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Dynamic critical behavior of random spin chains in a field

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The dynamic critical behavior of one-dimensional random spin systems in a field at zero temperature is investigated by a generalized transfer-matrix scaling technique. Three spin models are considered: Heisenberg in a longitudinal field, XY in a transverse field, and Ising in a transverse field, with either spin-glass or random-field disorder. Near the transition, i.e., for small values of the reduced field $\Delta=(H-H_c)$, where H_c is the critical field, one finds that the dispersion relation for the low-frequency ω, small-wave-vector k spin-wave excitations takes the scaling form $\omega=k^z f(\Delta/k^{\varphi})$, and exact results are given for the dynamic exponent z and crossover exponent φ. This form contains a crossover of the frequency ω between two different asymptotic behaviors: k^z for $\Delta \ll k^{\varphi}$ and h^{δ} for $\Delta \gg k^{\varphi}$, where the field exponent is $\delta=z/\varphi$. In the case of the Heisenberg systems the spin-glass disorder gives rise to the nontrivial dynamic exponent $z=\frac{3}{2}$, whereas in the random-field case it is the exponent associated to the field that becomes nontrivial, taking the value $\frac{4}{3}$. For the transverse XY systems the random-field disorder implies nontrivial values for both the dynamic and field exponents, $\frac{3}{2}$ and $\frac{3}{4}$, respectively, whereas the spin-glass disorder does not affect the dynamics of the system, which behaves like a pure transverse XY ferromagnet. Finally, for the transverse Ising systems neither the random field nor the spin-glass disorder affects the dynamics, which is the same as for a pure transverse Ising ferromagnet.

I. INTRODUCTION

In this paper we study the dynamic critical behavior of various random spin chains in a magnetic field at zero temperature, by a real-space renormalization-group technique. Near a critical point the long-wavelength dynamics of the spin-fluctuation modes becomes anomalous due to the divergence of a characteristic length, which for the cases considered here is associated with the field inducing the transition. Length-scaling methods are then required to handle the critical effects on the dynamics. Real-space methods are, in particular, well suited to treat strong local disorder, and standard techniques have been applied to the dynamics of random media (for a review see Stinchcombe[1]). The application of such techniques, however, typically involves approximations because of simplifications that are necessary to deal with the randomness, even in one dimension. Recently, a new, exact, transfer-matrix scaling technique was introduced by the authors in the study of the spin-wave dynamics of a one-dimensional spin glass.[2] This technique is here generalized to treat spin dynamics in a field, providing an exact investigation of a class of random systems exhibiting rich critical phenomena.

In particular, we shall consider the following systems in one dimension: (i) Heisenberg ferromagnet in a random field, (ii) Heisenberg spin glass in a uniform field, (iii) XY and (iv) Ising ferromagnets in transverse random fields, and (v) transverse XY and (vi) transverse Ising spin glasses. Random fields are most commonly generated by impurities in the materials.[3,4] Another realization of random fields occurs in randomly diluted uniaxial antiferromagnets in an external uniform field.[5] Spin glasses are traditionally magnetic systems involving competing ferro-antiferromagnetic exchange couplings, arising from spatial or substitutional disorder.[6,7] Spin-glass-like behavior is, however, also observed in nonmagnetic systems such as mixed ferro-antiferroelectrics which are described as a spin glass in a transverse field.[8]

For the systems (i) and (ii), which both involve Heisenberg spins in a magnetic field h, the Hamiltonian is

$$H=-\sum_{i=1}^{N}\mathbf{h}_i\cdot\mathbf{S}_i-\sum_{i=1}^{N-1}J_{i,i+1}\mathbf{S}_i\cdot\mathbf{S}_{i+1}\ , \tag{1}$$

where in the case of (i) the field is a random variable with a "plus-minus" distribution, $h_i=\pm h$, whereas in the case of (ii) the randomness is in the exchange couplings, which are random variables with a "plus-minus" distribution, $J_{ij}=\pm J$. It should be noted that since (ii) is a one-dimensional spin glass with nearest-neighbor interactions only it has no frustration.[9]

