Electromagnetic response of layered superconductors

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The electromagnetic surface impedance of a layered superconductor has been examined within the framework of the BCS theory. The dependences on the coherence length and the mean free path of the electrons, the ambient temperature, and the frequency of the incident microwaves have been calculated for a *c*-axis normal-oriented film.

I. INTRODUCTION

Microwave and far-infrared properties of superconductors have played an important role in our understanding of the origin of superconductivity in conventional superconductors.[1] Recently, results of microwave impedance measurements on the high-T_c superconducting oxides have begun to appear.[2,3] Therefore, it seems appropriate to carry out a theoretical investigation of the microwave impedance of this class of materials. Lacking a detailed microscopic theory for the superconducting oxides, we perform our calculation based on the BCS theory of *s*-wave pairing.[4]

The microwave properties of the conventional superconductors are usually discussed for two limiting cases. In the clean limit where the penetration depth is much shorter than both the coherence length and the mean free path, the response of the current to the field is nonlocal, and one is led to consider the extreme anomalous skin effect.[5] Mattis and Bardeen showed that in this limit the microwave impedance can be written in terms of a frequency-dependent complex conductivity which they calculated.[6] In the dirty limit where the penetration depth of the external field is larger than the coherence length and the mean free path, the current response to the external field is local, and one has the London limit.[7] Between these two limits, one has to take into consideration the wave-vector dependence, as well as the frequency dependence, of the response kernel $K(q,\omega)$, and the problem becomes more complicated.[8,9]

The superconducting copper oxides differ from conventional superconductors in many ways. In addition to having significantly higher transition temperatures, they are anisotropic with the one-electron transfer in the c direction, being much weaker than that in the a and b directions.[10] Furthermore, the coherence length in these materials is of the order of tens of angstroms,[11] which is much shorter than the penetration depth of the magnetic field which is of the order of several thousand angstroms.[11] This is in contrast to conventional clean superconductors in which the coherence length is typically much larger than the penetration depth.[12] In view of these observations, we have studied a simple model of a c-axis normal-oriented film. As a zero-order approximation, we have neglected the electron transfer between the planes. In this case, only the component of the wave vector of the electromagnetic field parallel to the layers enters, and it is effectively zero for the micrometer and millimeter waves of interest. The electromagnetic response kernel relating the current density J to the vector potential A is therefore independent of the wave vector and only depends on the frequency ω. Thus, the electromagnetic response of the layered superconductor is described by a local London relation.

Our calculation is carried out within the framework of the BCS theory. It allows a detailed study of the dependence of the surface impedance on the coherence length and the mean free path of the electrons, as well as the ambient temperature and the frequency. We find that when the mean free path is of the order of the coherence length, the microwave surface resistance *increases* with increasing mean free path. This curious effect, previously noted by Halbritter,[9] occurs when the reactive part of the surface impedance is large compared to the resistive part and the microwave frequency is smaller than the scattering rate.

II. CALCULATION PROCEDURE AND RESPONSE KERNEL

The procedure of our investigation is simple. We first calculate the response kernel $K(\omega)$ for one sheet of a two-dimensional (2D) superconductor. We assume the electronic structure in the sheet is isotropic and that the electronic dispersion relation near the Fermi surface is described by an effective mass. The kernel $K(\omega)$ is an intrinsic property of the 2D superconductor. Once the kernel is known, the microwave impedance can be calculated using Maxwell equations along with appropriate boundary conditions.

The kernel which describes the response of the current density (current per unit length for a two-dimensional sheet) to the vector potential is defined by the equation

$$J(\omega)=-\frac{K(\omega)}{\mu_0}A(\omega)\ . \tag{1}$$

It can be easily shown to have the following form for the two-dimensional superconducting sheet:[13]

$$K(\omega)=\frac{ne^2\mu_0}{2m^*}F(\omega)\ . \tag{2}$$

Here n is the number of electrons per unit area, and

$$F(\omega)=\int_{\Delta}^{\infty}dy\left[[1-2f(y+\hbar\omega,T)]\left[-\frac{g(y)-1}{\epsilon_{+}+\epsilon_1-i\hbar/\tau}+\frac{g(y)+1}{\epsilon_{+}-\epsilon_1+i\hbar/\tau}\right]-[1-2f(y,T)]\left[\frac{g(y)-1}{\epsilon_{+}+\epsilon_1+i\hbar/\tau}+\frac{g(y)+1}{\epsilon_{+}-\epsilon_1+i\hbar/\tau}\right]\right]+\int_{\Delta-\hbar\omega}^{\Delta}dy[1-2f(y+\hbar\omega,T)]\left[\frac{g(y)+1}{\epsilon_{+}-\epsilon_1+i\hbar/\tau}-\frac{g(y)-1}{\epsilon_{+}+\epsilon_1-i\hbar/\tau}\right]\ , \tag{3}$$

