Magnetophonon effect in the energy relaxation rate of electrons in a GaAs heterostructure

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The magnetophonon effect in the energy relaxation rate for a two-dimensional electron gas (2D EG) is investigated within an electron-temperature model. The relative contribution of the acoustic phonons and LO phonons is analyzed. The behavior of the contribution of acoustic phonons is studied as a function of the effective thickness of the 2D EG. A lower limit on the thickness of the 2D EG is found, below which the continuum Debye model for acoustic phonons is no longer valid. Experimental conditions are determined for the observation of the magnetophonon effect in the energy relaxation rate for a low-density electron gas. The effect of the screening of the electron-phonon interaction on the energy relaxation rate is investigated within the random-phase approximation. We find that all the temperature dependence of the energy relaxation rate is contained in the phonon occupation numbers and that the magnetophonon peak positions are shifted to lower magnetic fields with increasing electron-sheet density.

I. INTRODUCTION

Transverse and longitudinal magnetoresistance are the most frequently investigated electronic transport coefficients in which the magnetophonon effect[1] is manifested. This effect occurs when the LO-phonon energy matches the separation between two Landau levels: $n\hbar\omega_c = \hbar\omega_{\rm LO}$, $n = 1,2,\ldots$, where ω_c is the cyclotron frequency ($\omega_c = eB/cm^*$), m^* the electron (hole) effective mass, $\hbar\omega_{\rm LO}$ the LO-phonon energy, and B is the applied magnetic field. Recently, both the linear or normal[2] and the nonlinear or hot-electron[3,4] magnetophonon effect in two-dimensional (2D) electronic systems embedded in heterojunctions and quantum wells have been studied extensively. Much less attention has been paid to magnetophonon resonances (MPR's) in the energy relaxation of hot electrons. The magnetophonon effect causes a resonant cooling of the electrons. Apart from information on phonon frequencies and band structure, the energy relaxation rate (ERR) potentially contains useful information on the electron-phonon interaction processes.[5]

In the last five years a considerable amount of work has been done on the interpretation of hot carrier ERR as obtained from photoexcitation and photoluminescence measurements in 2D GaAs systems in zero magnetic field (see e.g., Refs. 5–11). Strongly reduced ERR's as compared to the three-dimensional case were found, but the various experimental results did not agree among each other on the dependence of the ERR on the thickness of the 2D system. Although the controversial experimental results seem to be reconciled somewhat by taking into account the combined effect of nonequilibrium LO phonons[6,8] and the 2D excitation density instead of the 3D one,[11] further study is required. Hollering *et al.*[12] and Turberfield *et al.*[13] performed photoexcitation studies of the ERR in the case of quantizing magnetic field. Evidently they encounter the same problem of interpretation as in the zero-magnetic-field case.

There exist two other experimental techniques by which the hot-electron ERR can be measured. In contradistinction with the photoexcitation technique, here the carriers are heated by the application of an external electric field. Since very small electric fields can be applied, typical electron densities and electron temperatures are small compared to the corresponding values for the photoexcitation method. In this sense, they form a different class of experiments. First, one has the steady-state conductivity measurements,[14] and secondly, there are the bolometer phonon measurements[15] where one measures directly the number of phonons emitted by the heated 2D electron gas (2D EG). In the absence of a magnetic field, the ERR for this situation has been studied theoretically, e.g., by Price,[16,17] who also examined the validity of the electron-temperature model, and by Karpus.[18]

In Ref. 4 we studied the LO-phonon contribution to the ERR of the 2D EG subjected to crossed electric and magnetic fields. Within an electron-temperature model the amplitude of the MPR's in the ERR was found to be much larger than in the resistivity. The ERR turned out to be rather sensitive to the broadening parameters of the electronic density of states. It was assumed that the electrons interact with the bulk GaAs LO phonon-phonon modes in a GaAs-$Ga_{1-x}Al_xAs$ single heterostructure. This was recently confirmed by the cyclotron-resonance measurements of Langerak *et al.*[19] together with the model of Wu *et al.*[20] In the model calculation for the ERR in Ref. 4, we did not examine the effect of acoustic phonons. Therefore, and in order to make a more relevant comparison with the experimental data on the ERR, the acoustic phonons will be included here. So far it has not been unambiguously shown whether (or under which conditions) the electrons in the 2D systems interact with 3D rather than with 2D acoustic-phonon modes. The experiment of Neugebauer and Landwehr[21] did not give conclusive evidence. Other studies of the acoustic-phonon part of the ERR (e.g., Hirakawa *et al.*[14]) do not treat this problem, but instead assume a deformation-potential constant which is twice the value for bulk GaAs, leading to a factor of 4 higher ERR. Of course this can only be viewed as a phenomenological way of interpreting the experimental data. This shows that there is a need for further detailed experimental and theoretical investigation of these phenomena.

