Mechanism of quasilinear temperature dependence of the surface magnetization in a semi-infinite ferromagnet

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A semiclassical method involving surface spin waves is used to calculate the spontaneous magnetization M near the surface of a semi-infinite ferromagnet. The magnitude of the surface anisotropy constant K_s introduced by Néel is assumed to be negligible compared to the magnitude of the surface anisotropy constant K_{ss} introduced by the present author. It is shown that under these conditions any positive value of K_{ss} causes the dependence of M on the temperature T to be quasilinear rather than proportional to $T^{3/2}$ and the dependence of M on position to be exponential. This surface-anisotropy-based mechanism contains a weakened-surface-exchange model as a special case. The theoretical predictions are found to provide a possible interpretation of recent experimental results on MnF_2-covered Fe(110).

I. INTRODUCTION

This paper gives a full account of a theory which we published previously as an abstract[1] only. The motivation for our work was a series of experiments[2,3] by Walker and his co-workers on the temperature dependence of the spontaneous magnetization M at a (110) surface of iron. Their experiments showed that M_0-M_T, the deviation of the value of M at a temperature T from its value at $T=0$, is proportional to $T^{3/2}$ or approximately proportional to T, depending on the material with which the iron samples are covered. These results led us to suggest that the form of the temperature dependence of M_0-M_T depends on the magnetic surface anisotropy of the ferromagnetic material under investigation. We predicted[1] that the quantity M_0-M_T is proportional to $T^{3/2}$ or quasilinear in T, depending on the range of values characterizing the surface anisotropy constants K_s and K_{ss}. These lowest-order constants were introduced by us on the basis of symmetry considerations[4] more general than the model-based arguments proposed by Néel[5] which led to K_s only. We have postponed publishing the details of our calculations until the results of additional experiments[6-9] on Fe(110) surfaces become available.

The method that we use includes generalizing our semiclassical spin-wave calculation[10] of 1957 by the inclusion of surface anisotropy and surface waves. Our earlier calculation[10] involves volume spin waves in a continuous medium and is relatively simple. Its two central predictions are that the T dependence of M near the ferromagnetic surface is proportional to $T^{3/2}$ and that the value of $(M_0-M_T)/M_0$ at the surface is twice that in the interior. Both of these predictions were confirmed theoretically a decade later in a quantum-mechanical treatment of a semi-infinite Heisenberg ferromagnet by Mills and Maradudin.[11] Moreover, these predictions were also confirmed experimentally by Stern *et al.*[7] in MgO-covered Fe(110) and discussed in conjunction with nonzero but small values of surface anisotropy by Rado and Walker.[12] Quite recently, Mathon and Ahmed[13] showed in a calculation based on the assumption of a weakened surface exchange that the value of $(M_0-M_T)/M_0$ at the surface can be somewhat larger than twice that in the interior, as observed by Walker *et al.*[3] in Ag-covered Fe(110) and by Pierce *et al.*[14] in the amorphous ferromagnet $Ni_{40}Fe_{40}B_{20}$.

The generalization of our 1957 calculation[10] reported in the present paper has two main objectives. Firstly, we wish to show that for a certain range of relative values of the surface anisotropy constants K_s and K_{ss} the temperature dependence of M near the surface of a semi-infinite ferromagnet can be quasilinear in T rather than being necessarily proportional to $T^{3/2}$. Secondly, we wish to determine the form of the position dependence of M near the surface. We note that a quasilinear T dependence of M was first observed experimentally[2,3] in MnF_2-covered Fe(110). We further note that the possibility of a quasilinear T dependence of M has already been suggested theoretically[15] but only for temperatures near the Curie temperature. Various effects of zero and infinite pinning fields have also been examined.[16] Our theoretical model (see Sec. II) applies to temperatures low compared to the Curie temperature and to situations where the exchange interactions are independent of position rather than being weakened near the surface. We show, in fact, that a quasilinear temperature dependence of $(M_0-M_T)/M_0$ does not require a weakened surface exchange but can nevertheless be consistent with it. We further show in Sec. II that our model calculations lead to an explicit formula [see Eq. (42)] for the temperature and position dependence of $(M_0-M_T)/M_0$ and that this formula is useful for interpreting experimental results. In Sec. III we first present an approximate extension of our formula [Eq. (42)] to the case of weakened surface exchange. This is followed by a digression on volume spin waves, some comments on the roles of surface anisotropy, and a discussion of the applicability of Eqs. (42) and (43) to the interpretation of experimental results.