where

$$\epsilon_1\equiv\begin{cases}(\mathrm{sgn}y)(y^2-\Delta^2)^{1/2} & \text{for } |y|>\Delta\ ,\\ -i(\Delta^2-y^2)^{1/2} & \text{for } |y|<\Delta\ ,\end{cases}$$
$$\epsilon_{+}\equiv[(y+\hbar\omega)^2-\Delta^2]^{1/2}\ , \tag{4}$$
$$g(y)\equiv\frac{y(y+\hbar\omega)+\Delta^2}{\epsilon+\epsilon_1}\ ,$$

and $f(y,T)$ is the usual Fermi function $(e^{y/k_BT}+1)^{-1}$. In addition to the normalization factor $ne^2\mu_0/2m^*$, we must specify ω/Δ_0 and l/ξ_0, the ratio of the mean free path ($l=v_F\tau$), and the zero-temperature coherence length $\xi_0\equiv\hbar v_F/\pi\Delta_0$.

Often, instead of using the kernel $K(\omega)$, it is convenient to introduce the complex conductivity related to the kernel by[12]

$$\sigma(\omega)=-i\frac{K(\omega)}{\mu_0\omega}=\sigma_1-i\sigma_2\ . \tag{5}$$

In Fig. 1, we show the real and imaginary parts of the conductivity as functions of the mean free path normalized to $\pi\xi_0$ for two different frequencies.[14] In our numerical calculation, we have used a frequency- and temperature-independent electron lifetime. This implies that the mean free path of the electrons is limited by elastic impurity scatterings rather than an intrinsic dynamic interaction. The frequencies are chosen so that for a superconductor with a critical temperature $T_c=125$ K and $2\Delta_0/k_BT_c=3.52$, they correspond to 10 and 100 GHz. Figure 1(a) shows that the real part of the conductivity σ_1 increases as the mean free path increases until the mean free path is so high that $\omega\tau$ is of order unity. When $\omega\tau>1$, σ_1 begins to decrease due to the fact that normal electrons also become effective in screening the external field. Figure 1(b) shows that the imaginary part of the conductivity σ_2 increases with increasing mean free path and reaches an asymptotic value in the extreme clean limit. Since the penetration depth is proportional to the inverse of the square root of σ_2, this simply reflects the fact