The aim of the present paper is to investigate the magnetophonon resonance oscillations in the energy relaxation rate of a 2D EG and to find an answer to the following questions. (1) Under which conditions is the MPR effect observable in the ERR? (2) What is the relative contribution of the acoustic phonons and the LO phonons? (3) How does the ERR due to the contribution of acoustic phonons behave as the effective thickness of the 2D EG is decreased? (4) What is the effect of the Pauli exclusion principle on the ERR? (5) What is the effect of screening on the ERR? The piezoelectric phonon contribution to the ERR is typically 1 order of magnitude smaller than the deformation-potential acoustic-phonon part [in GaAs (Refs. 14 and 17) for electron temperatures T_e larger than about 5 K] and will be neglected in this paper. The ERR for LO phonons and deformation-potential acoustic phonons depends on the magnetic field, on the electron density (via screening and the 2D form factor), on lattice and electron temperature, and on the amount of broadening of the density of states, characterized by the broadening parameter Γ_0. Numerical calculations of these dependences of ERR will be performed in the present paper for a 2D EG in a single-interface GaAs system. We will concentrate on the main trends of the acoustic-phonon part of the ERR, on a comparison of these trends with the 3D case, and on the effect of statistics and screening.

The organization of this paper is as follows. In Sec. II, the energy relaxation of the 2D EG subjected to crossed electric and magnetic fields is investigated without consideration of screening effects (low electron-density limit). The separate contribution from interactions of electrons with LO phonons and with acoustic phonons to the ERR is studied as a function of all the relevant parameters mentioned above. Section II ends with a discussion of the effect of the Pauli exclusion principle (Fermi-Dirac statistics versus Boltzmann statistics) on the MPR oscillations in the ERR. The effect of screening on the ERR due to electron–LO-phonon interaction is treated in Sec. III within the random-phase approximation (RPA). Our conclusions are presented in Sec. IV.

II. THE ENERGY RELAXATION RATE IN A 2D EG WITHOUT SCREENING

In this section the electron-temperature model of Ref. 4 (hereafter referred to as I) will be adopted as a model for the ERR. For clarity, the different approximations are briefly listed below. (1) Hot-phonon effects are neglected. (2) Only the central valley (parabolic band) and only the lowest electric subband (Fang-Howard wave function) are assumed to be occupied. (3) We consider an effective-mass approximation for the electrons. (4) The LO phonons are taken to be the bulk phonons (3D) of GaAs. (5) The first-order Born approximation for weak electron-phonon interaction is used. (6) Screening of the electron-phonon interaction is not included (the effect of screening will be analyzed in the next section). The approximations (1) and (2) are expected to be reasonable for steady-state measurements (absence of hot phonons) of the ERR on low electron-density samples in an external electric field such that the electron energy, as measured from the bottom of the lowest Landau level, is smaller than the intersubband energy distance. This is the case for both the conductivity measurements (Ref. 14) and the bolometer phonon-transport measurements (Ref. 15). Also for the description of ERR in photoexcitation measurements this model is expected to be qualitatively correct when only very few electrons are excited with near-band-gap light. However, in the experiments of Refs. 12 and 13, high electron densities are excited high up in the band so that, e.g., intervalley scattering is important; or even so high up in energy that in effect one observes properties of a 3D EG and no longer of a 2D EG. For most of our numerical results, Boltzmann statistics will be assumed. In order to check the validity of that assumption, the effect of retaining Fermi-Dirac statistics will be analyzed.

The ERR is given by the energy balance equation

$$e\mathbf{v}[\mathbf{E}+(\mathbf{v}\times\mathbf{B})]=W(\mathbf{v}) , \tag{1}$$

where $\mathbf{E}(E_x,E_y,0)$ is the total electric field, $\mathbf{B}(0,0,B)$ is the magnetic field, $\mathbf{v}$ is the average electron velocity, $-e$ is the electron charge, and $W(\mathbf{v})$ is the average energy relaxation rate of the electron due to the interaction with phonons. Within the electron-temperature model only terms up to zero order in v are retained, and W becomes independent of the electron velocity,

$$W=\frac{2}{N_e}\sum_{\mathbf{q}}\frac{\omega_{\mathbf{q}}}{\hbar}|V_{\mathbf{q}}|^2[n(\omega_{\mathbf{q}})-n_e(\omega_{\mathbf{q}})]\mathrm{Im}[D^r(\mathbf{q},\omega_{\mathbf{q}})] , \tag{2}$$

with $\mathrm{Im}[D^r(\mathbf{q},\omega)]$ being the imaginary part of the retarded density-density correlation function. $V_{\mathbf{q}}$ is the electron-phonon interaction Fourier coefficient, N_e the sheet electron density, and $\omega_{\mathbf{q}}$ the phonon frequency with phonon wave vector $\mathbf{q}$. Furthermore, $n(\omega_\mathbf{q})$ and $n_e(\omega_\mathbf{q})$ are the phonon occupation numbers at the lattice temperature T and at the electron temperature T_e, respectively. Inserting the expression for the retarded density-density correlation function with Gaussian broadening of the electronic density of states in Eq. (2) and using the interaction with 3D LO phonons and Boltzmann statistics, one recovers Eq. (15) of I. Application of Eq. (2) to the case of 3D acoustic phonons ($\omega_\mathbf{q}=uq$ with u the longitudinal velocity of sound) in the deformation-potential approximation (with Debye cutoff) and Boltzmann statistics leads to the following expression for the ERR:

$$W_{ac}=A\frac{\varepsilon_D^2 e^{\beta_e\mu}}{2\pi l_B^2 N_e u \Gamma_G}\sum_{n,m=0}^{\infty} e^{-\beta_e(\varepsilon_n+\varepsilon_m)/2} \times\int_0^{k_D} dq_\perp q_\perp J_{n,m}(l_B^2 q_\perp^2/2)\int_{-(k_D^2-q_\perp^2)^{1/2}}^{(k_D^2-q_\perp^2)^{1/2}} dq_z q^2 \frac{b^6}{(b^2+q_z^2)^3} e^{-(\varepsilon_m-\varepsilon_n+\hbar\omega_\mathbf{q})^2/\Gamma_G^2} \times\frac{e^{\beta_e\hbar uq}(e^{2(\beta-\beta_e)\hbar uq}-1)}{e^{2\beta\hbar uq}-1}, \qquad (3)$$

with $A=4.054\times10^{-12}$ W, $q^2=q_z^2+q_\perp^2$, μ is the chemical potential, $\beta^{-1}=k_BT$, $\beta_e^{-1}=k_BT_e$, ε_D is the deformation potential, $\varepsilon_n=\hbar\omega_c(n+\frac{1}{2})$ the energy of the nth Landau level, and $l_B=(\hbar c/eB)^{1/2}$ the magnetic length. The Fang-Howard wave-function parameter is defined by $b=[48\pi N_b e^2/(m^*\hbar^2\epsilon_s)]^{1/3}$ and $N_b=N_d+\frac{11}{32}N_e$ with N_d the depletion charge density and ϵ_s the static dielectric constant. Here we introduced $J_{n,n+j}(x)=[n!/(n+j)!]x^j e^{-x}[L_n^j(x)]^2$, with $L_n^j(x)$ the associated Laguerre polynomial, the Debye cutoff wave vector k_D defined by $k_BT_D=\hbar uk_D$ with the Debye temperature T_D and Gaussian broadening of the electronic density of states with broadening parameter $\Gamma_G=(2\hbar\omega_c\Gamma_0/\pi)^{1/2}$. The component of the wave vector in the direction parallel (perpendicular) to the 2D electron layer is denoted by $q_\perp$ (q_z). On the right-hand side of Eq. (3) all quantities are in dimensionless units, e.g., energy is in units of $\hbar\omega_{\text{LO}}$.

For our numerical analysis we took the parameter values[22] for GaAs: $\varepsilon_D=8$ eV, $T_D=340$ K, $u=5\times10^5$ cm/s, and mass density $\rho=5.32$ g/cm^3. Figure 1 displays the energy relaxation rate due to the electron-phonon interaction for the ideal 2D EG (zero thickness of the electron gas) (I 2D EG) and quasi-2D EG (Q 2D EG) as a function of the cyclotron frequency for fixed lattice and electron temperatures $T=4.2$ K and $T_e=100$ K, respectively, and for fixed $\Gamma_0=7.5$ meV. The contribution of polar LO phonons, as obtained from Eq. (15) of I, is shown separately from that of longitudinal-acoustic phonons (AC). Note that the energy relaxation rate is additive in the contribution of different scattering processes (LO and AC). The contribution of acoustic phonons to the ERR is displayed for different depletion charge densities N_d, for which the corresponding average distance of the electrons to the interface is indicated ($L_z=1/b$, which is a measure for the thickness of the 2D EG). The MPR effect is clearly present in the curves for the LO-phonon part of the ERR. From Fig. 1 it appears that for the ideal 2D EG the contribution of acoustic phonons to the ERR is of the same order of magnitude as the contribution of the LO phonons. For the quasi-2D EG, however, the contribution of the acoustic phonons to the ERR is about 1 or 2 orders of magnitude smaller than the contribution of the LO phonons. We found that the contribution of the acoustic phonons to the ERR is very sensitive to the extension (L_z or b) of the electronic wave function in the z direction (see Fig. 1), or equivalently to the form factor or the electron density (N_e). This requires further discussion.

In Eq. (3) there are three constraints on the magnitude of the z component of the phonon wave vectors which can contribute to the interaction. In the following, the $q_\perp$ dependence will be neglected since the term $J_{n,m}(l_B^2q_\perp^2/2)$ in Eq. (3) allows only very small values of $q_\perp$: the factor $\exp(-l_B^2q_\perp^2/2)$ leads to the constraint $q_\perp<\sqrt{2}/l_B$, which