II. THEORY

Throughout this paper we use the Cartesian coordinate systems shown in Fig. 1. The axes of the x,y,z system are parallel, respectively, to the [100], [010], and [001] axes of a cubic crystal, and the ξ and η axes of the ξ,η,z system are parallel, respectively, to the cubic [110] and [$\bar{1}$10] axes. We assume that the crystal is bounded by the planes $\eta=0$ and $\eta=l$ along the η axis, by the planes $\xi=0$ and $\xi=L$ along the ξ, and by the planes $z=0$ and $z=L$ along the z axis. We further assume that L is so large that along the ξ and z axes the crystal is effectively unbounded. For the sake of definiteness we assume the crystal to be bcc iron and the equilibrium orientation of the spontaneous magnetization $\mathbf{M}$ to be along the $+z$ direction. For a different crystal or a different configuration the calculation would be analogous to those presented below.

Next we introduce the unit vector

$$\mathbf{u}=\mathbf{M}/M=\mathbf{i}_\xi u_\xi+\mathbf{i}_\eta u_\eta+\mathbf{i}_z u_z\ , \tag{1}$$

where $\mathbf{i}_\xi,\mathbf{i}_\eta,\mathbf{i}_z$ are unit vectors along the ξ,η,z directions, respectively. Since we expect the thermally excited deviations of $\mathbf{u}$ from its equilibrium orientation $\mathbf{i}_z$ to be small, we shall use the approximations

$$|u_\xi|\ll 1,\quad |u_\eta|\ll 1,\quad u_z\approx 1\ , \tag{2}$$

which justify the relation

$$1-u_z\approx\tfrac{1}{2}(u_\xi^2+u_\eta^2)\ . \tag{3}$$

The quantity we wish to calculate is $(M_0-M_T)/M_0$, where M_T and M_0 are the position-dependent equilibrium values of M at temperatures $T=T$ and $T=0$, respectively. Because the approximation (3), we have

$$(M_0-M_T)/M_0=\frac{1}{2}\left\langle\sum\langle\langle u_\xi^2+u_\eta^2\rangle\rangle\right\rangle\ , \tag{4}$$

where $\langle\langle\cdots\rangle\rangle$ denotes averaging over all the energy levels of a given spin-wave mode, $\sum$ denotes summing (actually integrating) over all the spin-wave modes, and $\langle\cdots\rangle$ denotes performing a spatial average over a volume $V'=L^2d\eta$ which is not the volume $V=L^2l$ of the entire crystal but the volume of a slab of thickness $d\eta$ located at a distance η from the sample surface $\eta=0$.

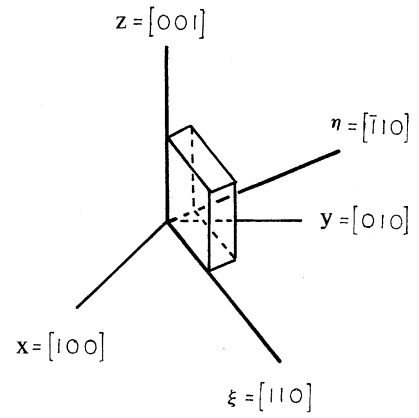

FIG. 1. Orientations of the Cartesian coordinate systems used in the calculations. The crystal shown is bounded by the planes $\xi=0$, $\xi=L$, $\eta=0$, $\eta=l$, and $z=0$, $z=L$.

To calculate u_ξ and u_η we use the Landau-Lifshitz[17] equation of motion and write it in the truncated form

$$(1/\gamma)\partial\mathbf{u}/\partial t=\mathbf{u}\times[H_{\rm dem}\mathbf{i}_\eta+H_K\mathbf{i}_z+(2A/M)\nabla^2\mathbf{u}]\ , \tag{5}$$

where $\gamma=\gamma_e g/2$ is the magnetochemical ratio, $\gamma_e=2\pi\times(2.80)$ MHz/Oe is the value of γ for a free electron, and g is the spectroscopic splitting factor. Here $H_{\rm dem}$ is the demagnetizing field and may be written as $-4\pi M u_\eta$ because the dimensions of the crystal in the ξz plane are assumed to be infinite. The quantity H_K represents the magnetocrystalline volume anisotropy by means of an effective "anisotropy field." This is justified by the fact that the inequalities (2) cause the deviations of $\mathbf{u}$ from a magnetically easy direction (in this case from the $\mathbf{i}_z$ direction) to be confined to small values. The remaining quantity in the brackets of Eq. (5) is the exchange term and contains the exchange stiffness constant A.