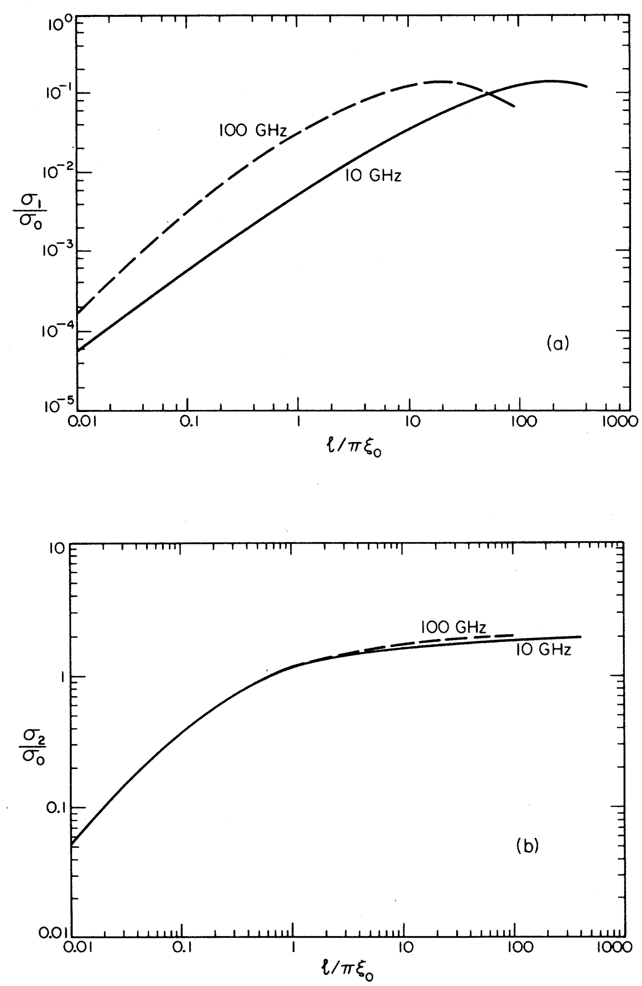

FIG. 1. Mean-free-path dependence of (a) σ_1 and (b) σ_2, the real and imaginary part of the conductivity for frequencies $f=10$ (solid curve) and 100 GHz (dashed curve). The mean free path is normalized to π times the coherence length ξ_0, and the σ is normalized to $\sigma_0=ne^2/2m^*\omega$.

that the penetration depth is smaller for cleaner superconductors. Figure 1(b) also shows that σ_2 is only weakly dependent on frequency when the frequency is small compared to $2\Delta(T)/h$.

The surface impedance is given by

$$Z(\omega)=\frac{E(0)}{H(0)} , \tag{6}$$

where $E(0)$ and $H(0)$ are the electric and magnetic fields at the surface. As usual, we will neglect the displacement current. Then, for a semi-infinite stack of two-dimensional superconducting sheets separated by a constant spacing d, Ampère's law gives

$$\frac{A_{n+1}+A_{n-1}-2A_n}{d^2}=-\frac{\mu_0}{d}J_n . \tag{7}$$

Using Eq. (1) for J_n and assuming that the penetration depth is large compared to the film thickness d, Eq. (7) is well approximated by

$$\frac{\partial^2 A}{\partial z^2}=KA , \tag{8}$$

where n appearing in K, Eq. (2), is now the usual density of electrons per unit volume (the previous density of electrons per unit area divided by d). Applying the boundary condition $A(z=\infty)=0$, one obtains

$$A(z,\omega)=A(0,\omega)e^{-z/\lambda(\omega)} , \tag{9}$$

with $\lambda(\omega)=K^{-1/2}(\omega)$. Then the surface impedance, Eq. (6), is

$$Z(\omega)=i\mu_0\omega\lambda(\omega)=\left[\frac{i\mu_0\omega}{\sigma(\omega)}\right]^{1/2} , \tag{10}$$

which is the same result calculated from the integral equations for specular reflection or diffusive scattering. One expects both cases to give the same result in the present consideration because the kernel for the two-dimensional superconductor is independent of the wave vector, and hence we are in the local London limit.

III. MICROWAVE SURFACE IMPEDANCE $Z(\omega)$

In this section, we present the result of our calculation showing the dependences of $Z(\omega)$ on the mean free path, temperature, and frequency.

A. Mean-free-path dependence of $Z(\omega)$

The dependence of the surface resistance $R_s=\mathrm{Re}Z(\omega)$ on the mean free path is shown in Fig. 2(a) for three different frequencies. The reduced temperature is 0.5, and we have taken the ratio $2\Delta_0/k_BT_c=3.52$. The scale factor R_0 is

$$R_0\equiv\left[\frac{2m\mu_0}{ne^2}\right]^{1/2}\left[\frac{k_BT_c}{\hbar}\right] \tag{11}$$

and is chosen to be independent of frequency, temperature, and mean free path. Using typical values[15] for the parameters, R_0 is of order 1 $\Omega/\square$. The curves a, b, and c correspond to frequencies of 1, 10, and 100 GHz, respectively. It is interesting to observe that there is a region in which R_s *increases with increasing mean free path.* Furthermore, the region for this behavior is larger for smaller frequencies. The difference of the maximum and minimum of the surface resistance also increases with decreasing frequency. This curious effect of a cleaner superconductor having more microwave loss has also been noted for three-dimensional superconductors and indeed has been observed in lead-bismuth alloys, although the observed effect is small.[9] Here it follows quite generally from Eq. (10). At low temperature, when $\sigma_2\gg\sigma_1$,

$$R_s\simeq\frac{1}{2}\left[\frac{\mu_0\omega}{\sigma_2}\right]^{1/2}\frac{\sigma_1}{\sigma_2} . \tag{12}$$

