Size of exciton bound to a neutral impurity

M. Suffczynski
Institute of Physics, Polish Academy of Sciences, PL-02-668 Warsaw, Poland

L. Wolniewicz
Institute of Physics, Nicolaus Copernicus University, PL-87-100 Toruń, Poland
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Variational calculations of the ground-state energy of an exciton bound by Coulomb forces to a neutral impurity have been performed in the approximation of spherical nondegenerate energy bands with an envelope function exponentially dependent on the three particle distances from the impurity center. With the envelope function optimized for the electron– to hole–effective-mass ratios between 0.01 and 10, the expectation values of the interparticle distances have been computed. In semiconductors doped with magnetic ions the computed values of the interparticle distances allow an estimate of the number of magnetic ions confined within the bound-exciton complex. The estimate reasonably agrees with the number of magnetic ions inferred from optical observation of magnetic bound polarons.

I. INTRODUCTION

Shallow excitons bound to impurities or defects in semiconductors are usually described in the effective-mass approximation.[1–4] The ground-state energy of an exciton bound to a neutral impurity can be calculated by a variational method. The minimization of the ground-state energy of the bound-exciton complex determines the optimized bound-exciton envelope. The optimized envelope can be used to compute the expectation values of the interparticle distances in the complex. This makes it possible to estimate in semiconductors doped with magnetic ions the number of magnetic ions confined within the orbit of electrons or holes in the bound-exciton complex. These numbers can be compared with corresponding numbers inferred from optical observation of bound magnetic polarons.[5–12]

II. EXCITON BOUND TO A NEUTRAL DONOR

Shallow excitons bound by Coulomb forces to a neutral donor, (D^0,X), or acceptor, (A^0,X), can be described in the effective-mass approximation. In actual calculations of bound excitons a spherically symmetric effective mass m_e from the bottom of the conduction band is ascribed to the electron, and an average spherically symmetric effective mass m_h to the hole. The Coulomb interaction is screened by the static isotropic dielectric constant ϵ. The ground state of an exciton bound to a neutral donor is described by a variational wave function $\phi(r_{ij})=\psi(kr_{ij})$ used by Stébé and Munschy,[1]

$$\psi(r_{ij})=\sum_{m,n,p,q} c_{mnpq}(1+P_{12})|mnpq\rangle , \quad (1)$$

$$|mnpq\rangle = r_1^m r_{1h}^n r_h^p r_{12}^q \exp(-\alpha r_1-\beta r_2-r_h) , \quad (2)$$

with positive integers m,n,p,q and the linear parameters c_{mnpq}. The operator P_{12} permutes indices 1 and 2. The nonlinear parameters α and β and the scaling factor k are optimized for each value of the effective-mass ratio $\sigma=m_e/m_h$. r_1 and r_2 are distances of the electrons 1 and 2 from the donor center, r_h and r_{1h} of the hole from the donor center and from electron 1, respectively, and r_{12} the interelectron distance, all measured in units of the donor Bohr radius $a_D=\epsilon\hbar^2/m_e e^2$. The unit of energy is twice the donor Rydberg energy, $2E_D=m_e e^4/\epsilon^2\hbar^2$, proportional to the electron effective mass.

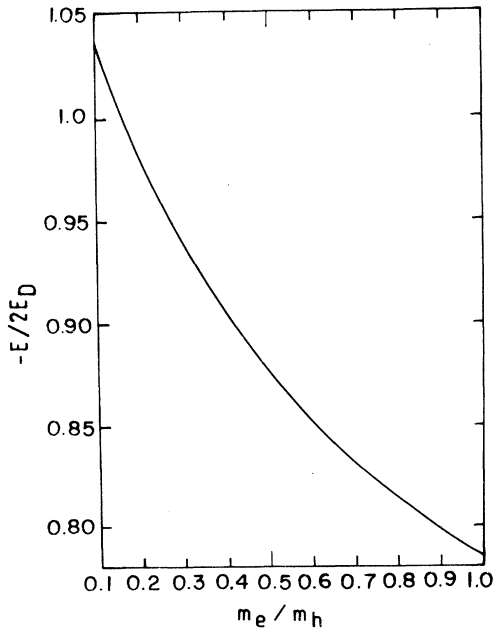

FIG. 1. Ground-state energy E of the (D^0,X) complex, in units of twice the donor Rydberg energy, $2E_D$, computed with a 70-term envelope function.