For reasons to be discussed in Sec. III, we choose for the solution of Eq. (5) the elliptically polarized surface spin waves

$$u_\xi=u_{\xi0}(\sin\omega t)\exp(-p\eta)F(\xi,z)\ , \tag{6}$$

$$u_\eta=u_{\eta0}(\cos\omega t)\exp(-p\eta)F(\xi,z)\ , \tag{7}$$

where $F(\xi,z)$ is defined by

$$F(\xi,z)=\cos(k_\xi\xi)\cos(k_z z)\ , \tag{8}$$

and each of the quantities p,k_ξ,k_z is assumed to be real and positive. By substituting Eqs. (6)–(8) into Eq. (5) and using the approximation (2), we obtain two linear homogeneous equations for $u_{\xi0}$ and $u_{\eta0}$. The requirement that $u_{\xi0}$ and $u_{\eta0}$ be nonvanishing then yields

$$(\omega/\gamma)^2=[H_K+4\pi M+(2A/M)(k_t^2-p^2)]\times[H_K+(2A/M)(k_t^2-p^2)]\ , \tag{9}$$

where the transverse wave number k_t is defined by

$$k_t^2=k_\xi^2+k_z^2\ . \tag{10}$$

Equation (9) is the dispersion relation for the surface spin waves (6) and (7).

To obtain the attenuation parameter p we apply the Rado-Weertman[18] boundary condition in a form similar to Eq. (2.7) of Ref. 4. At the $\eta=0$ plane, therefore, we use

$$\mathbf{u}\times[\nabla_u E_{\rm surf}-2A(\partial\mathbf{u}/\partial\eta)]=\mathbf{0}\ , \tag{11}$$

where $E_{\rm surf}$ is the surface anisotropy energy density. In the ξ,η,z coordinate system we have

$$\nabla_{\rm u}=\mathbf{i}_\xi(\partial/\partial u_\xi)+\mathbf{i}_\eta(\partial/\partial u_\eta)+\mathbf{i}_z(\partial/\partial u_z)\ , \tag{12}$$

$$E_{\rm surf}=\tfrac{1}{2}K_s(u_\xi^2-u_\eta^2)+K_{ss}u_z^2\ . \tag{13}$$

Equation (12) is a definition, and Eq. (13) arises from transforming to the ξ,η,z coordinate system the expression

$$E_{\text{surf}}=K_s u_x u_y + K_{ss} u_z^2 , \tag{14}$$

which we proposed[4] previously on the basis of symmetry. We note that Eq. (14) contains a K_{ss} term and that this term is absent from Néel's[5] expression for E_{surf}. As shown in the following, the K_{ss} term is indispensible for obtaining the central result [Eq. (42)] of the present paper.

Equations (11)–(13) and the approximation (2) lead to

$$(-K_s+2K_{ss})u_\xi + 2A\,\partial u_\xi/\partial\eta = 0 , \tag{15}$$

$$(K_s+2K_{ss})u_\eta + 2A\,\partial u_\eta/\partial\eta = 0 , \tag{16}$$

which we combine with Eqs. (6) and (7) to obtain

$$-K_s+2K_{ss}-2Ap=0 , \tag{17}$$

$$K_s+2K_{ss}-2Ap=0 . \tag{18}$$

These two equations evidently contradict each other. To deal with this difficultly we could, of course, simply abandon the assumed solutions (6) and (7). We find it preferable, however, to proceed by introducing the approximation

$$|K_s| \text{ negligible compared to } 2|K_{ss}| \tag{19}$$

because it leads to the useful results derived in the following. In other words, we confine the applicability of our theory to situations in which the approximation (19) is valid. We note, in this connection, that E_{surf} (and hence K_s and K_{ss}) refer not just to surfaces but also to interfaces, e.g., an MnF_2-covered Fe crystal.