Thus the surface resistance is *directly proportional* to σ_1, the real part of the conductivity, which, as shown in Fig. 1, increases with increasing mean free path for $\omega\tau<1$. Therefore, when the mean free path is increased, the surface resistance increases if σ_1 increases sufficiently fast

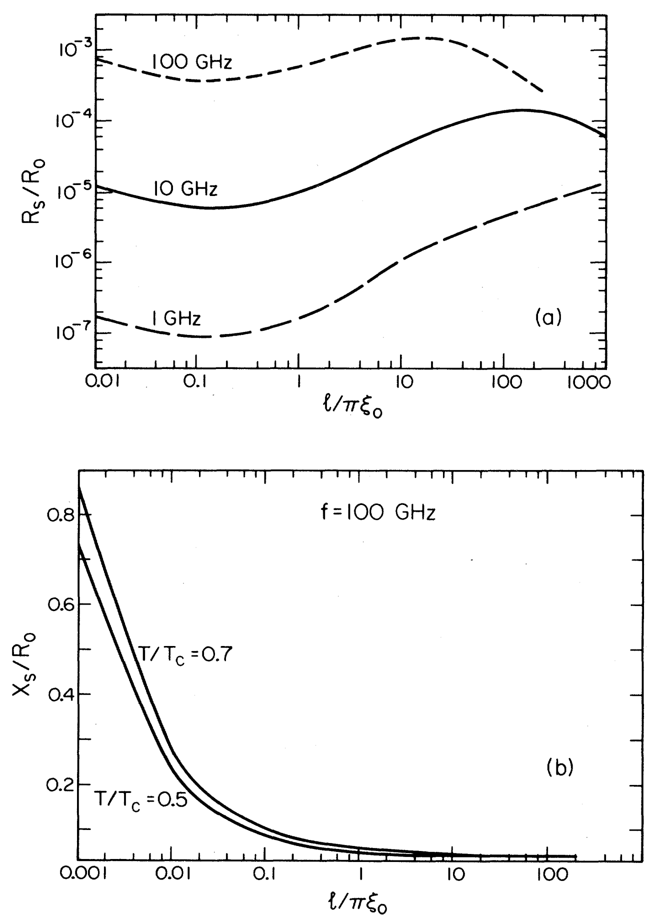

FIG. 2. Surface resistance (a) and surface reactance (b) as functions of the normalized mean free path. The normalization factor for the surface impedance R_0 is given by Eq. (9) and is of order $1\Omega/\square$.

relative to σ_2. This phenomenon is similar to the power loss of a parallel LR circuit with $\omega L \ll R$ fed by an ac current source. In such a circuit, the power loss increases when R is decreased because of the increasing amount of current which flows through the resistor.

The surface reactance versus the normalized mean free path is shown in Fig. 2(b) for $f=100$ GHz and several reduced temperatures. It decreases with increasing mean free path. Since X_s is directly proportional to the penetration depth, Eq. (10), this is simply a reflection of the well-known fact that clean superconductors have shorter penetration depths than impure superconductors.

B. Temperature dependence of $Z(\omega)$

Figures 3 and 4 show R_s and X_s normalized to R_0 versus T/T_c for $f=10$ and 100 GHz, respectively. Here we take $2\Delta_0/k_B T_c=3.52$. The value of the mean free path for each curve is labeled. R_s decreases rapidly as the temperature is reduced below T_c. When the ratio of $2\Delta_0/k_B T_c$ is increased, the rate of reduction is even faster. The mean-free-path dependence of R_s previously discussed is responsible for the ordering of the curves.

The surface reactance $X_s(\omega)$ is weakly dependent on the temperature except near T_c, where it increases rapidly. Because it is a monotonically increasing function of the mean free path, there are no crossings of the curves.