TABLE I. The nonlinear parameters, in units of a_D^{-1}, and the ground-state energy E and the binding energy W, in units of $2E_D$, computed for the (D^0,X) complex with the 70-term envelope function.

m_h/m_e	α	β	k	$-E$	$-W$
0.1	4.295	12.880	0.082 05	0.573 79	0.028 33
0.125	3.838	11.343	0.093 24	0.584 17	0.028 61
0.2	2.469	6.730	0.156 43	0.612 80	0.029 47
0.25	2.127	5.489	0.190 98	0.629 93	0.029 93
0.3	1.894	4.626	0.225 45	0.645 75	0.030 37
0.4	1.613	3.559	0.289 98	0.674 08	0.031 22
0.5	1.446	2.942	0.347 56	0.698 73	0.032 07
0.6	1.329	2.545	0.398 67	0.720 43	0.032 93
0.625	1.305	2.460	0.411 63	0.725 47	0.033 16
0.6667	1.271	2.331	0.432 94	0.733 55	0.033 54
0.7	1.246	2.237	0.449 93	0.739 75	0.033 86
0.7922	1.191	2.020	0.494 71	0.755 77	0.034 75
0.8	1.187	2.004	0.498 37	0.757 05	0.034 83
0.8182	1.177	1.968	0.506 84	0.760 01	0.035 00
0.9385	1.124	1.773	0.558 33	0.778 27	0.036 20
1.0	1.102	1.695	0.582 13	0.786 81	0.036 81
1.0656	1.080	1.618	0.607 80	0.795 42	0.037 48
1.2222	1.033	1.472	0.663 94	0.814 06	0.039 07
1.25	1.025	1.450	0.673 41	0.817 13	0.039 35
1.2623	1.021	1.440	0.677 82	0.818 46	0.039 47
1.5	0.938	1.283	0.757 17	0.841 89	0.041 89
1.6667	0.909	1.198	0.807 93	0.856 04	0.043 53
2.0	0.907	1.071	0.895 98	0.879 95	0.046 62
2.5	0.860	0.945	1.010 16	0.908 12	0.050 97
3.3333	0.808	0.816	1.163 92	0.941 93	0.057 32
3.956	0.777	0.739	1.278 30	0.960 39	0.061 28
4.3077	0.766	0.717	1.317 64	0.969 22	0.063 42
4.6875	0.758	0.702	1.347 64	0.977 71	0.065 63
4.8098	0.756	0.700	1.352 91	0.980 27	0.066 33
5.0	0.753	0.697	1.360 85	0.984 05	0.067 39
6.6667	0.713	0.611	1.543 54	1.009 40	0.074 62
8.0	0.701	0.595	1.594 47	1.024 00	0.079 56
10.0	0.683	0.570	1.672 97	1.039 93	0.085 38
20.0	0.640	0.493	1.947 31	1.077 37	0.101 18
50.0	0.613	0.450	2.164 54	1.106 20	0.116 00
100.0	0.604	0.440	2.238 34	1.117 70	0.122 65

Wolniewicz has computed the ground-state energy of an exciton bound to a neutral donor with a 70-term envelope function of the form (1) for mass ratio $\sigma = m_e/m_h$ between 0.01 and 10; the computed ground-state energy E together with optimized values of the nonlinear parameters and the binding energy,[1,2,13]

$$W = E + E_D + E_X = E + E_D[1 + (1+\sigma)^{-1}] , \quad (3)$$

are given in Table I for selected values of mass ratio. The optimized envelope function was used to compute the expectation values of the interparticle distances in the donor-bound–exciton complex. The necessary integrals were computed by the method of Perkins.[14] The ground-state energy of the complex and the interparticle distances depend monotonically on the mass ratio σ, see Figs. 1–9.

A hole of mass larger than the effective electron mass gets nearer to the donor center and the binding energy becomes strongly dependent on the mass ratio. For a hole lighter than the electron, the hole distance from the donor center increases and the hole has less influence on the binding; thus the energy of the complex gradually becomes independent of the mass ratio. For masses $m_h < 1.47 m_e$ the interelectron repulsion is larger than the hole–donor-center repulsion, $\langle r_{12}^{-1} \rangle > \langle r_h^{-1} \rangle$; for $m_h < 0.7 m_e$ the interelectron repulsion is larger in absolute value than the electron-hole attraction, $\langle r_{12}^{-1} \rangle > \langle r_{1h}^{-1} \rangle$. The electrons, attracted by the donor center, stay at a distance from the donor center almost independent of the mass ratio.

