Larmor-clock transmission times for resonant double barriers

C. R. Leavens and G. C. Aers
Division of Physics, National Research Council of Canada, Ottawa, Canada K1A 0R6
(Received 1 March 1989)

Recent generalizations of Büttiker's analysis of the Larmor clock to any region $z_1 \le z \le z_2$ within an arbitrary one-dimensional barrier lead to two local transmission "times" for an incident electron of energy $E = \hbar^2 k^2/2m$. These are conveniently regarded as the real and (minus) the imaginary parts of a complex time $\tau_T^V(k;z_1,z_2)$. In this paper the properties of $\tau_T^V(k;z_1,z_2)$ are investigated for the double-rectangular-barrier potential $V(z) = V_1\Theta(z)\Theta(a-z) + V_2\Theta(z-b)\Theta(d-z)$. Results are presented for the dependence of the real and imaginary parts of the Larmor-clock transmission time on incident energy and barrier and well widths. The local transmission time $\tau_T^V(k;z_1,z_2)$ is real for *symmetric* double barriers when the energy of the incident electron is exactly *on resonance*. Remarkably, the behavior of the real quantity $\tau_T^V(k;0,z \ll a)$ for this special case provides further evidence for the importance of the imaginary part of the local Larmor-clock transmission time for the corresponding (isolated) single barrier $V(z) = V_1\Theta(z)\Theta(a-z)$. At resonance, for symmetric double barriers, (real) local transmission speeds $v_T(z)$ can be very much in excess of the speed of light in the well region for z very close to a quasinode of the Schrödinger stationary-state wave function. It is proven that $v_T(z) \le c$ for a symmetric double barrier at resonance when the Dirac equation is used in place of the Schrödinger equation. On the other hand, it is shown that application of the Dirac equation to a single opaque rectangular barrier does not alter the well-known difficulty that $a/\mathrm{Re}[\tau_T^V(k;0,a)]$ exceeds c for sufficiently large barrier width a.

I. INTRODUCTION

Over the years there have been many attempts[1-30] to answer the fundamental question "how long does it take on average for an incident particle of energy E to tunnel through a potential barrier?" Most of the approaches for calculating the tunneling time τ_T lead either to the result of Bohm[1] and Wigner[2] or that of Büttiker and Landauer.[9-11] Consequently, there has been considerable controversy and confusion surrounding this question, particularly in the last year or two. It appears that resolution of the controversy will require answers to three intertwined questions: (1) Is the tunneling time determined by the sensitivity of the transmission probability amplitude $T \equiv |T| \exp(i\phi_T)$ to the incident energy E or to the average barrier height $\bar{V}$, or perhaps some other quantity?[31] (2) Is it the sensitivity of the phase ϕ_T or of the modulus $|T|$ that is most important?[9-11] (3) Is the tunneling time a real or a complex quantity?[12,13,15,16,31,32] Before becoming entangled in these questions it is reasonable to ask whether the tunneling times that result from the competing approaches are significantly different from a practical point of view. Consider an opaque ($|T|^2 \ll 1$) rectangular barrier of height V_0 and width d. The Bohm-Wigner result, which involves the sensitivity of the phase to the incident energy E, is virtually independent of d and diverges as $E^{-1/2}$ as E approaches zero, while the Büttiker-Landauer result, which involves the sensitivity of the modulus $|T|$ to the average barrier height $\bar{V}$, varies linearly with d and remains finite as E approaches zero. For typical metal-insulator-metal junction parameters ($V_0 = 10$ eV, $d = 10$ Å) the two tunneling times differ by a factor of 4 for $E = V_0/2$ corresponding to a typical Fermi energy E_F of 5 eV. For a semiconductor heterojunction with $E_F \ll V_0$ the difference can be even larger.

For a one-dimensional tunneling problem there are actually three times of interest:[10] the dwell time $\tau_D(k;0,d)$ is the mean time spent by an incident particle of energy $E_k \equiv \hbar^2 k^2/2m$ in the barrier region $0 \le z \le d$ regardless of whether it is ultimately transmitted or reflected; the transmission or tunneling time $\tau_T(k;0,d)$ is the corresponding mean time if the particle is finally transmitted; and the reflection time $\tau_R(k;0,d)$ the mean time if it is finally reflected. If these precise definitions of the times τ_D, τ_T, and τ_R are to be interpreted *literally* then, in the assumed absence of inelastic scattering and absorption, they must satisfy[31,33,15]

$$\tau_D(k;0,d) = |T(k)|^2 \tau_T(k;0,d) + |R(k)|^2 \tau_R(k;0,d) , \quad (1)$$

where $|T(k)|^2$ and $|R(k)|^2$ are the transmission and reflection probabilities, respectively. Throughout this paper the potential energy has been taken to be zero outside the barrier region $0 \le z \le d$, so that $|T(k)|^2 + |R(k)|^2 = 1$. Furthermore, it has been assumed that the incident beam of particles approaches the otherwise arbitrary barrier $V(z)$ from the left, so that exterior to the barrier region the stationary-state scattering wave function is given by $\psi_k(z) = e^{ikz} + R(k)e^{-ikz}$ for $z < 0$ and $T(k)e^{ikz}$ for $z > d$. Strictly speaking, various quantities such as $R(k)$ and $\tau_R(k;0,d)$ should have the label "$L \rightarrow R$" affixed,[24] but this has been dropped for simplicity; the argument k will often be dropped as well.

In a recent paper Leavens and Aers[33] used rigorous results from the paper by Hauge *et al.*[17] to prove that Büttiker's expression,

$$\tau_D(k;0,d)=\int_0^d dz\,|\psi_k(z)|^2 \Big/ \left[\frac{\hbar k}{m}\right], \tag{2}$$

for the dwell time is correct, despite claims to the contrary, and that the phase times for transmission and reflection of Bohm[1] and Wigner[2] do not satisfy Eq. (1). This leaves the Büttiker-Landauer results $\tau_T^{\rm BL}(k;0,d)$ and $\tau_R^{\rm BL}(k;0,d)$ as serious contenders for the transmission and reflection times as defined above.

It has recently been shown that Büttiker's analysis[10] of the Larmor clock[5,6] to obtain transmission and reflection times for a rectangular barrier can be readily generalized not only to an arbitrary barrier,[31] but to any region $z_1\le z\le z_2$ within the barrier.[32] The (generalized) Büttiker-Landauer result for the mean time spent in the region $z_1\le z\le z_2$ by an incident particle of energy E_k that is ultimately transmitted through the entire barrier can be written

$$\tau_T^{\rm BL}(k;z_1,z_2)\equiv|\tau_T^V(k;z_1,z_2)| , \tag{3}$$

where

$$\tau_T^V(k;z_1,z_2)\equiv i\hbar\left[\frac{\partial \ln T(E,V,\Delta V)}{\partial \Delta V}\right]_{E,V}\Bigg|_{\Delta V=0}, \tag{4}$$

and $T(E,V,\Delta V)$ is the transmission probability amplitude for the auxiliary barrier,

$$\tilde V(z)\equiv V(z)+\Delta V\,\theta(z-z_1)\theta(z_2-z) , \tag{5}$$

with ΔV independent of z. The complex "time" $T_T^V(k;z_1,z_2)$ is additive: $\tau_T^V(k;z_1,z_3)=\tau_T^V(k;z_1,z_2)+\tau_T^V(k;\,z_2,z_3)$.[32] The real quantity $\tau_T^{\rm BL}(k;z_1,z_2)$ obviously does not have this desirable property.

In Ref. 32 the complex "time" $\tau_T^V(k;z_1,z_2)$ did not appear naturally as a complex quantity. It was constructed from the two real "times,"

$$\tau_{xT}(z_1,z_2)=\lim_{\omega_L\to 0}\left[\frac{\hbar\omega_L}{2}\right]^{-1}\langle S_x\rangle_T = -\hbar\left[\frac{\partial \ln|T(E,V,\Delta V)|}{\partial \Delta V}\right]_{E,V}\Bigg|_{\Delta V=0}, \tag{6}$$

$$\tau_{zT}(z_1,z_2)=-\lim_{\omega_L\to 0}\left[\frac{\hbar\omega_L}{2}\right]^{-1}\langle S_z\rangle_T = -\hbar\left[\frac{\partial \phi_T(E,V,\Delta V)}{\partial \Delta V}\right]_{E,V}\Bigg|_{\Delta V=0}, \tag{7}$$

according to $\mathrm{Re}[\tau_T^V(z_1,z_2)]=\tau_{zT}(z_1,z_2)$ and $\mathrm{Im}[\tau_T^V(z_1,z_2)]=-\tau_{xT}(z_1,z_2)$. ω_L is the Larmor frequency. $\langle S_x\rangle$ and $\langle S_z\rangle$ are the x and z components of the local average spin $\langle \mathbf{S}\rangle\equiv\tilde\psi^\dagger\sigma\tilde\psi/\tilde\psi^+\tilde\psi$ per electron in the presence of an infinitesimal uniform magnetic field $\mathbf{B}=B\Theta(z-z_1)\Theta(z_2-z)\hat{\mathbf{x}}$ within the barrier when the incident beam is fully spin polarized in the y direction ($\langle S_x\rangle_I=0$, $\langle S_y\rangle_I=\hbar/2$, and $\langle S_z\rangle_I=0$). The nonzero value of $\langle S_x\rangle_T$ arises because the magnetic field changes the effective barrier potential in the region $z_1\le z\le z_2$ by the Zeeman energy $\mp\hbar\omega_L/2$ for electrons with $S_x=\pm\hbar/2$, so that there is, in general, differential transmission of spin-up ($S_x=+\hbar/2$) and spin-down ($S_x=-\hbar/2$) electrons; the nonzero value of $\langle S_z\rangle_T$ arises from the Larmor precession of the spin in the y-z plane whenever the electron is in the region $z_1\le z\le z_2$. The corresponding results for reflected electrons are obtained by replacing T by R in the above.

For an opaque barrier $\tau_{xT}(0,d)=-\mathrm{Im}[\tau_T^V(0,d)]$ is much larger than $\tau_{zT}(0,d)=\mathrm{Re}[\tau_T^V(0,d)]$, so that it is sensitivity of the modulus $|T|$ to the average barrier height $\bar V\equiv d^{-1}\int_0^d dz\,V(z)$, leaving $V(z)-\bar V$ unchanged, that makes the dominant contribution to the Büttiker-Landauer transmission time $\tau_T^{\rm BL}(k;0,d)$. For transmission above the barrier it is sensitivity of the phase that eventually dominates.[11]

In a previous paper[31] the present authors expressed some reservation regarding the identification of $\tau_T^{\rm BL}\equiv|\tau_T^V|$ and $\tau_R^{\rm BL}\equiv|\tau_R^V|$ with the actual transmission and reflection times as defined precisely (in words) above because they do not satisfy Eq. (1). In an important paper Sokolovski and Baskin[15] used the Feynman path-integral technique[34] to obtain the complex transmission and reflection times

$$\tau_T^{\rm SB}(k;0,d)\equiv i\hbar\int_0^d \frac{\delta \ln T[V(z)]}{\delta V(z)}dz ,$$
$$\tau_R^{\rm SB}(k;0,d)\equiv i\hbar\int_0^d \frac{\delta \ln R\,[V(z)]}{\delta V(z)}dz , \tag{8}$$

involving logarithmic functional derivatives of the transmission and reflection probability amplitudes with respect to the potential energy. These results are completely equivalent to $\tau_T^V(k;0,d)$ and $\tau_R^V(k;0,d)$, respectively.[15,32] Furthermore, the obvious generalization of (8) to an arbitrary region $z_1\le z\le z_2$ within the barrier by replacing 0 by z_1 and d by z_2 leads to equivalent local transmission and reflection times.