Use of the approximation (19) in either Eq. (17) or Eq. (18) yields

$$p=K_{ss}/A , \tag{20}$$

which must be positive since p should describe a positive attenuation. This means that if A is positive, as in Fe, then we must also have

$$K_{ss}>0 . \tag{21}$$

Next we define $\mathbf{u}_t$ by

$$\mathbf{u}_t=\mathbf{i}_\xi u_\xi+\mathbf{i}_\eta u_\eta , \tag{22}$$

so that Eqs. (6), (7), and (20) lead to

$$\mathbf{u}_t=[\mathbf{i}_\xi u_{\xi 0}\sin(\omega t)+\mathbf{i}_\eta u_{\eta 0}\cos(\omega t)] \times\exp(-K_{ss}\eta/A)F(\xi,z) , \tag{23}$$

From Eq. (23) we then obtain

$$\langle\langle u_t^2\rangle\rangle=\langle\langle u_{t0}^2\rangle\rangle\exp(-2K_{ss}\eta/A)F^2(\xi,z) , \tag{24}$$

provided we replace each of the quantities $\sin^2\omega t$ and $\cos^2\omega t$ by its time average value $\frac{1}{2}$.

Turning now to the derivation of an expression for $\langle\langle u_{t0}^2\rangle\rangle$, we begin by writing the exchange energy density in the usual form

$$E_{\text{ex}}^{(\omega)}/V=-\tfrac{1}{2}\mathbf{M}\cdot(2A/M^2)\nabla^2\mathbf{M}=-A\,\mathbf{u}_t\cdot\nabla^2\mathbf{u}_t . \tag{25}$$

With the use of Eqs. (22) and (10), Eq. (25) yields

$$E_{\text{ex}}^{(\omega)}=A[k_t^2-(K_{ss}/A)^2]\int_V u_t^2\,dV , \tag{26}$$

which we equate to the quantized form

$$E_{\text{ex}}^{(\omega)}=n\hbar\omega_{\text{ex}}=n\hbar(2A\gamma/M)[k_t^2-(K_{ss}/A)^2] . \tag{27}$$

Here n is the excitation quantum number of the spin-wave mode under consideration and ω_{ex} is that part of the ω of Eq. (9) which arises solely from exchange interactions. Next we take the thermal average of $E_{\text{ex}}^{(\omega)}$ in each of the Eqs. (26) and (27). Use of Eq. (24) then leads to

$$\langle\langle u_{t0}^2\rangle\rangle=\frac{(2\gamma\hbar/M)[\exp(\hbar\omega/k_BT)-1]^{-1}}{\int_V\exp(-2K_{ss}\eta/A)F^2(\xi,z)\,dV} , \tag{28}$$

where the reciprocal of the bracketed quantity represents $\langle\langle n\rangle\rangle$ and k_B it Boltzmann's constant. Substitution of Eq. (28) into Eq. (24) yields $\langle\langle u_t^2\rangle\rangle$ which may be combined with Eqs. (4) and (22) to give

$$\frac{M_0-M_T}{M_0}=\sum\frac{(\gamma\hbar/M_0V')\int_{V'}\exp(-2K_{ss}\eta/A)F^2(\xi,z)\,dV'}{[\exp(\hbar\omega/k_BT)-1]\int_V\exp(-2K_{ss}\eta/A)F^2(\xi,z)\,dV} . \tag{29}$$

Here the sum $\sum$ and the spatial average $\langle\ \rangle$ were interchanged, and the latter was expressed as $(1/V')$ times an integral over V'. It was further assumed, and will continue to be assumed, that M can be replaced by M_0 with sufficient accuracy.

We note parenthetically that if V' were equal to V, then Eq. (29) would yield

$$M_0-M_T=\gamma\hbar\langle\langle n\rangle\rangle/V , \tag{30}$$

which is a standard result.[19] It should be recalled that $\gamma\hbar$ is identical to $g\mu_B$, where μ_B is the Bohr magneton.

Returning to our surface magnetization problem, we see that Eq. (29) yields

$$\frac{M_0-M_T}{M_0}=\left[\frac{2\gamma\hbar K_{ss}}{AM_0L^2}\right]\exp(-2K_{ss}\eta/A)\sum[\exp(\hbar\omega/k_BT)-1]^{-1} , \tag{31}$$

because the integrals over ξ and z cancel. In obtaining Eq. (31) we assumed that $\exp(-2K_{ss}l/A)$ is negligible compared

to unity, i.e., that the penetration depth $A/(2K_{ss})$ of the surface wave is sufficiently small compared to the crystal thickness l.

