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Classical magnetic resonance equation for ordered-state systems

Stuart L. Hutton and John E. Drumheller
Department of Physics, Montana State University, Bozeman, Montana 59717
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The so-called rectangular approach to ferromagnetic resonance equations of Baselgia *et al.* has been generalized to include magnetic systems of several sublattices. The result is a form of a resonance equation which can be expressed in terms of a Hessian matrix of a general free-energy expression. Explicit expansions of this resonance equation for the cases of one-, two-, and three-sublattice systems in terms of Hessian and interaction matrices are presented. Examples of the reduction of this formalism to some well-known cases are presented.

I. INTRODUCTION

Since the development of the original equations for ferromagnetic resonance (FMR) and antiferromagnetic resonance (AFMR) by Kittel and others,[1-10] useful theoretical developments of FMR and AFMR led to the very important free-energy approach of Smith and Beljers.[11] This latter formalism has been the standard approach to FMR for over 30 years.[12-16] More recently, however, Baselgia *et al.* explored several of the shortcomings of the Smit and Beljers formulation,[17] which showed a possible divergence at an important laboratory angle. Also, for some symmetries, a direct substitution of field orientation angles for magnetization orientation angles failed to produce correct results. Baselgia *et al.* also presented a new formulation, the "rectangular" method,[17,18] which overcame the problems observed with the Smit and Beljers method. Until now, the rectangular approach has only been applied to the ferromagnetic (one-sublattice) case.

Because of recent interest in magnetic resonance in systems which can be described with more than one or two sublattices,[19-24] it would be useful to have a general method which can deal with these higher sublattice systems. Therefore, in this work, we generalize the Baselgia approach to include any number of sublattices and any free-energy model. In the following section, we begin with a short review of the Baselgia work and follow that by a generalization to multiple sublattices. We will show that the result can be reduced to a simple resonance equation expressed in terms of Hessian matrices[25] which contain the essentials of the free-energy model. It is this approach to the multisublattice resonance problem that we call ordered-state resonance (OSR).[26] In Sec. III, we show how these results reduce to the Baselgia result for ferromagnets and also derive explicit results for several higher sublattice systems.

II. ORDERED-STATE RESONANCE

A. The rectangular formulation (Baselgia *et al.*)

In the Baselgia or rectangular formulation,[17] the torque equation is expressed in local coordinates as

$$d\mathbf{M}/dt=\gamma\mathbf{M}\times\mathbf{H}^{\text{eff}}\,, \tag{1}$$

where $\mathbf{H}^{\text{eff}}$ is the effective field acting upon the sublattice designated by the magnetization $\mathbf{M}$. By expanding Eq. (1) in a Taylor expansion to first order, the ferromagnetic resonance equation was shown to become

$$(\omega/\gamma)^2=(MF_{M_1M_1}-F_{M_3})(MF_{M_2M_2}-F_{M_3})-(MF_{M_1M_2})^2, \tag{2}$$

where F is the free energy and the subscripts denote differentiation of a free-energy expansion with respect to components of the local Cartesian coordinates of magnetization (M_1,M_2,M_3). M_3 is parallel to the static orientation of M. Equation (2) differs from the previous resonance equations by the appearance of the first derivative terms. One important feature of this new approach is that effective fields are obtained from a free-energy expression by

$$\mathbf{H}^{\text{eff}}=-\nabla_M F\,. \tag{3}$$

The orientation of the magnetization M is then given by solutions to the condition that the 1 and 2 components of this effective field vanish. This condition is nearly equivalent to the equilibrium conditions from the Smit and Beljers approach, which were $F_\theta=0$ and $(1/\sin\theta)F_\Phi=0$. In the rectangular formulation, the connection between local and laboratory coordinates is obtained through the definition of an orthogonal transformation matrix, so that each component of magnetization

appearing in the free-energy expansion can be replaced by

$$M_j = \sum_{k=1}^{3} b_{jk} M_k \ , \tag{4}$$

where the elements b_{jk} are obtained from the orthogonal transformation matrix B defined by Eq. (B8) of Ref. 17.

