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Polaron cyclotron-resonance mass in a single GaAs quantum well

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The cyclotron-resonance mass of a quasi-two-dimensional electron gas in a single GaAs quantum well is investigated as a function of the magnetic field strength, the electron density, and the quantum-well width. The screening of the electron-phonon interaction is taken into account within the static random-phase approximation. The theoretical results are compared with available experimental data.

During the past few years, the cyclotron resonance of a quasi-two-dimensional (Q2D) electron gas interacting with longitudinal-optical (LO) phonons (e.g., polarons) has been extensively studied, both experimentally[1–9] and theoretically.[10–14] In this paper we present a theoretical study of the cyclotron-resonance mass of a quasi-2D polaron gas confined in a single quantum well. This work, which is a generalization of our previous study[14] of polaron cyclotron resonance in semiconductor heterostructures, is motivated by two recent cyclotron-resonance experiments performed on quantum wells.[8,9] In Ref. 8, Zeismann *et al.* performed a cyclotron-resonance experiment on an InAs quantum well ($n_e = 8.8 \times 10^{11}$ cm^{-2}) where no apparent polaron effects were found for the magnetic fields investigated ($B < 10$ T). This result can be understood in light of existing experimental data from GaAs heterostructures where it was found that polaron effects for large electron densities are considerably reduced in comparison with those in 3D systems.[1–9,15] In Refs. 16 and 17 it was shown that most of the earlier theoretical papers, where the nonzero width of the 2D electron layer and many-particle effects were neglected, have to be modified in order to correctly describe the experimental findings.

More recently, Singleton *et al.*[2] extended their work on GaAs heterostructures to the case of a single GaAs quantum well. The polaron effects were found to be almost negligibly enhanced over the predicted 3D values. In the present paper, we will discuss these experimental results in detail.

In Ref. 16 the present authors developed a theory to calculate the magnetoabsorption spectrum from which the polaron correction to the cyclotron-resonance mass can be deduced. In this theory the nonzero width of the 2D electron layer and many-particle effects are taken into account for the case of GaAs-$Al_xGa_{1-x}As$ heterostructures. We found[17] that by including these effects together with the band nonparabolicity our theory agrees very well with the experimental results for different electron densities. In the present paper we extend the calculation of Ref. 16 to the case of a *single quantum well.* A detailed account of the dependence of the polaron cyclotron-resonance mass on the magnetic field, the electron density, and the quantum-well width will be given. Also, a comparison with available experiments will be made.

There are three recent theoretical papers concerning the cyclotron resonance of a 2D electron gas in quantum wells where the interaction with phonons[18–20] was taken into account. However, (1) the screening of the electron-phonon interaction was not included in these papers, (2) in Refs. 18 and 19, severe approximations were made in order to obtain a closed form for their results, and (3) only the cyclotron-resonance linewidth due to electron–acoustic-phonon interaction was calculated in Ref. 19.

Before we present the theoretical results, a brief outline of the theoretical approach will be given. For simplicity, the electron energy band will be taken as parabolic. Nonparabolicity can be included within a local parabolic band approximation as shown in Ref. 16. This approximation was justified for GaAs as shown by Larsen.[21] Within the local parabolic band approximation, the electron energy band nonparabolicity is first calculated and the resulting effective mass is then used as an input for the calculation of polaron effects. The electrons are assumed to occupy only the lowest electric subband, which is a good approximation as long as the electron density is not too high. The experiments are typically performed at liquid-helium temperature, which implies that it is a good approximation to take the temperature equal to zero in our calculation. The electrons are assumed to interact with *bulk LO phonons* and the interaction is described by the standard Fröhlich coupling. The electron–LO-phonon interaction will be treated within perturbation theory since the Fröhlich coupling constant for GaAs is $\alpha = 0.068 \ll 1$. The screening of the electron–LO-phonon interaction will be considered within a static random-phase-approximation (RPA) screening approximation. The validity of this latter approximation is well established, since, as long as the broadening of the Landau levels is neglected (the same approximation will also be used in the present paper), a dynamical screening theory will give almost the same numerical results as the static screening theory in the resonant polaron region, which is the region of most interest to us. This was shown by the present authors in Ref. 22.