In the analysis of Sokolovski and Baskin the complex nature of these times arose naturally through the Feynman weight factor $\exp(iS/\hbar)$, where S is the action. Moreover, they proved that Eq. (1) is satisfied exactly by the complex times $\tau_T^{\rm SB}$ and $\tau_R^{\rm SB}$ and hence by τ_T^V and τ_R^V. Since τ_D is real, $\mathrm{Im}(|T|^2\tau_T+|R|^2\tau_R)$ must be identically equal to zero. For τ_T and τ_R of the form $\tau_T\equiv i\hbar\partial\ln T/\partial X$ and $\tau_R\equiv i\hbar\partial\ln R/\partial X$, with X an arbitrary real quantity, it is trivial to show that $\mathrm{Im}(|T|^2\tau_T+|R|^2\tau_R)\propto\partial(|T|^2+|R|^2)/\partial X=0$. Hence it is the real parts of τ_T and τ_R that determine the proper choice of X, Sokolovski and Baskin have shown that $X=\bar V$ is a suitable choice. Hence the first of the three questions posed above has been answered at least tentatively. It is the sensitivity of the transmission and reflection probability amplitudes to average barrier height $\bar V$, not the incident energy E, that leads to transmission and reflection times τ_T and τ_R that are consistent with τ_D.

The last two questions are intimately related. Since the real parts of τ_T^V and τ_R^V satisfy Eq. (1), one might consider dismissing the imaginary parts as meaningless and making the identification $\tau_T = \mathrm{Re}(\tau_T^V) = -\hbar\partial\phi_T/\partial\Delta V|_{\Delta V=0}$ and $\tau_R = \mathrm{Re}(\tau_R^V) = -\hbar\partial\phi_R/\partial\Delta V|_{\Delta V=0}$. This provides an answer to questions (2) and (3) at one stroke. There is, however, a major objection to the identification of τ_T with $\mathrm{Re}(\tau_T^V)$. For an opaque rectangular barrier of height V_0, $\mathrm{Re}(\tau_T^V)$ is virtually independent of barrier thickness d.[10] Hence, for a sufficiently thick barrier the mean transmission "speed" $v_T = d/\mathrm{Re}(\tau_T^V)$ is greater than the speed of light c. For the parameters $k=\kappa=1$ Å^{-1} $[\kappa^2 \equiv 2m(V_0-E)/\hbar^2]$ typical of metal electrodes this happens for $d \gtrsim 260$ Å, far beyond the practical tunneling regime [$d = O(10$ Å$)$]. However, the situation is much worse if one considers the local transmission speed

$$v_T(z) = \left[\frac{\partial \tau_T(0,z)}{\partial z} \right]^{-1} . \tag{9}$$

$\mathrm{Re}[\tau_T^V(0,z)]$ varies appreciably with z only within a decay length κ^{-1} of the $z=0$ and d interfaces, and its dependence on z is exponentially small [$\propto \exp(-2\kappa d)$] in the central portion of an opaque rectangular barrier.[32] Consequently, for $k=\kappa=1$ Å^{-1}, $v_T(z=d/2) > c$ for $d \gtrsim 6$ Å if $\mathrm{Re}[\tau_T^V(0,z)]$ is identified with $\tau_T(0,z)$.[32]

It is encouraging that two very different approaches lead to the same results for the transmission and reflection times and that these satisfy Eq. (1). The fact that these times are complex is, of course, disturbing and further work is clearly required to understand their physical meaning and significance, if any. Although the real and imaginary parts are measurable in principle,[10,15] Sokolovski and Hänggi[16] maintain that the complex nature of the Larmor-clock transmission and reflection times τ_T^V and τ_R^V is "an inherent property of quantal motion" and that "in general it is unlikely that any physical meaning can be ascribed to their real and imaginary parts separately." It should be noted that the complex nature of quantum dynamics has been known for a long time [for example, Feynman and Hibbs[34] show that the correlation function $\langle z(t)z(t')\rangle$ contains a small pure imaginary term]. Is it possible to construct from the real and imaginary parts of τ_T^V a real quantity with physically reasonable properties that can be identified with a tunneling time accessible to experiment? It has already been argued above that the most obvious choices, namely $\tau_T = \mathrm{Re}(\tau_T^V)$ and $\tau_T = |\tau_T^V|$ are not acceptable. Neither is $\tau_T = \mathrm{Im}(\tau_T^V)$, because it is not consistent with Eq. (1).

The main objective of the work presented in Sec. II is to take a step back from the above difficult question and to study the properties of the Larmor-clock transmission time $\tau_T^V(k;z_1,z_2)$ for a nontrivial situation in which it is a real quantity. The double barrier is ideal for this purpose. For a *symmetric* double barrier *at resonance* $[E = E_r^{(n)} \equiv \hbar^2 k_r^{(n)2}/2m^*$ with n the number of quasinodes of the wave function $\Psi_{k_r^{(n)}}(z)$ in the well region], the transmission probability is unity and, hence,

$$\mathrm{Im}[\tau_T^V(k_r^{(n)};z_1,z_2)] = \hbar \left. \left. \left(\frac{\partial \ln|T(E_r^{(n)},V,\Delta V)|}{\partial \Delta V} \right) \right|_{E_r^{(n)},V} \right|_{\Delta V=0} = 0$$

for all z_1 and z_2. This is, in general, not true for an unsymmetric double barrier at resonance because $|T|^2 < 1$.

The behavior of $\tau_T^V(k;z_1,z_2)$ can be quite complicated very near resonance and hence it is important to be precisely on resonance. Furthermore, there are severe numerical difficulties associated with carrying out the partial derivatives with respect to ΔV when E is very close to an extremely sharp resonance. Therefore, in order to be able to carry out all calculations analytically only the double rectangular barrier (DRB) $V(z) = V_1\Theta(z)\Theta(a-z) + V_2\Theta(z-b)\Theta(d-z)$ is considered. Since $|T(E_r^{(n)})|^2 = 1$ the Larmor-clock transmission time $\tau_T^V(k_r^{(n)};z_1,z_2)$ is equal to the dwell time $\tau_D(k_r^{(n)};z_l,z_2)$, providing a useful check.

Figure 1 shows the auxiliary barrier $\tilde{V}(z)$ employed in the calculation of the local Larmor-clock transmission time $\tau_T^V(k;0,z \le a)$ for the first barrier.

Focusing on the first barrier, it is assumed that at resonance, where all the incident particles are transmitted, there are two dominant types of important Feynman trajectories.[34] The first type of trajectory describes a particle tunneling completely through the first barrier into the well, a type which is conjectured to be very similar to the dominant type of trajectory for a particle that is transmitted through the corresponding (isolated) single rectangular barrier. The second type of trajectory describes the many excursions of the particle temporarily trapped in the well region back into the first barrier. This type is, of course, not of importance for the corresponding single barrier. It is *conjectured* that near the edge of the first barrier furthest from the well (i.e., for $0 \le z \ll a$) the contribution to $\tau_T^V(0,z)$ from the first type of trajectory dominates and is intimately related to $\tau_T^V(0,z)$ at precisely the same energy in the absence of the second barrier, provided that the direct trajectories for the SRB and the first barrier of the DRB are, in fact, similar. In that case

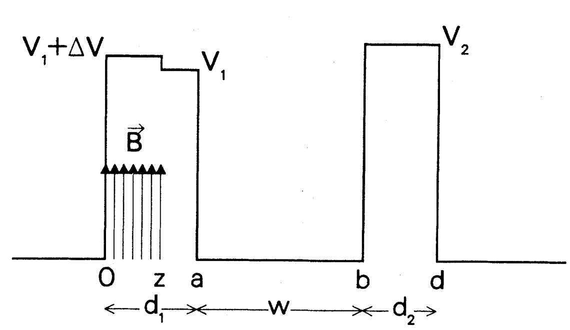

FIG. 1. Potential-energy profile used in the calculation of the local Larmor-clock transmission time $\tau_T^V(0,z)$ for the DRB $V(z) = V_1\Theta(z)\Theta(a-z) + V_2\Theta(z-b)\Theta(d-z)$. An infinitesimal transverse magnetic field $\mathbf{B} = B\hat{\mathbf{x}}$ is confined to the region $[0,z]$ indicated by arrows, raising (lowering) the effective barrier for down- (up-) spin electrons locally by $\Delta V = \hbar\omega_L/2$.

comparison of the real quantity $\tau_T^V(0, z \ll a)$ for the symmetric DRB at resonance with the real and imaginary parts of $\tau_T^V(0, z \ll a)$ for the isolated (first) barrier should give an indication of their relative importance. Such comparisons appear to support the claim that the imaginary part (i.e., Büttiker's $\tau_{xT} \cong \tau_T^{BL}$) dominates for an opaque single barrier.

For the symmetric DRB at resonance the (real) local transmission speed $v_T(z) \equiv [\partial \tau_T^V(0,z)/\partial z]^{-1}$ can exceed the speed of light c when z is very close to a local minimum (i.e., a quasinode) of $|\psi_k(z)|^2$ in the well region. Such cases are reexamined using the Dirac equation in place of the Schrödinger equation.

For completeness, Sec. III is devoted to the off-resonance behavior of the real and imaginary parts of τ_T^V as a function of incident energy, barrier widths, and well widths. It is also shown how to construct Larmor-clock transmission and reflection times for suitable wave packets. In Sec. IV the Larmor-clock result for the mean time spent by a reflected particle of energy E in the region $z_1 \le z \le z_2$ on the far side of a single rectangular barrier (SRB) is calculated and provides either a failure of the Larmor-clock approach or an interesting example of quantum nonlocality. The results are summarized briefly in Sec. V.

II. SYMMETRIC DOUBLE RECTANGULAR BARRIERS AT RESONANCE

At resonance, the transmission probability $|T|^2$ for a symmetric double barrier is unity[35] and therefore stationary with respect to any infinitesimal perturbation of the barrier, including that used in the calculation of the local Larmor-clock transmission time, namely $\Delta V(z) = \Delta V\,\Theta(z-z_1)\Theta(z_2-z)$ with $0 \le z_1 \le z_2 \le d$. Hence for this special case, $\mathrm{Im}[\tau_T^V(z_1,z_2)] = \hbar(\partial \ln|T|/\partial \Delta V)|_{\Delta V=0}$ is zero and the local Larmor-clock transmission time is a real quantity. This special case is important for two reasons: (1) the behavior of the Larmor-clock transmission time can be studied for a nontrivial situation involving tunneling without the complication of it being a complex quantity; (2) comparison of $\tau_T^V(0,z)$ for small z with the real and imaginary parts of the complex transmission time for the corresponding (isolated) single-barrier problem may actually shed light on that complication.

Analytical expressions for the local Larmor-clock transmission times for the barrier and well regions are given in the Appendix. Except for the calculations with the Dirac equation (see below), barrier parameters roughly typical of GaAs/Al_xGa_{1-x}As/GaAs heterostructures are used including, for simplicity, a z-independent effective mass $m^* = 0.067m$ where m is the free-electron mass.