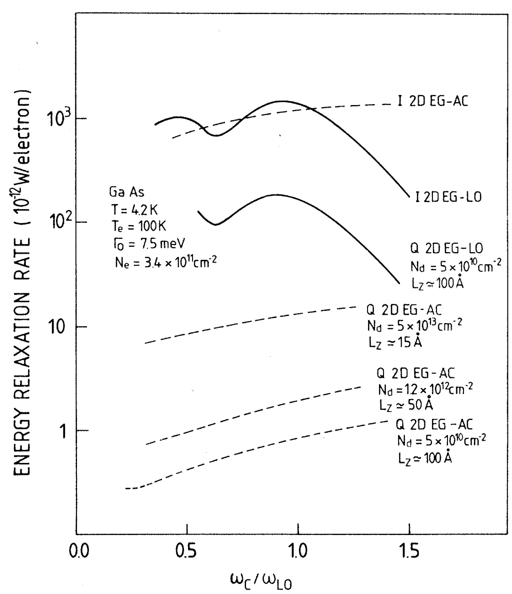

FIG. 1. Energy relaxation rate due to the electron-phonon interaction for the ideal 2D EG (I 2D EG) and quasi-2D EG (Q 2D EG) as a function of the cyclotron frequency. The contribution of polar LO phonons (LO) is shown separately from the contribution of longitudinal-acoustic phonons (AC). The contribution of acoustic phonons to the energy relaxation rate is displayed for different electron densities. Results are given for fixed $\Gamma_0=7.5$ meV, $T=4.2$ K, and $T_e=100$ K.

corresponds to $q_\perp < k_D/33$ for GaAs in the range of high magnetic fields considered here. The first constraint on q_z is the broadened energy-conserving term $\exp[-(\varepsilon_m - \varepsilon_n + \hbar\omega_q)^2/\Gamma_G^2]$. For intra-Landau-level processes $(n=m)$ this term reduces to $\exp\{-[\hbar u(q_z^2+q_\perp^2)^{1/2}/\Gamma_G]^2\}$. So the dominant contribution to this energy-conserving term comes from wave vectors for which $\hbar u q_z < \Gamma_G$ or equivalently $q_z < k_D(\Gamma_G/k_B T_D)$. For a reasonable value of $\Gamma_G = 3$ meV this limits the maximum value of the z component to $q_z < k_D/10$, which is still very large. In an analogous way one derives for inter-Landau-level processes $(N = n - m \neq 0)$ the constraint $q_z < k_D[(\Gamma_G + N\hbar\omega_c)/k_B T_D]$ which is weaker than that for intra-Landau-level processes. The second limiting factor is

$$\exp(\beta_e uq)\{\exp[2(\beta-\beta_e)uq]-1\}/[\exp(2\beta uq)-1] \approx (1-\beta_e/\beta)\exp(\beta_e uq)$$

for high electron temperatures $T_e \gg T$. The dominant contribution here comes from $q_z < k_D T_e/T_D$. For $T_e = 100$ K, as in our numerical results, this leads to the constraint $q_z < k_D/4$. Only the third factor is related to the extension of the wave function in the z direction: $1/[1+(q_z/b)^2]^3$ and expresses the fact that the largest contribution comes from wave vectors satisfying $q_z < b$. This leads to the constraint $q_z < k_D/100$ for $L_z = 100$ Å. Compared to this last restriction, the previous two are unimportant.

Consequently in the case of a Q 2D EG with very small spatial extension in the z direction ($1/b = L_z$ not much larger than $10/k_D \approx 15$ Å) and 3D acoustic phonons, the electrons will interact with phonons with relatively large q_z. In this situation the z component of the phonon wave vector is exclusively constrained by the form factor of the wave function in the z direction (typically $q_z < 1/L_z$) and by the Debye wave vector k_D. Consequently for the I 2D EG case this leads to the unphysical result that only k_D limits the magnitudes of the q_z which contribute to the scattering process. In that case the continuum (Debye) approximation for the acoustic phonons will no longer be valid (this statement is independent of temperature). It turns out that this is not a severe restriction since $L_z = 15$ Å corresponds to a sheet electron density $N_e = 10^{14}$ cm^{-2}, which is much larger than usual experimental densities.

The energy relaxation rate is shown in Fig. 2, as a function of electron temperature at a fixed magnetic field $B = 22$ T for $T = 4.2$ K and $\Gamma_0 = 7.5$ meV. The separate contribution of polar LO phonons and longitudinal-acoustic phonons is shown for the ideal 2D EG and the quasi-2D EG. For the Q 2D EG the contribution of polar LO phonons starts to dominate over the contribution from the longitudinal-acoustic phonons for $T_e > 40$ K, in agreement with the $B = 0$ case[5] and with the 3D EG case. For the I 2D EG, the crossover appears at higher electron temperature due to the larger relative contribution to the ERR of the longitudinal-acoustic phonons.