B. The generalized equation

The derivation of a general resonance equation is analogous to that of the ferromagnetic resonance equation described above. We begin with the torque equations, but instead of describing the motion of the magnetization vector for the entire system, they refer here to the motion or perturbation of the magnetization vector of each of the sublattices. Thus, the generalized torque equations are expressible as

$$d\mathbf{M}^\sigma/dt = \gamma \mathbf{M}^\sigma \times \mathbf{H}^\sigma \ , \tag{5}$$

where $\mathbf{M}^\sigma$ represents the magnetization vector associated with sublattice σ, and $\mathbf{H}^\sigma$ represents the effective field acting upon sublattice σ. The relevant coordinates are local Cartesian coordinates of magnetization, defined by $\langle M_1^\sigma, M_2^\sigma, M_3^\sigma \rangle$ with M_3^σ parallel to $\mathbf{M}^\sigma$, the sublattice magnetization. As with the ferromagnetic case, the components M_1^σ and M_2^σ are considered to be small perturbations of the sublattice magnetization away from the steady-state direction M_3^σ. Since no significant motion away from the M_3^σ direction is assumed, each additional sublattice increases the dimensionality of the problem by two. As in the ferromagnetic case, these torque equations describe the motions of magnetization vectors when placed in the presence of an external magnetic field. The torque acting upon sublattice σ is given by

$$\mathbf{\Gamma}^\sigma = \gamma \mathbf{M}^\sigma \times \mathbf{H}^\sigma \ , \tag{6}$$

where Eq. (6) is simply the right-hand side of Eq. (5). In order to obtain resonance equations for the motion of sublattice σ, the torque acting upon sublattice σ is expanded to first order in a Taylor series about the direction of the effective field acting upon that sublattice. By contrast to the one sublattice case, however, it is now essential that this Taylor expansion take into account the effects of all sublattices in the system since, in the general case, all other sublattices contribute to the effective field experienced at sublattice σ. For the ith component of the torque appearing in Eq. (6), the Taylor expansion appears as

$$\Gamma_i^\sigma \approx {}^0\Gamma_i^\sigma + \sum_{\alpha=1}^{\tau} \sum_{j=1}^{3} \Delta M_j^\alpha (\partial \Gamma_i^\sigma / \partial M_j^\alpha)|_{\langle M_1^\delta = 0, M_2^\delta = 0, M_3^\delta = M^\delta \rangle} + \cdots \tag{7}$$

and α refers to a sum over sublattices. The unusual notation of combining Greek and Latin indices is useful in order to clearly separate sublattices from coordinates. In equilibrium, the constant components of torque not in the $\hat{3}^\delta$ direction vanish. The notation used to represent the derivatives of the effective fields will be

$$A_{i,j}^{\alpha,\delta} \equiv -(\partial H_i^\alpha / \partial M_j^\delta) \ . \tag{8}$$

It is necessary to look at each component of torque appearing in Eq. (7). Thus, the $\hat{1}^\sigma$ and $\hat{2}^\sigma$ components of torque can be represented to first order as

$$\Gamma_1^\sigma \approx M^\sigma \{ M_1^\sigma A_{2,1}^{\sigma,\sigma} + M_2^\sigma [A_{2,2}^{\sigma,\sigma} + (H_3^\sigma / M^\sigma)] \} + M^\sigma \sum_{\alpha \neq \sigma} (M_1^\alpha A_{2,1}^{\sigma,\alpha} + M_2^\alpha A_{2,2}^{\sigma,\alpha}) \tag{9}$$

and

$$\Gamma_2^\alpha \approx -M^\sigma \{ M_1^\sigma [A_{1,1}^{\sigma,\sigma} + (H_3^\sigma / M^\sigma)] + M_2^\sigma A_{1,2}^{\sigma,\sigma} \} - M^\sigma \sum_{\alpha \neq \sigma} (M_1^\alpha A_{1,1}^{\sigma,\alpha} + M_2^\alpha A_{1,2}^{\sigma,\alpha}) \ . \tag{10}$$

Equations (9) and (10) may be expressed in matrix notation which we will utilize below.