Figure 2 shows the local Larmor-clock transmission times $\tau_T^V(0, 0 \le z \le a)$ and $\tau_T^V(b \le z \le d, d)$ as a function of z for the first and second barrier regions of the DRB, $V(z) = V_1\Theta(z)\Theta(a-z) + V_2\Theta(z-b)\Theta(d-z)$, with $V_1 = V_2 = 0.16$ eV, $d_1 \equiv a = d_2 \equiv d - b = 100$ Å, $w = 50$ Å, and $E = E_r^{(0)} \approx 0.067\,69$ eV ($E_r^{(0)}$ was actually determined

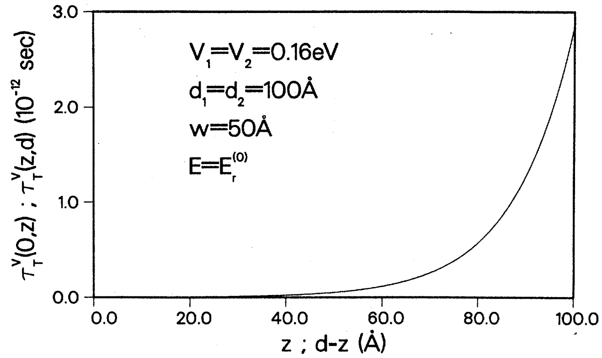

FIG. 2. Local Larmor-clock transmission times *at resonance*, $\tau_T^V(0,z) = \mathrm{Re}[\tau_T^V(0,z)]$ with $0 \le z \le a$ and $\tau_T^V(z,d) = \mathrm{Re}[\tau_T^V(z,d)]$ with $b \le z \le d$, for the first and second barriers, respectively, of the symmetric DRB with $V_1 = V_2 = 0.16$ eV, $d_1 = d_2 = 100$ Å, $w = 50$ Å, and $m^* = 0.067m$. The incident energy is $E = E_r^{(0)} \approx 0.067\,69$ eV.

to many more significant figures, but compatibility to this accuracy with the results of others obviously depends, for example, on the exact values used for such constants as $\hbar$; hence only enough figures are quoted to identify the resonance). The two times coincide for this symmetrical situation when the former is plotted as a function of z and the latter as a function of $d-z$. At resonance an incident particle spends a very long time in the well region $a \le z \le b$ and the behavior of $\tau_T^V(0,z)$ and $\tau_T^V(z,d)$ shown in the figure reflects the very many small tunneling excursions of the trapped particle into the barrier regions before its eventual escape through the second barrier.

In terms of Feynman paths,[34] one might expect important contributions to the mean time spent in the first barrier from two very different types of trajectories: (1) those in which the incident particle tunnels completely through the first barrier into the well region, and (2) those in which the particle oscillating back and forth in the well repeatedly tunnels back into the first barrier. Despite the small average penetration depth $(2\kappa)^{-1}$, for the latter type of trajectory, it completely dominates in Fig. 2 because of the very large average number of oscillations before escape. In order to see the contribution to $\tau_T^V(0,z)$ from the former type of trajectory, it is necessary to take advantage of the expected exponential falloff with $a-z$ of the latter type and to focus on that part of the first barrier furthest from the well, i.e., $0 \le z \ll a$. Intuitively, one might expect that $\tau_T^V(0, 0 \le z \ll a)$ would be closely related to the same quantity for the (isolated) single barrier, $V(z) = V_1\Theta(z)\Theta(a-z)$, at precisely the same incident energy. However, the Larmor-clock transmission time $\tau_T^V(z_1,z_2)$ for the single barrier is a complex quantity or, alternatively,[10] some combination of the two real times τ_{xT} and τ_{zT}, e.g., $(\tau_{xT}^2 + \tau_{zT}^2)^{1/2}$. The imaginary part has been the subject of much controversy: there are those who maintain that it should be discarded and those who argue that it is the dominant component[7–11] in the "deep" tunneling regime $\kappa a \gg 1$. Comparison of the real quantity $\tau_T^V(0, 0 \le z \ll a)$ for the symmetric DRB, $V(z) = V[\Theta(z)\Theta(a-z) + \Theta(z-b)\Theta(b+a-z)]$, at resonance with the real and imaginary parts of $\tau_T^V(0, 0 \leq z \ll a)$ for the SRB, $V(z)=V\Theta(z)\Theta(a-z)$, at the same incident energy offers a unique opportunity to shed some light on this controversial issue. Such a comparison is made in Fig. 3 for both the Larmor transmission time and inverse speed using the parameters of Fig. 2 [note the 3-orders-of-magnitude change of scale between Figs. 2 and 3(a)]. It is clear that as z becomes very small, the real transmission time $\tau_T^V(0,z)$ for the symmetric DRB at resonance approaches (minus) the imaginary part of $\tau_T^V(0,z)$ for the corresponding SRB much more closely than it does either the real part, i.e., τ_{zT}, or the modulus, i.e., τ_T^{BL} (not shown). It is also important to note that $\tau_T^V(0,z)$ and $v_T^{-1}(z)$ for the DRB lie above $-\mathrm{Im}[\tau_T^V(0,z)]$ and $-\mathrm{Im}[v_T^{-1}(z)]$, respectively, for the corresponding SRB. This is in keeping with the above conjecture of contributions from two very different types of trajectory for the double barrier, one of which is absent for the single barrier. For completeness, it is important to consider a situation in which $\mathrm{Re}[v_T^{-1}(z=0)]$ is considerably larger than $-\mathrm{Im}[v_T^{-1}(z=0)]$. From the analytic expression for $\tau_T^V(z_1,z_2)$ given in Ref. 32 for a SRB, it follows that $-\mathrm{Im}[v_T^{-1}(z=0)]/\mathrm{Re}[v_T^{-1}(z=0)] \approx \kappa/k$ when $\kappa a \gg 1$. Hence in order to have $\mathrm{Re}[v_T^{-1}(z=0)]$ considerably larger than $-\mathrm{Im}[v_T^{-1}(z=0)]$, we need an incident energy significantly greater than half the barrier height. Figure 4 repeats the comparison of Fig. 3 for a symmetric DRB with w increased by a factor of 2 so that its second resonance $E_r^{(1)}$ is large enough for $\mathrm{Re}[v_T^{-1}(z=0)]$ to be considerably larger than $-\mathrm{Im}[v_T^{-1}(z=0)]$ for the corresponding SRB. For very small z the real transmission time $\tau_T^V(0,z)$ for the symmetric DRB at resonance again merges with (minus) the imaginary part of $\tau_T^V(0,z)$ for the corresponding SRB rather than the real part. Moreover, although $\tau_T^V(0,z)$ and $v_T^{-1}(z)$ for the DRB are now less than $\mathrm{Re}[\tau_T^V(0,z)]$ and $\mathrm{Re}[v_T^{-1}(z)]$, respectively, for the corresponding SRB for very small z, they never fall below the SRB results for $-\mathrm{Im}[\tau_T^V(0,z)]$ and $-\mathrm{Im}[v_T^{-1}(z)]$. This is true for all the cases that we have studied.

The increasing spread between the solid and dashed curves of Figs. 3 and 4 with increasing z is due to increased sampling of those trajectories for which a particle trapped in the well makes repeated excursions of varying length, with short trips exponentially more probable than long ones, back into the first barrier. On the other hand, for energies far above the top of the SRB, $|T|^2 \approx 1$, the imaginary part of $\tau_T^V(0,z)$ is negligibly small, and there is no doubt that $\mathrm{Re}[\tau_T^V(0,z)] \approx m^*z/\hbar k$ is the dominant

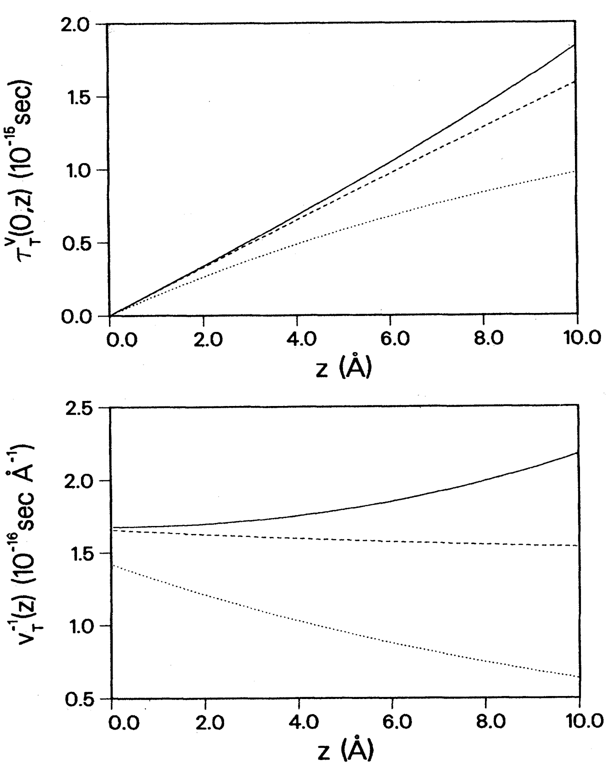

FIG. 3. Top: (a) comparison for small z of the local Larmor-clock time *at resonance*, $\tau_T^V(0,z)=\mathrm{Re}[\tau_T^V(0,z)]$, for the symmetric DRB of Fig. 2 (——) with the real (· · · ·) and (minus) the imaginary (— — —) parts of the local Larmor-clock time for the isolated SRB $V(z)=V_1\Theta(z)\Theta(a-z)$ at the same incident energy. Bottom: the same comparison for the inverse local mean transmission speed $v_T^{-1}(z) \equiv \partial\tau_T^V(0,z)/\partial z$.

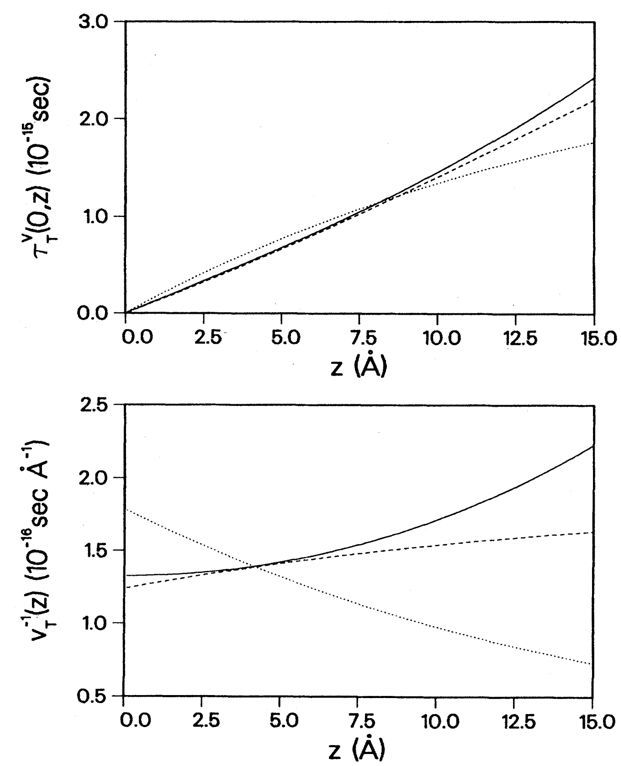

FIG. 4. Top: a comparison for small z of the local Larmor-clock time *at resonance*, $\tau_T^V(0,z)=\mathrm{Re}[\tau_T^V(0,z)]$, for the symmetric DRB having $V_1=V_2=0.16$ eV, $d_1=d_2=100$ Å, $w=100$ Å, $m^*=0.067m$, and $E=E_r^{(1)} \approx 0.107\,95$ eV (——) with the real (· · · ·) and (minus) the imaginary (— — —) parts of the local Larmor-clock time for the isolated SRB $V(z)=V_1$ $\Theta(z)\Theta(a-z)$ at the same incident energy. Bottom: the same comparison for the inverse local mean speed $v_T^{-1}(z) \equiv \partial\tau_T^V(0,z)/\partial z$.

component. Hence, as the incident energy approaches the top of the barrier from below the increasing importance of $\mathrm{Re}[\tau_T^V(0,z)]$ should become more and more apparent. There is some indication of this in Fig. 4(b), where the solid curve bends slightly away from the dashed curve in the direction of the dotted one as z approaches zero.