The dependence of the ERR on the broadening parameter Γ_0 is shown in Fig. 3, for the quasi-2D EG as a function of the cyclotron frequency. The lattice temperature

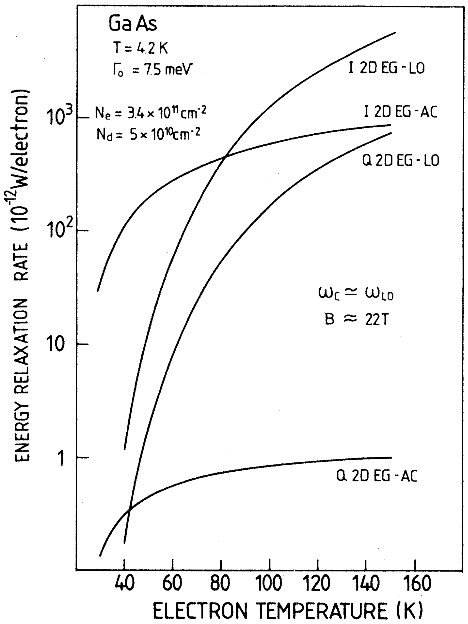

FIG. 2. Energy relaxation rate as a function of electron temperature at a fixed magnetic field $B = 22$ T for $T = 4.2$ K and $\Gamma_0 = 7.5$ meV. The separate contribution of polar LO phonons and longitudinal-acoustic phonons is shown for the ideal 2D EG and the quasi-2D EG.

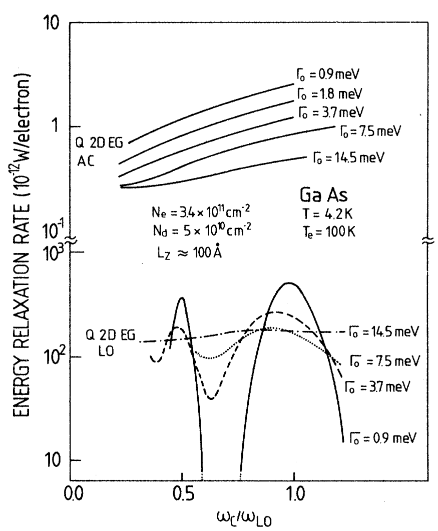

FIG. 3. Energy relaxation rate for the quasi-2D EG as a function of the cyclotron frequency for different values of the broadening parameter Γ_0. The lattice temperature T, the electron temperature T_e, and the electron density are fixed here. The top part shows the contribution of longitudinal-acoustic phonons. The bottom part displays the contribution of polar LO phonons.

T, the electron temperature T_e, and the electron density are fixed here. The top part of Fig. 3 shows the contribution of the longitudinal-acoustic phonons. The bottom part displays the contribution of polar LO phonons. Evidently the broadening parameter Γ_0 plays a much more dramatic role for the LO-phonon contribution to the ERR than for the acoustic-phonon part, due to the resonant character of the LO-phonon part. The question under which conditions the MPR in the ERR should be observable cannot be answered straightforwardly. First, it depends on the experimental method: the present model is only expected to be valid for the situation of photoexcitation experiments if the excitation density and the carrier heating are low, as indicated above. Our model is also suited for the description of ERR measurements where an external electric field is applied. Although this type of experiment results in a nonzero average electron velocity v (which is neglected in the electron-temperature model) the ERR in this situation is still governed by the electron temperature: the first-order term in v is only a perturbation to the zeroth-order term as long as $v \ll v_{\mathrm{LO}}$. Second, before experimental conditions can be specified, the effect of statistics (Pauli exclusion principle) on the ERR needs to be estimated.

In Fig. 4 the energy relaxation rate due to electron LO-phonon interaction for the ideal 2D EG is displayed as a function of the filling factor at $T=4.2$ K, $T_e=100$ K, and $\Gamma_0=1.7$ meV. A comparison is made between the results for Fermi statistics [circles, from Eq. (14c) of I] and Boltzmann statistics [crosses, from Eq. (15) of I]. As indicated by arrows, two sets of curves are for fixed magnetic field ($B=22$ T and $B=10$ T) while changing the sheet electron density (different samples or by shining light). Another set of curves is for fixed electron density ($N_e=3.4\times10^{11}$ cm^{-2}). This last set of results corresponds to the experimental situation where the magnetic field is swept. We checked that the MPR peak positions are not affected by inclusion or omission of the Pauli exclusion principle (only the magnitude of the ERR is reduced). The results of this paragraph indicate that resonant cooling will only be observed in photoexcitation measurements if only few carriers are excited. Experimentally this requires, e.g., a precise matching of the excitation energy with the band-gap energy (see Refs. 12 and 13). For the electric field technique we expect the MPR effect to be observable for $T_e>50$ K if $\Gamma_0 \ll 10$ meV and if $N_e<4\times10^{11}$ cm^{-2} so that the filling factor is smaller than 2 for $\omega_{\mathrm{LO}}/\omega_c=3$.

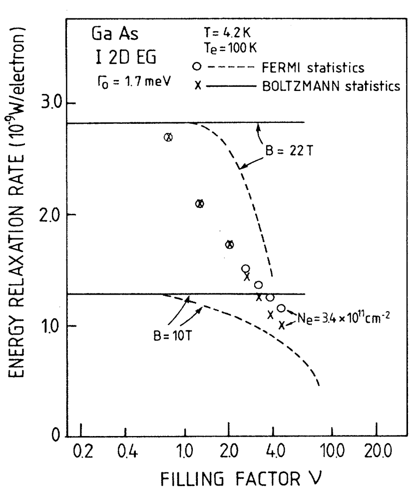

FIG. 4. Energy relaxation rate due to electron–LO-phonon interaction for the ideal 2D EG as a function of filling factor at $T=4.2$ K, $T_e=100$ K, and $\Gamma_0=1.7$ meV. A comparison is made between the results for Fermi statistics (circles) and Boltzmann statistics (crosses). As indicated by arrows two sets of curves are for fixed magnetic field ($B=22$ and 10 T) and one set of curves is for fixed electron density ($N_e=3.4\times10^{11}$ cm^{-2}).

