{S}_{ij}(\omega,\Omega)[f(\omega-\Omega)-f(\omega)] , \quad (6a)$$

where $f(\omega)$ is the Fermi function $f(\omega)=(e^{\beta\omega}+1)^{-1}$ and

$$\tilde{S}_{ij}(\omega,\Omega)=\tfrac{3}{2}[\langle\hat{\mathbf{k}}_i\hat{\mathbf{k}}_j\mathrm{Re}(\tilde{\omega}'_+/\xi'_{0+})\rangle\langle\mathrm{Re}(\tilde{\omega}_+/\xi_{0+})\rangle+\langle\hat{\mathbf{k}}_i\hat{\mathbf{k}}_j\mathrm{Re}(\tilde{\omega}_+/\xi_{0-})\rangle\langle\mathrm{Re}(\tilde{\omega}'_+/\xi'_{0+})\rangle] . \quad (6b)$$

The angular brackets denote an average over the Fermi surface. Note the subscript $-$ on ξ_0 in the second term of (6b). We note that the strict criterion for the validity of this simple expression is that the off-diagonal component of the impurity self-energy Σ_{12} vanish. For s-wave scattering and many simple order-parameter symmetries, $\langle\Delta_{\hat{\mathbf{k}}}\rangle_{\hat{\mathbf{k}}}=0$ is a sufficient condition.

This expression deserves a few comments. First, we note that, since

$$\left\langle\mathrm{Re}\frac{\tilde{\omega}_+}{\xi_{0+}}\right\rangle_{\hat{\mathbf{k}}}=N(\omega)/N_0\equiv\hat{N}(\omega) ,$$

where $N(\omega)$ is the quasiparticle density of states, Eq. (6) is a kind of angle-dependent generalization of the Mattis-Bardeen expression.[13] It involves a convolution of $N(\omega)$ with a moment of the angle-resolved density of states $\langle\hat{\mathbf{k}}_i\hat{\mathbf{k}}_j\mathrm{Re}(\tilde{\omega}'_+/\xi'_{0+})\rangle$ instead of the $N(\omega)N(\omega')$ of Mattis and Bardeen. In addition, we note that the "anomalous densities of states" $D(\omega)\equiv 1/\pi\langle\mathrm{Re}(\Delta_k/\xi_0)\rangle_{\hat{\mathbf{k}}}$, which lead to an additional convolution $D(\omega)D(\omega')$ in the Mattis-Bardeen result, do not occur in the regime considered (see Appendix B).

The full result (5) is valid for all scattering rates and phase shifts. In particular, we consider below the "gapless" limit, which we take to mean the existence of a finite density of quasiparticle excitations of zero energy. In the isotropic BCS case, with $c \gg 1$, this requires scattering rates $\Gamma_N \equiv \Gamma/(1+c^2) \geq \Delta$, as is well known (see, e.g., Ref. 14). For anisotropic states with lines of nodes, $N(0)>0$ for infinitesimal Γ, but does not become appreciable relative to N_0 until $\Gamma_N \sim \Delta_0$ again.[19] All superconducting states become gapless with small concentrations of impurities if the scattering is strong ($c \ll 1$). In isotropic states this takes the form of a low-energy bound state in the gap ("Kondo-resonance"), as first discussed by Maki[20] and Shiba[21] for the s wave, and by Buchholtz and Zwicknagl[22] for the p-wave case. This bound state is smeared and acquires a finite lifetime in the anisotropic superconducting state, but still gives rise to zero-energy excitations at surprisingly small impurity concentrations.[16,17] One expects that these effects should manifest themselves in two-particle properties as well.[16–18]

III. ISOTROPIC STATE

We consider here a superconducting order parameter which gives rise to isotropic quasiparticle excitations, but whose average over the Fermi surface vanishes, leaving its symmetry unperturbed by impurity scattering. A prototype for such a state is the Balian-Werthamer (BW) state of superfluid ^{3}He.[23] For such an order parameter, Eq. (6) reduces to

$$\sigma(\Omega)/\sigma_N(\Omega)=\frac{1}{\Omega}\int_{-\infty}^{\infty} d\omega \frac{\omega\,|\Omega-\omega|}{(\omega^2-\Delta^2)^{1/2}[(\omega-\Omega)^2-\Delta^2]^{1/2}} \times[f(\omega-\Omega)-f(\omega)] \; . \tag{7}$$

This is similar to the Mattis-Bardeen result[13] except for a term $-\Delta^2$ in the numerator under the integral, missing because $D(\omega)=0$ in the odd-parity BW state.

At $T=0$, (7) takes the simple form

$$\sigma(\Omega)/\sigma_N(\Omega)=\frac{1}{\Omega}\int_0^{\Omega} d\omega\, \hat{N}(\omega)\hat{N}(\omega-\Omega) \tag{8}$$

alluded to above.

The result (8) is approximately valid for finite, but small impurity concentration n_i, such that $\Gamma \ll \Delta$. [The actual condition is

$$\mathrm{Im}(\tilde{\omega}_+)\mathrm{Im}(1/\xi_{0+}) \ll \mathrm{Re}(\tilde{\omega}_+)\mathrm{Re}(1/\xi_{0+}) \; ,$$

as seen from (6b).] The density of states is then given by the general expression for $\hat{N}(\omega)$ given above.

However, even for $\Gamma \ll \Delta$, the conductivity $\sigma(\Omega)$ does not necessarily resemble the known BCS result. We may calculate the density of states by solving the self-consistent equation for $\tilde{\omega}$

$$\tilde{\omega}_+ = \omega + i\Gamma \frac{\tilde{\omega}_+(\tilde{\omega}_+^2-\Delta^2)^{1/2}}{c^2(\tilde{\omega}_+^2-\Delta^2)+\tilde{\omega}_+^2} \; . \tag{9}$$

For weak impurity scattering strength (phase shift $\delta \ll 1$) one finds a density of states close to the usual BCS density of states

$$\hat{N}_{\mathrm{BCS}}(\omega)=\mathrm{Re}[\omega/(\omega^2-\Delta^2)^{1/2}] \; .$$

Hence, at zero temperature there is no absorption, i.e., $\sigma(\Omega)=0$ for $\Omega < 2\Delta$. Contrary to the singlet isotropic pairing case, however, where $\sigma(\Omega)$ is found to increase linearly with $\Omega-2\Delta$ above the threshold, here $\sigma(\Omega)$ jumps to a finite value at $\Omega=2\Delta+i0$. The linear slope of $\sigma(\omega)$ as a function of $\Omega-2\Delta_0$ usually found in calculations of the absorption arises only as a result of cancellations resulting from the presence of the anomalous convolution $D(\omega)D(\omega')$, which vanishes here. As the impurity concentration is increased, the threshold moves to lower Ω and broadens.

In Fig. 1, we have evaluated Eq. (5) for $\sigma(\Omega)/\sigma_N(\Omega)$ numerically for the BW state in the presence of weak scatterers at $T=0$. The crucial difference from the Mattis-Bardeen case is the discontinuous jump in absorption from zero to a finite value at $\omega=2\Delta_0$ in the clean limit. Otherwise the progressive "smearing" of the gap edge at 2Δ with increasing scattering rate Γ and eventual gapless behavior at $\Gamma_N \approx \Delta$ (recall the gap magnitude Δ is also depressed by pair breaking) is reminiscent of the BCS case.[14]

For strong impurity scattering, such that $\Gamma > \Delta \cos^2\delta$, the behavior of $\sigma(\Omega)$ is quite different. In this case the scattering amplitude has a pole, which, as discussed in Sec. II, gives rise to a resonance peak in the density of states in the gap region, located at $\omega=0$ for unitary scattering ($\delta=\pi/2$) but centered at a finite frequency ω_r for $\delta \neq \pi/2$ (see Fig. 3). We examine first the case of resonant scattering, $c=0$. Given the presence of low-energy bound states, the small absorption features shown in Fig. 2 below $\Omega=\Delta$ are perhaps not surprising; they correspond to scattering of quasiparticles from filled bound states just below the Fermi surface to empty ones just above. The most unusual feature in the figure is the large absorption edge at $\Omega=\Delta$ ($c=0$). This may be crudely understood in terms of Eq. (8), which at $\Omega=\Delta$ involves products of the BCS-type density of states function at the

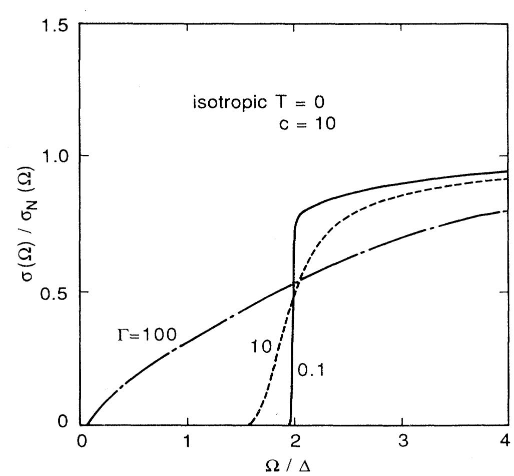

FIG. 1. Absorption in a Balian-Werthamer (isotropic) state at $T=0$ for weak scattering ($c=10$). Solid line: $\Gamma/T_c=0.1$; dashed: $\Gamma/T_c=10.0$; dashed-dotted: $\Gamma/T_c=100$.