C. Frequency dependence of $Z(\omega)$

Figures 5 and 6 show R_s and X_s versus frequency for $T/T_c=0.5$ and 0.7, respectively. All curves for R_s increase with increasing frequency except for the very clean case in which they saturate at high frequencies as shown. This is due to the large density of states near the gap edge which leads to an R_s which is a decreasing function of frequency at high frequencies for clean superconductors. Again the crossing of the curves due to the mean-free-

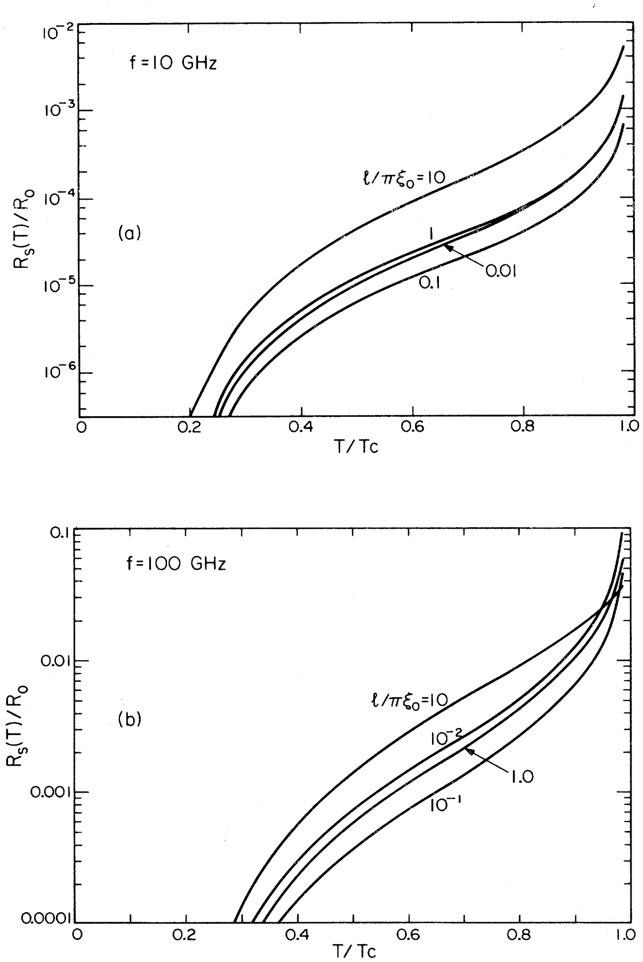

FIG. 3. Normalized surface resistance vs reduced temperature for microwave frequencies (a) $f=10$ GHz, and (b) $f=100$ GHz. The different curves correspond to different mean free paths as indicated.

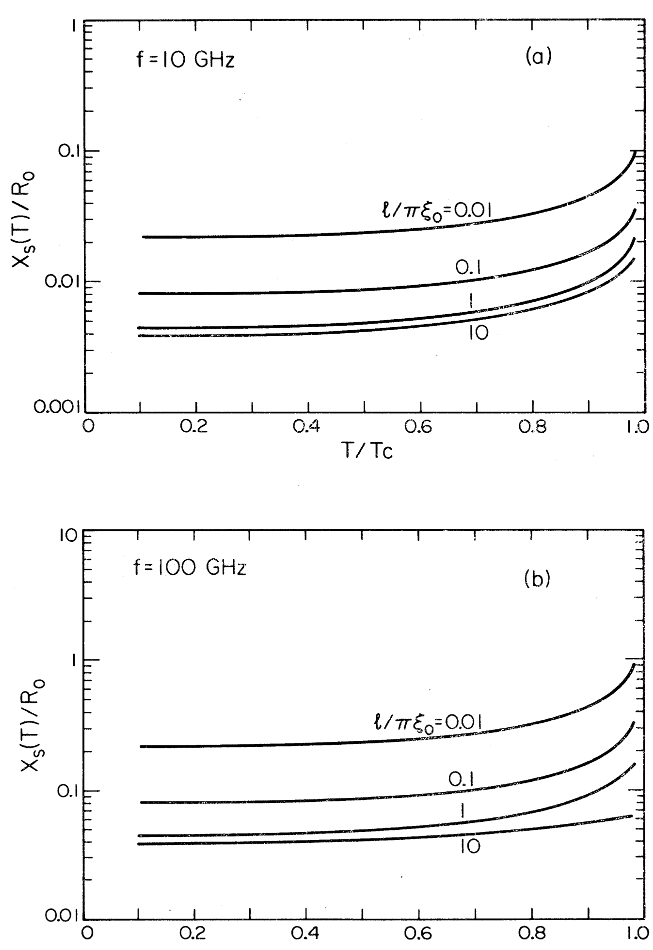

FIG. 4. Normalized surface reactance vs reduced temperature for microwave frequence (a) $f=10$ GHz, and (b) $f=100$ GHz. The different curves correspond to different mean free paths as indicated.