Figure 5 shows the local transmission time $\tau_T^V(a,z)$ and inverse speed $v_T^{-1}(z)$ for the well region $a \le z \le b$ of the DRB considered in Figs. 2 and 3. $\tau_T^V(a,z)$ increases most rapidly with z in the center of the well where the probability of finding a trapped particle is largest for the lowest resonant energy $E_r^{(0)}$. In Fig. 6 the same quantities are shown for the well region of a symmetric DRB when the incident energy coincides with $E_r^{(1)}$. In this case $\tau_T^V(a,z)$ increases most slowly with z in the center of the well, where the probability of finding a particle is smallest because of a quasinode in the wave function $\psi_k(z)$ at the center of the well. Because the tunneling current is finite, the wave function does not have an actual node anywhere and hence $v_T^{-1}(z)$ is never zero. However, in the immediate vicinity of the point at which $|\psi_k(z)|^2$ is a minimum $v_T^{-1}(z)$ is so small that $v_T(z)$ is much greater than the speed of light. Since $v_T(z)$ was calculated using the Schrödinger equation, this is not necessarily a cause for concern. Since the question of whether or not $v_T(z)$ is bounded by the speed of light c is of fundamental importance, the effective-mass approximation is not used in the following discussion. The tunneling of a free electron is considered with the barrier parameters chosen in a range appropriate to tunneling between typical metal rather than semiconducting electrodes, and assuming that the dielectric constant ϵ is unity.

For a symmetric double barrier at resonance, $|T|^2=1$ and the Larmor-clock transmission time $\tau_T^V(z_1,z_2)$ is equal to the dwell time $\tau_D(z_1,z_2)$. The latter is, in turn, equal to the average number of electrons in the region $z_1 \le z \le z_2$ divided by the particle flux of the incident beam, i.e.,

$$\tau_D(z_1,z_2)=\int_{z_1}^{z_2} dz\, \rho_k(z)/j_k^I$$

$$=\int_{z_1}^{z_2} dz\, \psi_k^*(z)\psi_k(z) \Big/ \left[\frac{\hbar k}{m}\right], \qquad (10)$$

where $\psi_k(z)$ is the scattering solution of the time-independent Schrödinger equation.[10] The relativistic generalization of Büttiker's expression, Eq. (10), is obtained by replacing the nonrelativistic (Schrödinger) ex-

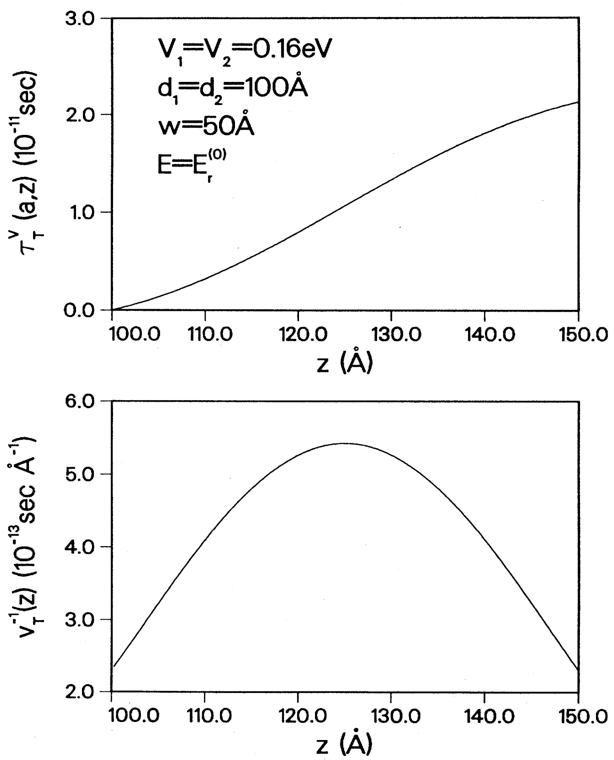

FIG. 5. Top: local Larmor-clock time *at resonance*, $\tau_T^V(a,z)=\mathrm{Re}[\tau_T^V(a,z)]$ with $a \le z \le b$, for the well region of the symmetric DRB with $V_1=V_2=0.16$ eV, $d_1=d_2=100$ Å, $w=50$ Å, and $m^*=0.067m$. The incident energy is $E=E_r^{(0)}\approx 0.067\,69$ eV. Bottom: the inverse local mean speed $v_T^{-1}(z)\equiv\partial\tau_T^V(0,z)/\partial z$ at resonance for the well region.

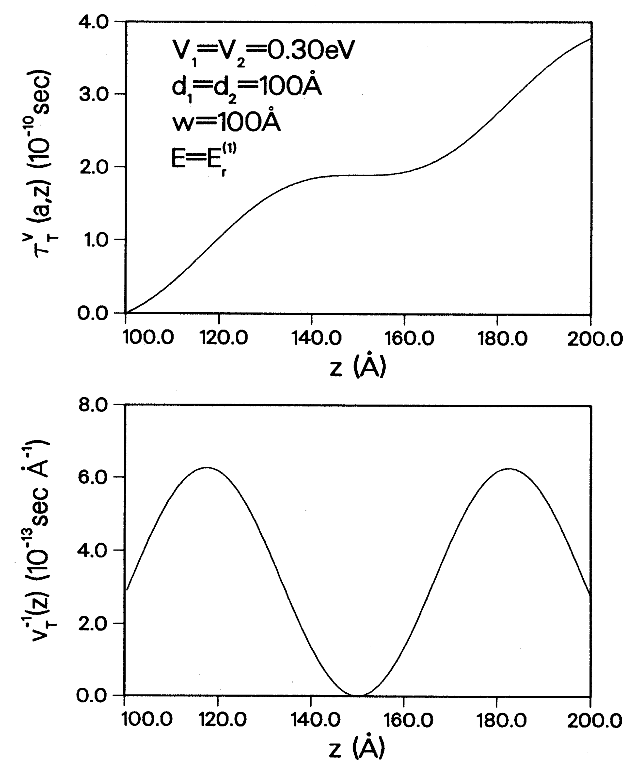

FIG. 6. Top: local Larmor-clock time *at resonance*, $\tau_T^V(a,z)=\mathrm{Re}[\tau_T^V(a,z)]$ with $a \le z \le b$, for the well region of the symmetric DRB with $V_1=V_2=0.30$ eV, $d_1=d_2=100$ Å, $w=100$ Å, and $m^*=0.067m$. The incident energy is $E=E_r^{(1)}\approx 0.132\,59$ eV. Bottom: the inverse local mean speed $v_T^{-1}(z)\equiv\partial\tau_T^V(0,z)/\partial z$ at resonance for the well region.

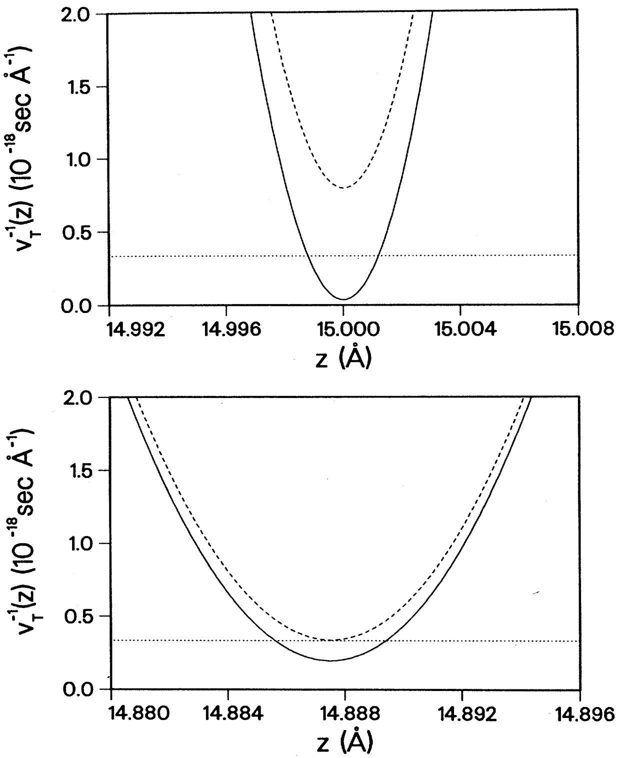

FIG. 7. Comparison of Dirac (— — —) and Schrödinger (——) calculations of $v_T^{-1}(z)$ for the symmetric DRB with z in the immediate neighborhood of a local minimum of the eigenstate density $\rho_k(z)$ in the well. The horizontal dotted line denotes c^{-1}. The barrier parameters are $V_1=V_2=10$ eV, $d_1=d_2=10$ Å, $w=10$ Å, and $E_r\approx 9.772$ eV for the top panel, and $V_1=V_2=10$ eV, $d_1=d_2=10$ Å, $w=9.775$ Å, and $E_r\approx 9.997$ eV for the bottom panel.

pressions for the probability density $\rho_k(z)$ and probability current density $j_k(z)$ with the corresponding relativistic (Dirac) expressions:[36,37]

$$\rho_k \equiv \tilde{\psi}_k^\dagger(z)\tilde{\psi}_k(z)$$
$$= \psi_k^{(1)*}(z)\psi_k^{(1)}(z)+\psi_k^{(3)*}(z)\psi_k^{(3)}(z) \ , \tag{11}$$

$$j_k(z) \equiv c\tilde{\psi}_k^\dagger(z)\alpha_z\tilde{\psi}_k(z)$$
$$= c[\psi_k^{(1)*}(z)\psi_k^{(3)}(z)+\psi_k^{(3)*}(z)\psi_k^{(1)}(z)] \ , \tag{12}$$

where

$$\tilde{\psi}_k(z)=\begin{bmatrix}\psi_k^{(1)}(z)\\ 0\\ \psi_k^{(3)}(z)\\ 0\end{bmatrix} \tag{13}$$

for (positive-energy) spin-up Dirac electrons, and

$$\alpha_z=\begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & -1\\ 1 & 0 & 0 & 0\\ 0 & -1 & 0 & 0\end{bmatrix} . \tag{14}$$

The nonzero components of the stationary-state scattering wave function $\tilde{\psi}_k(z)$ outside the barrier region $0\le z\le d$ are given by[37]

$$\psi_k^{(1)}(z)=e^{ikz}+Re^{-ikz}, \quad \psi_k^{(3)}(z)=\xi(k)(e^{ikz}-Re^{-ikz}), \quad z\le 0 \tag{15}$$

$$\psi_k^{(1)}(z)=Te^{ikz}, \quad \psi_k^{(3)}(z)=\xi(k)Te^{ikz}, \quad z\ge d \tag{16}$$

with $c\hbar k\equiv(\tilde{E}^2-m^2c^4)^{1/2}$, $\xi(k)\equiv c\hbar k/(\tilde{E}+mc^2)$, and $\tilde{E}\equiv E+mc^2$. The incident particle flux j_k^I is obtained by substituting $\psi_k^{(1)}(z)=e^{ikz}$ and $\psi_k^{(3)}(z)=\xi(k)e^{ikz}$ into Eq. (12) for $j_k(z)$. The result is $j_k^I=2c^2\hbar k/(\tilde{E}+mc^2)$. Hence the relativistic expression for the dwell time is

$$\tau_D(z_1,z_2)=\left[1+\frac{E}{2mc^2}\right]\frac{m}{\hbar k}\int_{z_1}^{z_2}dz[|\psi_k^{(1)}(z)|^2+|\psi_k^{(3)}(z)|^2] \ . \tag{17}$$