Following Ref. 16, the dynamical conductivity of the system can be written as

$$\sigma(\omega)=\frac{in_e e^2/m_b}{\omega-\omega_c-\Sigma(\omega)}\ , \tag{1}$$

where ω is the frequency, $\omega_c=eH/cm_b$ the unperturbed cyclotron frequency, m_b the electron band mass, and $\Sigma(\omega)$ the so-called memory function. Within the above approximation the polaron cyclotron-resonance frequency is given by the solution ($\omega=\omega_c^*=eH/cm^*$) of

$$\omega-\omega_c-\mathrm{Re}\Sigma(\omega)=0\ , \tag{2}$$

and the real part of the memory function can be written as

$$\mathrm{Re}\Sigma(\omega)=\sum_k \frac{k_\parallel^2}{n_e m_0\omega}|V_k|^2\frac{\omega^2}{\pi}\int_{-\infty}^{\infty}dx\frac{[1+n(x)]\mathrm{Im}\Pi^R(k_\parallel,x)}{(x+\omega_0)^2-\omega}$$
$$+n(\omega_0)\sum_k \frac{k_\parallel^2}{n_e m_b\omega}|V_k|^2\tfrac{1}{2}[\mathrm{Re}\Pi^R(k_\parallel,\omega+\omega_0)+\mathrm{Re}\Pi^R(k_\parallel,\omega-\omega_0)-2\,\mathrm{Re}\Pi^R(k_\parallel,\omega_0)]\ , \tag{3}$$

where $\Pi=D/\hbar$ is the polarization function of the 2D electron gas, ω_0 is the LO-phonon frequency, $n(x)=1/(e^{\beta\hbar x}-1)$, and $k_\parallel^2=k_x^2+k_y^2$. For a quantum well with infinitely high barriers, the summation over k_z in Eq. (3) introduces a form factor

$$f(u)=\frac{2}{u}\left[1-\frac{1-e^{-u}}{u}\right]+\frac{u}{u^2+4\pi^2}+\frac{4}{u^2+4\pi^2}(1-e^{-u})$$
$$-\frac{2u^2}{(u^2+4\pi^2)^2}(1-e^{-u})\ , \tag{4}$$

where $u=ak_\parallel$ with a the quantum-well width. The ideal 2D limit is obtained by taking $a=0$. To recover a 3D result, all subbands have to be included. The form factor given by Eq. (4) is different from that used for GaAs-$\mathrm{Al}_x\mathrm{Ga}_{1-x}\mathrm{As}$ heterostructures in Ref. 16, where the electron confinement is described by the variational Fang-Howard wave function. In the numerical calculation, all physical parameters are taken for a GaAs quantum well. The theory can also be applied to quantum wells made out of other materials.

First, let us study the magnetic field dependence of the cyclotron-resonance mass. In Fig. 1 the polaron correction to the cyclotron-resonance mass is plotted as a function of the magnetic field for different values of the quantum-well width: $a=50$, 100, 200, and 400 Å. For a quantum-well width of $a=50$ Å we also show the one-polaron result, i.e., without Fermi-Dirac statistics (occupation effect) and screening (dashed-dotted curve), and for $a=50$ and 100 Å the results are also plotted when only the Fermi-Dirac exclusion principle is included (dashed curve).

Similar to the case for heterostructures,[16] the polaron correction to the cyclotron-resonance mass shows a resonant enhancement when the unperturbed cyclotron-resonance frequency approaches the LO-phonon frequency. The effect of the finite width of the quantum well is to reduce the polaron correction from that of an ideal 2D system. From Fig. 1, it is clear that with increasing

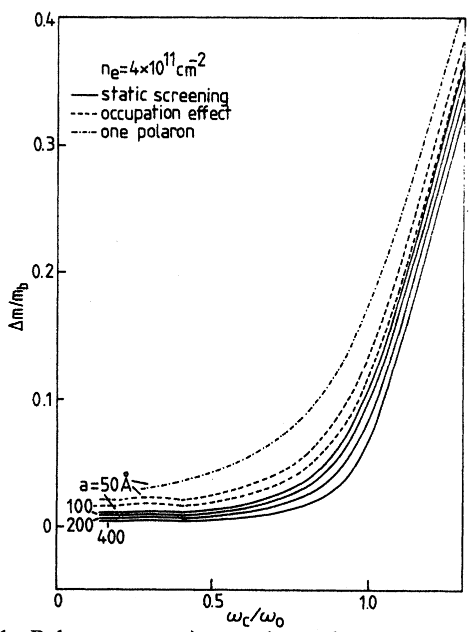

FIG. 1. Polaron correction to the cyclotron mass as a function of the magnetic field for different values of the quantum-well width: $a=50$, 100, 200, and 400 Å. Different approximations are considered: (i) the one-polaron results are given by the dashed-dotted curve, (ii) the dashed curve represents the case where only the occupation effect is taken into account, and (iii) screening of the electron-phonon interaction is included for the full curves.

quantum-well width the polaron correction to the cyclotron-resonance mass decreases. In Fig. 1, the electron density is chosen as 4×10^{11} cm^{-2}. One may conclude that the static screening further reduces the polaron corrections. Similar conclusions were reached in the study of the polaron cyclotron-resonance mass in GaAs heterostructures.[16] Note that at low magnetic fields the polaron correction to the cyclotron-resonance mass is almost independent of the magnetic field, which is similar to the results found for heterostructures, and which is due to the occupation effect.