III. THE EFFECT OF SCREENING ON THE ERR IN A 2D EG

The following model for the ERR due to LO phonons includes the effect of dynamical screening of the 2D EG and electron–LO-phonon interaction within the well-known RPA approximation:

$$W=\frac{\hbar\omega_{\mathrm{LO}}}{\tau_0}[n(\omega_{\mathrm{LO}})-n_e(\omega_{\mathrm{LO}})]P\ , \tag{4}$$

with

$$\frac{1}{\tau_0}=\frac{2\alpha\hbar\varepsilon_\infty}{e^2}\left[\frac{\hbar}{2m^*\omega_{\mathrm{LO}}}\right]^{1/2}(\omega_{\mathrm{LO}})^2\ , \tag{5}$$

$$P=\frac{1}{2\pi N_e}\int_0^\infty dq\, q\,\mathrm{Im}\frac{-1}{\varepsilon(q,\omega_{\mathrm{LO}})}\ , \tag{6}$$

and where $\epsilon(q,\omega)$ is the dielectric function of the 2D EG, ε_∞ is the high-frequency dielectric constant, and α is the Fröhlich coupling constant. The dielectric function as calculated in the RPA approximation is expressed as $\varepsilon(q,\omega)=1-v(q)\Pi^0(q,\omega)$, where $v(q)$ is the Fourier-transformed electron-electron interaction matrix element and $\Pi^0(q,\omega)$ is the polarizability of the unperturbed 2D EG,[23]

$$\Pi^0(q,\omega)=\frac{m^*\omega_c}{2\pi\hbar}\sum_{n,m=0}^{\infty}J_{n,m}(l_B^2q^2/2)\Pi^0_{n,m}(\omega)\ , \tag{7}$$

with

$$\Pi^0_{n,m}(\omega)=-2\frac{f(\varepsilon_m)-f(\varepsilon_n)}{\hbar\omega-\varepsilon_m+\varepsilon_n+i\Gamma_L}\ , \tag{8}$$

where $f(x)$ is the electron-energy distribution function, taken here as a Boltzmann distribution function, and where Lorentz-type broadening was introduced by adding a nonzero imaginary part (Γ_L) in the denominator. Equation (7) involves transitions between Landau levels with index n and m. The summation over Landau-level indices can be exactly converted into an integral (see Glasser[24]), and this results in

$$\Pi^0(q,\omega)=\frac{-2}{\pi\hbar}(1-e^{-2\pi\Gamma_L/\hbar\omega_c+i2\pi\omega/\omega_c})^{-1}\times\int_0^{2\pi}dt\,\sin(a\,\sin t)e^{a(\cos t-1)}\times G_j[2a(1-\cos t)]e^{-t\Gamma_L/\hbar\omega_c+it\omega/\omega_c}\,, \tag{9}$$

where $a=q^2/2l_B^2$, j is the largest integer contained in $\mu/\hbar\omega_c$, and $G_n(x)=L_n(x)+2L_{n-1}^1(x)$ [see Eq. (27) of Ref. 24]. The temperature dependence of the dielectric function is included along the lines of Ref. 25.

In Fig. 5 numerical results for P from Eq. (4) are displayed as a function of cyclotron frequency for two different values of the electron temperature. The electron densities and the broadening parameter Γ_L are fixed here. Apparently there is almost no dependence of P on the electron temperature T_e. So the effect of screening is to a good approximation independent of T_e. Consequently, all the T and T_e dependence of the ERR is in the phonon occupation factors in Eq. (4), in agreement with our previous results of Ref. 4 (see also Fig. 2). The magnitude of the ERR is comparable to results of Sec. II and the results of Ref. 4 for comparable values of the parameters.

The dependence of P on the sheet electron density N_e and broadening parameter Γ_L is shown in Fig. 6. Electron temperature T_e and depletion charge density N_d are fixed here. The effect of screening on the magnitude of the ERR is clearly not very large. Even for a sheet electron density $N_e=1.4\times10^{12}$ cm^{-2} (for fixed $\Gamma_L=1.8$ meV) the magnitude of the $N=1$ MPR peak is still 76% of the magnitude of the corresponding peak for $N_e=0.4\times10^{12}$ cm^{-2}. The peak positions of all the curves are clearly shifted down in magnetic field from the normal MPR peak positions at $\omega_c/\omega_{\mathrm{LO}}=1/n$ with $n=1,2$. One observes that this shift increases with increasing N_e, if the broadening parameter is fixed. On increasing the broadening parameter Γ_L with a factor of 3 while keeping N_e fixed, the relative shift of the peak position is smaller than 0.1%, but the magnitude of P is strongly reduced (with a factor 3).