Considerable attention has been given to the study of the effects of quenched random fields, which couple linearly to the order parameter, on static critical phenomena.[3,10–13] It is clear that if the field is strong enough the spins will align along the direction of the field, destroying the order. However, the order is also destroyed for infinitesimal fields if the system reaches a lower free energy by breaking up into domains. It has been shown that random-field fluctuations imply an effective reduction of the dimensionality of the pure system, so that long-range order is lost for (integer) dimensionalities $d\le 4$ (instead of $d\le 2$) for systems with continuous symmetry (Heisenberg or XY),[3] and for $d\le 2$ (instead of $d\le 1$) for Ising-like systems.[14] As noted by Mattis,[15] for nonfrustrated spin-glass models one can define a transformation of spins, by which a spin glass with random coupling $\pm J$ in a uniform field, as in (ii), is mapped onto a uniform ferromagnet in a random field $\pm h$, as in (i). This implies that the statics of the two models are equivalent, and on this basis Aharony and Imry[16] studied the static critical properties of (nonfrustrated) spin glasses in a uniform field. The Mattis transformation does not, however, imply the equivalence of the dynamics of the two models. It will be shown that (i) and (ii) have distinct nontrivial dynamic critical behavior.

The Hamiltonian for the systems (iii)–(vi), describing spins in a transverse field Γ, can be written in the general form

$$H=-\sum_{i=1}^{N}\Gamma_i^z S_i^z - \sum_{i=1}^{N-1} J_{i,i+1}[(1+\eta)S_i^x S_{i+1}^x+(1-\eta)S_i^y S_{i+1}^y] \qquad (2)$$

with $\eta=1,0$ corresponding to the transverse Ising and isotropic transverse XY models, respectively. The different random systems are characterized as follows: in (iii) and (iv), the transverse field is a random variable with a "plus-minus" distribution, $\Gamma_i=\pm\Gamma$, whereas in (v) and (vi), the exchange couplings are random variables which have a "plus-minus" distribution, $J_{ij}=\pm J$. Again, for the same reason as for the spin-glass chain (ii), the transverse spin-glass chains (v) and (vi) have no frustration.

The one-dimensional pure transverse Ising and isotropic transverse XY models have been exactly solved,[17–19] with the result that they exhibit a "quantum" phase transition at zero temperature, in which long-range order is lost for transverse fields above a critical value, for which $\Gamma_c/2J=1$ (for spins normalized to unity). The transverse Ising model, which is the simplest, has been extensively used as a basis for the study of quantum critical phenomena, in particular the static critical behavior of quantum systems with random fields (which couple to the order parameter),[20] and of quantum spin glasses.[21] The static critical behavior of classical transverse Ising spin glasses has been investigated by Pirč *et al.*[8] Also, in the presence of a transverse field, the Mattis transformation implies the equivalence between the statics of a nonfrustrated spin glass with random couplings $\pm J$ in a transverse uniform field, as in (v) and (vi), and the statics of a uniform ferromagnet in a transverse random field $\pm\Gamma$, as in (iii) and (iv). Again, however, the Mattis transformation does not imply the equivalence of the dynamics of those models. We will be particularly interested in the dynamic behavior of the systems (iii)–(vi), close to the transition induced by the transverse field.

In the dynamics of systems (i)–(vi), one observes a crossover in the frequency ω, of spin-wave excitations, from one asymptotic regime to another, which results from the competition between the characteristic length k^{-1} of the spin excitations and the controlling length of the behavior of the system which is measured by the inverse of the reduced field $\Delta=(H-H_c)$, where H is h or Γ and H_c is the critical field; the two asymptotic regimes are determined by the dominance of either k or Δ. So, for Δ, k, and $\omega\to 0$, the dispersion relation takes the scaling form

$$\omega=k^z f(\Delta/k^{\varphi}) , \qquad (3)$$

where z is a dynamic exponent characterizing the spin-wave dynamics near the transition, φ is a crossover exponent, and f is a scaling function governing the crossover which occurs at $\Delta\sim k^{\varphi}$. For the systems (i)–(vi) we will show that due to the randomness the exponents z and φ can take nontrivial values. While randomness is treated exactly, any quantum effects are ignored in this work since in each section the starting point is the linearized equation of motion provided by usual "mean-field" approximations.