The expectation values of the interparticle distances $\langle r \rangle$ have been computed with a 35-term envelope function; the expectation values of the inverse distances $\langle r^{-1} \rangle$ which determine the potential energy in the complex have been computed with a 70-term envelope function. The products $\langle r \rangle \langle r^{-1} \rangle$ have values between

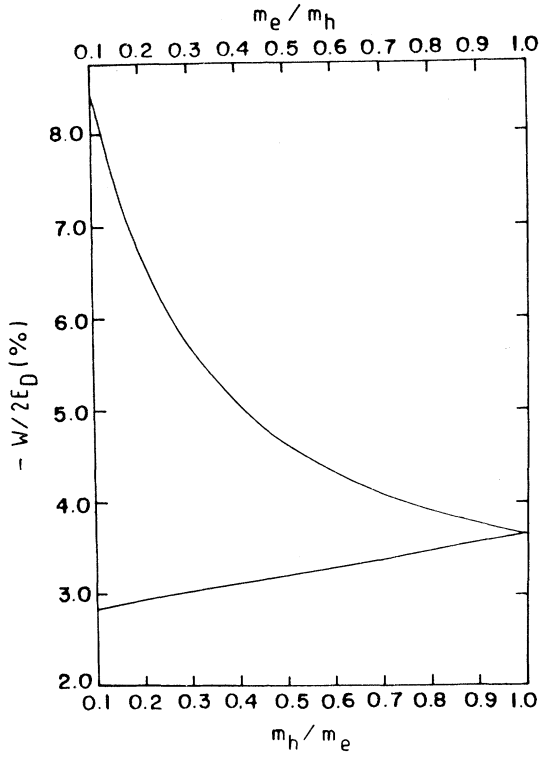

FIG. 2. Binding energy W of the (D^0,X) complex, in units of twice the donor Rydberg energy, $2E_D$, vs electron-hole effective-mass ratio. Lower abcissa refers to lower part of plot, upper abcissa to upper part.

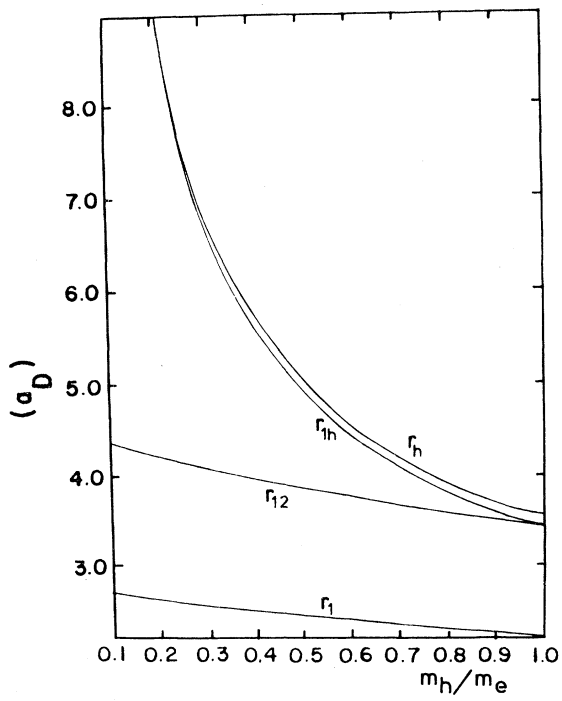

FIG. 3. Expectation values of the interparticle distances in the (D^0,X) complex for $m_h < m_e$.

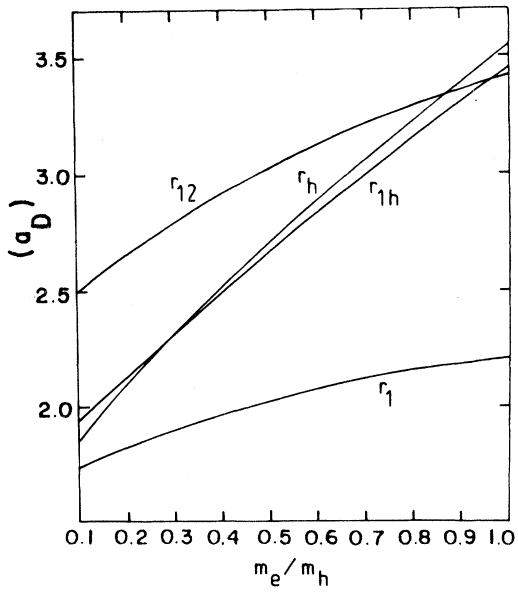

FIG. 4. Expectation values of the interparticle distances in the (D^0,X) complex for $m_e < m_h$.