It is now necessary to find the normal mode solutions in which the $\hat{1}$ and $\hat{2}$ components of magnetization have harmonic time dependence. We assume

$$M_i^\sigma = {}^0M_i^\sigma e^{i\omega t} \ , \tag{11}$$

where, as was also the case in the Baselgia formulation, the $\hat{3}^\sigma$ component of magnetization is constant in time and along the effective field. Now, however, this field is not the effective field for the entire system but rather that which is acting only upon sublattice σ. If one then proceeds as before for sublattice σ, two coupled equations of motion result. These equations of motion are given by

$$0 = M_1^\sigma (A_{1,1}^{\sigma,\sigma} + H_3^\sigma / M^\sigma) + M_2^\sigma (A_{1,2}^{\sigma,\sigma} + i\omega/\gamma M^\sigma) + \sum_{\alpha \neq \sigma} (M_1^\alpha A_{1,1}^{\sigma,\alpha} + M_2^\alpha A_{1,2}^{\sigma,\alpha}) \tag{12}$$

and

$$0 = M_1^\sigma (A_{2,1}^{\sigma,\sigma} - i\omega/\gamma M^\sigma) + M_2^\sigma (A_{2,2}^{\sigma,\sigma} + H_3^\sigma / M^\sigma) + \sum_{\alpha \neq \sigma} (M_1^\alpha A_{2,1}^{\sigma,\alpha} + M_2^\alpha A_{2,2}^{\sigma,\alpha}) \ . \tag{13}$$

In the Appendix, effective fields in the general resonance problem are obtained. If the effective fields, which are obtained through application of the sublattice-specific operator [Eq. (A1)] to a general free-energy model, are substituted into the equations of motion we obtain

$$0 = M_1^\sigma (F_{1,1}^{\sigma,\sigma} - F_3^\sigma / M^\sigma) + M_2^\sigma (F_{1,2}^{\sigma,\sigma} + i\omega/\gamma M^\sigma) + \sum_{\alpha \neq \sigma} (M_1^\alpha F_{1,1}^{\sigma,\alpha} + M_2^\alpha F_{1,2}^{\sigma,\alpha}) \tag{14}$$

and

$$0 = M_1^\sigma (F_{2,1}^{\sigma,\sigma} - i\omega/\gamma M^\sigma) + M_2^\sigma (F_{2,2}^{\sigma,\sigma} - F_3^\sigma / M^\sigma) + \sum_{\alpha \neq \sigma} (M_1^\alpha F_{2,1}^{\sigma,\alpha} + M_2^\alpha F_{2,2}^{\sigma,\alpha}) \ . \tag{15}$$

The final step is to express these τ equations in the form of a matrix. Thus, for a system comprised of τ sublattices, one would have a matrix which is block diagonal, termed the *frequency matrix*, and a second matrix which is the *Hessian matrix*. The first matrix is given by

$$\begin{bmatrix} F_3^1/M^1 & -i\omega/\gamma M^1 & 0 & 0 & \cdots & 0 & 0 \\ i\omega/\gamma M^1 & F_3^1/M^1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & F_3^2/M^2 & -i\omega/\gamma M^2 & & 0 & 0 \\ 0 & 0 & i\omega/\gamma M^2 & F_3^2/M^2 & & 0 & 0 \\ \vdots & \vdots & & & \ddots & & \\ 0 & 0 & 0 & 0 & & F_3^\tau/M^\tau & -i\omega/\gamma M^\tau \\ 0 & 0 & 0 & 0 & & i\omega/\gamma M^\tau & F_3^\tau/M^\tau \end{bmatrix} . \tag{16}$$