This paper is organized as follows. In Sec. II we consider the Heisenberg spin chains in a field, and in Secs. II A and II B we study the critical spin-wave dynamics for the cases of a ferromagnet in a random field and of a spin glass in a uniform field, respectively. The transverse XY and transverse Ising spin chains are considered in Secs. III and IV, respectively; the critical spin-wave dynamics of these systems in the presence of a transverse random field or with spin-glass disorder are treated in Secs. III A, III B, IV A, and IV B, respectively. Finally, in Sec. V, we summarize the conclusions of this work.

II. DYNAMICS OF HEISENBERG SPINS IN A FIELD

The linearized equations of motion governing the transverse spin dynamics of a Heisenberg spin system in a longitudinal field at zero temperature, are

$$\omega S_i^+ = h_i S_i^+ + \sum_j J_{ij}(S_i^+\langle S_j^z\rangle - \langle S_i^z\rangle S_j^+) , \qquad (4)$$

where S_i^+ is the usual combination $S_i^+=S_i^x+iS_i^y$ of transverse spin components at site i, $\langle\cdots\rangle$ denotes the expectation value taken in the ground state, and the summation is taken over the nearest neighbors of i. We shall now discuss the dynamics for the cases where random-field or spin-glass disorder occur, in the scaling regime where $h,k,\omega\to 0$.

A. Heisenberg ferromagnet in a random field

In this system the couplings J_{ij} are uniform, $J_{ij}=J>0$, and the fields h_i are random independent variables with a "plus-minus" distribution, so that one can write $h_i=\zeta_i h$, where $\zeta_i=\pm 1$ with equal probability. It is easy to see that the ferromagnetic ground state (all spins "up") is unstable with respect to the formation of "reversed" domains of length (in number of spins) $l\geq 2J/h$ induced by the field. However, the formation of such "reversed" domains only occurs for configurations of the random fields with l neighboring sites with fields "opposite" to the ferromagnetic order, and these occur with probability $p\leq(\frac{1}{2})^l$. Thus in the limit $h\to 0$ (for finite J), only very large "reversed" domains can occur and with a very small probability. In the following study of the dynamics we will assume that the ground state has local ferromagnetic order ($\langle S_i^z\rangle=1$, say) over lengths larger then the characteristic lengths of the spin waves considered. This implies, however, that our results are valid only for spin waves with characteristic lengths k^{-1} up to the size of the smallest domain, i.e., for $k^{-1}<2J/h$. Since $J/h\gg 1$, these still lie in the scaling region. In this situation, (4) takes the form

$$(2+\zeta_i h-\omega)S_i^+=S_{i-1}^+ +S_{i+1}^+ \;. \tag{5}$$

Here ω and h are frequency and field divided by exchange constant J, respectively. Equation (5) can also be written in the following form:

$$\begin{aligned}\begin{bmatrix} S_{i+1}^+ \\ S_i^+ \end{bmatrix} &= \begin{bmatrix} 2+\zeta_i h-\omega & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} S_i^+ \\ S_{i-1}^+ \end{bmatrix} \\ &= T_i(\omega,h) \begin{bmatrix} S_i^+ \\ S_{i-1}^+ \end{bmatrix} ,\end{aligned} \tag{6}$$

which involves the transfer matrix T_i. We now proceed to the application of the generalized transfer-matrix scaling technique[2] which incorporates the additional dependence on a field. For a chain of $N+1$ spins with periodic boundary conditions $S_i^+=S_{i+N}^+$, the allowed frequencies are determined by $\det(\Lambda_N-1)=0$, or equivalently (since $\det\Lambda_N=1$) by $\mathrm{Tr}\Lambda_N=2$, where

$$\Lambda_N(\omega,h)=\prod_{i=1}^{N} T_i(\omega,h) \tag{7}$$

is the transfer matrix across the whole chain. Thus under a lattice rescaling by a dilation factor b, the dynamics is preserved if ω and h are transformed to ω' and h', respectively, so that

$$\mathrm{Tr}\Lambda_N(\omega,h)=\mathrm{Tr}\Lambda_{N/b}(\omega',h') \;. \tag{8}$$