$\langle r_1 \rangle \langle r_1^{-1} \rangle = 1.83$ for $\sigma = 10$ and $\langle r_h \rangle \langle r_h^{-1} \rangle = 1.056$ for $\sigma = 0.01$, see Figs. 9(a) and 9(b). These values are reasonably consistent with the fact[12,15] that a hydrogenic atom with nuclear charge Ze and Bohr radius a_B has in the ground state the expectation value of the electron distance from the nucleus

$$\langle r \rangle = 3a_B/(2Z) \tag{4}$$

and

$$\langle r^{-1} \rangle = Za_B^{-1} . \tag{5}$$

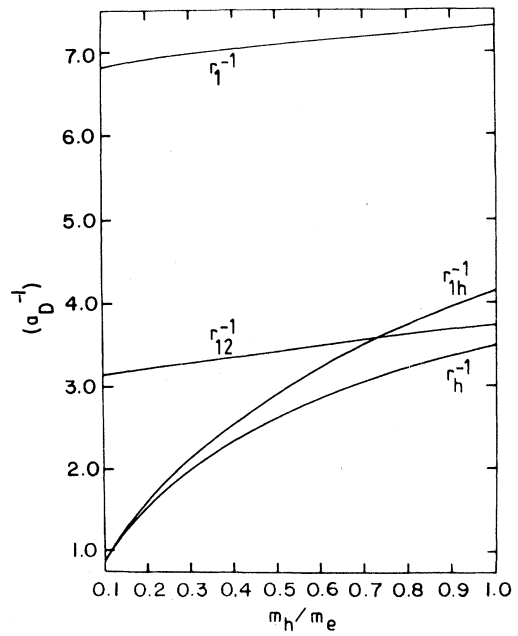

FIG. 5. Expectation values ($\times 10$) of the inverse interparticle distances in the (D^0,X) complex for $m_h < m_e$.

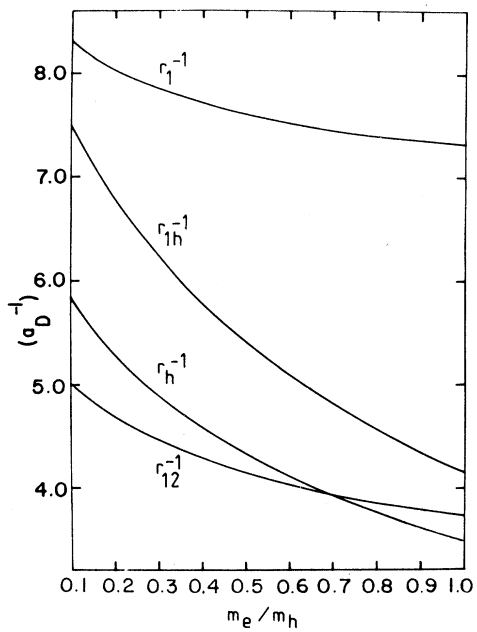

FIG. 6. Expectation values ($\times 10$) of the inverse interparticle distances in the (D^0,X) complex for $m_e < m_h$.

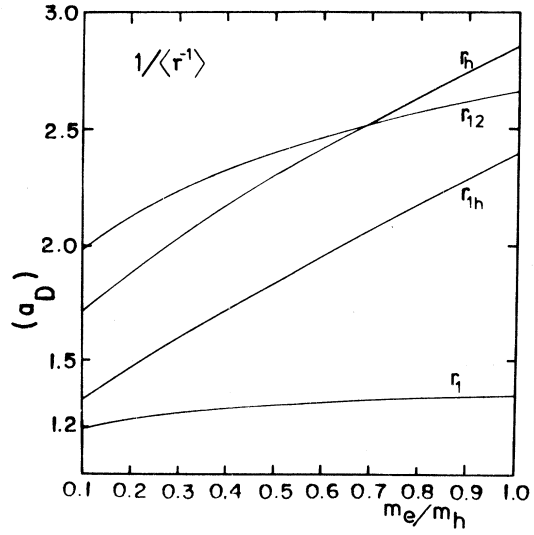

FIG. 8. Reciprocals of the expectation values of the inverse interparticle distances in the (D^0,X) complex for $m_e < m_h$.

Table II gives our calculated expectation values of the interparticle distances and inverse distances in units of the donor Bohr radius for selected values of the mass ratio.

Our expectation values of distances agree with the values computed by Dujardin and Stébé[16] and reasonably interpolate between the values for the hydrogen negative ion[17,18] H^- ($\sigma=\infty$) and the hydrogen molecule[19] H_2 ($\sigma=\frac{1}{918}$) limiting cases listed in Table III. Our distance $\langle r_1 \rangle$ does not agree with the value for the positronium hydride PsH ($\sigma=1$) reported by Ho.[20]

