Distribution and localization of the harmonic magnon modes in a one-dimensional Heisenberg spin glass

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The harmonic magnon modes in a one-dimensional Heisenberg spin glass having nearest-neighbor exchange interactions of fixed magnitude and random sign are investigated. The Lyapounov exponent is calculated for chains of 10^7–10^8 spins over the interval $0 \leq \omega \leq 4J$. In the low-frequency regime, $\omega \lesssim 0.1J$, an anomalous behavior for the density of states $\rho(\omega) \sim \omega^{-1/3}$ is established, consistent with earlier results obtained by Stinchcombe and Pimentel using transfer-matrix techniques; at higher frequencies, gaps appear in the spectrum. At low frequencies, the localization length diverges as $\omega^{-2/3}$. A formal connection is established between the spin glass and the one-dimensional discretized Schrödinger equation. By making use of the connection, it is shown that the theory of Derrida and Gardner, which was developed for weak potential disorder, can account quantitatively for the distribution and localization of the low-frequency magnon modes in the spin-glass model.

I. INTRODUCTION

In a recent paper, Stinchcombe and Pimentel[1] studied the harmonic spin dynamics in a one-dimensional Heisenberg spin glass with nearest-neighbor exchange interactions of fixed magnitude and random sign ($J_{i,i+1} = \pm J$). Using a new transfer-matrix scaling technique, they were able to show that the system has anomalous dynamics in the low-frequency–long-wavelength regime. They found behavior consistent with the relation $\omega_c \propto k^{3/2}$, in contrast to the conventional hydrodynamic picture where the characteristic frequency, ω_c, is proportional to the wave vector, i.e., $\omega_c \propto k$. The purpose of this paper is to extend the investigation of the one-dimensional spin glass that was begun in Ref. 1. Using mode-counting techniques, we determine the distribution and localization of the modes over the entire spectrum. By exploiting an analogy with the discretized one-dimensional Schrödinger equation, we are able to establish a connection between the anomalous spin dynamics and the density of electronic states at the band edge in the weak-disorder limit.

Our starting point is the set of linearized equations for the spins, which can be written in a form similar to that displayed in Eq. (5) of Ref. 1:

$$(2-\xi_n\omega)u_n = u_{n+1} + u_{n-1} . \quad (1)$$

Here ω is the frequency in units of J and $\xi_n = (+/-)1$ depending on whether the nth spin points up or down in the classical ground state. The amplitude u_n is related to the transverse spin operator S_n^+ through the equation $u_n = \xi_n S_n^+$. Equation (1) is analogous to the discretized one-dimensional Schrödinger equation with a random potential λV_n. This equation assumes the form[2]

$$(E - \lambda V_n)\psi_n = \psi_{n+1} + \psi_{n-1} . \quad (2)$$

Comparing (2) with (1), it becomes apparent that the one-dimensional Heisenberg spin glass is a special case of (2) corresponding to $E = 2$ and $\lambda V_n = \xi_n \omega$.[3] Because of this analogy, one can apply results obtained for the electronic problem to the spin glass. In particular, the low-frequency regime studied in Ref. 1 corresponds to the weak-disorder limit of the random-potential model at $E = 2$, the band edge of the system when $\lambda = 0$.

II. LYAPOUNOV EXPONENT

Insight into the spin-glass dynamics can be obtained from a study of the complex Lyapounov exponent,[2] $\gamma(\omega)$, which is defined in terms of the amplitude ratio. With

$$R_n = u_n / u_{n-1} \quad (u_0 = 1) , \quad (3)$$

one has

$$\gamma(\omega) = \frac{1}{N} \sum_{n=1}^{N} \ln R_n , \quad (4)$$

where N is the number of spins in the chain. Separating γ into its real and imaginary parts, one obtains

$$\mathrm{Re}\gamma(\omega) = \frac{1}{N} \sum_{n=1}^{N} \ln|R_n| , \quad (5)$$

and

$$\mathrm{Im}\gamma(\omega) = \pi S_N / N , \quad (6)$$

where S_N is the number of negative signs in the sequence $R_1, R_2, \ldots, R_N$.

As pointed out by Thouless,[4] $\mathrm{Re}\gamma$ is the inverse localization length (in units of the reciprocal of the lattice constant). In the context of the electronic problem, $\mathrm{Im}\gamma/\pi$ is the integrated density of states, while in the case of the spin glass, arguments similar to those developed for the disordered antiferromagnet[5,6] show that $\mathrm{Im}\gamma(\omega)/\pi$ is the number of magnon modes in the interval between 0 and ω.[7]

In Ref. 2, analytic calculations of the Lyapounov exponent in the weak-disorder limit are outlined that can be used to determine the behavior of $\gamma(\omega)$ in the low-frequency regime.[8] For $\mathrm{Re}\gamma$ one obtains

$$\mathrm{Re}\gamma(\omega)=0.2893\ldots\omega^{2/3}, \tag{7}$$

whereas for $\mathrm{Im}\gamma$ one has

$$\mathrm{Im}\gamma(\omega)/\pi=0.1595\ldots\omega^{2/3}. \tag{8}$$

Differentiating (8) with respect to ω, one finds for the magnon density of states

$$\rho(\omega)=0.1063\ldots\omega^{-1/3}, \tag{9}$$

in which the functional dependence is seen to be in agreement with Ref. 1.