Expanding $\mathrm{Tr}\Lambda_N$ in powers of ω and h leads to

$$\begin{aligned}\mathrm{Tr}\Lambda_N(\omega,h)=2-N\sum_{i=1}^{N}(\omega-\zeta_i h) &\\ +\sum_{i=1}^{N}\sum_{j=1}^{i-1}[\omega^2-\omega h(\zeta_i+\zeta_j)+h^2\zeta_i\zeta_j] &\\ \times[N(i-j)-(i-j)^2]+\cdots \;. &\end{aligned} \tag{9}$$

Since (9) depends on the random variables ζ_i the matching (8) is in fact generating the scaling of their distribution. From (9) we find, for any odd moment of $\mathrm{Tr}\Lambda_N(\omega,h)$, a form $F_0(N^2\omega)$, and for any even moment a form $F_e(N^2\omega,N^{3/2}h)$, which by (8) leads to the scaling transformations, $\omega'=b^2\omega$ and $h'=b^{3/2}h$. Such scaling transformations together with the scaling of the inverse characteristic length $k'=bk$ (resulting directly from the lattice dilation) imply a dispersion relation of the form

$$\omega=k^2 f(h/k^{3/2}) \;, \tag{10}$$

where the crossover exponent takes the nontrivial value $\varphi=\frac{3}{2}$ compared to $\varphi=2$ for the case of a uniform field. The form of the dispersion relation in the two asymptotic regimes is given by

$$\omega\sim\begin{cases} k^2 & \text{for } (h/k^{3/2})\ll 1 \;, \\ h^{4/3} & \text{for } (h/k^{3/2})\gg 1 \;. \end{cases} \tag{11}$$

In (11) the usual (zero field) ferromagnetic dispersion is trivially recovered in the limit where h is negligible, whereas in the other limit the dynamics is characterized by a nontrivial exponent associated with the field. It is important to note that the crossover in (10) occurs at a characteristic length $k^{-1}\sim h^{-2/3}$, which lies in the range of lengths for which our results apply ($k^{-1}<h^{-1}$), and hence it can in fact be observed.

B. Heisenberg spin glass in a uniform field

We consider a spin glass in which the couplings are random independent variables with a "plus-minus" distribution, i.e., $J_{ij}=\pm J$ with equal probability, in the presence of a uniform field $h_i=h$.

As noted before, if one performs a Mattis transformation of spins $\mathbf{S}_i\to\boldsymbol{\mu}_i=\zeta_i\mathbf{S}_i$, where $\zeta_i=\pm 1$ with equal probability, the statics of this spin-glass system becomes equivalent to the statics of the random-field system studied in the previous section, II A. Thus, all the considerations that we made then regarding the stability of the ferromagnetic ground state apply now for the ground state defined by $\langle S_i^z\rangle=\zeta_i$, in which the spins are aligned parallel or antiparallel according to the configuration of the random couplings. We will consider such a ground state in the following study of the dynamics, which, in a similar way to before, implies that our results are valid only for spin waves with characteristic length $k^{-1}<2J/h$. We note that we have ignored real quantum effects in taking the classical ground state, which is appropriate for sufficient large spin. Equation (4) can then be written as

$$[2-\zeta_i(\omega-h)]\mu_i^+=\mu_{i-1}^+ +\mu_{i+1}^+ \;. \tag{12}$$

Comparing this equation with (5) clearly shows that the dynamics of the two systems are distinct. Equation (12) has a similar form to the one found in our previous work[2] for the spin-glass chain in zero field, but where ω has been replaced by $(\omega-h)$. Written in a matrix form (12) becomes