For the symmetric DRB, $V(z)=V[\Theta(z)\Theta(a-z)+\Theta(z-b)\Theta(d-z)]$ with $d=b+a$, the nonzero components of $\tilde{\psi}_k(z)$ within the barrier are given by[37]

$$\begin{aligned}
&\psi_k^{(1)}(z)=A_1e^{\kappa z}+B_1e^{-\kappa z}, \quad \psi_k^{(3)}(z)=\eta(\kappa)(A_1e^{\kappa z}-B_1e^{-\kappa z}), \quad 0\le z\le a\\
&\psi_k^{(1)}(z)=Ce^{ikz}+De^{-ikz}, \quad \psi_k^{(3)}(z)=\xi(k)(Ce^{ikz}-De^{-ikz}), \quad a\le z\le b\\
&\psi_k^{(1)}(z)=A_2e^{\kappa z}+B_2e^{-\kappa z}, \quad \psi_k^{(3)}(z)=\eta(\kappa)(A_2e^{\kappa z}-B_2e^{-\kappa z}), \quad b\le z\le d
\end{aligned} \tag{18}$$

with $c\hbar\kappa\equiv -i[(\tilde{E}-V)^2-m^2c^4]^{1/2}$ and $\eta(\kappa)\equiv -ic\hbar\kappa/(\tilde{E}-V+mc^2)$. The coefficients R, A_1, B_1, C, D, A_2, B_2, and T are obtained by requiring continuity of $\psi_k^{(1)}(z)$ and $\psi_k^{(3)}(z)$ across the interfaces at $z=0$, a, b, and d. For the symmetric DRB at resonance ($E=E_r$) the local mean speed for transmitted particles is given by

$$v_T(z)=2c\xi(k)[|\psi_{k_r}^{(1)}(z)|^2+|\psi_{k_r}^{(3)}(z)|^2]^{-1} \ . \tag{19}$$

After a straightforward evaluation of C and D, it is found that

$$|\psi_{k_r}^{(1)}(z)|^2+|\psi_{k_r}^{(3)}(z)|^2=(2\gamma^2\delta^2)^{-1}([2\gamma^2\delta^2+(\gamma^2+\delta^2)^2\sinh^2(\kappa_r a)](1+\gamma^2)$$
$$+(\gamma^2+\delta^2)\sinh(\kappa_r a)\{(\gamma^2-\delta^2)\sinh(\kappa_r a)\cos[2k_r(z-b)]$$
$$-2\gamma\delta\cosh(\kappa_r a)\sin[2k_r(z-b)\}(1-\gamma^2)),\quad a\le z\le b$$

where $\gamma\equiv(\hbar/mc)k_r/(2+E/mc^2)$ and $\delta\equiv(\hbar/mc)\kappa_r/[2+(E-V)/mc^2]$. The resonant energies used in the relativistic calculations are obtained for the symmetric DRB by solving $|T(E_r)|^2=1$. The result is

$$\cot(k_r w)=[(\gamma^2-\delta^2)/2\gamma\delta]\tanh(\kappa_r a)\ ,$$

where $w=b-a$. In the limit that the speed of light c approaches infinity, this reduces to the nonrelativistic expression of Hauge *et al.*[17]

For the barrier parameters of interest [i.e., $V_1=V_2=O(10\text{ eV})$] the relative difference between the Schrödinger and Dirac calculations of $v_T(z)$ is negligible, except in the immediate vicinity of a local minimum of $\rho_k(z)$ where, as shown in Fig. 7, it can be very large.

It is easy to prove for the symmetric double barrier $V(z)[\Theta(z)\Theta(a-z)+\Theta(z-b)\Theta(b+a-z)]$, with $V(z)$ arbitrary, that the Dirac result for $v_T(z)$ is bounded by the speed of light c. It follows from Eqs. (11) and (18) that in the well region the local extrema in $\rho_{k_r^{(n)}}(z)$ occur for $z_m^{(n)}$ given by $\exp(i4k_r^{(n)}z_m^{(n)})=C^*D/D^*C$, subject to the condition that $a\le z_m^{(n)}\le b$. At one of these extrema

$$\rho_{k_r^{(n)}}(z_m^{(n)})=(|C|\pm|D|)^2+\xi(k_r^{(n)})^2(|C|\mp|D|)^2\ ,\quad (20)$$

where the upper (lower) sign corresponds to a maximum (minimum). Hence, for a local minimum in $\rho_{k_r^{(n)}}(z)$ within the well,

$$v_T(z_m^{(n)})=\frac{2c\xi(k_r^{(n)})}{1+2|D|^2-2|D|(1+|D|^2)^{1/2}+\xi^2(k_r^{(n)})[1+2|D|^2+2|D|(1+|D|^2)^{1/2}]}\ ,$$

where $|C|^2-|D|^2=1$ has been used [this follows from the requirement of current conservation, using (12), (16), (18), and $|T|^2=1$]. Substituting $v_T(z_m^{(n)})=c/(1+\epsilon)$ and solving the resulting quadratic equation for $\xi(k_r^{(n)})$, one obtains

$$\xi(k_r^{(n)})=\frac{1+\epsilon\pm(2\epsilon+\epsilon^2)^{1/2}}{1+2|D|^2+2|D|(1+|D|^2)^{1/2}}\ .$$

Since $\xi(k)\equiv c\hbar k/(2mc^2+E)$ is real, the quantity $2\epsilon+\epsilon^2$ must be non-negative. There are two possibilities (1) $\epsilon\ge 0$, in which case $v_T(z_m^{(n)})\le c$; (2) $\epsilon<-2$, in which case $v_T(z_m^{(n)})\le -c$, which must be rejected because $v_T(z)$ is a speed. Hence the local Larmor-clock transmission speeds in excess of c obtained with the Schrödinger equation for the symmetric double barrier at resonance are eliminated when the calculation is carried out with the Dirac equation. The Schrödinger results for the local minima and maxima in $\rho_{k_r^{(n)}}$ are $(|C|-|D|)^2_{c=\infty}$ and $(|C|+|D|)^2_{c=\infty}$ respectively. When $(|C|-|D|)^2_{c=\infty}$ is very small, $(|C|+|D|)^2_{c=\infty}$ is very large in comparison. The primary role of $\xi(k_r^{(n)})^2$ in Eq. (20) is to mix in the large quantity $(|C|+|D|)^2$. This explains why the Schrödinger and Dirac results for $v_T(z_m^{(n)})$ can differ by many orders of magnitude when $(|C|-|D|)^2\approx(|C|-|D|)^2_{c=\infty}$ is extremely small, even though $\xi(k_r^{(n)})^2\simeq E/2mc^2$ is itself very small.

It is important to note that using the Dirac equation to calculate τ_T^V for the SRB does not change the conclusion that $d/\mathrm{Re}(\tau_T^V)>c$ for sufficiently large d. The relativistic results for the transmission and reflection probability amplitudes of the SRB $V(z)=V_0\Theta(z)\Theta(d-z)$ are

$$T=2\gamma\delta\frac{2\gamma\delta\cosh(\kappa d)+i(\gamma^2-\delta^2)\sinh(\kappa d)}{4\gamma^2\delta^2+(\gamma^2+\delta^2)^2\sinh^2(\kappa d)}\exp(-ikd)\ ,$$

$$R=-i(\gamma^2+\delta^2)\sinh(\kappa d)T\exp(ikd)/2\gamma\delta\ ,$$

with $\gamma=c\hbar k/(2mc^2+E)$ and $\delta=c\hbar\kappa/(2mc^2+E-V_0)$. It follows readily from Eq. (4) for $\tau_T^V(k;z_1,z_2)$ and the corresponding result for $\tau_R^V(k;z_1,z_2)$ that $\tau_T^V(k;0,d)=i\hbar\partial\ln T/\partial V_0$ and $\tau_R^V(k;0,d)=i\hbar\partial\ln R/\partial V_0$ for the special case of a SRB. Using the above expressions for T and R, it is not difficult to show that

$$\tau_D(k;0,d)=\mathrm{Re}[\tau_T^V(k;0,d)]$$
$$=\mathrm{Re}[\tau_R^V(k;0,d)]$$
$$=\left[1+\frac{E}{2mc^2}\right]\gamma^2(m/\hbar k)\frac{\kappa^{-1}(1+\delta^2)(\gamma^2+\delta^2)\sinh(2\kappa d)-2(1-\delta^2)(\gamma^2-\delta^2)d}{4\gamma^2\delta^2+(\gamma^2+\delta^2)^2\sinh^2(\kappa d)}\ .$$

For an opaque SRB ($\kappa d \gg 1$) it follows that $\mathrm{Re}[\tau_T^V(k;0,d)]$ is virtually independent of d and, in particular, that $d/\mathrm{Re}[\tau_T^V(k;0,d)] > c$ for barrier widths such that

$$\kappa d > (1+E/2mc^2)(2mc/\hbar k)\gamma^2(1+\delta^2)/(\gamma^2+\delta^2) .$$

III. COMPLEX LARMOR-CLOCK TRANSMISSION TIMES FOR DOUBLE-RECTANGULAR BARRIERS

The investigation of local Larmor-clock transmission times $\tau_T^V(z_1,z_2)$ for double barriers in Sec. II was restricted to symmetric barriers at resonance. If either of these two restrictions is removed, $\tau_T^V(z_1,z_2)$ can develop an imaginary part, the physical meaning and significance of which are controversial. The reader who cannot accept complex transmission times may prefer to equate the real part of τ_T^V with the time τ_{zT} associated with Larmor precession of the electron spin in the plane perpendicular to the magnetic field[5,6] and (minus) the imaginary part with the analogous time τ_{xT} introduced by Büttiker[10] to describe the spin polarization of the transmitted beam in the magnetic field direction.

Figure 8 shows the real and imaginary parts of $\tau_T^V(0,z)$ as a function of z with $0 \le z \le d$ for the symmetric DRB of Figs. 2, 3, and 5 for two nonresonant energies $E = E_r^{(0)}/2$ and $E = 3E_r^{(0)}/2$. For the off-resonance energies shown, most of the variation of $\mathrm{Re}[\tau_T^V(0,z)]$ with z occurs within a decay length κ^{-1} of the outside barrier edges at $z=0$ and $z=d$. ($\kappa^{-1} \cong 21$ Å for $E = E_r^{(0)}/2$ and $\kappa^{-1} \cong 31$ Å for $E = 3E_r^{(0)}/2$.) This is just the behavior found in Ref. 32 for the (isolated) single barrier. For both cases, there is significant Larmor precession only within a

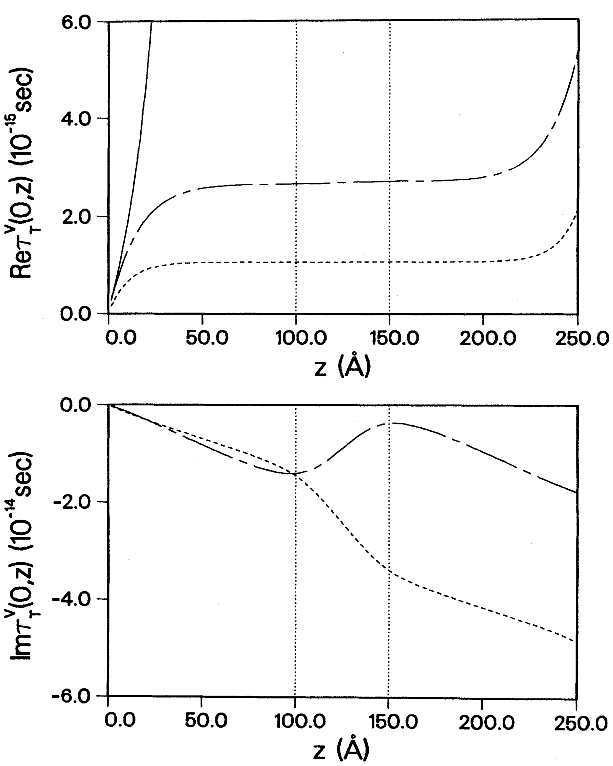

FIG. 8. The real and imaginary parts of the local Larmor-clock time $\tau_T^V(0,z)$ as a function of z with $0 \le z \le d$ for the symmetric DRB having $V_1 = V_2 = 0.16$ eV, $d_1 = d_2 = 100$ Å, $w = 50$ Å, and $m^* = 0.067m$, and for three incident electron energies: $E = E_r^{(0)} \approx 0.067\,69$ eV (——); $E = E_r^{(0)}/2$ (— — —); $E = 3E_r^{(0)}/2$ (—·—·—·). The vertical dotted lines indicate the well region.