Next, the electron-density dependence of the polaron correction to the cyclotron-resonance mass will be studied. In Fig. 2 the polaron correction to the cyclotron-resonance mass is plotted as a function of the electron density for different values of the quantum-well width. The result with inclusion of the occupation effect is compared with the results with inclusion of static screening of the electron-phonon interaction. The polaron corrections decrease as the electron density increases. The magnetic field is chosen in such a way that we are close to the resonant polaron region. The reduction of the polaron correction at high electron density is consistent with the generally accepted physical picture that for larger electron densities the screening of the electron-phonon interaction is larger, and consequently, the effective coupling strength is smaller. Note also that screening further reduces the polaron corrections. It is clear from Fig. 2 that with increasing quantum-well width the polaron correction decreases considerably over the whole range of the electron densities under study here.

The polaron correction to the cyclotron-resonance mass as function of the quantum-well width is most clearly seen in Fig. 3, where the polaron correction is plotted as a function of the quantum-well width for a fixed value of the electron density and of the magnetic field strength. Again, the static screening further reduces the polaron effects. In the $a\rightarrow0$ limit the polaron correction reduces to the results for a 2D EG with zero width.

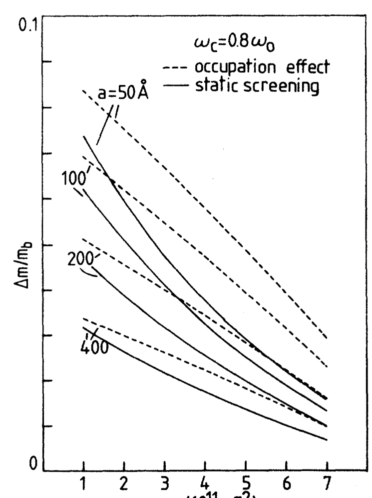

FIG. 2. Same as Fig. 1, but now as a function of the electron density for a fixed magnetic field.

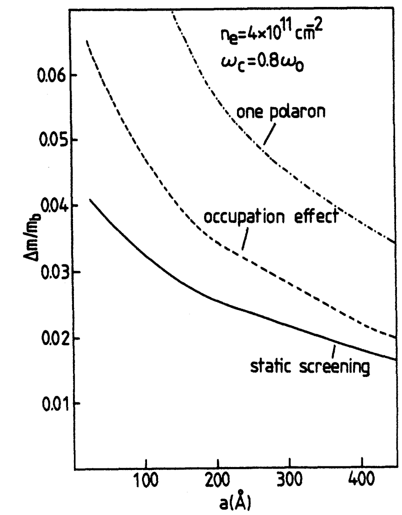

FIG. 3. Same as Fig. 1, but now as a function of the quantum-well width for fixed electron density and fixed magnetic field.

Now let us turn to the experimental results of Ref. 9. The experiment in Ref. 9 was performed on a GaAs quantum well with barriers consisting of $Al_xGa_{1-x}As$ with $x=0.36$. The electron density was almost 4 orders of magnitude lower (i.e., $n_e\approx10^7$ cm^{-2}) than that usually studied in a GaAs-$Al_xGa_{1-x}As$ heterostructure. This low density implies that a one-polaron theory should be applicable. The measured cyclotron-resonance mass increases as the magnetic field strength increases, as in the case of GaAs heterostructures, and its magnetic field dependence is obviously nonlinear, which is clear evidence of the presence of polaron effects.[1-14] But note that the mass is rather large compared with the data of GaAs heterostructures.[4,5] At a magnetic field of about 10 T (at this magnetic field the electrons are in the lowest Landau level), the measured cyclotron-resonance mass in Ref. 9 is $0.0865m_e$, while in Refs. 4 and 5, for GaAs heterostructures, it ranges from $0.069m_e$ to $0.070m_e$ depending on the experimental electron density.

It is known that the increase of the cyclotron-resonance mass, as a function of the magnetic field or the laser frequency, is due to two effects: (1) the electron energy band nonparabolicity, which is practically linear in the magnitude field, and (2) the polaron mass renormalization,[1-9] which is linear in B for small magnetic fields but has a resonant behavior for $\omega_c\approx\omega_{LO}$ that results in a strong nonlinear component. Let us first study the electron band nonparabolicity. Within a two-band or three-band $\mathbf{k}\cdot\mathbf{p}$ theory[11,23] the Landau-level energy is

$$E_n=-\frac{E_g}{2}+\frac{E_g}{2}\left[1+4\frac{E_n^0}{E_g}\right]^{1/2}, \qquad (5)$$

with $E_n^0=(n+\frac{1}{2})\hbar\omega_c+E_z$, where E_z is calculated for a potential well with height $V_0=260$ meV and width $a=22$ Å, and a 3D band mass of $0.066m_e$ for the electron (the electron band mass is obtained from the analysis of cyclotron-resonance experimental data of Refs. 4 and 5).