$$\begin{aligned}\begin{bmatrix} S_{i+1}^+ \\ S_i^+ \end{bmatrix} &= \begin{bmatrix} 2-\zeta_i(\omega-h) & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} S_i^+ \\ S_{i-1}^+ \end{bmatrix} \\ &= T_i(\omega,h) \begin{bmatrix} S_i^+ \\ S_{i-1}^+ \end{bmatrix} .\end{aligned}$$

Then from the results obtained in our previous work it follows that for the present problem the scaling transformation is $(\omega'-h')=b^{3/2}(\omega-h)$. This scaling transformation yields a dispersion relation of the form

$$\omega=k^{3/2} f(h/k^{3/2}) \;, \tag{13}$$

where the nontrivial spin-glass dynamic exponent $z=\frac{3}{2}$ occurs. In (13) the crossover variable $h/k^{3/2}$, which is the same as in (10), drives a crossover in ω between the two asymptotic forms

$$\omega\sim\begin{cases} k^{3/2} & \text{for } (h/k^{3/2})\ll 1 \;, \\ h & \text{for } (h/k^{3/2})\gg 1 \;. \end{cases}$$

Here the spin-glass dispersion is naturally found in the limit of negligible field, and the other limit is trivial.

III. DYNAMICS OF *XY* SPINS IN A TRANSVERSE FIELD

The linearized equations of motion for the transverse spin dynamics of an isotropic XY spin system in a transverse field at zero temperature (given that the average spin component in the xy plane is along the x direction) are

$$\omega S_i^y = \Gamma_i S_i^y - \sum_j J_{ij} \langle S_i^z \rangle S_j^y \ , \tag{14}$$

where, as usual, $\langle \cdots \rangle$ denotes the expectation value taken in the ground state and the summation is taken over the nearest neighbors of i. We shall now study the dynamics in the cases where the transverse field is random or the exchange couplings have spin-glass disorder, in the scaling regime, i.e., for $(\Gamma-\Gamma_c)$, k, $\omega \to 0$.

A. *XY* ferromagnet in a transverse random field

This system is characterized by uniform couplings $J_{ij}=J>0$ and a transverse field which is a random independent variable with a "plus-minus" distribution, i.e., $\Gamma_i=\zeta_i\Gamma$, where $\zeta_i=\pm 1$ with equal probability.

In the ground state the spin components in the transverse direction align parallel to the field and hence manifest its randomness, i.e., $\langle S_i^z\rangle=\zeta_i\langle S^z\rangle$. Thus, in this case, and considering that near the transition $\langle S^z\rangle=1$, (14) takes the form

$$(2+\Delta-\zeta_i\omega)S_i^y = S_{i-1}^y + S_{i+1}^y \ , \tag{15}$$

where Δ and ω are, respectively, the reduced field $(\Gamma-\Gamma_c)$ and frequency divided by exchange constant J. Writing (15) in matrix form gives

$$\begin{aligned} \begin{bmatrix} S_{i+1}^y \\ S_i^y \end{bmatrix} &= \begin{bmatrix} 2+\Delta-\zeta_i\omega & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} S_i^y \\ S_{i-1}^y \end{bmatrix} \\ &= T_i(\omega,\Delta) \begin{bmatrix} S_i^y \\ S_{i-1}^y \end{bmatrix} . \end{aligned} \tag{16}$$