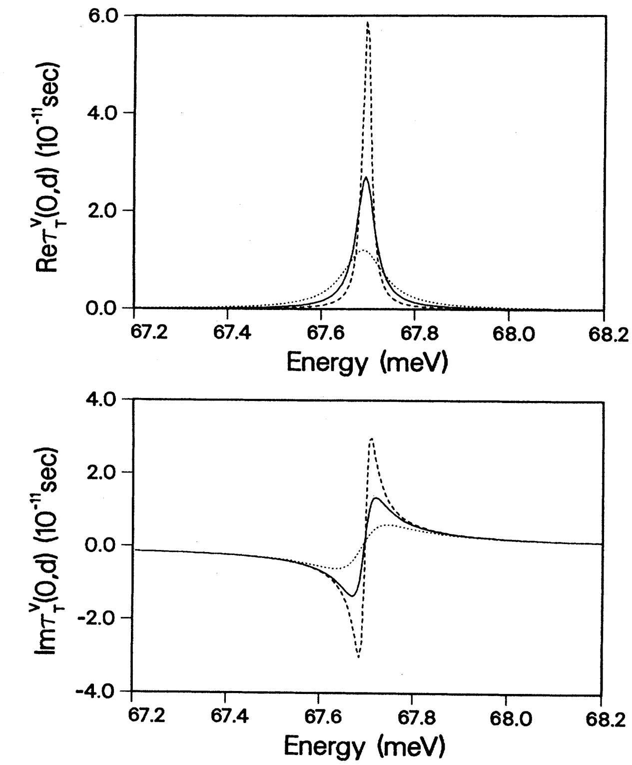

FIG. 9. The real and imaginary parts of the Larmor-clock time $\tau_T^V(0,d)$ for the entire width d of a symmetric DRB with $V_1 = V_2 = 0.16$ eV, $w = 50$ Å, and $m^* = 0.067m$ as a function of incident energy E near resonance for $d_1 = d_2 = 90$ Å (· · · ·), 100 Å (——), and 110 Å (— — —). At resonance, $\mathrm{Im}[\tau_T^V(0,d)] = 0$.

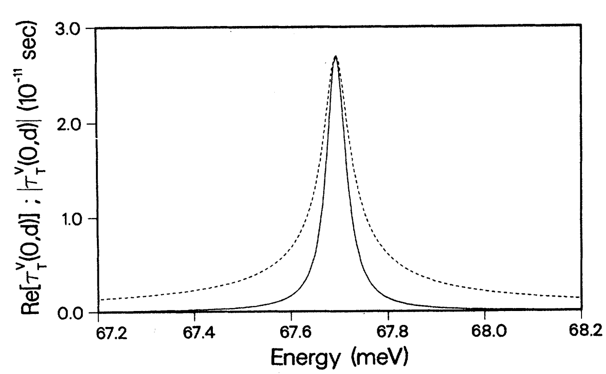

FIG. 10. The near-resonance behavior of $\mathrm{Re}[\tau_T^V(0,d)]$ (——) and $|\tau_T^V(0,d)|$ (— — —) for the symmetric DRB with $V_1 = V_2 = 0.16$ eV, $d_1 = d_2 = 100$ Å, and $w = 50$ Å.

decay length of the outside edges of the barrier. It is also clear from the figure that $|\tau_T^V(0,z)|$ is not a monotonically increasing function of z for $E=3E_r^{(0)}/2$. Such behavior is also possible for the SRB, but only for transmission above the barrier.[32]

The near-resonance ($E \approx E_r^{(0)}$) behavior of the real and imaginary parts of $\tau_T^V(0,d)$ is shown in Fig. 9 as a function of incident energy E for a symmetric DRB with $V_1=V_2=0.16$ eV, $w=50$ Å, and three values (90, 100, and 110 Å) of the common barrier width $d_1=d_2$. The very small shift to lower energy in $E_r^{(0)}$ should be noted for the thinnest barrier. Although the imaginary part of $\tau_T^V(0,d)$ is zero for $E=E_r^{(0)}$, it becomes comparable in magnitude to the real part very quickly as E moves away from $E_r^{(0)}$ in either direction. This is illustrated in Fig. 10, where $\mathrm{Re}[\tau_T^V(0,d)]$ is compared with $|\tau_T^V(0,d)|$ for $d_1=d_2=100$ Å. The full width at half maximum of the resonance peak in $|\tau_T^V(0,d)|$ is almost double that for $\mathrm{Re}[\tau_T^V(0,d)]$.

The dependence of the real and imaginary parts of $\tau_T^V(0,d)$ on energy for the entire tunneling regime $0 \le E \le V_1 = V_2$ is shown in Fig. 11 for three symmetric DRB's with $d_1=d_2=50$, 100, and 150 Å. Note the change in the vertical scale by a factor of 10^{-3} from Fig. 9, so that the behavior far from resonance is visible. For E sufficiently small that the effect of the resonance is negligible, $\tau_T^V(0,d)$ for the DRB has qualitative features in common with the same quantity for the SRB:[10] (1) $\mathrm{Re}[\tau_T^V(0,d)] \propto E^{1/2}$ and $\mathrm{Im}[\tau_T^V(0,d)] \propto E^0$ in the limit that E approaches zero; (2) for opaque barriers $\mathrm{Re}[\tau_T^V(0,d)]$ is almost independent of barrier thickness.

The dependence of the low-energy off-resonance behavior of the real and imaginary parts of $\tau_T^V(0,d)$ on the well width w is shown in Fig. 12 for a symmetric DRB with $d_1=d_2=100$ Å and $V_1=V_2=0.16$ eV. The resonance energy $E_r^{(0)}$ is very sensitive to w, and only the curve for $w=150$ Å has a resonance peak below 0.02 eV, the maximum value of E considered in the figure. It is important to note that, despite the large variation in $d=d_1+d_2+w$ the curves for $\mathrm{Re}[\tau_T^V(0,d)]$ show negligible dependence on the well width w in the energy range well below the lowest resonance energy. This is a reflection of the fact that, in the low-energy off-resonance regime of interest here, the real part of the Larmor time $\tau_T^V(a,b)$ for the entire well region is completely negligible compared to the free-particle time, $\tau_T^{\mathrm{free}}(a,b) \equiv m(b-a)/\hbar k \equiv mw/\hbar k$, to

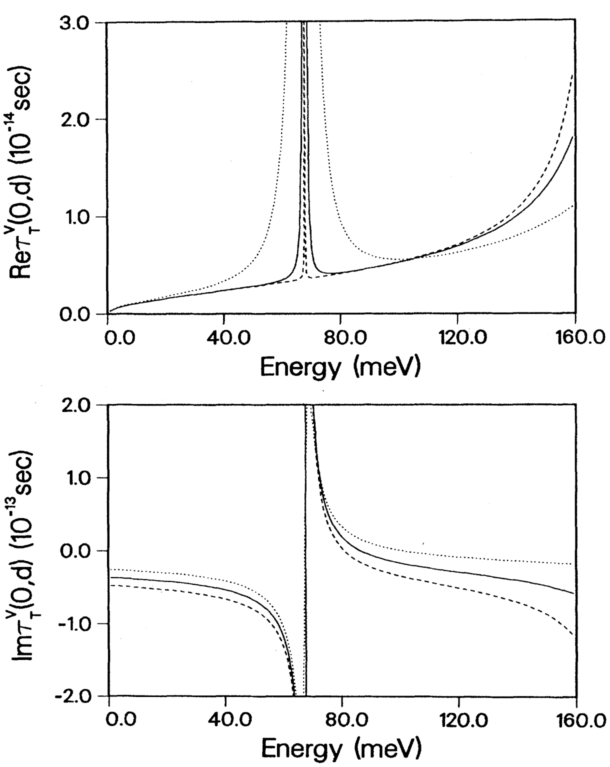

FIG. 11. The real and imaginary parts of the Larmor-clock time $\tau_T^V(0,d)$ for the entire width d of a symmetric DRB with $V_1=V_2=0.16$ eV, $w=50$ Å, and $m^*=0.067m$ as a function of incident energy E over the entire tunneling regime $0 \le E \le V_1 = V_2$ for $d_1=d_2=50$ Å (· · · ·), 100 Å (——), and 150 Å (— — —). Note the difference in scale between the top panel of this figure (10^{-14} s) and that of Fig. 9 (10^{-11} s).

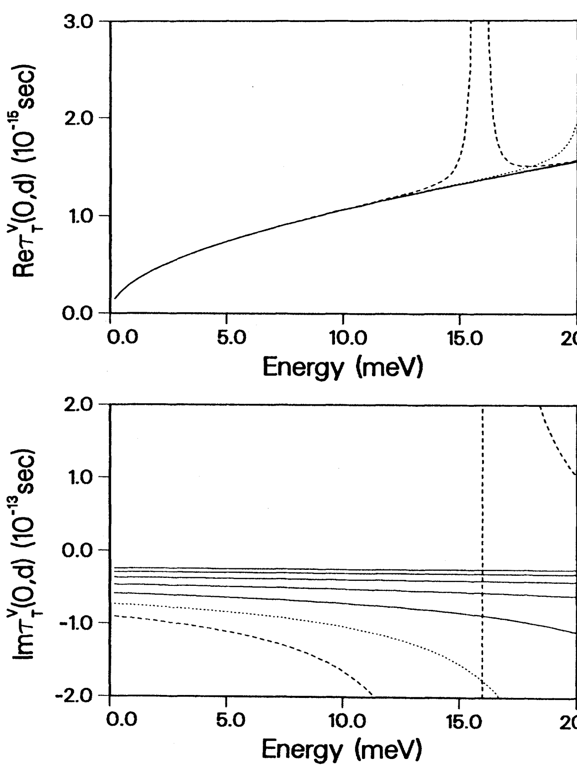

FIG. 12. The real and imaginary parts of the Larmor-clock time $\tau_T^V(0,d)$ for the entire width d of a symmetric DRB with $V_1=V_2=0.16$ eV, $d_1=d_2=100$ Å, and $m^*=0.067m$ as a function of incident energy E over the low-energy range $0 \le E \le 20$ meV for $w=0$, 25, 50, 75, 100 Å (——), 125 Å (· · · ·), and 150 Å (— — —). In the top panel the results for $w=0$–100 Å coincide to within plotting accuracy, while in the bottom panel they are ordered (for $E \le 15$ meV) in terms of increasing w from top to bottom.