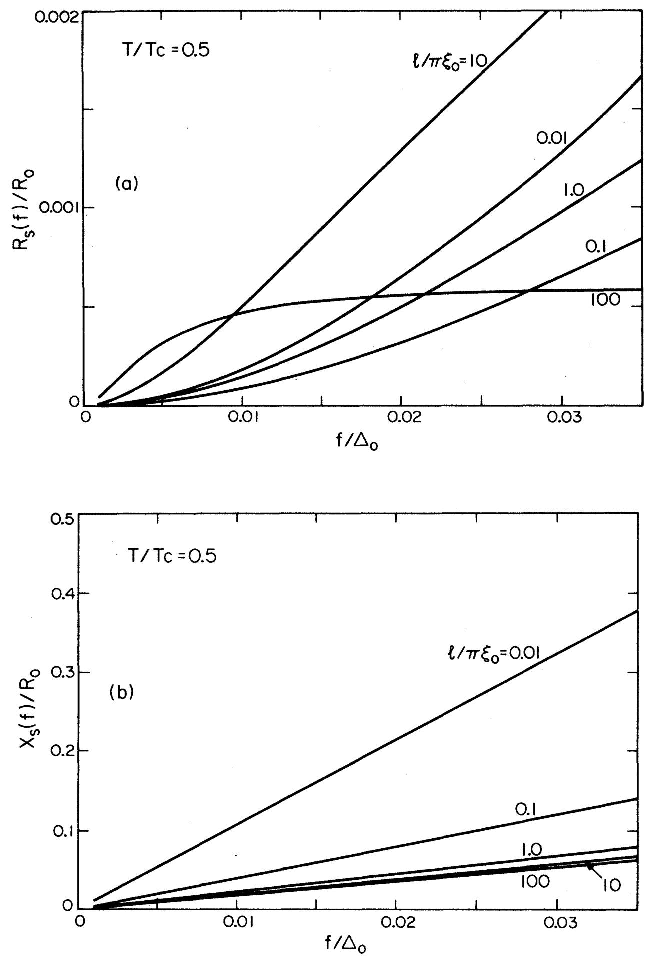

FIG. 5. Frequency dependences of the normalized (a) surface resistance and (b) surface reactance for a reduced temperature $T/T_c=0.5$. The different curves correspond to different values of the mean free path.

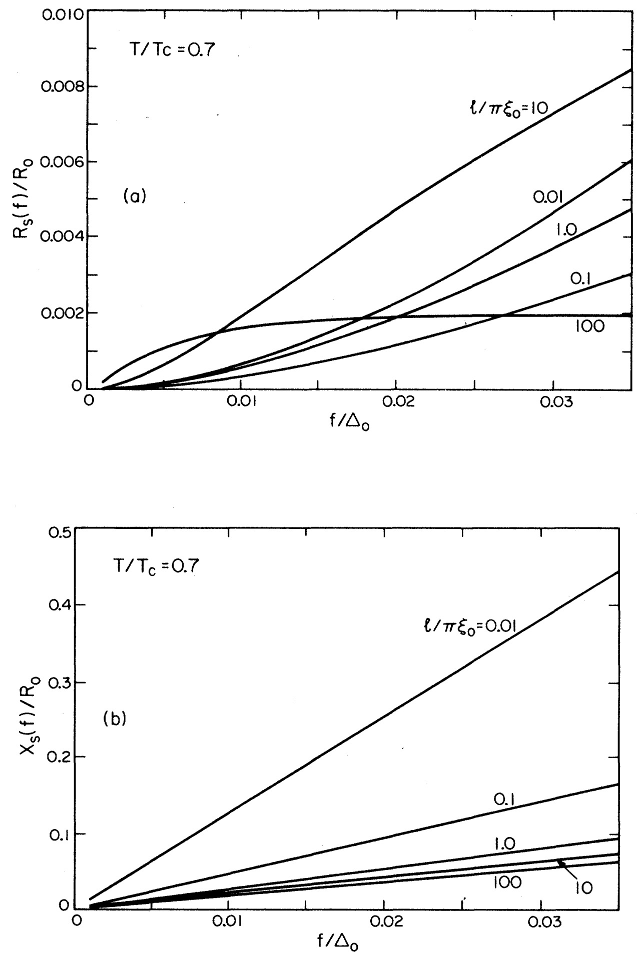

FIG. 6. Frequency dependences of the normalized (a) surface resistance and (b) surface reactance for a reduced temperature $T/T_c=0.7$. The different curves correspond to different value indicated for the mean free path.

path dependence is clearly visible. The dependence of X_s on the frequency is linear. This implies that the penetration depth is independent of the frequency in the range $\omega \ll 2\Delta/\hbar$ considered here.