This first matrix is designated by $\Omega(1,2,\ldots,\tau)$ and it is a $2\tau\times 2\tau$ block-diagonal matrix. The second matrix is the Hessian matrix with respect to the local coordinates M_1^σ and M_2^σ and is designated by $H(1,2,\ldots,\tau)$. Thus, the Hessian matrix of the system is given by

$$\begin{bmatrix} F_{1,1}^{1,1} & F_{1,2}^{1,1} & F_{1,1}^{1,2} & F_{1,2}^{1,2} & \cdots & F_{1,1}^{1,\tau} & F_{1,2}^{1,\tau} \\ F_{2,1}^{1,1} & F_{2,2}^{1,1} & F_{2,1}^{1,2} & F_{2,2}^{1,2} & \cdots & F_{2,1}^{1,\tau} & F_{2,2}^{1,\tau} \\ F_{1,1}^{2,1} & F_{1,2}^{2,1} & F_{1,1}^{2,2} & F_{1,2}^{2,2} & & F_{1,1}^{2,\tau} & F_{1,2}^{2,\tau} \\ F_{2,1}^{2,1} & F_{2,2}^{2,1} & F_{2,1}^{2,2} & F_{2,2}^{2,2} & & F_{2,1}^{2,\tau} & F_{2,2}^{2,\tau} \\ \vdots & \vdots & & & \ddots & & \\ F_{1,1}^{\tau,1} & F_{1,2}^{\tau,1} & F_{1,1}^{\tau,2} & F_{1,2}^{\tau,2} & & F_{1,1}^{\tau,\tau} & F_{1,2}^{\tau,\tau} \\ F_{2,1}^{\tau,1} & F_{2,2}^{\tau,1} & F_{2,1}^{\tau,2} & F_{2,2}^{\tau,2} & & F_{2,1}^{\tau,\tau} & F_{2,2}^{\tau,\tau} \end{bmatrix} . \tag{17}$$

If we now define the column vector whose elements are the $\hat{1}^\delta$ and $\hat{2}^\delta$ components of the magnetization vectors for each sublattice as

$$\mathbf{M}\equiv \begin{bmatrix} M_1^1 \\ M_2^1 \\ M_1^2 \\ M_2^2 \\ \vdots \\ M_1^\tau \\ M_2^\tau \end{bmatrix} , \tag{18}$$

then it is easy to see that the general resonance equation can be written in the form

$$H(1,2,\ldots,\tau)\cdot\mathbf{M}-\Omega(1,2,\ldots,\tau)\cdot\mathbf{M}=0 . \tag{19}$$

The solutions for frequency are given by

$$\det[H(1,2,\ldots,\tau)-\Omega(1,2,\ldots,\tau)]=0 , \tag{20}$$

which is the general resonance equation.

III. EXAMPLES

A. Reduction to ferromagnetic resonance

The are many simple examples for the application of Eq. (20), the simplest of which would be the one-sublattice system, i.e., ferromagnetic resonance. In this case, Eq. (20) can be written as

$$\det\left[\begin{bmatrix} F_{1,1}^{1,1} & F_{1,2}^{1,1} \\ F_{2,1}^{1,1} & F_{2,2}^{1,1} \end{bmatrix}-\begin{bmatrix} F_3^1/M^1 & -i\omega/\gamma M^1 \\ i\omega/\gamma M^1 & F_3^1/M^1 \end{bmatrix}\right]=0 , \tag{21}$$

resulting in the single-sublattice ferromagnetic resonance equation,

$$(\omega/\gamma)^2=(MF_{M_1M_1}^{1\,1}-F_{M_3}^1)(MF_{M_2M_2}^{1\,1}-F_{M_3}^1)-(MF_{M_1M_2}^{1\,1})^2 .$$

This reproduces the result of the Baselgia approach. As mentioned above, this equation differs from the Smit and Belgers resonance equation only in the existence of the first derivative terms which arise because the coordinate system is now oriented along the direction of the magnetization vector.