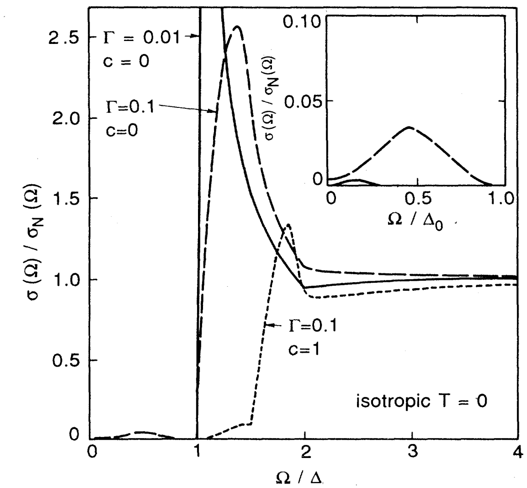

FIG. 2. Absorption in a Balian-Werthamer state at $T=0$ for strong scattering. Solid line: $\Gamma/T_c=0.01$, $c=0$; long dashed: $\Gamma/T_c=0.1$, $c=0$; short dashed: $\Gamma/T_c=0.1$, $c=1$.

gap edge $\omega=\Delta$ with the density of states at zero energy. Whereas for weak scattering the latter is zero, for resonant scattering the weight in $N(\omega)$ due to the bound state is folded in with the singularity in $N(\omega)$ at $\omega=\Delta$, giving a large absorption. Physically this corresponds to scattering a quasiparticle from the bound state at the Fermi surface into unoccupied states near the gap edge.

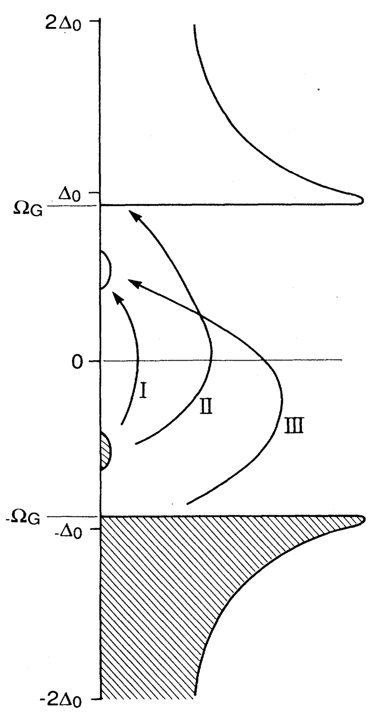

FIG. 3. Schematic of absorption in Fig. 2, $\Gamma/T_c=0.1$, $c=1$. I: scattering processes from filled to empty resonant states; II: scattering from filled resonance to gap edge; III: scattering from filled states at gap edge to unoccupied resonance.

For nonresonant scattering phase shifts, the bound state moves away from zero energy.[22] Thus it is easy to understand the short-dashed curve in Fig. 2 ($c=1$) in terms of the schematic of Fig. 3. Below $\Omega\simeq\Delta$, no phase space is available for scattering. At $\Omega\simeq\Delta$, quasiparticles may be scattered from bound state to bound state with small amplitude (type-I process). Above $\Omega\simeq 1.5\Delta_0$, processes of types II and III contribute, scattering quasiparticles from gap edge to bound state and vice versa. Pair-breaking processes of the usual type also occur above $\Omega\simeq 2\Omega_G$, where $2\Omega_G$ is the density-of-states energy gap in the presence of impurities, but do not serve to determine the absorption threshold.

In ordinary superconductors with Kondo impurities, where few measurements have been made, observation of an absorption threshold at $\Omega=\Delta_0$ would be an interesting and useful confirmation of the single-particle picture we have presented. [While we have not treated the gap renormalization correctly for this case, we expect that the qualitative features, particularly the threshold in $\sigma(\Omega)$, should be preserved in a more complete calculation.]

IV. ANISOTROPIC STATES

The general result (5) of the absorptive part of the bare current response is correct for anisotropic order parameters only insofar as we may ignore vertex corrections. These are of two types: (a) pair interaction vertex corrections, which correspond to collective modes of the tensor order parameter, which may couple to electromagnetic waves, and (b) impurity dressings of the bare current vertex. In order to maintain a fully conserving approximation to σ, one should solve the coupled diagrammatic equations which result from consideration of both types of corrections together, as attempted recently by May[24] in the context of sound propagation in heavy-fermion systems (cf. discussion in Sec. V). Even in the clean limit, solution of the full collective-mode problem is an arduous task, and has only recently been solved by Hirashima and Namaizawa for some special cases.[25] In Sec. V we will calculate the contribution of collective modes to the electromagnetic absorption for the simplest case of a quasi-isotropic order parameter (Balian-Werthamer state[23]), in the clean limit. Even at low frequencies, however, impurity vertex corrections will in general contribute to $\sigma(\Omega)$. For certain choices of order-parameter symmetry, these vanish altogether, as discussed in Ref. 18. There is, for example, no coupling of any even-parity order parameter with the odd-parity bare current vertex.[18] Even in the case of odd-parity pairing, the largest tensor component of any current-current correlation function ($q=0$) is unrenormalized by impurity vertex corrections. In what follows we focus on exactly these components, since they are expected to dominate the qualitative behavior of the conductivity except for certain special directions. In Appendix A, we give a prescription for calculating impurity vertex corrections and discuss how they might affect the observed conductivity anisotropy at low frequencies, for example.

In the following we present results of an evaluation of $\sigma(\Omega)$ as given by (5). To simplify our discussion we calculate $\sigma(\Omega)$ for two model states representative of broader classes of states with nodes on the Fermi surface. We consider (1) an axial state $\Delta_k=\Delta_0(\hat{\mathbf{k}}_x+i\hat{\mathbf{k}}_y)$ with point nodes on the Fermi surface, and (2) a polar state $\Delta_k=\Delta_0\hat{\mathbf{k}}_z$, with a single line of nodes. In the spirit of Sec. III, it is very instructive to examine the collisionless limit, where Eq. (6) holds. The required quantities $\langle\xi_0^{-1}\rangle_{\hat{k}}$ have been calculated in Ref. 18 for various order parameters. In particular, for an axially symmetric gap one finds

$$\langle\hat{\mathbf{k}}_z^2\mathrm{Re}(1/\xi_0)\rangle_{\hat{\mathbf{k}}}=\frac{1}{2\Delta_0}\left[z-\frac{z^2-1}{2}\ln\left|\frac{z+1}{z-1}\right|\right], \quad (10a)$$

where $z=\omega/\Delta_0$, and

$$\langle\mathrm{Re}(1/\xi_0)\rangle_{\hat{\mathbf{k}}}=\frac{1}{2\Delta_0}\ln\left|\frac{z+1}{z-1}\right| . \quad (10b)$$

For a polar gap one gets

$$\langle\mathrm{Re}(1/\xi_0)\rangle_{\hat{\mathbf{k}}}=\frac{1}{\Delta_0}\sin^{-1}\frac{1}{z} \quad (10c)$$

and

$$\langle\hat{\mathbf{k}}_z^2\mathrm{Re}(1/\xi_0)\rangle_{\hat{\mathbf{k}}}=\frac{1}{2\Delta_0}\left[-(z^2-1)^{1/2}+z^2\sin^{-1}\frac{1}{z}\right] . \quad (10d)$$

In the limit of small frequencies $\Omega\ll\Delta_0$, one finds at $T=0$

$$\frac{\sigma_{\parallel,\perp}(\Omega)}{\sigma_N(\Omega)}=a_{\parallel,\perp}\left[\frac{\Omega}{\Delta_0}\right]^{p_{\parallel,\perp}} , \quad (11)$$

where $p_\parallel=4$ and $p_\perp=6$ and $a_\parallel=\frac{1}{10}$, $a_\perp=\frac{2}{105}$ for the axial state, while for the polar state $p_\parallel=4$, and $p_\perp=2$ and $a_\parallel=3\pi^2/160$, $a_\perp=\pi^2/16$. Note that in this regime $\sigma_N(\Omega)\sim\Omega^{-2}$, so that, e.g., $\sigma_\perp(\Omega)\sim$ const and $\sigma_\parallel(\Omega)\propto\Omega^2$ for a polar state, in contrast to the result of Maekawa *et al.*[15] These power laws are clearly visible in the solid and short-dashed lines of Figs. 4 and 5 for the polar $\perp$ and axial $\parallel$ components of σ at $T=0$.

On the other hand, the behavior of σ in the strong scattering case is dominated down to the lowest frequencies by the resonance analogous to the bound state described in Sec. III. For $\Omega\lesssim\Gamma^{1/2}$, we may perform a low-frequency expansion appropriate to the gapless regime for the polar state generally, and for the axial state provided $\Gamma/\Delta_0>2c^2/\pi$ (see Ref. 18). Using the result obtained in Ref. 18 for the renormalized frequency $\tilde{\omega}=iv_0+a\omega$ for $\omega\to0$ we expand (5) in powers of ω with the result

$$\frac{\sigma_{ij}(\Omega)}{\sigma_N(0)}=\frac{1}{\Delta_0\tau}\bar{S}_{ij}\left[y_0^2+\frac{a^2}{6}\left[\frac{\Omega}{\Delta_0}\right]^2\right] , \quad (12a)$$

where

$$\bar{S}_{ij}=\tfrac{3}{2}\Delta_0^3\left\langle\frac{\hat{\mathbf{k}}_i\hat{\mathbf{k}}_j}{(v_0^2+\Delta_k^2)^{3/2}}\right\rangle_{\hat{\mathbf{k}}} , \quad (12b)$$

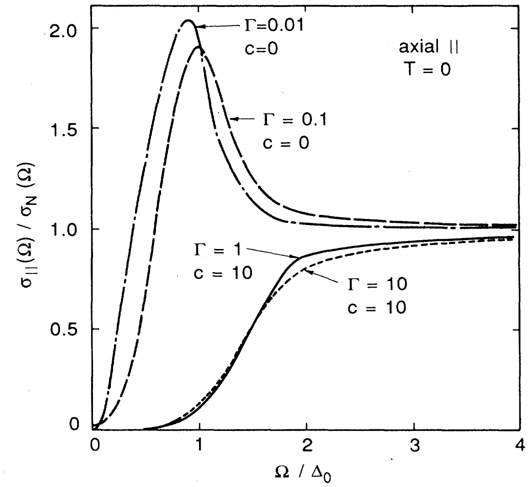

FIG. 4. Absorption in an axial state at $T=0$. Solid line: $\Gamma/T_c=1$, $c=10$; short dashed: $\Gamma/T_c=10$, $c=10$; long dashed: $\Gamma/T_c=0.1$, $c=0$; dashed dotted: $\Gamma/T_c=0.01$, $c=0$.