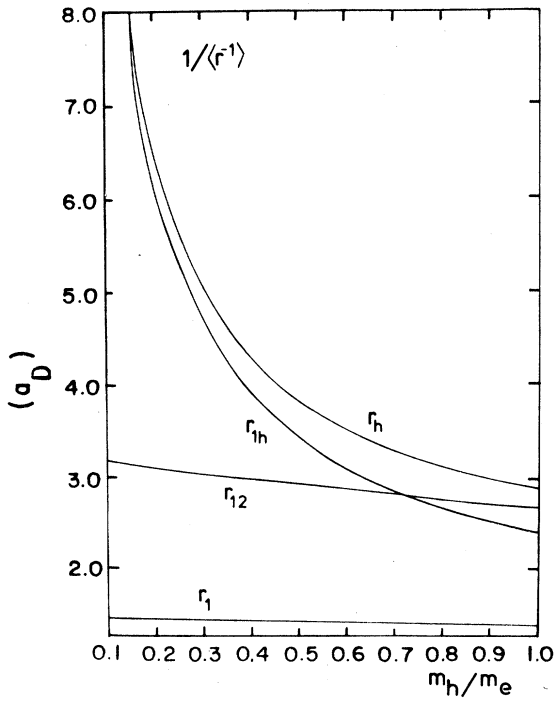

FIG. 7. Reciprocals of the expectation values of the inverse interparticle distances in the (D^0,X) complex for $m_h < m_e$.

III. EXCITON BOUND TO A NEUTRAL ACCEPTOR

In the approximation of nondegenerate spherically symmetric band extrema the calculation of the binding energy and the envelope function of an exciton bound to a neutral donor can be applied to an exciton bound to a neutral acceptor if the electrons are interchanged with the holes.[1,2] The Rydberg energy of an acceptor, $E_A = m_h e^4/2\epsilon^2\hbar^2$, is proportional to the hole effective mass m_h. The ground-state energies of the acceptor–bound-exciton complex $E_{(A^0,X)}(\sigma)$ and the donor–bound-exciton complex $E_{(D^0,X)}(\sigma^{-1})$ are related by

$$E_{(A^0,X)}(\sigma)/E_A = E_{(D^0,X)}(\sigma^{-1})/E_D \ . \tag{6}$$

In most of the semiconductors the effective mass of the hole is larger than the effective mass of the electron, particularly for the heavy hole. Thus, in the exciton bound to a neutral acceptor the electron is much further from the acceptor center than the holes, and the binding energy and the hole distance depend rather weakly on the electron– to hole–effective-mass ratio.

IV. BOUND MAGNETIC POLARONS

In semiconductors doped with magnetic ions, such as $Cd_{1-x}Mn_xTe$ and $Cd_{1-x}Mn_xSe$, the exchange interaction between the carrier spins and the ion magnetic moments causes the formation of magnetically aligned complexes in the vicinity of carriers bound to impurities: the

bound magnetic polarons (BMP's).[5-12,21-34] The formation, the radiative recombination, and the size of BMP's have been studied in reflectance[33] and luminescence experiments[5,23-32] and by spin-flip Raman scattering.[7-11,34] Magnetic polarons bound to acceptors, A^0-BMP's, to donors, D^0BMP's, or to exciton-neutral impurity complexes are observed. Photoexcitation of semiconductors creates electron-hole pairs which form excitons. Excitons attach themselves to impurities forming bound-exciton complexes. In semiconductors doped with magnetic ions

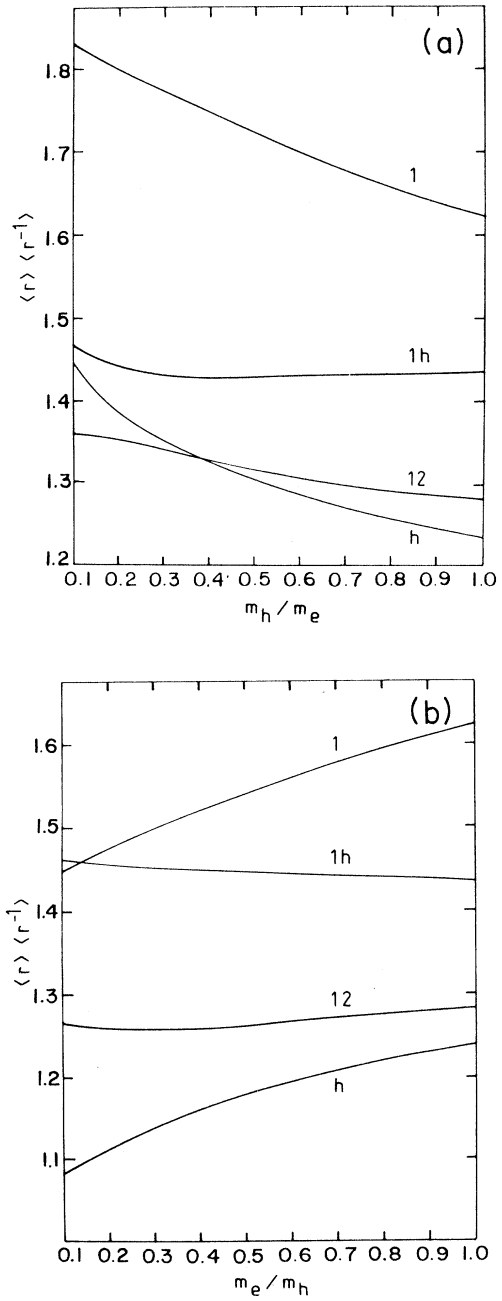

FIG. 9. Products of the expectation values $\langle r\rangle\langle r^{-1}\rangle$ computed for the (D^0,X) complex: (a) for $m_h < m_e$ and (b) for $m_e < m_h$.