B. Two-sublattice systems

Application of Eq. (20) to systems of several sublattices is faciliated by some general expansions for the case of two-sublattice magnets, i.e., ferromagnets, antiferromagnets, or ferrimagnets. Such expansions are not necessary for the computerized applications of Eq. (20) since algorithms exist which can numerically evaluate Eq. (20). For the two-sublattice system, however, much information is obtainable without computerization. The expansion for two-sublattice systems requires definition of several matrices. First the *reduced Hessian matrix H_r is defined by*

$$H_r(1,2,\ldots,\tau)\equiv H(1,2,\ldots,\tau)-\Omega_d(1,2,\ldots,\tau) , \tag{22}$$

where Ω_d is the matrix with only the diagonal elements

of $\Omega(1,2,\ldots,\tau)$. The other matrix, which is useful to define, is termed the *two-sublattice interaction matrix*, and it is given by

$$X(\alpha,\delta)\equiv\begin{bmatrix} F^{\alpha\,\delta}_{M_1M_1} & F^{\alpha\,\delta}_{M_1M_2} \\ F^{\alpha\,\delta}_{M_2M_1} & F^{\alpha\,\delta}_{M_2M_2} \end{bmatrix} . \tag{23}$$

If one also defines the quantity ξ by

$$\xi_\delta\equiv(1/\gamma M^\delta) , \tag{24}$$

then the expansion of the resonance equation for a two-sublattice system can be expressed as

$$\omega^4\xi_1^2\xi_2^2-\omega^2\sum_{\delta=1}^{2}[\xi_\delta^2|H_r(\delta+1)|+\xi_\delta\xi_{\delta+1}|X(\delta,\delta+1)|]+|H_r(1,2)|=0 , \tag{25}$$

where the index δ is cyclic in that $\delta=3$ means $\delta=1$. Under the assumption that all sublattice magnetizations are the same, this expansion reduces to

$$(\omega\xi)^4-(\omega\xi)^2\sum_{\delta=1}^{2}[|H_r(\delta+1)|+|X(\delta,\delta+1)|]+|H_r(1,2)|=0 . \tag{26}$$

The solution to Eq. (26) is given by

$$(\omega\xi)^2=\tfrac{1}{2}\sum_{\delta=1}^{2}[|H_r(\delta+1)|+|X(\delta,\delta+1)|]\pm\frac{1}{2}\left[\left\{\sum_{\delta=1}^{2}[|H_r(\delta+1)|+|X(\delta,\delta+1)|]\right\}^2-4|H_r(1,2)|\right]^{1/2} . \tag{27}$$

For the case of an antiferromagnet in the antiferromagnetic state, evaluation of Eq. (27) is required.

An example of recent interest is the possible two-sublattice ferromagnet.[27] Such a system can possibly occur in layered magnetic compounds in which each layer orders ferromagnetically but with weaker exchange coupling the layers. For such two-sublattice ferromagnets, the generalized equation predicts two modes, that is, predicts that two resonances will occur. More interesting is that the difference between them is twice the weak interplanar exchange, providing a possible experimental determination of exchange by magnetic resonance. The details of this particular case are to be published elsewhere, however, we outline them here as an example of the two-sublattice case.

The free-energy model for a uniaxial ferromagnet with the external magnetic field applied parallel to the x axis is given by

$$F=\tfrac{1}{2}K_z[(M_z^1)^2+(M_z^2)^2]+\epsilon\mathbf{M}^1\cdot\mathbf{M}^2-H_x(M_x^1+M_x^2) , \tag{28}$$

where, in the mean-field approximation, the simple ferromagnetic interaction between the two sublattices is represented by the second term. Here the $\hat{\mathbf{z}}$ axis represents the easy axis, and $K_z=-|K_z|$ so that in zero applied field, the magnetization vectors $\mathbf{M}^1$ and $\mathbf{M}^2$ will lie parallel to the $\hat{\mathbf{z}}$ direction. Also, ϵ represents a ferromagnetic interaction, then $\epsilon=-|\epsilon|$ so that if all other fields vanished, $\mathbf{M}^1$ would be parallel to $\mathbf{M}^2$ in the lowest-free-energy state. The next step to obtain resonant frequencies is to determine the elements of the Hessian matrix given by Eq. (17). The explicit derivatives are easily obtained. Since the interaction ϵ is assumed to be ferromagnetic, the two sublattices are parallel in their static orientations. Therefore, many of the aforementioned derivatives are duplicated or they vanish. The nonzero elements of the Hessian matrix are thus