and $y_0=v_0/\Delta_0$ is a solution of the equation

$$\frac{\Gamma}{\Delta_0}f(y)-c^2=y^2f^2(y) , \quad (13)$$

with $f(y)=\langle[y^{-2}+(\Delta_{\hat{k}}/\Delta_0)^2]^{-1/2}\rangle_{\hat{k}}$. For the axial state $f(y)=\tan^{-1}(1/y)$ and for the polar state $f(y)=\sinh^{-1}(1/y)$. The finite frequency correction coefficient a is given by

$$a=1-\left[\frac{\Delta_0}{\Gamma}\right]h[y^2-c^2/f^2(y)] , \quad (14)$$

where $h=\Delta_0\langle\Delta_{\hat{k}}^2(v_0^2+\Delta_{\hat{k}}^2)^{-3/2}\rangle_{\hat{k}}$. Note that in the limit

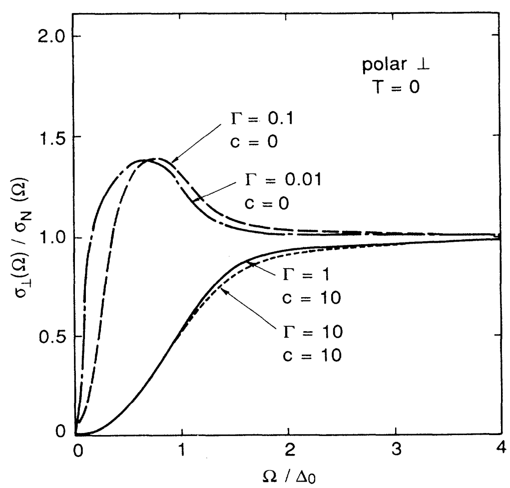

FIG. 5. Absorption in a polar state at $T=0$. Solid line: $\Gamma/T_c=1$, $c=10$; short dashed: $\Gamma/T_c=10$, $c=10$; long dashed: $\Gamma/T_c=0.01$, $c=0$; dashed dotted: $\Gamma=0.1$, $c=0$.

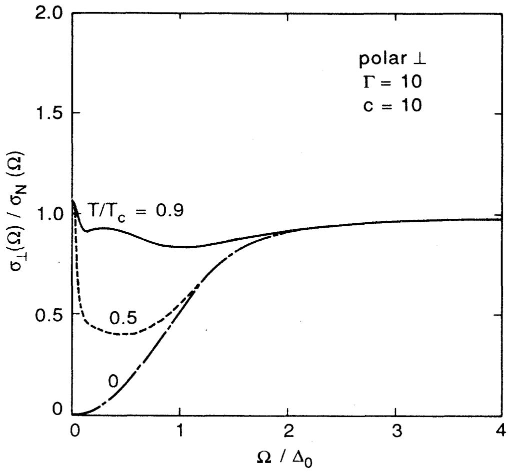

FIG. 6. Finite temperature absorption in a polar state (weak scattering, $\Gamma/T_c=10$, $c=10$). Solid line: $T/T_c=0.9$; dashed: $T/T_c=0.5$; dashed dotted: $T/T_c=0$.

$\Gamma/\Delta_0 \gg 1$,

$$v_0 \rightarrow \Gamma_N \equiv (1/2\tau) = \Gamma/(1+c^2)$$

and $S_{ij} \rightarrow \delta_{ij}(\Delta_0/\Gamma_N)^3$, so that $\sigma_{ij}(0)/\sigma_N(0) \rightarrow 1$.

At any finite temperatures, some absorption will be caused by scattering from thermally excited quasiparticles. In Figs. 6 and 7 we show results for $\sigma(\Omega)$ in a polar state at various temperatures for weak and strong scattering, respectively. Obviously when the gap magnitude is sufficiently reduced by temperature, anisotropic states become more or less indistinguishable.

V. COLLECTIVE-MODE CONTRIBUTION TO THE ELECTROMAGNETIC ABSORPTION IN THE BALIAN-WERTHAMER STATE

The frequency-dependent conductivity given in Eq. (5) neglects absorption resulting from collective modes of the

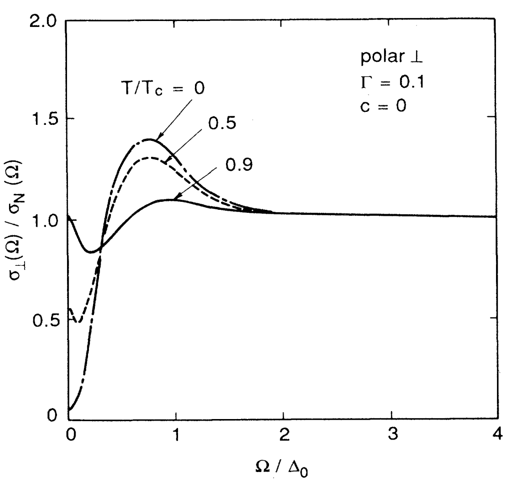

FIG. 7. Finite temperature absorption in a polar state (strong scattering, $\Gamma/T_c=0.1$, $c=0$). Solid line: $T/T_c=0.9$; dashed: $T/T_c=0.5$; dashed dotted: $T/T_c=0$.

order parameter. The role of collective modes in the electromagnetic response of superconductors is an old subject which originated with questions raised about the gauge invariance of the original BCS theory. Anderson's 1958 paper[26] provided a gauge-invariant formulation of the pairing theory and elucidated the role of the collective modes in superconductors. These modes may be classified into (i) Goldstone modes, associated with spontaneously broken continuous symmetries, and (ii) exciton modes, a generic title assigned to modes that involve a time-dependent deformation of the order parameter. Exciton modes have a dispersion relation with a gap at $q=0$. In conventional superconductors, those in which only gauge symmetry is broken, the only Goldstone mode is the phase oscillation of Δ_k. However, the phase mode oscillates at the plasma frequency $\Omega_{\rm pl}^2=4\pi ne^2/m$, because of the coupling to long-wavelength charge fluctuations, and so is uninteresting for experimental superconductivity. Many authors, beginning with Anderson[26] and Bogoliubov, Shirkov, and Tolmachev,[27] predicted the existence of exciton modes with energies $\Omega < 2\Delta_k$, corresponding to excited, bound states of Cooper pairs.[28] Exciton modes have never been definitively observed in any superconductor to our knowledge. The probable reason is that the exciton modes are always nested close to the gap edge of 2Δ for any conventional superconductor. For an exciton mode to exist with energy below the gap edge in a conventional s-wave superconductor, there must be pairing interaction V_l, binding Cooper pairs with relative angular momentum l, that is nearly as attractive as the s-wave pairing interaction V_0 that binds pairs in the ground state. This was shown theoretically to be the case by Tsuneto,[29] and Bardasis and Schrieffer.[30] The energies Ω_l for excitons of angular momentum l are functions of $|1/V_0-1/V_l|$ and are located near 2Δ except for $V_l - V_0$. The existence of two nearly degenerate pairing channels is an unlikely occurrence for conventional superconductors in which the pairing is mediated by the electron-phonon interaction, except perhaps in highly anisotropic conventional superconductors.

In unconventional superconductors, exciton modes with energies well separated from the gap edge do not require the existence of a second, nearly degenerate pairing channel. The reason is that, in contrast to an s-wave superconductor described by a scalar order parameter, an unconventional superconductor is described by an order parameter that breaks the rotational symmetry of the normal state, in addition to gauge symmetry. Rotational symmetry breaking implies that the order parameter belongs to a nontrivial representation of the symmetry group of the normal state. That Δ_k belongs to a higher dimensional representation implies that there is a spectrum of pairing states belonging to this representation, and thus bound by the same pairing interaction. This feature of the collective mode spectrum is well known in superfluid ^{3}He, which is an unconventional superfluid with an order parameter that breaks rotational symmetry in both spin and orbital space.[31]

If heavy-fermion superconductors are unconventional superconductors, then collective modes of the order parameter should play a more important role in the electromagnetic response of these superconductors,[32] at least in the limit of high purity where $\omega\tau_{\mathrm{imp}} \gg 1$. For this reason, several authors[25,33] have examined the spectrum of collective modes for various models of the superconducting ground state. What is common to all of these studies is that unconventional superconducting phases of either space parity (i.e., even-parity singlet states or odd-parity triplet states) exhibit a spectrum of order parameter collective modes including the gauge mode and excitonic modes with excitation energies lying below the gap edge.

Here we calculate the contribution that excitonic modes make to the absorptive part of the electromagnetic response of an unconventional superconductor. We choose the odd-parity triplet Balian-Werthamer state as our model unconventional superconducting phase simply because the isotropic gap of the BW phase allows us to carry out the calculations of the absorption analytically. The main features of this calculation are also qualitatively, and semiquantitatively, correct for anisotropic unconventional superconductors with exciton modes lying in the gap. Those results that are not general are discussed below.

Heavy-fermion metals (UPt_3, UBe_{13}) are type-II superconductors with field penetration lengths Λ that are large compared to their coherence lengths ξ_0. This is important because it implies that an electromagnetic wave penetrates relatively deep into the superconductor at a vacuum-superconducting interface, and probes the bulk order parameter by exciting currents far from the interface where the order parameter is distorted from its bulk form.