IV. THIN STACK OF LAYERED SUPERCONDUCTORS

Finally, we consider a c-axis normal oriented film of thickness s on a semi-infinite dielectric substrate. By applying Maxwell equations and matching the electric and magnetic fields at the interface, where both must be continuous, one obtains for the effective surface impedance[15]

$$Z_{\text{eff}}(\omega)=Z_f\frac{\coth(s/\lambda)+Z_f/Z_{\text{sub}}}{1+(Z_f/Z_{\text{sub}})\coth(s/\lambda)} . \tag{13}$$

Here Z_{sub} and Z_f are, respectively, the semi-infinite surface impedance of the substrate and the layered superconductor, and $\lambda(\omega)$ is the ω-dependent penetration depth obtained from X_s. In the usual case in which the material that forms the substrate has a high surface impedance relative to Z_f and $s/\lambda \gg Z_f/Z_{\text{sub}}$,

$$Z_{\text{eff}}=Z_f\coth(s/\lambda) . \tag{14}$$

For films with thickness s small compared to λ, but large compared to $\lambda Z_f/Z_{\text{sub}}$,

$$Z_{\text{eff}}\cong Z_f\lambda/s . \tag{15}$$

When s is small compared to $\lambda Z_f/Z_{\text{sub}}$, Z_{eff} is equal to Z_{sub}.

V. SURFACE IMPEDANCE IN THE INFRARED REGION

We have extended our calculation to study the surface impedance in the infrared region where the frequency can be of order 2Δ. At these frequencies

$$\omega\tau=(\omega/\Delta_0)(l/\xi_0)$$

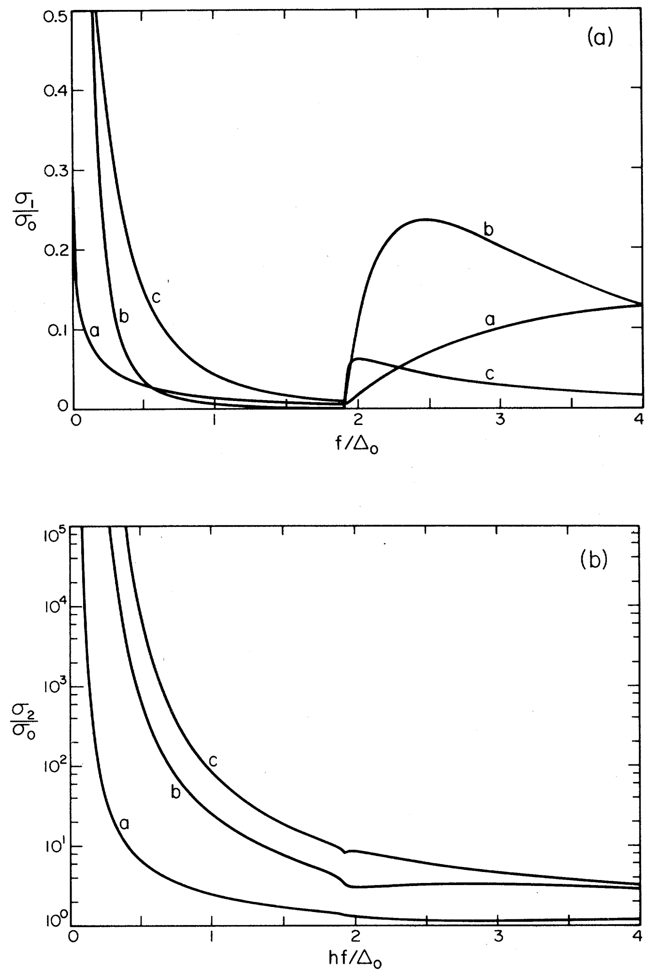

FIG. 7. Frequency dependence of (a) σ_1 and (b) σ_2 for three different values of the normalized mean free path $l/\pi\xi_0$. Here σ_1 and σ_2 are normalized with respect to $\bar{\sigma}_0 = ne^2/2m^*\Delta_0$.