$$F_{11}^{11}=K(\sin^2\theta_1)=F_{11}^{22} \tag{29}$$

and

$$F_{11}^{12}=F_{11}^{21}=F_{22}^{12}=F_{22}^{21}=\epsilon , \tag{30}$$

so that the Hessian matrix appears as

$$H(1,2)=\begin{bmatrix} F_{11}^{11} & 0 & \epsilon & 0 \\ 0 & 0 & 0 & \epsilon \\ \epsilon & 0 & F_{11}^{11} & 0 \\ 0 & \epsilon & 0 & 0 \end{bmatrix} . \tag{31}$$

The next step is to obtain the elements of the frequency matrix [Eq. (16)]. The derivatives which give effective fields are

$$F_3^1=F_3^2=K_zM^1(\cos\theta_1)^2+\epsilon M^1-H_x\sin\theta_1\cos\Phi_1 . \tag{32}$$

The equilibrium conditions are also required for this problem if we wish to look at solutions outside of the high-field limiting cases. To obtain a first approximation to the solutions, we assume that the external field is large enough so that the effective field does indeed lie in the direction of the external field. Then, the angles θ_1 and Φ_1 in the above expressions can be simply replaced by the external field angles θ and Φ. Since the magnetic field is applied along the $\hat{\mathbf{x}}$ axis, $\theta=\pi/2$ and $\Phi=0$. With this simplification, the derivatives reduce to

$$F_{11}^{11}=K \quad\text{and}\quad F_3^1=\epsilon M^1-H_x . \tag{33}$$

The resonance frequency is then obtained by evaluating the determinant of the difference between the Hessian matrix and the frequency matrix. Thus,

$$(\omega/\gamma)^2=[H+KM_s-(\epsilon\pm\epsilon)M_s][H-(\epsilon\pm\epsilon)M_s] , \quad (34)$$

where M_s is the magnetization associated with a sublattice. It is clear that two resonance modes are obtained, and it is straightforward to show that these two modes are separated by twice the intersublattice exchange field. This result can be seen by looking at the two solutions to Eq. (34). For the lower-sign solution, the resonance frequency is given by $(\omega/\gamma)^2=(H+KM_s)H$. Inversion of Eq. (34) then yields

$$H_f=-(KM_s/2)+[(KM_s/2)^2-(\omega/\gamma)^2]^{1/2} .$$

From Eq. (34), if the upper sign solution is now chosen, the field in Eq. (34) can be replaced by $H\equiv H_f-2\epsilon M_s$. Inversion of Eq. (34) then shows that $H_{af}=2\epsilon M_s+H_f$. Since the exchange is ferromagnetic here, H_{af} will be at a lower magnetic field than H_f. The separation of twice the exchange field is also obtained for an antiferromagnetic system in the "collapsed" state:[28] i.e., the paramagnetic state at higher fields than the spin-flop state.

The two modes correspond to an in-phase resonance mode, where M^1 and M^2 precess in phase about their effective field and to an out-of-phase mode, where the two magnetization vectors precess 180° out of phase about their effective field. In order to determine resonance modes, it is necessary to substitute the frequencies obtained in Eq. (34) into the system of equations given by Eq. (19) and then to solve for the components of M^1 and M^2. For the frequency designated by the lower sign, the resonance frequency is given by

$$(\omega/\gamma)^2=H(H+KM_s) , \quad (35)$$

similar to the value obtained in the simple ferromagnetic case. To obtain the resonance mode corresponding to Eq. (35), it is necessary to substitute this frequency into Eq. (19) and solve for M_1^q and M_2^q. The result yields solutions in terms of the quantities $M_1^1+M_1^2$ and $M_2^1+M_2^2$, which corresponds to an in-phase precession about the direction of the effective field, and $M_1^1-M_1^2$ and $M_2^1-M_2^2$, which corresponds to the sublattices precessing out of phase. If the frequency designated by the upper sign is desired, the result is given by

$$(\omega/\gamma)^2=(H-2\epsilon M_s)(H-2\epsilon M_s+KM_s) , \quad (36)$$

which is unlike a ferromagnetic resonance frequency in several respects. The sublattices precess about the effective field, but they have a phase difference of 180°. Also the resonance frequency explicitly contains a reference to an exchange field, which provides the possibility of measuring this intersublattice exchange with resonance techniques.