Equation (9) is consistent with a dispersion relation

$$\omega=A(ka)^{3/2}, \tag{10}$$

where a is the lattice constant and $A\approx 1$. With this interpretation, Eq. (7) implies that the localization length of a mode of wave vector k is inversely proportional to k itself.[9]

III. NUMERICAL RESULTS AND DISCUSSION

In this section, we first report the results of tests of the Derrida-Gardner predictions, Eqs. (7) and (8), for chains of 10^7 spins. Approximate power-law behavior was observed for $\omega\lesssim 0.1$. Numerical data were obtained for $0.0001\le\omega\le 0.001$ at intervals of 0.0001, for $0.001\le\omega\le 0.01$ at intervals of 0.001, and for $0.01\le\omega\le 0.1$ at intervals of 0.01. The data were fit to the functional form $C\omega^x$, with C and x as adjustable parameters. For $\mathrm{Re}\gamma$, we obtained $C=0.2847$ and $x=0.665$, whereas for $\mathrm{Im}\gamma/\pi$ the values were $C=0.1620$ and $x=0.668$, both pairs of parameters being in good agreement with the theory, for which $C=0.2893$, $x=0.667$ ($\mathrm{Re}\gamma$) and $C=0.1595$, $x=0.667$ ($\mathrm{Im}\gamma/\pi$).

By taking the difference between successive values of $\mathrm{Im}\gamma(\omega)/\pi$ one obtains a histogram of the distribution of positive frequency modes. The results of such a calculation are shown in Fig. 1, while the corresponding inverse localization lengths are displayed in Fig. 2. The irregular structure in the density of states is independent of the particular sequence of ξ_n and thus appears to be a characteristic feature of the infinite system.

From Fig. 1 it is evident that there are gaps in the spectrum for $2.0<\omega<2.1$, $3.0<\omega<3.1$, and $3.4<\omega<3.5$. A higher-resolution study carried out for 10^8 spins in steps of 0.01 shows that on the finer scale the gaps are restricted to $2.00<\omega<2.08$, $3.00<\omega<3.05$, and $3.41<\omega<3.44$. Additional gaps over smaller intervals are also present. For example, in chains of 10^8 spins, no modes were detected in the intervals $3.73<\omega<3.74$ and $3.80<\omega<3.81$. In contrast to the density of states, the inverse localization length (Fig. 2) shows a rather smooth increase with ω, apart from cusplike behavior at high frequencies.

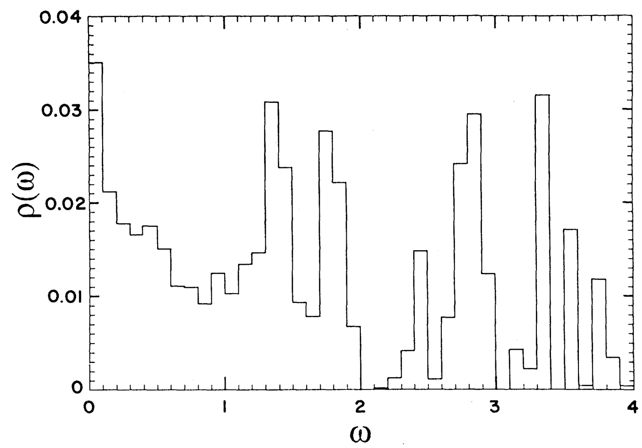

FIG. 1. Histogram of the density of states, $\rho(\omega)$, for a chain of 10^7 spins. Frequency in units of J, $\Delta\omega=0.1$. $\rho(\omega)$ is the difference between $\mathrm{Im}\gamma(\omega)/\pi$ and $\mathrm{Im}\gamma(\omega-\Delta\omega)/\pi$.

Finally, we note that the analysis presented here involves only the distribution and localization of the modes. The response of the system to finite wavelength disturbances is also of interest. To probe this behavior, we are undertaking a numerical calculation of the dynamic structure factor.

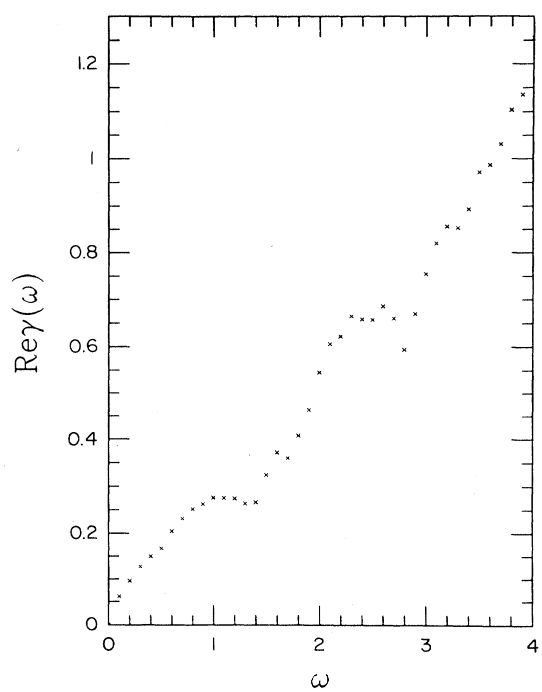

FIG. 2. $\mathrm{Re}\gamma(\omega)$, inverse localization lengths, vs ω for a chain of 10^7 spins. Frequency in units of J; inverse localization length in units of the reciprocal of the lattice constant.

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[7]In the interpretation of the solutions of the eigenvalue equation, positive and negative eigenvalues are associated with distinct magnon eigenstates. In the large N limit, the distribution of eigenvalues is symmetric about $\omega=0$.

[8]Ref. 2, Eqs. (41) and (42), with $\lambda^2\langle V^2\rangle$ replaced by ω^2.

[9]Strictly speaking, Stinchcombe and Pimentel showed that the characteristic frequency of functions like the spatial and temporal Fourier transform of the dynamic spin-spin correlation function varied with wave vector as $k^{3/2}$. The picture of weakly damped excitations implicit in (10) is for illustrative purposes only.