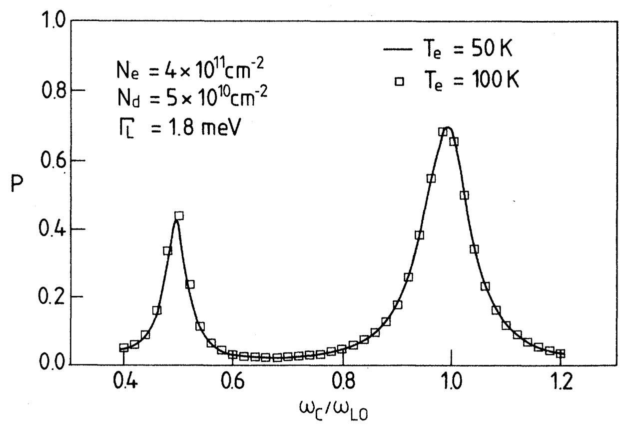

FIG. 5. P [from Eq. (4)] as a function of cyclotron frequency for two different values of the electron temperature. Electron densities and broadening parameter Γ_L are fixed here.

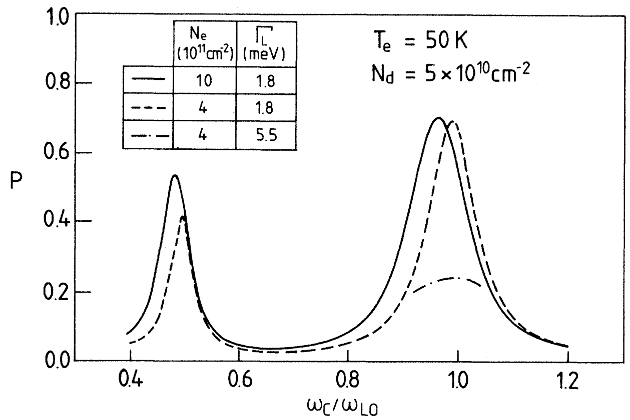

FIG. 6. Same as Fig. 5, but now for different values of sheet electron density N_e and broadening parameter Γ_L. Electron temperature T_e and depletion charge density N_d are fixed here.

IV. CONCLUSIONS

In this paper magnetophonon oscillations were investigated in the energy relaxation rate for a two-dimensional electron gas subjected to crossed electric and magnetic fields, with emphasis on the role of the acoustic phonons in the energy relaxation. The ERR was calculated from the energy balance equation within an electron-temperature model.

In the first part of this paper many-body effects were neglected. For the Q 2D EG it was found that (i) the contribution of the longitudinal-acoustic phonons to the ERR starts to dominate that of the polar LO phonons for $T_e<40$ K, in agreement with the $B=0$ case[5] and with the 3D EG case; (ii) the density-of-states broadening parameter evidently plays a much more pronounced role for the LO-phonon contribution to the ERR than for the acoustic-phonon part; (iii) the Pauli exclusion principle does not affect the MPR peak positions and only starts to diminish the MPR amplitudes for filling factors larger than 2; and (iv) for low-excitation photoexcitation and magnetotransport measurements we expect the MPR effect to be observable for $T_e>50$ K if $\Gamma_0\ll10$ meV and for $N_e<4\times10^{11}$ cm^{-2}.

In a separate section the effect of screening on the ERR has been investigated within the RPA approximation[24,25] and introducing an effective Lorentzian broadening of the electronic polarizability. It turns out that (i) all the temperature dependence is contained in the phonon occupation numbers, in agreement with the results of I, (ii) the Lorentzian broadening does not shift the MPR peak positions, but only reduces the MPR amplitude, and (iii) the MPR peak positions are shifted to lower magnetic fields with increasing electron sheet density.

We found that the contribution of the acoustic phonons to the ERR (within the 3D continuum model) is very sensitive to the extension of the electronic wave function in the z direction and that in the case of a Q 2D EG with $1/b=L_z$ not much larger than $10/k_D\approx15$ Å the electrons predominantly interact with phonons with large q_z. In that case the continuum (Debye) approximation for the acoustic phonons will not be valid, irrespective of temperature. In Ref. 26 a qualitative remark in this direction was made. It turns out that this is not a severe restriction since $L_z = 15$ Å corresponds to a sheet electron density $N_e = 10^{14}$ cm^{-2}, which is much larger than usual experimental densities. As in Ref. 21, also in Ref. 26 an attempt was made to identify the nature of acoustic phonons (i.e., bulk or surface) with which the electrons interact predominantly in silicon metal-insulator-semiconductor structures, by measuring the energy relaxation time, but no clear evidence was found. It is clear[27] that the purely 2D acoustic-phonon model of Ref. 28 does not suffice to describe real confined systems, but to our knowledge further work on this subject is lacking. However, some theoretical work (see, e.g., Refs. 29 and 30) has been done on the confinement of polar LO phonons in single- and multiple-quantum-well structures: due to the mismatch of the dielectric and elastic properties of the adjacent layers the propagation of the phonons is limited. In Ref. 30 both intra- and intersubband magnetophonon resonance data on a GaAs quantum well are compared with the results of a model which incorporates the confined phonons of Ref. 29. It would be worthwhile to study experimentally the energy relaxation rate for a quantum well and a single heterojunction with the same effective thickness of the electron gas, since then one would be able to single out the effect of possible phonon confinement.