The previous results for the frequency-dependent conductivity omit any excitonic contribution to the current response. To include the collective-mode contributions to the current, the bare paramagnetic response function K_p discussed in Sec. II is generalized to include vertex corrections resulting from polarization of the medium by the field. In addition, at any finite $\mathbf{q}$, $K_p(\mathbf{q},\Omega)$ is renormalized by vertex corrections due to impurity scattering. To treat the response in the presence of impurities in a fully gauge invariant, conserving random-phase approximation (RPA) one should, in principle, include[24] all the diagrams shown in Fig. 8. For simplicity, we focus on the collisionless regime, where impurity vertex corrections may be neglected and collective mode contributions are included by solving a transport equation for the distribution function $\delta g(\hat{\mathbf{k}},\omega;q,\Omega)$ for electrons propagating along the direction $\hat{\mathbf{k}}$ with excitation energy ω in the presence of a field $\mathbf{A}$. The details of this calculation are lengthy and omitted here. The current response is given in terms of the distribution function as

$$\mathbf{j} = -eN_0 \int \frac{d\Omega}{4\pi} \mathbf{v}_{\hat{\mathbf{k}}} \int d\omega\, \delta g(\hat{\mathbf{k}},\omega;\mathbf{q},\Omega) \;, \tag{15}$$

where $\delta g(\hat{\mathbf{k}},\omega;\mathbf{q},\Omega)$ is the solution of the transport equation and is given by

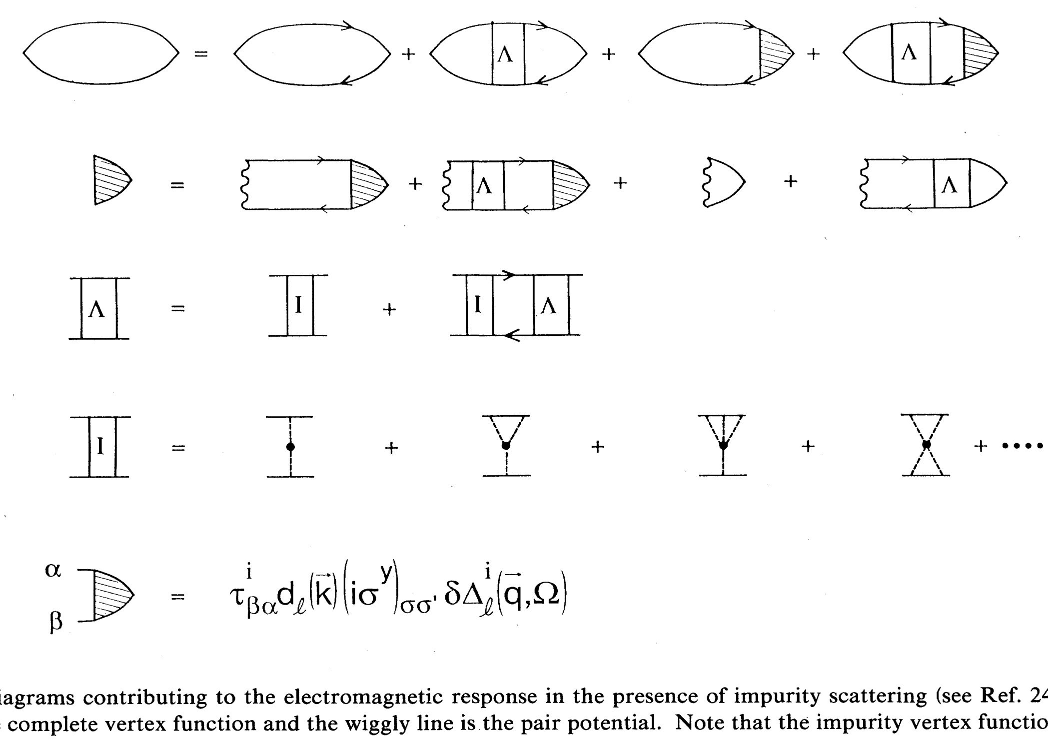

FIG. 8. Diagrams contributing to the electromagnetic response in the presence of impurity scattering (see Ref. 24). The shaded triangle is the complete vertex function and the wiggly line is the pair potential. Note that the impurity vertex function Λ and the irreducible vertex I are discussed in Appendix A.

$$\int d\omega\, \delta g(\hat{\mathbf{k}},\omega;\mathbf{q},\Omega)=\frac{2e}{c}\lambda(\mathbf{v}_{\hat{\mathbf{k}}}\cdot\mathbf{A})-2ie\left[\frac{\Omega}{\Omega^2-(\mathbf{q}\cdot\mathbf{v}_{\hat{\mathbf{k}}})^2}\right](1-\lambda)(\mathbf{v}_{\hat{\mathbf{k}}}\cdot\mathbf{E})+\lambda\left[\frac{\mathbf{q}\cdot\mathbf{v}_{\hat{\mathbf{k}}}}{\Delta^2}\right][\mathbf{\Delta}\cdot\mathbf{d}(\mathbf{k})]\ , \tag{16}$$

where λ is the finite-q generalization of the pair response function introduced by Tsuneto:[29]

$$\lambda(\Omega,q)=|\Delta_{\hat{\mathbf{k}}}|^2\int\frac{d\omega}{2\pi i}\frac{2\omega\Omega(\beta_+-\beta_-)+\eta^2(\beta_++\beta_-)}{(4\omega^2-\eta^2)(\Omega^2-\eta^2)+4|\Delta_{\hat{\mathbf{k}}}|^2\eta^2}\ ,$$
$$\beta(\omega)=2\pi i\frac{\mathrm{sgn}\omega}{(\omega^2-|\Delta_{\hat{\mathbf{k}}}|^2)^{1/2}}\tanh\frac{\omega}{2T}\theta(\omega^2-|\Delta_{\hat{\mathbf{k}}}|^2)\ . \tag{17}$$

Here $\beta_\pm=\beta(\omega\pm\Omega/2)$, $\eta=\mathbf{v}_{\hat{\mathbf{k}}}\cdot\mathbf{q}$, and $\mathbf{\Delta}\equiv-\frac{1}{2}\mathrm{tr}(i\underline{\sigma}^2\underline{\boldsymbol{\sigma}}\Delta_{\hat{\mathbf{k}}})$ is the vector representation of the spin-triplet gap matrix $\underline{\Delta}_{\hat{\mathbf{k}}}$. The term proportional to $\mathbf{d}(\mathbf{k})\equiv\mathrm{Im}\delta\mathbf{\Delta}_{\mathbf{k}}$ represents the contribution to the distribution function due to excitation of the order parameter, and $|\Delta_k|^2=\Delta^2$ for the BW state. This expression is clearly gauge invariant since under an infinitesimal transformation $\mathbf{A}\rightarrow\mathbf{A}+i\mathbf{q}\xi$, the order parameter transforms as

$$\Delta_{\hat{\mathbf{k}}}\rightarrow\Delta_{\hat{\mathbf{k}}}e^{-i\frac{2e}{c}\xi}\approx\Delta_{\hat{\mathbf{k}}}\left[1-i\frac{2e}{c}\xi\right]\ . \tag{18}$$

For the transverse gauge, $\mathbf{q}\cdot\mathbf{A}=0$, $\mathbf{E}=i\Omega\mathbf{A}/c$, in which case

$$\int d\omega\,\delta g(\hat{\mathbf{k}},\omega;\mathbf{q},\Omega)=\frac{2e}{c}\left[\lambda+\frac{\Omega^2}{\Omega^2-(\mathbf{q}\cdot\mathbf{v}_{\hat{\mathbf{k}}})^2}(1-\lambda)\right](\mathbf{v}_{\hat{\mathbf{k}}}\cdot\mathbf{A})+\lambda\left[\frac{\mathbf{q}\cdot\mathbf{v}_{\hat{\mathbf{k}}}}{\Delta^2}\right][\mathbf{\Delta}_{\hat{\mathbf{k}}}\cdot\mathbf{d}(\mathbf{k})]\ . \tag{19}$$

The response of the order parameter, $\mathbf{d}(\mathbf{k})$, is given by the time-dependent gap equation. The result calculated from the quasiclassical transport equations may be written

$$\mathbf{d}(\hat{\mathbf{k}})=\frac{1}{2}\int\frac{d\Omega_{\mathbf{k}'}}{4\pi}V_t(\hat{\mathbf{k}}\cdot\hat{\mathbf{k}}')\left[\left[\gamma+\frac{\lambda}{2\Delta^2}[\Omega^2-4\Delta^2-(\mathbf{q}\cdot\mathbf{v}_{\hat{\mathbf{k}}'})^2]\right]\mathbf{d}(\hat{\mathbf{k}}')+\frac{2\lambda}{\Delta^2}\mathbf{\Delta}_{\hat{\mathbf{k}}'}[\mathbf{\Delta}_{\hat{\mathbf{k}}'}\cdot\mathbf{d}(\hat{\mathbf{k}}')]-\frac{2e\lambda}{c\Delta^2}(\mathbf{q}\cdot\mathbf{v}_{\hat{\mathbf{k}}'})(\mathbf{v}_{\hat{\mathbf{k}}'}\cdot\mathbf{A})\mathbf{\Delta}_{\hat{\mathbf{k}}'}\right]\ , \tag{20}$$

where $V_t=3V_1(\hat{\mathbf{k}}\hat{\mathbf{k}}')$ is the p-wave pairing interaction and $\gamma=2/V_1$.