their exchange interaction with electrons and holes aligns the magnetic ions within the carrier orbit, leading to a formation of the exciton–magnetic-polaron (EMP) complexes.[5,6,12,25,26,30-32] In particular, the EMP bound to a neutral acceptor, (A^0,X), is observable[5,6,30-32] at concentrations of Mn^{2+} ions even below $x=1\%$. The p-d exchange constant of the hole is typically 3–4 times larger than the s-d constant of the electron.[10,27] The hole mass is 2–5 times larger than the electron effective mass, making the hole localization easier. Therefore, the hole affects the magetic polaron energy more strongly than the electron. An acceptor-bound EMP complex has two strongly localized holes in the two available states with angular momentum component $m_j=-\frac{3}{2}$ and $-\frac{1}{2}$, and an electron with extended wave function determined primarily by the Coulomb interaction with the holes.[12,20,30,31]

In the exciton–neutral-donor complex, (D^0,X), two electrons are in the singlet state; therefore, only the hole can align the magnetic ions within its orbit.[26,27,30] An important parameter characterizing a bound magnetic polaron is the number of magnetic ions contained within the electron or hole orbit.[5,12] In Refs. 5, 6, 8–11, 21–28, 30, 31, and 34 envelope functions of a single carrier coordinate have been used to describe in a self-consistent potential the free-energy functional[8,10,11,21-25,27] of BMP's. The expectation values of the electron and hole distance from the impurity center computed with our correlated envelope function allow one, in the lightly doped semiconductors, to estimate the number of magnetic ions confined within the electron or the hole orbit of the Coulomb bound-exciton complex. The estimate is valid for low concentrations of the magnetic ions since only the Coulomb interaction is considered; contribution of the carrier-exchange interaction with the magnetic ions, including fluctuation effects,[21-24] gives an excess binding energy dependent on magnetic ion concentration and on temperature,[21-25,27] but is not taken into the calculation of our correlated envelope function. To estimate in a simple way the increase in binding of the exciton complex due to self-consistent exchange interaction with magnetic ions, one could reduce the radius of neutral donor or acceptor, but this requires introduction of a phenomenological or model-dependent factor of reduction.[27,34] The Mn^{2+} magnetic ions which are at the nearest-neighbor lattice sites, and are thus closer than 3–5 Å, form several types of clusters[35] and at low temperatures order antiferromagnetically.[21-24,27-35] This severely complicates the estimate of their resulting magnetic moment from their average concentration.

Proper calculation of the exchange-energy contribution from magnetic ions contained within an electron or hole orbit requires appropriate integration of the squared modulus of the envelope function with a corresponding Brillouin function,[5,23,24] but such integration of the correlated envelope function[14] is cumbersome and has not been done. In the exciton–neutral-donor complex, (D^0,X), the number $N(r_e)$ of magnetic ions, at the fractional concentration x, contained within the electron orbit can be computed roughly by the formula[36]

$$N(r_1)=(4\pi/3)x(\tfrac{2}{3}\langle r_1\rangle a_1)^3/v_1\ , \tag{7}$$

TABLE II. Expectation values of distances and inverse distances in (D^0,X) in units of a_D and a_D^{-1}, respectively.