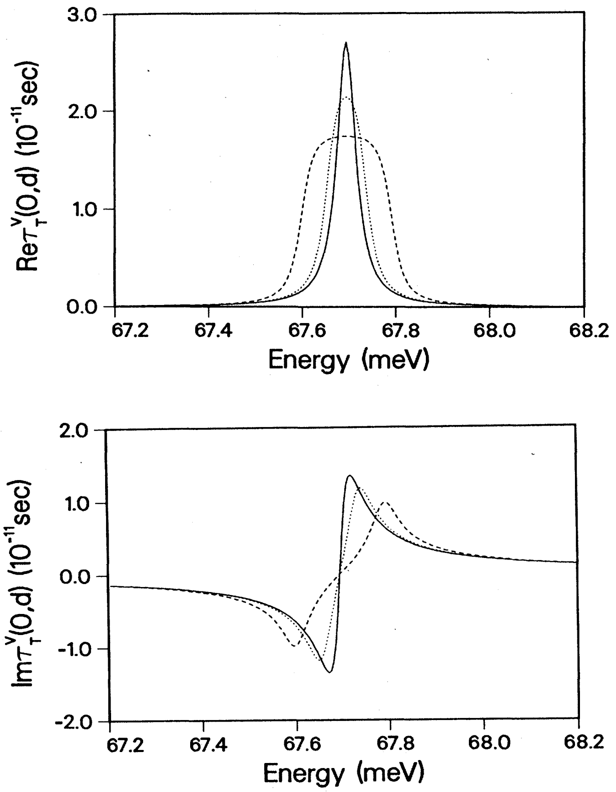

FIG. 13. The real and imaginary parts of the Larmor-clock wave-packet transmission time $\tau_T^V(0,d)$ as a function of $E_0 \equiv \hbar^2 k_0^2/2m^*$, with E_0 near $E_r^{(0)}$, for wave packets described by $|\phi(k)|^2 = (3\pi/2\,\Delta k)[1-(k-k_0)^2/(\Delta k)^2]\Theta(k-k_0+\Delta k)\Theta(k_0+\Delta k-k)$ incident on the symmetric DRB of Figs. 2, 3, and 5 (i.e., $V_1 = V_2 = 0.16$ eV, $d_1 = d_2 = 100$ Å, and $w = 50$ Å). The half-widths for the three curves are $\Delta k = 0.0$ Å^{-1} (——), 1.0×10^5 Å^{-1} (· · · ·), and 2.5×10^{-5} Å^{-1} (— — —).

cross the well. On the other hand, over the same energy range, $\mathrm{Im}[\tau_T^V(0,d)]$ has a much stronger dependence on well width and $-\mathrm{Im}[\tau_T^V(a,b)]$ has a much more reasonable magnitude in comparison with $\tau_T^{\mathrm{free}}(a,b)$. This is additional evidence for the importance of the imaginary part of the Larmor transmission time for off-resonance energies.

It is possible to construct complex Larmor-clock transmission (and reflection) times for suitable wave packets incident on arbitrary barriers. These are needed in the next section. From the work of Hauge, Falck, and Fjeldly[17] it follows that the mean time $\bar{t}_D(0,d)$ spent in the barrier region $0 \le z \le d$ by an electron with time-dependent wave function $\psi(z,t)$ is given by[33]

$$\bar{t}_D(0,d) \equiv \int_{-\infty}^{\infty} dt \int_0^d dz\, |\psi(z,t)|^2 = \int_0^{\infty} \frac{dk}{2\pi} |\phi(k)|^2 \tau_D(k;0,d) \;, \tag{21}$$

provided the Fourier transform

$$\phi(k) \equiv \int_{-\infty}^{\infty} dz\, \psi(z, t=0) e^{-ikz} \quad \left[\int_{-\infty}^{+\infty} \frac{dk}{2\pi} |\phi(k)|^2 = 1 \right] \tag{22}$$

of $\psi(z,t=0)$ is negligibly small for negative values of k. For the moment, the k dependence of the dwell time is shown explicitly. Substitution of Eq. (1) into (21) gives

$$\begin{aligned} \bar{t}_D(0,d) &= \int_0^{\infty} \frac{dk}{2\pi} |\phi(k)|^2 |T(k)|^2 \tau_T^V(k;0,d) \\ &\quad + \int_0^{\infty} \frac{dk}{2\pi} |\phi(k)|^2 |R(k)|^2 \tau_R^V(k;0,d) \\ &\equiv \bar{t}_T(0,d) + \bar{t}_R(0,d) \;. \end{aligned} \tag{23}$$

Hence the real time $\bar{t}_D(0,d)$ splits quite naturally into the sum of two terms, one describing transmission and the other reflection, with no interference term involving both transmission and reflection. Although each of the two terms is, in general, complex their sum is real.

Figure 13 shows the real and imaginary parts of the Larmor-clock wave-packet transmission time (normalized by the transmission probability for the packet)

$$\tau_T^V(0,d) \equiv \bar{t}_T(0,d) \Big/ \int_0^{\infty} \frac{dk}{2\pi} |\phi(k)|^2 |T(k)|^2$$

as a function of $E_0 \equiv \hbar^2 k_0^2/2m^*$, with E_0 near $E_r^{(0)}$, for wave packets described by

$$|\phi(k)|^2 = (3\pi/2\,\Delta k)[1-(k-k_0)^2/(\Delta k)^2] \times \Theta(k-k_0+\Delta k)\Theta(k_0+\Delta k-k)$$

incident on the symmetric DRB of Figs. 2, 3, and 5. Clearly, well below the resonance the imaginary component of $\tau_T^V(0,d)$ dominates just as it did in the plane-wave case.

IV. A BREAKDOWN OF THE LARMOR-CLOCK APPROACH OR AN EXAMPLE OF QUANTUM NONLOCALITY?

In this section the Larmor clock is used to calculate the mean time spent by a transmitted (reflected) particle of energy E in a region $z_1 \le z \le z_2$ on the far side of an isolated SRB, $V(z) = V_0\Theta(z)\Theta(a-x)$. This can be calculated either directly or from the expression for the local Larmor-clock transmission (reflection) time for the DRB $V(z) = V_1\Theta(z)\Theta(a-z) + V_2\Theta(z-b)\Theta(d-z)$ by setting $V_1 = V_0$, $b = z_1$, and $d = z_2$, and then letting V_2 go to zero. Intuitively, one might expect that $\tau_T^V(z_1,z_2)$ is just the free-particle transmission time $m^*(z_2-z_1)/\hbar k$ and that $\tau_R^V(z_1,z_2)$ is zero. These guesses are obviously consistent with the dwell time $\tau_D(z_1,z_2) = |T|^2 m^*(z_2-z_1)/\hbar k$. The calculated results are, in general, complex:

$$\mathrm{Re}[\tau_T^V(z_1,z_2)]=\frac{m^*(z_2-z_1)}{\hbar k}+\frac{m^*(k^2+\kappa^2)\sinh(\kappa a)}{2\hbar k^2 D(k,\kappa)}[(k^2-\kappa^2)s(z_1,z_2)\sinh(\kappa a)-2k\kappa c(z_1,z_2)\cosh(\kappa a)]\;, \tag{24}$$

$$\mathrm{Im}[\tau_T^V(z_1,z_2)]=-\frac{m^*(k^2+\kappa^2)\sinh(\kappa a)}{2\hbar k^2 D(k,\kappa)}[(k^2-\kappa^2)c(z_1,z_2)\sinh(\kappa a)+2k\kappa s(z_1,z_2)\cosh(\kappa a)]\;, \tag{25}$$

$$\mathrm{Re}[\tau_R^V(z_1,z_2)]=\frac{2m^*\kappa^2}{\hbar}\frac{2k\kappa c(z_1,z_2)\cosh(\kappa a)-(k^2-\kappa^2)s(z_1,z_2)\sinh(\kappa a)}{(k^2+\kappa^2)D(k,\kappa)\sinh(\kappa a)}\;, \tag{26}$$

$$\mathrm{Im}[\tau_R^V(z_1,z_2)]=\frac{2m^*\kappa^2}{\hbar}\frac{2k\kappa s(z_1,z_2)\cosh(\kappa a)+(k^2-\kappa^2)c(z_1,z_2)\sinh(\kappa a)}{(k^2+\kappa^2)D(k,\kappa)\sinh(\kappa a)}\;,\quad a\le z_1\le z_2 \tag{27}$$

where

$$s(z_1,z_2)=\sin[2k(z_2-a)]-\sin[2k(z_1-a)]\;,$$
$$c(z_1,z_2)=\cos[2k(z_2-a)]-\cos[2k(z_1-a)]\;, \tag{28}$$

$$D(k,\kappa)\equiv(k^2+\kappa^2)^2\sinh^2(\kappa a)+4k^2\kappa^2\;. \tag{29}$$

The transmission time $\tau_T^V(z_1,z_2)$ is just the anticipated free-particle time plus a complex oscillatory term; the complex oscillatory reflection time $\tau_R^V(z_1,z_2)$ is exponentially small, proportional to $e^{-2\kappa a}$ for $\kappa a\gg 1$. If the barrier height V_0 is set equal to zero (this, of course, necessitates replacing κ by ik), the free-particle transmission time is recovered.

It should be noted that substitution of Eqs. (24)–(27) and the well-known expressions for $|T|^2$ and $|R|^2$ for the rectangular barrier into $|T|^2\tau_T^V(z_1,z_2)+|R|^2\tau_R^V(z_1,z_2)$ gives the dwell time $\tau_D(z_1,z_2)=|T|^2m^*(z_2-z_1)/\hbar k$, as required by Eq. (1).

There is an interesting symmetry in $\tau_T^V(z_1,z_2)$ for the isolated rectangular barrier: the mean time spent by a transmitted particle of energy E in the region $z_1\le z\le z_2\le 0$ in front of the barrier is exactly equal to the mean time spent in the symmetrically located region $a-z_2\le z\le a-z_1$ behind the barrier, i.e., $\tau_T^V(z_1,z_2)=\tau_T^V(a-z_2,a-z_1)$. There is, of course, no such symmetry for either the reflection time $\tau_R^V(z_1,z_2)$ or the dwell time $\tau_D^V(z_1,z_2)$.

Because of the stationary-state nature of the calculation (with the particle's energy and wave vector exactly specified) there is no overall attenuation of the oscillatory real and imaginary parts of $\tau_T^V(z_1,z_2)$ and $\tau_R^V(z_1,z_2)$ as z_1 and z_2, with z_2-z_1 fixed, move further and further to the right of $z=a$. However, for a wave packet of finite extent, convolution with $|\phi(k)|^2|T(k)|^2$ and $|\phi(k)|^2\,|R(k)|^2$, respectively, and integration over k leads to such attenuation because of increasing interference between oscillatory contributions of different wave vector. This is illustrated in Fig. 14, where the real parts of the (normalized) wave-packet transmission and reflection times $\tau_T^V(z_1,z_2)$ and $\tau_R^V(z_1,z_2)$ with $z_2-z_1=1$ Å are shown as a function of z_1 for

$$|\phi(k)|^2=(3\pi/2\Delta k)[1-(k-k_0)^2/(\Delta k)^2]$$
$$\times\Theta(k-k_0+\Delta k)\Theta(k_0+\Delta k-k)$$

with $\Delta k=0$ and 0.01 Å^{-1}.

The question of whether or not a reflected particle can spend any time in the region $z\ge a$ on the far side of the barrier $V(z)\Theta(z)\Theta(a-z)$ is clearly an important one, because a proof that it cannot would have serious consequences for the Larmor-clock approach. In fact, Hauge and Stövneng[38] have independently obtained Eqs.