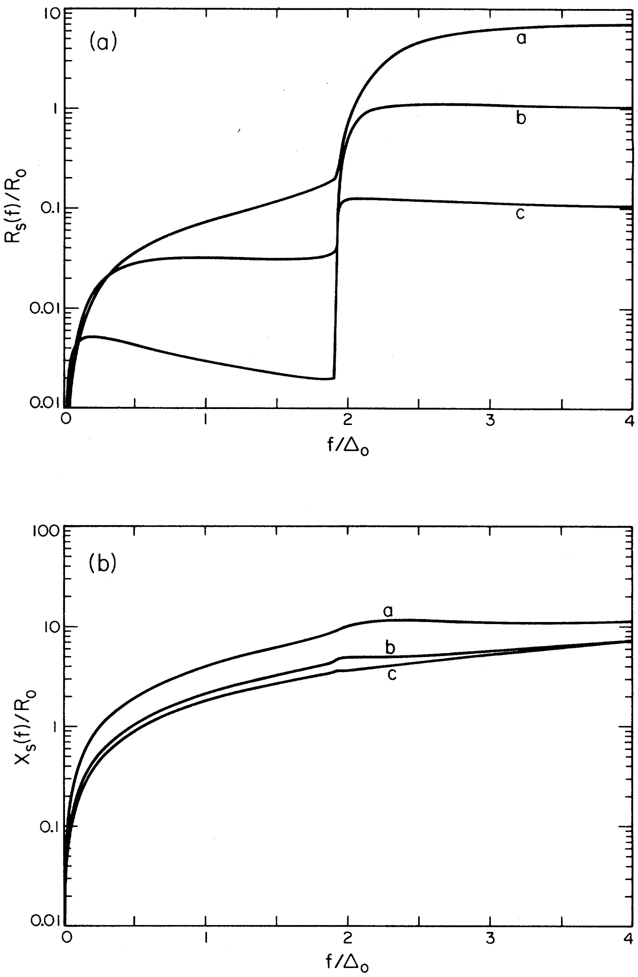

FIG. 8. Frequency dependences of the normalized (a) surface resistance and (b) surface reactance for a reduced temperature $T/T_c = 0.5$. The different curves correspond to different values of the mean free path: $l/\pi\xi_0 = 0.1$ (curve a), 1.0 (curve b), and 10 (curve c). This is the same as Fig. 5 except for the extended frequency range.

is of order unity and the normal electrons begin to participate in screening. The curious mean-free-path effect discussed in the preceding section becomes more complicated. In Fig. 7 we show the real and imaginary parts of the conductivity as functions of the frequency (normalized to Δ_0/h) of the radiation, with the normalized mean free path $l/\pi\xi_0 = 0.1$, 1.0, and 10. Note that the conductivities are normalized to a frequency independent value different from the σ_0 used in Fig. 1. Again, the three curves for the $\sigma_1(\omega)$ intersect. We also notice that for frequency $f > 2\Delta_0/h$ there is a peak structure in $\sigma_1(\omega)$ which moves to higher frequency and broadens as the mean free path l is reduced. Furthermore, this peak structure is more pronounced when $l \sim \pi\xi_0$. The corresponding surface resistance and surface reactance, both normalized to R_0, have been calculated and are shown in Fig. 8.

VI. DISCUSSION AND CONCLUSIONS

We have studied the microwave response of a layered superconductor in which the one-electron transfer between the layers is neglected. This can be considered to be the zero-order approximation for a c-axis normal oriented film of the superconducting oxides. It was found that due to the reduced dimensionality, the electromagnetic response of the layered superconductor is in the local London limit. It is important to emphasize that in the case we have investigated, we have assumed that the mean free path is limited by elastic impurity scattering. We have not attempted to treat the dynamic problem in which the interaction responsible for the pairing induces scattering. This greatly simplified the problem and allowed a detailed investigation of the mean-free-path dependence of the microwave impedance in addition to its frequency and temperature dependences. One curious result is the prediction of an increased surface resistance with increasing mean free path when the coherence length is comparable to or slightly smaller than the mean free path.

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