m_h/m_e	$\langle r_1 \rangle$	$\langle r_1^{-1} \rangle$	$\langle r_{12} \rangle$	$\langle r_{12}^{-1} \rangle$	$\langle r_{1h} \rangle$	$\langle r_{1h}^{-1} \rangle$	$\langle r_h \rangle$	$\langle r_h^{-1} \rangle$
0.1	2.667 54	0.686 03	4.335 97	0.314 91	16.3790	0.089 52	16.3185	0.088 65
0.2	2.603 17	0.692 46	4.208 26	0.321 90	9.180 94	0.157 17	9.171 60	0.151 79
0.25	2.571 01	0.695 68	4.143 53	0.325 50	7.759 42	0.185 21	7.774 18	0.176 46
0.3	2.539 30	0.698 80	4.079 24	0.329 11	6.813 24	0.210 41	6.848 01	0.197 82
0.4	2.478 70	0.704 72	3.955 41	0.336 36	5.631 06	0.254 08	5.694 88	0.233 09
0.5	2.423 55	0.710 23	3.842 19	0.343 48	4.919 05	0.290 85	5.000 88	0.261 23
0.6	2.374 80	0.715 35	3.742 19	0.350 31	4.441 44	0.322 44	4.534 11	0.284 40
0.7	2.328 98	0.720 19	3.648 16	0.357 03	4.093 63	0.350 16	4.189 99	0.304 17
0.8	2.287 20	0.724 75	3.562 57	0.363 50	3.828 68	0.374 74	3.924 59	0.321 37
1.0	2.218 23	0.733 00	3.422 07	0.375 28	3.453 23	0.416 40	3.544 43	0.349 89
1.25	2.151 44	0.741 98	3.287 13	0.388 06	3.146 16	0.458 17	3.227 93	0.377 98
1.5	2.102 26	0.749 63	3.189 43	0.398 66	2.938 58	0.491 61	3.012 19	0.400 03
2.0	2.019 80	0.762 81	3.025 81	0.417 39	2.663 78	0.543 45	2.709 20	0.435 24
2.5	1.965 48	0.773 16	2.921 07	0.431 42	2.496 08	0.581 01	2.520 69	0.460 69
3.956	1.867 42	0.794 67	2.737 57	0.459 70	2.235 96	0.650 37	2.209 37	0.509 61
5.0	1.831 08	0.804 93	2.673 30	0.471 38	2.144 35	0.679 79	2.104 07	0.529 97
8.0	1.764 73	0.824 79	2.558 23	0.494 06	2.001 24	0.730 57	1.920 07	0.568 64
10.0	1.740 81	0.833 25	2.518 71	0.502 66	1.952 36	0.749 77	1.858 98	0.583 53
20.0	1.681 47	0.856 13	2.423 01	0.525 63	1.845 41	0.796 14	1.708 82	0.624 17
50.0	1.638 02	0.876 54	2.356 72	0.543 72	1.774 67	0.830 54	1.607 12	0.658 02
100.0	1.621 42	0.885 42	2.332 31	0.550 73	1.749 00	0.843 47	1.572 12	0.671 66

where $\langle r_1 \rangle = \langle r_e \rangle$ is the expectation value of the donor-center–to–electron (attractive) distance in the complex, $a_1 = a_D$ is the neutral donor ground-state radius, and v_1 is the volume of one molecule of the semiconductor compound determined by its lattice constant. The number $N(r_h)$ of magnetic ions contained within the hole orbit is given by Eq. (7) with the expectation value $\langle r_h \rangle$ of the donor-center–to–hole (repulsive) distance in place of $\langle r_1 \rangle$. In the exciton–neutral-acceptor complex, (A^0,X), the number $N(r_h)$ of magnetic ions contained within the hole orbit can be computed by Eq. (7) with the expectation value $\langle r_h \rangle$ in place of $\langle r_1 \rangle$ and the neutral-acceptor ground-state radius a_A in place of a_1. The number $N(r_e)$ of magnetic ions contained within the electron orbit is given by Eq. (7) with the expectation value $\langle r_e \rangle$ of the acceptor-center–to–electron (repulsive) distance in place of $\langle r_1 \rangle$.

The number of magnetic ions contained within the orbit of mean radius $\langle r_1 \rangle$ can also be estimated from the computed expectation values $\langle r_1^{-1} \rangle$ of the inverse distances by the formula

$$N(r_1^-) = (4\pi/3)x(a_1/\langle r_1^{-1} \rangle)^3/v_1 \ . \tag{8}$$

Table IV gives our estimated numbers of Mn^{2+} ions in doped semiconductors, the typical representative of which is $Cd_{1-x}Mn_xTe$. For each compound the ion concentration is $x = 0.01$. We give the estimate first according to Eq. (7), and then according to Eq. (8), of the number of Mn^{2+} ions contained within the orbit of the center-attracted, and then center-repelled, carrier. The estimates practically do not differ for the orbits of the center-attracted carriers. For the large radii of the center-repelled carriers, numbers of Eq. (8) differ from those of Eq. (7) by a factor up to 2.4.

We have adopted Bohr radii of the acceptor and donor from experimental data,[2,7,12,21,37] which necessitates adoption of appropriate values of the dielectric constant for the acceptor and for the donor, respectively. In $Cd_{1-x}Mn_xTe$ the number of Mn^{2+} ions at $x = 1\%$ within the hole orbit of the acceptor-bound EMP inferred from the photoluminescence measurements[5,6,24] is $N(r_h) = 3$, in good agreement with our calculated value. In $Cd_{1-x}Mn_xSe$ the effective number of Mn^{2+} ions at $x = 1\%$ within the donor orbit is estimated as $N(r_D) = 60$ in Ref. 11 and $N(r_D) = 40$ in Ref. 21, and within the D^0-BMP orbit $N_e = 148$ in Ref. 11, which is between our estimates for a donor-bound EMP, $N(r_h) = 107$ and $N(r_h^-) = 255$, given in Table IV. The number within the acceptor orbit is $N(r_A) = 2$ in Refs. 27 and 29, in fair agreement with our estimate for an acceptor EMP, $N(r_h) = 1$, given in Table IV.