The solutions to the homogeneous equation for $q=0$ have been studied extensively in the context of ultrasonic absorption in superfluid ^{3}He-B. For $q=0$ the homogeneous equation is solved by expressing the components of d as $d_i(k)=d_{ij}k_j$, and assuming the simplest BW state $\Delta_k^i=\Delta\hat{\mathbf{k}}_i$. The eigenfunctions are then the spherical tensors $t_{ij}^{(JM)}$ with $J=0,1,2$ and $|M|=0,1,\ldots,J$. Only the collective modes with $J=0$ and 2 couple to the electromagnetic field. For these modes the eigenfunctions and frequencies are

$$t_{ij}^{(0,0)}=\frac{1}{\sqrt{3}}\delta_{ij},\quad\Omega=0\ ,$$
$$t_{ij}^{(2,0)}=\sqrt{3/2}(\hat{\mathbf{s}}_i\hat{\mathbf{s}}_j-\tfrac{1}{3}\delta_{ij}),\quad\Omega=\sqrt{12/5}\Delta\ ,$$
$$t_{ij}^{(2,\pm1)}=\mp\tfrac{1}{2}[\hat{\mathbf{s}}_i(\hat{\mathbf{u}}\pm i\hat{\mathbf{v}})_j+(\hat{\mathbf{u}}\pm i\hat{\mathbf{v}})_i\hat{\mathbf{s}}_j],\quad\Omega=\sqrt{12/5}\Delta\ , \tag{21}$$
$$t_{ij}^{(2,\pm2)}=\tfrac{1}{2}(\hat{\mathbf{u}}\pm i\hat{\mathbf{v}})_i(\hat{\mathbf{u}}\pm i\hat{\mathbf{v}})_j,\quad\Omega=\sqrt{12/5}\Delta\ ,$$

where (u,v,s) is a fixed set of orthonormal coordinate axes defining the orientation of the collective modes; the orientation and coupling to these modes is dictated by the external field $\mathbf{A}$ and the direction of propagation $\mathbf{q}$. To determine the collective-mode contributions to the current response we solve the inhomogeneous equation for d_{ij} by expanding in the solutions $t_{ij}^{(JM)}$ of the homogeneous equation; choosing the quantization axis of the modes to be $\mathbf{q}$, we have

$$d_{ij}=\sum_{J,M}\alpha_{JM}(\Omega,q)t_{ij}^{(JM)}\ . \tag{22}$$

The spherical tensors satisfy the orthogonality relations,

$$\mathrm{Tr}(t^{(JM)}t^{(J'M')\dagger})=\delta_{JJ'}\delta_{MM'}\ , \tag{23}$$

which we use to project out the amplitudes $\alpha_{JM}(\Omega,q)$ from Eq. (22):

$$[\Omega^2-\tfrac{1}{3}(qv_f)^2]\alpha_{00}-\tfrac{2}{15}(qv_f)^2(\sqrt{2}\alpha_{20})=0\ ,$$
$$[\Omega^2-\tfrac{12}{5}\Delta^2-\tfrac{7}{15}(qv_f)^2]\alpha_{20}-\tfrac{2}{15}(qv_f)^2(\sqrt{2}\alpha_{00})=\frac{4\Delta}{5}\left[\frac{2e}{c}\right]qv_f^2(\hat{q}_it_{ij}^{(2,0)*}A_j)\ ,$$
$$[\Omega^2-\tfrac{12}{5}\Delta^2-\tfrac{2}{5}(qv_f)^2]\alpha_{2\pm1}=\frac{4\Delta}{5}\left[\frac{2e}{c}\right]qv_f^2(\hat{q}_it_{ij}^{(2,\pm1)*}A_j)\ ,$$
$$[\Omega^2-\tfrac{12}{5}\Delta^2-\tfrac{1}{5}(qv_f)^2]_{\alpha\pm2}=\frac{4\Delta}{5}\left[\frac{2e}{c}\right]qv_f^2(\hat{q}_it_{ij}^{(2,\pm2)*}A_j)\ .$$

In the transverse gauge $\mathbf{q}\cdot\mathbf{A}=0$, only the modes with $J=2$, $|M|=1$ are excited by the electromagnetic field; all other matrix elements, $q_it_{ij}^{(JM)}A_j$, vanish. Note in particular that the phase mode, corresponding to $J=0$, does not contribute in the transverse gauge. This result is peculiar to isotropic states.[34,35] The resulting amplitude for the $J=2$, $|M|=1$ modes becomes

$$\alpha_{2,\pm1}=\left[\frac{\lambda_2-\lambda_4}{\lambda_0+\lambda_2}\right]\frac{8\Delta v_F^2 q}{\Omega^2-\Omega_{2,\pm1}^2(q)}\left[\frac{2e}{c}\right](\hat{q}_i t_{ij}^{(2,\pm1)*}A_j)\ , \tag{25}$$

where

$$\lambda_n=\int_{-1}^{1}\frac{dx}{2}x^n\lambda(\Omega,qv_Fx)\ .$$

These modes are excited resonantly by the electromagnetic field at frequencies

$$\Omega_{2,\pm1}^2=\frac{12}{5}\Delta^2+\left[\frac{\lambda_2-\lambda_4}{\lambda_0+\lambda_2}\right](v_Fq)^2+\frac{2\lambda_0-18\lambda_2+20\lambda_4}{\lambda_0+\lambda_2}\frac{4}{5}\Delta^2 \tag{26a}$$

$$\simeq\tfrac{12}{5}\Delta^2+\tfrac{2}{5}(qv_F)^2\equiv\Omega_1(q)^2\ . \tag{26b}$$

Note that the last term in (26a) is of order q^2.

The dispersion of these modes may be calculated by solving (26) self-consistently for $\Omega=\Omega_{2,\pm1}$, as shown in Fig. 9. The initial quadratic rise in $\Omega_{2,\pm1}$ is weakened by a level repulsion effect as $\Omega_{2,\pm1}$ gets close to the pair-breaking continuum above $\Omega=2\Delta$. The mode frequency reaches the pair-breaking edge at $q=q_c\simeq1.2(2\Delta/v_F)$. Beyond this point, the mode is damped by pair-breaking

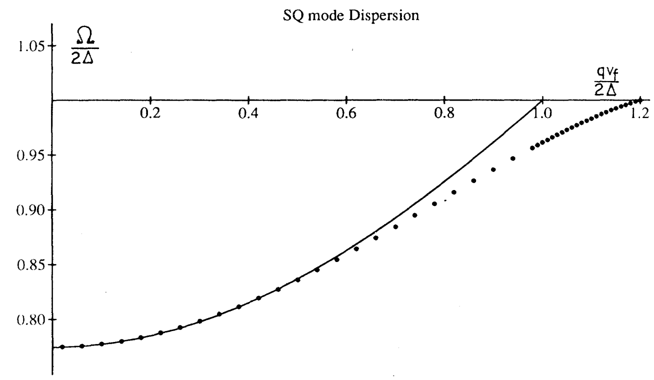

FIG. 9. Collective mode frequency $\Omega/2\Delta$ as a function of $v_Fq/2\Delta$. Points: exact solution of Eq. (26a). Solid line: approximate dispersion, Eq. (26b).

processes. These modes play an important role in the electromagnetic absorption as we show below. Impurity scattering modifies the response by broadening the collective mode. We can include this broadening qualitatively by replacing $\Omega\rightarrow\Omega+i/\tau$, where τ is of order the electron lifetime due to impurity scattering. However, for the estimates that follow we assume $1/\tau\ll\Delta$. Equation (1) can now be written in the form

$$j_i(\mathbf{q},\Omega)=(K_{ij}^{\text{ex}}+K_{ij}^{\text{mode}})A_j\ , \tag{27}$$

where K_{ij}^{ex} gives the current response of the single-particle excitations and K_{ij}^{mode} gives the current response from excitations of the collective modes with $J=2$ and $|M|=1$. The response of the single-particle excitations is given by $K_{ij}^{\text{ex}}=K^{\text{ex}}\delta_{ij}$:

$$K^{\text{ex}}=\left[\frac{ne^2}{mc}\right]\left[1+\tfrac{3}{2}\int_{-1}^{1}\frac{dx}{2}(1-x^2)[1-\lambda(\Omega,qv_fx)]x\Big/\left[\frac{\Omega}{qv_f}-x\right]\right]\ . \tag{28}$$

For temperatures above T_c, K^{ex} is simply proportional to the frequency-dependent conductivity for an impurity-free metal, and determines the power absorption in the anomalous skin region. The corrections to K^{ex} below the superconducting transition due to single particle excitations were discussed earlier in the long-wavelength limit in Secs. II–IV. The $|M|=1$ collective modes give $K_{ij}^{\text{mode}}A_j=K^{\text{mode}}A_i$ with

$$K^{\text{mode}}=12\left[\frac{ne^2}{mc}\right]\frac{(qv_f)^2I(q,\Omega)}{\Omega^2-\Omega_{2,\pm1}(q)^2}\ ,$$
$$I(q,\Omega)=\left[\frac{\lambda_2-\lambda_4}{\lambda_0+\lambda_2}\right]\int_{-1}^{1}\frac{dx}{4}x^2(1-x^2)\lambda(\Omega,qv_fx)\ . \tag{29}$$

In order to estimate the importance of the collective modes to the current response we calculate the power absorption $P(\Omega)$ from the collective modes and compare the result with the normal-metal power absorption in the anomalous skin region. Because the field penetrates only a distance of order the skin depth into the metal we are required to solve the boundary value problem of an electromagnetic field incident on a vacuum-metal interface and penetrating into the metal a distance of order Λ. Thus, we expect collective modes to be excited for wave vectors $q\lesssim1/\Lambda<1/\xi$. The power absorption at frequency Ω is given by integrating the Joule dissipation over the half-space of the metal,

$$P(\Omega)=-\tfrac{1}{2}\int_0^\infty dz\,\text{Re}(\mathbf{E}_\Omega^*\cdot\mathbf{j}_\Omega)\ . \tag{30}$$

The half-space boundary value problem is mapped onto a full-space boundary value problem by assuming specular boundary conditions at the vacuum-metal interface.[36] We specify the magnetic field $B_0(\Omega)$ on the vacuum side of the interface, and obtain from Maxwell's equations and this boundary condition,

$$A(q,\Omega)=\frac{2B_0(\Omega)}{q^2-\frac{4\pi}{c}K(q,\Omega)} . \tag{31}$$

The result for the power absorption becomes

$$P(\Omega)=-\frac{2\Omega\,|B_0(\Omega)|^2}{c}\int_0^{\infty}dq\frac{1}{2\pi}\frac{\mathrm{Im}[K(q,\Omega)]}{\left|q^2-\frac{4\pi}{c}K(q,\Omega)\right|^2} . \tag{32}$$