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[1]V. L. Gurevich and Yu. A. Firsov, Zh. Eksp. Teor. Fiz. **40**, 199 (1961) [Sov. Phys.—JETP **13**, 137 (1963)].

[2]R. J. Nicholas, Prog. Quantum Electron. **10**, 1 (1985).

[3]R. J. Nicholas, in Landau Level Spectroscopy, edited by E. I. Rashba and G. Landwehr (North-Holland, Amsterdam, in press).

[4]P. Warmenbol, F. M. Peeters, and J. T. Devreese, Phys. Rev. B **37**, 4694 (1988).

[5]J. Shah, in *The Physics of the Two-Dimensional Electron Gas,* edited by J. T. Devreese and F. M. Peeters (Plenum, New York, 1987).

[6]J. Shah, A. Pinczuk, A. C. Gossard, and W. Wiegmann, Phys. Rev. Lett. **54**, 2045 (1985).

[7]W. Pötz and P. Kocevar, Phys. Rev. B **28**, 7040 (1983).

[8]P. Lugli and S. M. Goodnick, Phys. Rev. Lett. **59**, 716 (1987).

[9]W. Cai, M. C. Marchetti, and M. Lax, Phys. Rev. B **35**, 1369 (1987).

[10]S. Das Sarma, J. K. Jain, and R. Jalabert, Phys. Rev. B **37**, 4560 (1988).

[11]K. Leo, W. W. Ruhle, H. J. Queisser, and K. Ploog, Phys. Rev. B **37**, 7121 (1988).

[12]R. W. J. Hollering, T. T. J. M. Berendschot, H. J. A. Bluyssen, H. A. J. M. Reinen, and P. Wyder, Phys. Rev. B **38**, 13 323 (1988); in *Proceedings of the 18th International Conference on the Physics of Semiconductors, Stockholm, 1986,* edited by O. Engström (World Scientific, Singapore, 1987), p. 1323.

[13]A. J. Turberfield, in Proceedings of the 5th International Conference on Hot Carriers in Semiconductors, Boston, 1987 [Solid State Electron. **31**, 387 (1988)]; J. F. Ryan, R. A. Taylor, A. J. Turberfield, and J. M. Worlock, Surf. Sci. **170**, 511 (1986).

[14]K. Hirakawa, H. Sakaki, and J. Yoshino, in *Proceedings of the 18th International Conference on the Physics of Semiconductors, Stockholm, 1986,* edited by O. Engström (World Scientific, Singapore, 1987), p. 461.

[15]L. J. Challis, A. J. Kent, and V. W. Rampton, in *Proceedings of the 9th International Conference on the Application of High Magnetic Fields in Semiconductors Physics, Würzburg, 1988, High Magnetic Fields in Semiconductors Physics II,* edited by G. Landwehr (Springer, Berlin, in press).

[16]P. J. Price, J. Appl. Phys. **53**, 6863 (1982).

[17]P. J. Price, Ann. Phys. (N.Y.) **133**, 217 (1981).

[18]V. Karpus, Fiz. Tekh. Poluprovodn. **20**, 12 (1986) [Sov. Phys.—Semicond. **20**, 6 (1986)].

[19]C. J. G. M. Langerak, J. Singleton, D. J. Barnes, R. J. Nicholas, P. J. van der Wel, M. A. Hopkins, J. A. A. J. Perenboom, and C. T. Foxon, Phys. Rev. B **38**, 13 133 (1988).

[20]X. Wu, F. M. Peeters, and J. T. Devreese, Phys. Rev. B **36**, 9760 (1987).

[21]T. Neugebauer and G. Landwehr, Phys. Rev. B **21**, 702 (1980).

[22]*Physics of Group IV Elements and III-V Compounds,* Vol. 17a of *Landolt-Börnstein,* edited by O. Madelung, M. Schulz, and H. Weiss (Springer, Heidelberg, 1982).

[23]A. L. Fetter and J. D. Walecka, *Quantum Theory of Many-Particle Systems* (McGraw-Hill, New York, 1971).

[24]M. L. Glasser, Phys. Rev. B **28**, 4387 (1983).

[25]P. F. Maldague, Surf. Sci. **73**, 246 (1978).

[26]V. T. Dolgopolov, A. A. Shashkin, S. I. Dorozhkin, and E. A. Vyrodov, Zh. Eksp. Teor. Fiz. **89**, 2113 (1985) [Sov. Phys.—JETP **62**, 1219 (1985)].

[27]Nguyen Van Trong, G. Mahler, and A. Fourikis, Phys. Rev. B **38**, 7674 (1988).

[28]S. Kawaji, J. Phys. Soc. Jpn. **27**, 906 (1969).

[29]N. Sawaki, Surf. Sci. **170**, 537 (1986).

[30]N. Mori, K. Taniguchi, C. Hamaguchi, S. Sasa, and S. Hiyamizu, J. Phys. C **21**, 1791 (1988).