TABLE III. Binding energy in atomic units and expectation values of interparticle distances in units of hydrogen-atom Bohr radius a_B.

	$\sigma = m_e/m_p$	$-W$	$\langle r_1 \rangle$	$\langle r_{12} \rangle$	$\langle r_{1p} \rangle$	$\langle r_p \rangle$	Reference
H^-		0.0277 51	2.710 178	4.412 694 52			17,18
PsH	1	0.0389 45	2.993	3.556	3.849	3.664	20
H_2	$\frac{1}{918}$	0.1645	1.5745	2.201	1.5745	1.4487	19

TABLE IV. Number of Mn^{2+} ions at $x=1\%$ in the exciton–neutral-acceptor and exciton–neutral-donor complexes [(A^0,X) and (D^0,X), respectively].

	Exciton–neutral-acceptor complex (A^0,X)											
	m_e/m_h	a_A (Å)	v_1 (Å^3)	$\langle r_h\rangle$	$N(r_h)$	$\langle r_h^{-1}\rangle$	$N(r_h^-)$	$\langle r_e\rangle$	$N(r_e)$	$\langle r_e^{-1}\rangle$	$N(r_e^-)$	$N(r_h)$ (Expt.)
ZnTe	0.2[a]	8.2[a]	56.8[a]	2.603 17	2	0.692 46	1	9.1716	93	0.151 79	116	
CdSe	0.232[b]	7.4[c]	64.9[a]	2.581 95	1	0.694 52	1	8.200 99	43	0.168 05	56	2[c]
CdTe	0.3[d]	12[d]	68[a]	2.5393	5	0.6988	3	6.848 01	101	0.197 82	137	3[f]
	Exciton–neutral-donor complex (D^0,X)											
	m_h/m_e	a_D (Å)	v_1 (Å^3)	$\langle r_e\rangle$	$N(r_e)$	$\langle r_e^{-1}\rangle$	$N(r_e^-)$	$\langle r_h\rangle$	$N(r_h)$	$\langle r_h^{-1}\rangle$	$N(r_h^-)$	$N(r_h)$ (Expt.)
CdS	4.8098[b]	21.8[b]	49.4[a]	1.836 28	16	0.8033	17	2.118 49	25	0.526 94	60	
CdSe	4.3077[b]	38[e]	64.9[a]	1.852 88	67	0.7985	70	2.166 06	107	0.517 56	255	148[g]

[a]Reference 37.
[b]Reference 2.
[c]Reference 29.
[d]Reference 12.
[e]Reference 21.
[f]References 5 and 24.
[g]Reference 11.

V. DISCUSSION

The size of the bound-exciton complex is an important parameter characterizing the complex. A reliable calculation of the size of the complex is difficult, since in this few-body problem the necessary approximations neglect several known complicating factors. The band degeneracy and anisotropy of holes is described by one average spherically symmetric effective mass, and the dielectric constant is taken as isotropic. Coulomb interaction between the constituent charged particles is assumed, and the magnetic exchange interaction with the magnetic ions in the doped semiconductor is not taken into account in the calculation of the binding energy. Minimization of the ground-state energy gives the optimized Coulomb-correlated envelope function of the complex, but does not necessarily ensure a comparable accuracy of the computed expectation values of the interparticle distances in the complex. Approximations involved in the use of Eqs. (7) and (8) are difficult to assess.

Recent progress in the luminescence measurements of the bound magnetic polarons offers a possibility of obtaining a magnetization map in the vicinity of the bound-exciton complex.[12,30] The spatial distances of electrons and holes can therefore be compared with expectation values computed with the optimized envelope function.

With all the reservations mentioned, it is important to compare the particle distances in the exciton complex with the average distance of magnetic ions. This allows an estimate of the number of magnetic ions contained within the electron and the hole orbit in the exciton-impurity complex. The estimates presented here based on a correlated envelope function compare reasonably with the numbers reported from optical and magnetic measurements of bound magnetic polarons at low concentrations.

The interparticle distances in the Coulomb bound-exciton complex are rather large, particularly for the lighter electrons or holes. Therefore, the bound-exciton complexes are sensitive to quantum wells.[38] Exciton-complex formation can be inhibited when the width of the quantum well is comparable to the diameter of the complex. The interval of widths in which an exciton complex still exists can furnish additional information about the size of the complex.

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