In the impurity-free normal metal the absorption becomes

$$P_N(\Omega)=\frac{1}{2\pi}\Omega\,|B_0(\Omega)|^2\Lambda^2 \times\int_0^{\infty}\frac{dq}{2\pi}\frac{\mathrm{Im}\tilde{K}_N(\Omega/qv_f)}{|(q\Lambda)^2+\tilde{K}_N(\Omega/qv_f)|^2} \tag{33}$$

where

$$\Lambda=\left[\frac{mc^2}{4\pi ne^2}\right]^{1/2}$$

is the $T=0$ London penetration length, and

$$\mathrm{Im}\tilde{K}_N(z)\equiv-\frac{3\pi}{4}z(1-z^2)\theta(1-|z|) ,$$
$$\mathrm{Re}\tilde{K}_N(z)\equiv\frac{3}{2}z\left[z-\frac{1}{2}(1-z^2)\ln\left|\frac{1-z}{1+z}\right|\right] . \tag{34}$$

In the limit $(\Omega\Lambda/v_f)\ll 1$ this reduces to the power absorption in the anomalous skin effect region

$$P_N(\Omega)=\frac{1}{8\sqrt{3}}\left[\frac{2}{3\pi}\right]^{1/3}v_f\,|B_0(\Omega)|^2(\Omega\Lambda/v_f)^{2/3} . \tag{35}$$

With increasing frequency, $P_N(\Omega)$ reaches a maximum at $\Omega\sim v_f/\Lambda$ and decreases as

$$P_N(\Omega)=\frac{3}{192\pi}v_f\,|B_0(\Omega)|^2(v_f/\Omega\Lambda)^2 \tag{36}$$

for $\Omega>v_f/\Lambda$. The result of a complete numerical evaluation of $P_N(\Omega)$ is shown in Fig. 10.

In the superconducting phase the absorption from the collective modes for $\Omega<2\Delta$ becomes

$$P_{\mathrm{mode}}(\Omega)=\frac{-2\Omega\,|B_0|^2}{c}\int_0^{\infty}dq\frac{1}{2\pi}\frac{\mathrm{Im}(\tilde{K}^{\mathrm{mode}})}{\left|q^2+\frac{4\pi}{c}K^{\mathrm{ex}}\right|^2} ,$$
$$\mathrm{Im}(\tilde{K}^{\mathrm{mode}})\simeq\pi A(q,\Omega)\delta[\Omega-\Omega_1(q)-\Delta\Omega_1(q)] , \tag{37}$$

where

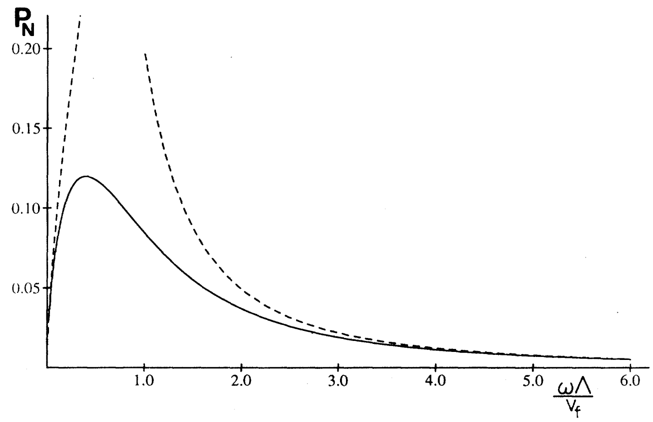

FIG. 10. Power absorption in the normal state $P_N(\Omega)/[3/16\pi\,|B_0(\Omega)|^2v_f]$ vs reduced frequency $\Omega\Lambda/v_f$. Dashed lines: asymptotic behavior given in Eqs. (35) and (36). Solid line: exact evaluation of Eq. (34).

$$A(q_1\Omega)=6(ne^2/mc)(qv_f)^2I(q_1\Omega)/\Omega .$$

Here,

$$\Delta\Omega_1=\frac{4\pi}{c}\frac{A(q,\Omega_1)}{q^2+\frac{4\pi}{c}K^{\mathrm{ex}}}$$

if a frequency shift is induced by the collective mode contribution in the denominator of the integrand of (32). It is convenient to introduce the wave vector $q_1(\Omega)$ at the mode crossing $\Omega=\Omega_1(q_1)$; thus,

$$q_1(\Omega)=\begin{cases}0, & \Omega<\Omega_0\\ \frac{1}{c_1}(\Omega^2-\frac{12}{5}\Delta^2)^{1/2}, & \Omega>\Omega_0 ,\end{cases} \tag{38}$$

where $\Omega_0\equiv\Omega_1(0)$ and $c_1\simeq\sqrt{2/5}v_f$ is the velocity that determines the dispersion of the collective mode. In the limit $qv_f\ll\Omega$, the denominator of $P_{\mathrm{mode}}(\Omega)$ may be the approximated $K^{\mathrm{ex}}\sim ne^2/mc$ and $\Delta\Omega_1(q)$ may be neglected, with the result

$$P_{\mathrm{mode}}(\Omega)=\frac{|B_0(\Omega)|^2}{100\pi}c_1\left[\frac{v_f}{\Lambda}\right]^2 \times\lambda_0(\Omega,q=0)\frac{\Omega(\Omega^2-\Omega_0^2)^{1/2}}{(\Omega^2-\Omega_0^2+c_1^2/\Lambda^2)^2} . \tag{39}$$

At $T=0$ the power absorption in the BW state is zero below the threshold $\Omega<\Omega_0$; at these frequencies only the supercurrent is excited by the field. The absorption increases rapidly for $\Omega>\Omega_0$ and reaches a maximum for

$$\Omega_*=\Omega_0+\frac{1}{6}\left[\frac{c_1^2}{\Lambda^2\Omega_0}\right]<2\Delta \quad\text{for}\quad \Lambda\gg\xi\sim\frac{v_f}{\pi\Delta} . \tag{40}$$

The width of this absorption peak is $\sim c_1/\Lambda$, and its total area is a factor of Λ/ξ_0 larger than the integrated power absorption in the normal state up to $\Omega\cong2\Delta$. With decreasing ratio Λ/ξ_0 the peak rapidly disappears. Above

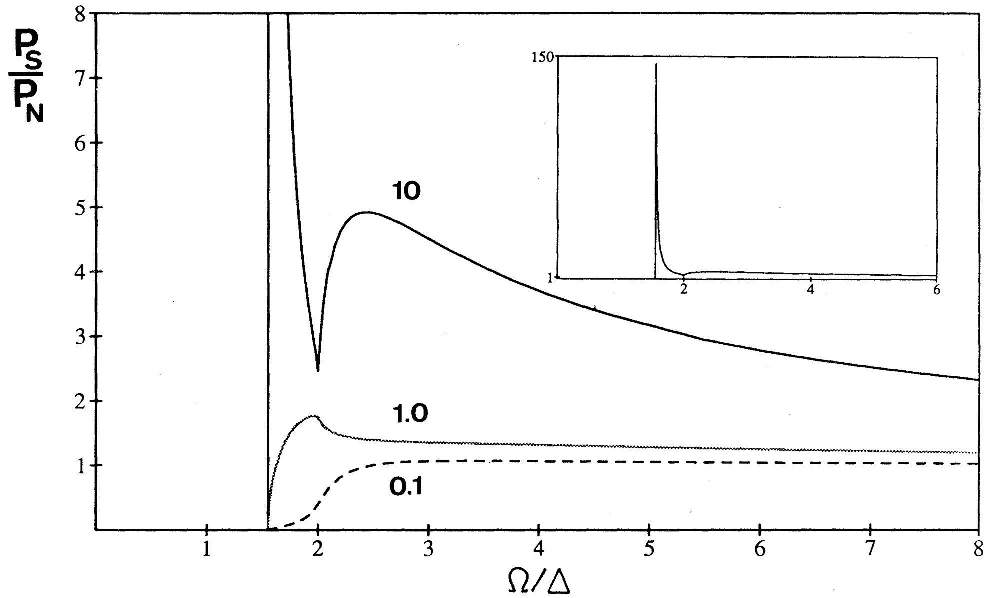

FIG. 11. Power absorption in the Balian-Werthamer state at $T=0$ normalized to $P_N(\Omega)$ for $\Lambda/\xi_0=0.1,1.0,10$ vs reduced frequency $(\Omega/2\Delta)$.

the gap edge $(\Omega>2\Delta)$ the single-particle excitations that are produced by pair breaking contribute to the power absorption. The qualitative behavior just discussed is borne out by a detailed numerical evaluation. Figure 11 shows the power absorption as a function of frequency, normalized to the absorption in the normal state, for several values of Λ/ξ_0. The collective mode peak is seen to dominate the absorption for large values of Λ/ξ_0 and is well separated from the pair breaking continuum above $\Omega=2\Delta$.

The main results of this calculation are that excitonic collective modes of the order parameter, which are a generic feature of clean unconventional superconductors, contribute significantly to the power absorption at frequencies well below the quasiparticle gap edge of 2Δ. In the BW state the threshold for exciting collective modes is $\Omega_0=1.55\Lambda$; however, this threshold frequency is sensitive to a variety of "real-metals" effects, including Fermi-surface anisotropy, impurity scattering, and Fermi-liquid effects. Impurity scattering in particular will broaden the collective modes, eventually overdamping them and washing out any structure in the absorption spectrum when $\tau\Delta<1$.