We note that the transfer matrix $T_i(\omega,\Delta)$ in (16) is formally identical to the transfer matrix $T_i(\omega,h)$ in (6), with the following change of parameters: $-\omega\to\Delta$ and $-h\to\omega$. Thus, using the results of Sec. II A, with the proper replacement of parameters, we obtain for the present system the scaling transformations, $\omega'=b^{3/2}\omega$ and $\Delta'=b^2\Delta$, which imply a dispersion relation of the form

$$\omega = k^{3/2} f(\Delta/k^2) \ , \tag{17}$$

where the dynamic exponent takes the nontrivial value $z=\frac{3}{2}$. The frequency appearing in (17) crosses over between the two asymptotic forms

$$\omega \sim \begin{cases} k^{3/2} & \text{for } (\Delta/k^2) \ll 1 \ , \\ \Delta^{3/4} & \text{for } (\Delta/k^2) \gg 1 \ . \end{cases} \tag{18}$$

It is interesting to note that in this system the randomness in the field implies nontrivial exponents for the dynamics in both regimes in (18), $\frac{3}{2}$ and $\frac{3}{4}$ compared to 2 and 1, respectively, in the case of a uniform field.

B. *XY* spin glass in a transverse uniform field

We consider a spin glass where the couplings are random independent variables with a "plus-minus" distribution, i.e., $J_{ij}=\pm J$ with equal probability, and the transverse field is uniform, $\Gamma_i=\Gamma$. Since, in the ground state, the transverse components of the spins are parallel to the uniform field this implies that they are also uniform, and hence $\langle S_i^z\rangle=\langle S^z\rangle$. Then, by a Mattis transformation of spins ($\mathbf{S}_i\to\boldsymbol{\mu}_i=\zeta_i\mathbf{S}_i$), (14) becomes

$$[(\Gamma/J)-\omega]\mu_i^y = \langle S^z\rangle(\mu_{i-1}^y+\mu_{i+1}^y) \ , \tag{19}$$

where ω is frequency divided by coupling constant J. It turns out that (19) is precisely of the form of the equation for an XY ferromagnet in a transverse uniform field. So the spin-glass randomness does not affect the dynamics. From the solution of (19) it follows that below the transition $(\Gamma/2J<1)$ the dispersion relation is $\omega=(\Gamma/J)(1-\cos k)$, and above the transition $(\Gamma/2J>1)$ it is $\omega=(\Gamma/J)-2\cos k$. In the scaling regime these expressions yield the form

$$\omega = k^2 f(\Delta/k^2) \ , \tag{20}$$

where Δ is as defined before, yielding the dynamic critical exponent $z=2$. In (20) a crossover occurs between the two asymptotic forms

$$\omega \sim \begin{cases} k^2 & \text{for } (\Delta/k^2) \ll 1 \ , \\ \Delta & \text{for } (\Delta/k^2) \gg 1 \ , \end{cases}$$

where the trivial exponents of a pure transverse XY ferromagnet occur.

IV. DYNAMICS OF ISINGS SPINS IN A TRANSVERSE FIELD

The linearized equations of motion for the transverse spin dynamics of an Ising spin system in a transverse field at zero temperature, are

$$\left[\Gamma_i^2 + \left[\sum_j J_{ij}\langle S_j^x\rangle\right]^2 - \omega^2\right] S_i^x = \Gamma_i\langle S_i^z\rangle \sum_j J_{ij} S_i^x S_j^x \ , \tag{21}$$

where, as usual, $\langle \cdots \rangle$ denotes the expectation value in the ground state and the summation is taken over the nearest neighbors of i. We shall now discuss the dynamics for the cases where random field or spin-glass disorder occur, again in the scaling regime where $(\Gamma-\Gamma_c)$ k, $\omega\to 0$.

A. Ising ferromagnet in a transverse random field

In this system the couplings are uniform $J_{ij}=J>0$ and the transverse field is a random variable with a "plus-minus" distribution, i.e., $\Gamma_i=\zeta_i\Gamma$ where $\zeta_i=\pm 1$ with equal probability. In the ground state the components of the spins in the transverse direction align parallel to the field and hence exhibit its randomness, $\langle S_i^z\rangle=\zeta_i\langle S^z\rangle$. In turn the ferromagnetic coupling determines that in the x direction the spin components are uniform, $\langle S_i^x\rangle=\langle S^x\rangle$. So, for the present system (21) takes the form