We found that the nonparabolicity alone will give a cyclotron-resonance mass of $0.081m_e$ at 10 T, which is well below the measured value of $0.0865m_e$. This leaves us with a 6.8% correction to the cyclotron-resonance mass that must be accounted for solely by the polaron effect.

In Refs. 9 and 20 another form for the band nonparabolicity was used,

$$E_n = E_n^0 \left[1 - \delta \frac{E_n^0}{E_g} \right] . \tag{6}$$

This expression can also be viewed as the series expansion in E_n^0/E_g of the expression of (5) with $\delta = 1$. Using δ as an adjustable parameter, we found that the experimental results can approximately be described for $\delta = 0.95$ (see Fig. 4). At higher magnetic fields, i.e., $B > 15$ T, the cyclotron mass is underestimated, and in this region polaron corrections are clearly needed. Because $\delta < 1$, one may conclude that band nonparabolicity effects are weaker in the 2D than in the corresponding 3D system.

When we include both the band nonparabolicity and polaron effects in the calculation of the cyclotron mass we find the best fit for $\delta = 0.857$ as shown in Fig. 4. In our calculation, the finite height of the potential is incorporated in the calculation of the form factor. It is apparent that the slope of the theoretical curve is larger than inferred from experiment. Because of the relative small quantum-well width, polaron effects are large. Theoretically, we expect the result close to the pure 2D, because the width $a = 22$ Å of the quantum well is a factor of 2 smaller than the polaron radius $r_{\mathrm{LO}} = (\hbar/2m^*\omega_{\mathrm{LO}})^{1/2} \approx 40$ Å. Thus, in this narrow quantum well, much larger polaron effects are expected than in typical heterostructures where the width of the 2D electron gas layer is typically of the order 50–100 Å. Note that the polaron interaction induces a strong nonlinear component (see, e.g., Fig. 1) that is not observed experimentally. Surprisingly, we found that the experimental results can well be fitted with $\delta = 0.905$, and an effective quantum-well width of $a = 200$ Å for the polaron effects. This suggests that polaron effects are seriously diminished. In heterostructures, this was also shown to be the case, but it was a combined effect of the nonzero width of the 2D electron gas and the screening of the electron-phonon interaction due to the relative large electron density. In the system under study, the electron density is so small that screening is unimportant. Another effect has to be present.

The experimental result tells us that coupling to the LO phonons of GaAs is severely weakened. This may be a consequence of the small width of the quantum well. Only four layers of GaAs are present. It may be possible that the system is so narrow that the LO phonon of GaAs is not well developed.[24] In the present study, we *assumed* that the electron–LO-phonon interaction is described by the standard Fröhlich model with the interaction matrix

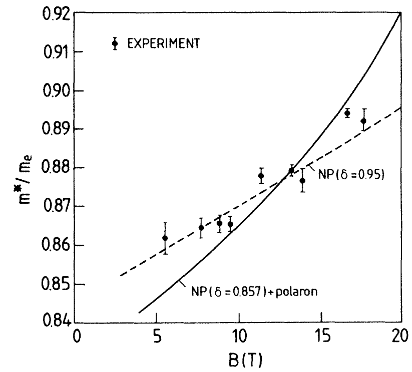

FIG. 4. Comparison between the experimental results of Singleton *et al.* (Ref. 9) and our theoretical results with (solid curve) and without (dashed curve) polaron effects. Note that for both curves the band nonparabolicity is different.

being the same as for the bulk material. This approximation is commonly adopted in the literature.[25] For a quantum-well structure, the phonon spectrum and the polarization field induced by the phonons is different from that of bulk material. However, a calculation of the electron mobility in the 2D plane, in which the effect of a realistic quantum-well structure on the phonon spectrum and on the electron-phonon interaction matrix is taken into account, gives almost the same result as if the standard Fröhlich model is employed.[26] This puzzle has not been solved yet.

In conclusion, we have investigated the magnetic field, the density, and the quantum-well width dependence of the polaron cyclotron-resonance renormalization in a single quantum well. A comparison with a recent experiment on a very low electron density and narrow single quantum well of GaAs suggests that the polaron effects are considerably weakened by an as yet unknown mechanism that is different from the well-established finite-width effect of the 2D layer and screening of the electron-phonon interaction. Unfortunately, because of the large scatter in the experimental data, no definite conclusion can be reached. More experiments are needed on narrow single quantum wells with different widths and different heights.

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