Although the BW state should be qualitatively representative of all unconventional superconductors in that it exhibits order parameter collective modes that couple to the electromagnetic field, the important distinction between the BW and most of the unconventional superconducting states that have been suggested as possible ground states for the heavy-fermion superconductors UPt_3, UBe_{12}, etc., is that the BW gap does not vanish in any direction in k space. Polar and axial states, which have been proposed as candidates for the ground state in various heavy-fermion superconductors, have gaps which vanish along lines or at points in k space. In consequence, the single-particle excitations are always present at finite temperature or frequency. These excitations always couple to the collective modes, producing a sizable width to the modes in these phases. Nevertheless, the collective modes have been clearly observed in ^{3}He-A, which is the prototype axial state, and there is no reason to believe that quasiparticle broadening will destroy the collective-mode structure in the EM absorption spectrum at least in a pure sample of a heavy-fermion superconductor with an axial- or polar-like gap. Finally, the existence of collective-mode peaks in the electromagnetic absorption with frequencies well below the gap edge of 2Δ may look qualitatively similar to the bound-state peak at $\Omega<\Delta$ that results in the strong-impurity-scattering limit.

VI. CONCLUSIONS

We consider the measurement of low-temperature microwave absorption in heavy-fermion superconductors extremely important from a number of points of view. First, such experiments provide a direct measurement of the energy gap, although this gap need not be identical to that deduced from tunneling measurements. Second, it will be useful to test the consistency of the anisotropic pairing hypothesis, coupled with the assumption of near-resonant scattering, which together have been qualitatively successful in explaining various transport measurements in the superconducting state.[17,18] Finally, electromagnetic absorption is the only probe suitable for exciting collective oscillations of the order parameter magnitude at frequencies of order Δ_0, possible for certain unconventional superconducting states. Although coupling to collective modes has been discussed theoretically in the context of sound absorption,[32,33,37] it is unlikely that sound experiments will be able to access the regime $\Omega\tau_{\rm mode}\gg 1$, $\Omega\sim\Delta$, and $q\ll\xi_0^{-1}$, where such effects are important, in the near future.

We note that, in order to account for experimentally observed pair breaking effects on T_c and superconducting transport properties, values of $(\tau T_c)^{-1}$ from 10^{-3} to 10^{-2} have been deduced in nominally pure samples.[16-18] On the other hand, normal-state resistivity measurements indicate a considerably higher scattering rate. This may be a consequence of two facts: (1) Part of the normal-state scattering is inelastic, and would be expected to disappear for $T\ll T_c$ and (2) the elastic part of the scattering may be very anisotropic, such that it is less effective in destroying the anisotropic superconducting coherence. For example, in a polar state, if the scattering is confined to the equator, the mean free path obtained from a resistivity measurement may be quite small, whereas the effect of the scattering on the polar gap will be minimal. Since the scattering probability must have the symmetry of the crystal lattice, this model with a single line of nodes is only possible in hexagonal or tetragonal symmetry. For example, in the hexagonal superconductor UPt_3 the gap deduced from comparison with transport properties,[16] and from a recent model of the pairing interaction[38] has such a line of nodes in the basal plane.

We note that in either case (1) or (2) the collective mode width $1/\tau$ mode discussed is determined by the longer of the two relaxation times. Thus small estimates for τ obtained from resistivity measurements need not preclude the observation of the collective mode.

Our calculations of $\sigma_{ij}(\Omega)$ show that for model anisotropic states with lines or points of nodes on the Fermi surface, electromagnetic radiation is always absorbed for energies down to $\Omega=0$. If scattering phase shifts are small, power laws in frequency may be deduced for the eigenvalues of $\sigma(\Omega)/\sigma_N(\Omega)$ in the collisionless limit. For the largest of these we find Ω^2 in a polar and Ω^4 for an axial state, while the smaller eigenvalue is reduced by Ω^2 in both cases. Thus one should, under this assumption, observe enormous anisotropy in $\sigma(\Omega)$ at the lowest frequencies in a single crystal; in a polycrystalline sample we expect the power laws quoted to dominate. In the resonant scattering limit of large phase shifts, on the other hand, the absorption at very low frequencies is gapless, reflecting similar behavior in the density of states. The large absorption feature caused by scattering of quasiparticles to or from the resonance near the Fermi surface is peaked around $\Omega=\Delta_0$, but in the highly anisotropic states considered here dominates the intermediate-frequency region as well due to smearing caused by the gap nodes. In this case distinguishing between states with lines and points of nodes may prove quite difficult.

In addition, identification of such a feature at $\Omega\cong\Delta_0$ will be complicated by the possibility that a collective mode may be excited near the same frequency. In such a case, measurements of doping dependence would be necessary to resolve the two effects. This is not the case for ordinary superconductors with Kondo impurities, where none of the collective modes couple to microwaves. Measurement of $\sigma(\Omega)$ in such a system would be an interesting test of some of the ideas discussed above.

Note added in proof. After submission of this manuscript we received a preprint from R. A. Klemm *et al.* which presents results for the surface impedance of anisotropic superconductors. During revisions, this work was published as Ref. 40.

ACKNOWLEDGMENTS

P.H., P.W., and J.S. are grateful to the Aspen Center for Physics for its hospitality during the summer of 1987. Research was partially supported by National Science Foundation (NSF) Grant No. DMR-8607941 and U.S. Defense Advanced Research Projects Agency (DARPA) Grant No. MDA-972-B5-T-1006 Projects Agency (P.W.), and NSF Grant No. DMR85-20280 (J.S.). Numerical calculations were performed on the San Diego Supercomputer Center (SDSC) Cray-XMP.

APPENDIX A: VERTEX CORRECTIONS TO $\sigma(\Omega)$

In this section we calculate the impurity vertex corrections to Eq. (5), assuming the scattering takes place entirely in the s-wave channel. Collective excitations are neglected. It is convenient to first examine Eq. (4) more closely. As the matrix spectral function at $q=0$ is given by

$$\underline{a}(k,\omega)=\underline{g}(k,\omega+i0)-\underline{g}(k,\omega-i0)\ ,$$

we may, assuming a spherical Fermi surface, write

$$I_0\equiv\sum_{\mathbf{k}}\hat{\mathbf{k}}_i\hat{\mathbf{k}}_j\mathrm{Tr}[\underline{a}(k,\omega)\underline{a}(k,\omega')]$$
$$=\frac{2\pi N_0}{\Gamma}\langle\hat{\mathbf{k}}_i\hat{\mathbf{k}}_j(L^{00}_{++}-L^{00}_{+-}-L^{00}_{-+}+L^{00}_{--})\rangle_{\hat{\mathbf{k}}}\ . \tag{A1}$$

Here,

$$L^{00}_{\alpha\beta}(\omega,\omega',\hat{\mathbf{k}})\equiv\frac{\Gamma_N}{\pi}\int d\xi\left[\frac{\tilde{\omega}_\alpha\tilde{\omega}_\beta+\tilde{\xi}_\alpha\tilde{\xi}'_\beta+\Delta_k^2}{(\tilde{\omega}^2_\alpha-\tilde{\xi}^2_\alpha-\Delta_k^2)[(\tilde{\omega}'_\beta)^2-\tilde{\xi}^2_\beta-\Delta_k^2]}\right] \tag{A2}$$

and the indices $\alpha,\beta=+,-$ indicate that the renormalized frequencies are to be evaluated at $\omega\pm i0$. The quantities $\tilde{\xi}_\alpha\equiv\xi+\Sigma_3(\omega+i0\,\mathrm{sgn}\alpha)$ are single-particle energies renormalized by τ^3 components of the impurity t matrix, as discussed in Ref. 18. These corrections are neglected in the text, but we include them here for completeness.

The expression (A2) gives a logarithmic divergence in Eq. (4), but it is easy to check that all divergences cancel in (A1), which may be rewritten as a sum of explicitly convergent terms,

$$I_0=\frac{2\pi N_0}{\Gamma}\left\langle\hat{\mathbf{k}}_i\hat{\mathbf{k}}_j\left[\sum_{\alpha,\beta}(\alpha\beta)\tilde{L}^{00}_{\alpha\beta}\right]\right\rangle_{\hat{\mathbf{k}}}\ , \tag{A3}$$

with $(\alpha\beta)\equiv\mathrm{sgn}\alpha\,\mathrm{sgn}\beta$ and

$$\tilde{L}^{00}_{\alpha\beta}\equiv\frac{\Gamma}{\pi}\int d\xi\left[\frac{\tilde{\omega}_\alpha\tilde{\omega}_\beta+(\tilde{\omega}'_\beta)^2+\tilde{\xi}_\alpha\tilde{\xi}'_\beta-(\tilde{\xi}'_\beta)^2}{(\tilde{\omega}^2_\alpha-\tilde{\xi}^2_\alpha-\Delta_k^2)[(\tilde{\omega}'_\beta)^2-(\xi'_\beta)^2-\Delta_k^2]}\right]\ . \tag{A4}$$

Note that the analogous procedure for the heat current correlation function (bare vertex $\omega\underline{\tau}^3$) leads to $\tilde{L}^{33}_{++}=\tilde{L}^{33}_{--}=0$ (18), but that retarded-retarded and advanced-advanced terms both contribute in the present case. As discussed in the text, the integrals (A4) may now be performed analytically, leading to the result (5) when $\Sigma_3=0$.