$$[(\Gamma/J)^2+4\langle S^x\rangle^2-\omega^2]S_i^z=(\Gamma/J)\langle S^z\rangle(S_{i-1}^z+S_{i+1}^z)\ , \tag{22}$$

where ω is frequency divided by coupling constant J. Equation (22) turns out to be identical to the one for an Ising ferromagnet in a uniform transverse field, thus implying that the randomness in the field does not affect the dynamics. Hence it follows that below the transition $(\Gamma/2J<1)$ the dispersion relation is $\omega^2=4-(\Gamma/J)^2\cos k$, and above the transition $(\Gamma/2J>1)$ it is $\omega^2=(\Gamma/J)^2-2(\Gamma/J)\cos k$. In the scaling regime these expressions lead to the scaling form

$$\omega=kf(\Delta/k^2)\ , \tag{23}$$

yielding the dynamic critical exponent $z=1$, and ω crosses over between the two asymptotic forms

$$\omega\sim\begin{cases}k & \text{for } (\Delta/k^2)\ll 1\ ,\\ \Delta^{1/2} & \text{for } (\Delta/k^2)\gg 1\ ,\end{cases} \tag{24}$$

corresponding to the trivial behavior of a pure transverse Ising ferromagnet.

B. Ising spin glass in a transverse uniform field

We consider a spin glass in which the couplings are random independent variables with a "plus-minus" distribution, i.e., $J_{ij}=\pm J$ with equal probability, in a transverse uniform field $\Gamma_i=\Gamma$. Thus, in the ground state, the spin components in the transverse direction are uniform $\langle S_i^z\rangle=\langle S^z\rangle$, whereas in the x direction the spin-glass coupling determines that the spin components have a configuration such that $\langle S_i^x\rangle=\pm\langle S^x\rangle$, according to the distribution of the random couplings. Introducing these conditions in (21) and performing a Mattis transformation of spins leads to an equation which is of the same form as (22). We therefore conclude that the transverse Ising spin glass has the same spin-wave dynamics as a transverse Ising ferromagnet. Hence the spin-glass randomness does not affect the dynamics which is then given by the results (23) and (24).

V. CONCLUSIONS

We have studied the dynamics of a number of one-dimensional random spin systems, in which critical behavior is induced by an applied magnetic field. We considered three different spin models, in a field: Heisenberg in a longitudinal field, XY in a transverse field, and Ising in a transverse field, and in each considered spin-glass or random-field disorder. The critical spin-wave dynamics of these systems was calculated by a transfer-matrix scaling technique which provides exact results for the dynamic critical exponents as well as the crossover exponents.

In the case of the Heisenberg systems one finds for both the spin-glass and the random-field systems the same nontrivial crossover exponent $\varphi=\frac{3}{2}$, but their dynamic exponents are different. So, for the spin glass in a uniform field z takes the nontrivial value $\frac{3}{2}$, the same as for the system in zero field, whereas for the ferromagnet in the random field z takes the typical value for ferromagnetic systems $(z=2)$. This implies that because of the randomness, in the spin-glass case the dynamic exponent is modified whereas in the random-field case it is the exponent associated to the field that becomes nontrivial taking the value $\frac{4}{3}$.

For the transverse XY systems we find again that the spin-glass and random-field systems have the same crossover exponent, which is also the same as for a pure ferromagnet in a transverse uniform field, but that their dynamic exponents are different. In the case of the random field z takes the nontrivial value $\frac{3}{2}$, which has before been associated to spin-glass disorder. It turns out that for this system the randomness in the field implies nontrivial values for both the dynamic and the field exponents, $\frac{3}{2}$ and $\frac{3}{4}$, respectively. On the other hand, the spin-glass randomness does not affect the dynamics of the system, for which one finds the trivial exponents of a pure transverse XY ferromagnet. Finally, for the transverse Ising systems we find that neither the random-field nor the spin-glass randomness affects the dynamics which is then the same as that of a pure transverse Ising ferromagnet.

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