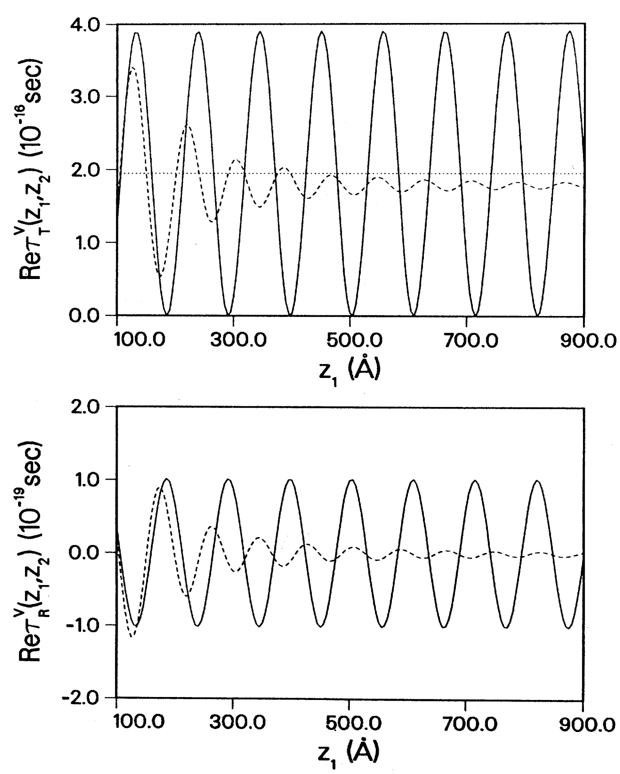

FIG. 14. Larmor-clock wave-packet transmission and reflection times $\tau_T^V(z_1,z_2)$ and $\tau_R^V(z_1,z_2)$ for a region $z_1\le z\le z_2$, with $z_2-z_1=1$ Å, on the far side of an isolated SRB, $V(z)=V\Theta(z)\Theta(a-z)$, with $V=0.16$ eV and $a=100$ Å. $E_0\equiv\hbar^2k_0^2/2m^*=50$ meV. The wave packet is described by $|\phi(k)|^2=(3\pi/2\Delta k)[1-(k-k_0)^2/(\Delta k)^2]\Theta(k-k_0+\Delta k)\Theta(k_0+\Delta k-k)$ with $\Delta k=0.0$ Å^{-1} (——) and 0.01 Å^{-1} (– – –). The horizontal dotted line denotes $\tau_D(k_0;\,z_1,z_2)/|T(k_0)|^2=m^*(z_2-z_1)/\hbar k_0$ with $z_2-z_1=1$ Å.

(24)–(29) and concluded that they provide clear evidence of the unreliability of the Larmor-clock approach. They reject the possibility that the common sense answer that a reflected particle spends exactly zero time on the far side of a barrier might in fact be incorrect. A study of Wigner trajectories[39] for an electron of energy E incident on a rectangular barrier might provide a conclusive answer.

V. SUMMARY

In this paper the Larmor-clock approach has been applied to the calculation of local transmission times for symmetric double rectangular barriers. The on-resonance results provide support for Büttiker's extension[10] of the original Larmor-clock analyses of Baz'[5] and Rybachenko[6] to include the differential effect of the Zeeman energy on the transmission probabilities for up- and down-spin electrons. It has also been shown that local transmission speeds $v_T(z)$ calculated with the Schrödinger equation can be very much in excess of the speed of light c for symmetric double barriers on resonance, but that $v_T(z) \le c$ when the Dirac equation is used. This satisfying result is in strong and significant contrast to that for the transmission speed calculated for the single rectangular barrier from the real part of τ_T^V. In this case, the conclusion that $v_T(z) > c$ for sufficiently opaque barriers is not altered when the Dirac equation is used in place of the Schrödinger equation. This provides strong grounds against identifying $\mathrm{Re}(\tau_T^V)$ with the transmission time τ_T. Results for the dependence of the Larmor-clock transmission time on incident energy, barrier widths, and well widths have been presented with an emphasis on the behavior off resonance. Finally, it has been shown that the Larmor-clock approach leads to a result contrary to the common sense notion that a reflected particle does not spend any time on the far side ($z > a$) of the potential barrier $V(z)\Theta(z)\Theta(a-z)$. Does this indicate a breakdown of the Larmor-clock method or does it provide a simple example of quantum nonlocality? It is hoped that this question and others raised in this paper will stimulate further activity to clarify the often controversial and confusing problem of "tunneling times."

APPENDIX: LOCAL TRANSMISSION TIMES FOR SYMMETRIC DOUBLE RECTANGULAR BARRIERS AT RESONANCE

The local Larmor-clock time for the first barrier ($0 \le z \le a$) is

$$\tau_T^V(0, 0 \le z \le a) = \frac{m^*}{\hbar k}\left[\frac{C_1(k,\kappa)(e^{2\kappa z}-1) - C_2(k,\kappa)(e^{-2\kappa z}-1)}{2\kappa} + C_3(k,\kappa)z\right],$$

where

$$\begin{aligned} C_1(k,\kappa) = \frac{e^{-2\kappa a}(k^2+\kappa^2)}{8k^2\kappa^4}\big([(k^2+\kappa^2)^2\sinh^2(\kappa a) + 2k^2\kappa^2] \\ -\sinh(\kappa a)\{[(k^2-\kappa^2)^2\sinh(\kappa a) + 4k^2\kappa^2\cosh(\kappa a)]\cos(2kw) \\ -2k\kappa(k^2-\kappa^2)[\sinh(\kappa a) - \cosh(\kappa a)]\sin(2kw)\}\big), \end{aligned}$$

$$\begin{aligned} C_2(k,\kappa) = \frac{e^{2\kappa a}(k^2+\kappa^2)}{8k^2\kappa^4}\big([(k^2+\kappa^2)^2\sinh^2(\kappa a) + 2k^2\kappa^2] \\ -\sinh(\kappa a)\{[(k^2-\kappa^2)^2\sinh(\kappa a) - 4k^2\kappa^2\cosh(\kappa a)]\cos(2kw) \\ +2k\kappa(k^2-\kappa^2)[\sinh(\kappa a) + \cosh(\kappa a)]\sin(2kw)\}\big), \end{aligned}$$

$$\begin{aligned} C_3(k,\kappa) = -\{(k^2-\kappa^2)[(k^2+\kappa^2)^2\sinh(\kappa a) + 2k^2\kappa^2] \\ -(k^2+\kappa^2)^2\sinh(\kappa a)[(k^2-\kappa^2)\sinh(\kappa a)\cos(2kw) + 2k\kappa\cosh(\kappa a)\sin(2kw)\}/16k^2\kappa^4. \end{aligned}$$

The local Larmor-clock time for the well ($a \le z \le b$) is

$$\begin{aligned} \tau_T^V(a, a \le z \le b) = \frac{m^*}{\hbar k}\Bigg[\frac{(k^2+\kappa^2)\sinh(\kappa a)}{2k}\big((k^2-\kappa^2)\sinh(\kappa a)\{\sin[2k(z-b)] - \sin[2k(a-b)]\} \\ +2k\kappa\cosh(\kappa a)\{\cos[2k(z-b)] - \cos[2k(a-b)]\}\big) \\ +[2k^2\kappa^2 + (k^2+\kappa^2)^2\sinh^2(\kappa a)](z-a)\Bigg]\Big/ 2k^2\kappa^2. \end{aligned}$$

The resonance condition is[17]

$$\cot(kw) = \left[\frac{k^2-\kappa^2}{2k\kappa}\right]\tanh(\kappa a) \ .$$

The local Larmor-clock time for the second barrier ($b \le z \le d$) of the symmetric DRB is symmetrically related to the time for the first barrier by

$$\tau_T^V(d-z, d) = \tau_T^V(0,z), \quad 0 \le z \le a \ .$$

[1] D. Bohm, *Quantum Theory* (Prentice-Hall, New York, 1951).
[2] E. P. Wigner, Phys. Rev. **98**, 145 (1955).
[3] F. T. Smith, Phys. Rev. **118**, 349 (1960).
[4] T. E. Hartman, J. Appl. Phys. **33**, 3427 (1962).
[5] A. I. Baz', Yad. Fiz. **4**, 252 (1966) [Sov. J. Nucl. Phys. **4**, 182 (1967)].
[6] V. F. Rybachenko, Yad. Fiz. **5**, 895 (1967) [Sov. J. Nucl. Phys. **5**, 635 (1967)].
[7] M. Jonson, Solid State Commun. **33**, 743 (1980).
[8] K. W. H. Stevens, J. Phys. C **16**, 3649 (1983).
[9] M. Büttiker and R. Landauer, Phys. Rev. Lett. **49**, 1739 (1982).
[10] M. Büttiker, Phys. Rev. B **27**, 6178 (1983).
[11] M. Büttiker and R. Landauer, IBM J. Res. Dev. **30**, 451 (1986).
[12] E. Pollak and W. H. Miller, Phys. Rev. Lett. **53**, 115 (1984).
[13] E. Pollak, J. Chem. Phys. **83**, 1111 (1985).
[14] P. Hedegård, Phys. Rev. B **35**, 533 (1987).
[15] D. Sokolovski and L. M. Baskin, Phys. Rev. A **36**, 4604 (1987).
[16] D. Sokolovski and P. Hänggi, Europhys. Lett. **7**, 7 (1988).
[17] E. H. Hauge, J. P. Falck, and T. A. Fjeldly, Phys. Rev. B **36**, 4203 (1987).
[18] J. P. Falck and E. H. Hauge, Phys. Rev. B **38**, 3287 (1988).
[19] N. Teranishi, A. M. Kriman, and D. K. Ferry, Superlatt. Microstruct. **3**, 509 (1987).
[20] Z. Kotler and A. Nitzan, J. Chem. Phys. **88**, 3871 (1988).
[21] W. Jaworski and D. M. Wardlaw, Phys. Rev. A **37**, 2843 (1988).
[22] Z. Huang, P. H. Cutler, T. E. Feuchtwang, R. H. Good, Jr., E. Kazes, H. Q. Nguyen, and S. K. Park (unpublished).
[23] A. P. Jauho and M. Nieto, Superlatt. Microstruct. **2**, 401 (1986).
[24] A. Tagliacozzo, Nuovo Cimento **10D**, 363 (1988).
[25] S. Collins, D. Lowe, and J. R. Barker, J. Phys. C **20**, 6213 (1987); **20**, 6233 (1987).
[26] A. B. Nasser, Phys. Rev. A **38**, 683 (1988).
[27] W. R. Frensley, Phys. Rev. B **36**, 1570 (1987).
[28] W. Jaworski and D. M. Wardlaw, Phys. Rev. A **38**, 5404 (1988).
[29] A. P. Jauho and M. Jonson (unpublished).
[30] J. A. Støvneng and E. H. Hauge (unpublished).
[31] C. R. Leavens and G. C. Aers, Solid State Commun. **63**, 1101 (1987).
[32] C. R. Leavens and G. C. Aers, Solid State Commun. **67**, 1135 (1988).
[33] C. R. Leavens and G. C. Aers, Phys. Rev. B **39**, 1202 (1989).
[34] R. P. Feynman and A. R. Hibbs, *Quantum Mechanics and Path Integrals* (McGraw-Hill, New York, 1965).
[35] B. Ricco and M. Ya. Azbel, Phys. Rev. B **29**, 1970 (1984).
[36] L. I. Schiff, *Quantum Mechanics* (McGraw-Hill, New York, 1968), Chap. 13.
[37] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964), Chap. 3.
[38] E. H. Hauge and J. A. Stövneng (unpublished).
[39] H.-W. Lee and M. O. Scully, Found. Phys. **13**, 61 (1983).