C. An expansion for three-sublattice systems

The expansion for three-sublattice systems shows similarities to both the one- and two-sublattice systems in that the polynomial to be solved for resonance frequency is expressible in terms of Hessian and interaction matrices. For these systems, the result is given by

$$\omega^6\xi_1^2\xi_2^2\xi_3^2-\omega^4\sum_{\delta=1}^{3}\xi_\delta^2[\xi_{\delta+1}^2|H_r(\delta+2)|+2\xi_{\delta+1}\xi_{\delta+2}|X(\delta+1,\delta+2)|]$$

$$+\omega^2\sum_{\delta=1}^{3}[\xi_\delta^2|H_r(\delta+1,\delta+2)|+2\xi_\delta\xi_{\delta+1}|X_3(\delta,\delta+1;\delta+2)|]-|H_r(1,2,3)|=0 , \quad (37)$$

where we have defined the *three-sublattice interaction matrix* as

$$X_3(\alpha,\beta;\delta)\equiv\begin{bmatrix} X(\alpha,\beta) & X(\alpha,\delta) \\ X(\delta,\beta) & H_r(\delta) \end{bmatrix} . \quad (38)$$

IV. DISCUSSION

It is clear that the general form of the resonance equation given by Eq. (20) can be applied to many systems. For example, this equation should find application to superlattice compounds of many magnetic entities. In more conventional systems, this equation can be applied to systems in which a complete description requires the use of more than the usual number of sublattices.

The earlier and important approach of Smit and Beljers can also be generalized in a similar manner to the generalization of the Baselgia formulation shown in this work. In this generalization of the Smit and Beljers form, the frequency matrix is still block diagonal but with zero elements along the diagonal. The resulting equations differ only in that it is convenient to use polar coordinates. As expected, no first derivative terms occur in this form. However, the problems observed with the Smit and Beljers form are also present in its generalization. The generalization of the Baselgia form thus seems the natural choice.

So far, very little mention has been made of the actual modes one observes. No generalization has yet been able to yield modes which fit for any free-energy model and the development of a method for obtaining resonances modes in general is open for investigation.

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APPENDIX: EFFECTIVE FIELDS IN ORDERED-STATE RESONANCE PROBLEMS

In the one-sublattice problem, the sublattices were assumed to be interacting with an effective field which could be expressed in terms of the gradient of a free energy.[17] For multisublattice systems, however, a single effective field is not adequate since each sublattice in general experiences different effective fields, although, it is likely that the crystal and magnetic symmetries would predict the effective fields to be the same for each sublattice. It is possible to define effective fields for each sublattice by definition of a sublattice-specific gradient operator:

$$\nabla_M^\sigma \equiv -[(\partial/\partial M_1^\sigma)\hat{1}^\sigma+(\partial/\partial M_2^\sigma)\hat{2}^\sigma+(\partial/\partial M_3^\sigma)\hat{3}^\sigma] , \tag{A1}$$

where now, in a magnetic system comprised of τ sublattices, one would also in general expect τ such distinct operators. The usual method for obtaining the effective fields which appear in the torque equations then is to allow this operator to act upon the free energy. It is apparent that the quantity $A_{ij}^{\alpha,\delta}$ can be expressed as the second derivatives of this free-energy expression, so, a revised definition for the $A_{ij}^{\alpha,\delta}$ is given by

$$A_{i,j}^{\alpha,\delta} \equiv (\partial^2 F/\partial M_i^\alpha \partial M_j^\delta) , \tag{A2}$$

where, for a conservative free energy,

$$(\partial^2 F/\partial M_i^\alpha \partial M_j^\delta) = (\partial^2 F/\partial M_j^\delta \partial M_i^\alpha) . \tag{A3}$$