The RPA expressions for the general impurity-dressed two-particle retarded-advanced function and the associated vertex function have been given in Ref. 18. It is now straightforward to write down the appropriate generalization applicable to the conductivity. We find

$$\mathrm{Im}[K^{ij}(0,\Omega)]=\frac{e^2k_F^2N_0}{2\Gamma}\int d\omega\left[\sum_{\alpha,\beta}(\alpha\beta)\chi^{ij}_{\alpha\beta}(\omega)\right][\tanh(\tfrac{1}{2}\beta\omega)-\tanh(\tfrac{1}{2}\beta\omega')]\ , \tag{A5}$$

where

$$\chi^{ij}_{\alpha\beta}=\langle\hat{\mathbf{k}}_i\hat{\mathbf{k}}_j\tilde{L}^{00}_{\alpha\beta}\rangle+\sum_{n=1,2}\langle\hat{\mathbf{k}}_iL^{0n}_{\alpha\beta}\rangle\Lambda^{\alpha\beta}_{nm}\langle\hat{\mathbf{k}}_jL^{m0}_{\alpha\beta}\rangle\ . \tag{A6}$$

Here we have introduced the generalized response functions

$$L^{mn}_{\alpha\beta}(\omega,\omega',\hat{\mathbf{k}})\equiv\frac{\Gamma_N}{8\pi}\mathrm{Tr}(\underline{\tau}^i\underline{\tau}^m\underline{\tau}^j\underline{\tau}^n)\int d\xi\,\mathrm{Tr}[\underline{\tau}^i\underline{g}_{\alpha}(k,\omega)]\mathrm{Tr}[\underline{\tau}^j\underline{g}_{\beta}(k,\omega')]\ , \tag{A7}$$

where

$$\underline{g}_{\alpha}(k,\omega)\equiv\frac{\tilde{\omega}_\alpha\underline{\tau}^0+\tilde{\xi}_\alpha\underline{\tau}^3+\Delta_k\underline{\tau}^1}{\tilde{\omega}_\alpha^2-\tilde{\xi}_\alpha^2-\Delta_k^2}\ . \tag{A8}$$

Note that only the functions

$$L^{01}_{\alpha\beta}=\frac{\Gamma_N}{\pi}\int d\xi\frac{\Delta_k(\tilde{\omega}_\alpha+\tilde{\omega}'_\beta)}{(\tilde{\omega}_\alpha^2-\tilde{\xi}_\alpha^2-\Delta_k^2)[(\tilde{\omega}'_\beta)^2-(\tilde{\xi}'_\beta)^2-\Delta_k^2]}=L^{10}_{\alpha\beta} \tag{A9a}$$

and

$$L^{02}_{\alpha\beta}=\frac{i\Gamma_N}{\pi}\int d\xi\frac{\Delta_k(\tilde{\xi}_\alpha-\tilde{\xi}'_\beta)}{(\tilde{\omega}_\alpha^2-\tilde{\xi}_\alpha^2-\Delta_k^2)[(\tilde{\omega}'_\beta)^2-(\tilde{\xi}'_\beta)^2-\Delta_k^2]}=-L^{20}_{\alpha\beta} \tag{A9b}$$

couple to the p-wave current vertex $\hat{\mathbf{k}}$, and then only under the assumption that the gap itself has p-wave symmetry. Furthermore, the second couples only through the intrinsic particle-hole asymmetry $\Sigma_3\neq0$ neglected in the text. It vanishes for $c\gg1$ or $c\ll1$. *All* vertex corrections in Eq. (A6) are seen to vanish identically if $\langle\hat{\mathbf{k}}_i\Delta_k\rangle=0$, e.g., for all even-parity gaps and for certain special symmetry directions for uniaxial p-wave gaps. In particular, it was shown in Ref. 18 that the largest eigenvalue of K_{ij} is unrenormalized; thus the evaluations of Eq. (5) plotted in Figs. 4–7 require no vertex correction.

The function $\Lambda^{\alpha\beta}_{mn}$ introduced in (A6) is the full s-wave impurity vertex function obeying

$$\Lambda^{\alpha\beta}_{ij}=I^{\alpha\beta}_{ij}+I^{\alpha\beta}_{ii'}\langle L^{i'j'}_{\alpha\beta}\rangle_{\hat{\mathbf{k}}}\Lambda^{\alpha\beta}_{j'j}\ , \tag{A10}$$

where $I^{\alpha\beta}_{ij}$ is the bare irreducible vertex $\frac{1}{2}\mathrm{Tr}(\underline{\tau}^i\underline{T}^\alpha\underline{\tau}^j\underline{T}^\beta)$. Under the assumption of no gap renormalization the T matrix is $T^\alpha=T^\alpha_0\underline{\tau}^0+T^\alpha_3\underline{\tau}^3$, with

$$T_3^+=\frac{-c}{c^2-G_0}=(T_3^-)^* \tag{A11}$$

and $T_3^+=\Sigma_0^+/\Gamma=(T_0^-)^*$ as given in the text.

Equation (A10) may now be solved for Λ, yielding

$$\Lambda^{\alpha\beta}_{11}=[(1-I_{11}\langle L^{22}\rangle)I_{11}-I_{12}^2\langle L^{22}\rangle]/R\ , \tag{A12a}$$

$$\Lambda^{\alpha\beta}_{12}=-\Lambda^{\alpha\beta}_{21}=[I_{12}+(I_{12}^2+I_{11}^2)\langle L^{12}\rangle]/R\ , \tag{A12b}$$

$$\Lambda^{\alpha\beta}_{22}=[(1-I_{11}\langle L^{11}\rangle)I_{11}-I_{12}^2\langle L^{11}\rangle]/R\ , \tag{A12c}$$

where

$$R\equiv(1-I_{11}\langle L^{22}\rangle+I_{12}\langle L^{12}\rangle)(1-I_{11}\langle L^{11}\rangle+I_{12}\langle L^{12}\rangle)+(I_{11}\langle L^{12}\rangle+I_{12}\langle L^{22}\rangle)(I_{12}\langle L^{11}\rangle+I_{11}\langle L^{12}\rangle)\ , \tag{A13}$$

and I_{ij} and L^{ij} in (A12) and (A13) are understood to carry indices $\alpha\beta$.

This completes the formal solution of the vertex problem and determines the response (A5) completely. In practical terms, we expect these corrections to play an important role in measurable quantities only to the extent that one is interested in anisotropy, e.g., $\sigma_\parallel/\sigma_\perp$ for p-wave states at low frequencies. Then one can show that the correction term in (A6) is formally of the same order as the bare bubble, and must be included. If the current $J_0\hat{\mathbf{q}}$ is taken to flow at an arbitrary angle with respect to the gap nodes $\hat{\mathbf{n}}$, these corrections are of order $|\hat{\mathbf{q}}\times\hat{\mathbf{n}}|^2$. One interesting sidelight is that, as in the case of the thermal conductivity,[18,39] the response need not be diagonal for axial-type states. This is seen immediately from (A6).

APPENDIX B: SHORT-WAVELENGTH ELECTROMAGNETIC RESPONSE

In this section we discuss the Pippard, or short-wavelength limit of the response function K. Following the same procedure as in Sec. II for spectral representation of the propagators and desingularization of the resulting expressions, we arrive at the following result for the imaginary part of the transverse response valid for arbitrary q:

$$\mathrm{Im}[K_\perp(\mathbf{q},\Omega)]=\frac{e^2k_F^2}{4m^2c}\int d\omega\frac{1}{4\pi}\int dx\frac{1-x^2}{2}\sum_{\alpha,\beta}(\alpha\beta)I_{\alpha\beta}(\mathbf{k},\omega,\omega',\mathbf{q})[\tanh(\tfrac{1}{2}\beta\omega)-\tanh(\tfrac{1}{2}\beta\omega')] , \quad \text{(B1)}$$

where $x=\cos\theta$, $(\alpha\beta)=\mathrm{sgn}\alpha\,\mathrm{sgn}\beta$, and

$$I_{\alpha\beta}=2\int d\xi\left[\frac{\tilde\omega_\alpha\tilde\omega'_\beta+(\tilde\omega'_\beta)^2+\xi_{k+}\xi_{k-}-\xi_{k-}^2}{(\tilde\omega_\alpha^2-\xi_{k+}^2-\Delta_k^2)[(\tilde\omega'_\beta)^2-\xi_{k-}^2-\Delta_k^2]}\right] . \quad \text{(B2)}$$

Here $k_\pm=k\pm q/2$, and $\xi_{k\pm}\cong\xi_k\pm\frac{1}{2}v_Fqx+\mathcal{O}(q^2/m)$. For the isotropic state alone, $\Delta_k=\Delta$ is independent of x, and one may see that in the Pippard limit $v_Fq\gg T_c$ the integrand in (B2) is dominated by values of $x\lesssim T_c/v_Fq$. We then may neglect x^2 in the numerator of (B1), and use the trick of Abrikosov *et al.*[13] to transform from variables (x,ξ) to (ξ_+,ξ_-). Since small values of x dominate, the limits of integration may be extended to infinity and the integrals performed directly. We then recover the MB-like result:

$$\mathrm{Im}[K_\perp(\mathbf{q},\Omega)]\to\frac{e^2n}{mc}\left[\frac{3\pi}{4v_Fq}\right]\int_0^\Omega d\omega[\tanh(\tfrac{1}{2}\beta\omega)-\tanh(\tfrac{1}{2}\beta\omega')]\left[\mathrm{Re}\left[\frac{\tilde\omega}{\xi_0}\right]\mathrm{Re}\left[\frac{\tilde\omega'}{\xi'_0}\right]+\mathrm{Re}\left[\frac{\Delta}{\xi_0}\right]\mathrm{Re}\left[\frac{\Delta}{\xi'_0}\right]\right] , \quad \text{(B3)}$$

where the notation is that of Sec. II. Note in this limit that the anomalous terms $D(\omega)D(\omega')$ occur despite the odd parity of the order parameter. As $T\to T_c$ the penetration depth Λ diverges and the short-wavelength result is no longer appropriate.

If $I_{\alpha\beta}$ depends on angles through Δ_k as well as through $\xi_{k\pm}$, the above transformation may not be carried through and no simple form results. For $T_c\ll V_Fq\ll\sqrt{T_cT_F}$, one may neglect the q^2 term in $\xi_{k\pm}$ and perform the ξ integration as in Sec. II. While the resulting expression may easily be evaluated numerically, we have been unable to cast it in a simple MB form.

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