One notational simplification is appropriate at this point. With derivatives denoted by subscripts as before, we let

$$F_{ij}^{\alpha,\delta} \equiv \partial^2 F/\partial M_i^\alpha \partial M_j^\delta \tag{A4}$$

and

$$F_{ij}^{\alpha,\delta}| \equiv \partial^2 F/\partial M_i^\alpha \partial M_j^\delta|_{\langle M_1^\delta=0, M_2^\delta=0, M_3^\delta=M\rangle} , \tag{A5}$$

where again Greek indices refer to sublattices.

The sublattice-specific gradient operator defined by Eq. (A1), which by analogy to the case of ferromagnetic resonance, yields effective fields, is defined with regard to local coordinates, while free-energy models must be defined with regard to laboratory coordinates. Under the assumption that the magnetization vector associated with sublattice σ orients in the direction of this effective field, one is able to obtain explicit equilibrium conditions for the angles Φ_σ and θ_σ by solving the equilibrium conditions. In ordered-state resonance problems, however, these equilibrium conditions are typically much more complicated. Also, the number of equations which must be simultaneously solved increases by two for each additional sublattice. Nevertheless, in principle, the equilibrium conditions can be solved. The solutions to the equilibrium conditions should apply to magnetic resonance experiments and also to magnetization measurements in mean-field approximations. Thus, we show explicitly how these conditions are obtained. If one applies Eq. (A1) to a free energy, it is seen that the effective field is given by

$$\mathbf{H}^\delta = -(\partial F/\partial M_1^\delta)\hat{1}^\delta-(\partial F/\partial M_2^\delta)\hat{2}^\delta-(\partial F/\partial M_3^\delta)\hat{3}^\delta . \tag{A6}$$

The sublattice magnetization is parallel to the $\hat{3}^\delta$ direction, and we see that the $\hat{1}^\delta$ and $\hat{2}^\delta$ components of magnetization must vanish. Hence, for sublattice σ, the equilibrium conditions are given by

$$(\partial F/\partial M_1^\sigma)|_{\langle M_1^\delta=0, M_2^\delta=0, M_3^\delta=M\rangle}=0 \tag{A7}$$

and

$$(\partial F/\partial M_2^\sigma)|_{\langle M_1^\delta=0, M_2^\delta=0, M_3^\delta=M\rangle}=0 , \tag{A8}$$

where δ ranges over all sublattices in the system. These two equations, though simple, become much more complicated for a system of τ sublattices since there are 2τ such equations which *usually* must be solved. In high-field limits this formulation permits direct substitution of magnetic field directions for magnetization directions and thus the solution to these equations is not needed in this limit. Before we proceed, however, a connection between laboratory and local coordinate systems must be obtained. In ordered-state resonance problems, the generalization of the transformation matrix is obvious and for sublattice σ, it is given by

$$\begin{bmatrix} x^\sigma \\ y^\sigma \\ z^\sigma \end{bmatrix} = \begin{bmatrix} \cos\theta_\sigma\cos\Phi_\sigma & -\sin\Phi_\sigma & \sin\theta_\sigma\cos\Phi_\sigma \\ \cos\theta_\sigma\sin\Phi_\sigma & \cos\Phi_\sigma & \sin\theta_\sigma\sin\Phi_\sigma \\ -\sin\theta_\sigma & 0 & \cos\theta_\sigma \end{bmatrix} \begin{bmatrix} 1^\sigma \\ 2^\sigma \\ 3^\sigma \end{bmatrix} . \tag{A9}$$

Thus, as before, each component of the sublattice magnetization M_j^σ in the free-energy expansion is replaceable by

$$M_j^\sigma = \sum_{k=1}^{3} b_{jk}^\sigma M_k^\sigma , \tag{A10}$$

with the elements b_{jk}^σ given by the transformation matrix above.

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