The remainder of the calculation consists in summing over k_ξ and k_z in Eq. (31). It is more convenient, however, to convert the sum into an integral. We assume periodic boundary conditions in the transverse plane and thus introduce integers N via the relation $k_t = N\pi/L$. The number of modes for which k_t is between k_t and $k_t + dk_t$ is then given by the infinitesimal "area"

$$\tfrac{1}{4}(2\pi)N\,dN = \tfrac{1}{4}(2L^2/\pi)k_t\,dk_t\ , \tag{32}$$

where the factor $\frac{1}{4}$ assures that k_ξ and k_z are indeed positive, as assumed in connection with Eq. (8). Thus Eq. (31) becomes

$$\frac{M_0 - M_T}{M_0} = \left[\frac{\gamma\hbar K_{ss}}{\pi M_0 A}\right]\exp(-2K_{ss}\eta/A)\int_0^\infty \frac{k_t}{\exp(\hbar\omega/k_B T)-1}\,dk_t\ , \tag{33}$$

which is seen to be independent of L. The functional dependence of ω on k_t is expressed by the dispersion relation (9) in conjunction with Eq. (20).

The integral in Eq. (33) is not simple but it can undoubtedly be worked out numerically as a function of temperature. We prefer, however, to make a suitable approximation which enables us to perform the integration by elementary analytical methods. This has the advantage of providing physical insight and exhibiting clearly the quasilinear nature of the temperature dependence of $(M_0 - M_T)/M_0$. Specifically we assume

$$\frac{H_K + 4\pi M}{(2A/M)|k_t^2 - (K_{ss}/A)^2|} \ll 1\ , \tag{34}$$

which means that the sum of the anisotropy field and the demagnetizing field is assumed to be small compared to the exchange field. A straightforward calculation shows that use of the approximation (34) in Eq. (9) yields

$$\frac{\hbar\omega}{k_B T} = \Lambda^2 k_t^2 + \frac{g\mu_B}{k_B T}(H_K + 2\pi M_0) - \Lambda^2\left[\frac{K_{ss}}{A}\right]^2\ , \tag{35}$$

where we replaced M by M_0 and $\gamma\hbar$ by $g\mu_B$. The quantity Λ denotes the characteristic length

$$\Lambda = \left[\frac{2Ag\mu_B}{M_0 k_B T}\right]^{1/2} \tag{36}$$

introduced by us previously.[10] We note that Λ equals $1/(2\pi)$ times the wavelength of those volume spin waves for which one quantum of exchange energy, namely, $\hbar\omega = (2A\gamma h/M_0)k^2$, just equals $k_B T$.

Next we introduce the dimensionless variable

$$v = \Lambda^2 k_t^2 \tag{37}$$

and the dimensionless parameters

$$\beta = (g\mu_B/k_B T)(H_K + 2\pi M_0)\ , \tag{38}$$

$$\alpha = \Lambda^2(K_{ss}/A)^2\ . \tag{39}$$

The integral in Eq. (33), to be denoted by J, can now be written as

$$J = \frac{1}{2\Lambda^2}\int_0^\infty dv \frac{1}{\exp(v+\beta-\alpha)-1}\ , \tag{40}$$

which can be worked out easily. We multiply the numerator and denominator of the integrand by $\exp[-(v+\beta-\alpha)]$ and convert the resulting fraction into an infinite series. After having to integrate only a simple exponential, we recognize this series as the expansion of

$$J = -\tfrac{1}{2}\Lambda^{-2}\ln[1-\exp(\alpha-\beta)]\ . \tag{41}$$

By combining Eq. (41) with Eqs. (36), (38), and (39) and then substituting it into Eq. (33), we obtain our principal result

$$\frac{M_0 - M_T}{M_0} = -\frac{K_{ss}k_B T\exp(-2K_{ss}\eta/A)}{4\pi A^2}\ln\left\{1-\exp\left[-\frac{g\mu_B}{k_B T}\left[H_K + 2\pi M_0 - \frac{2K_{ss}^2}{M_0 A}\right]\right]\right\}\ . \tag{42}$$

Among the central predictions of Eq. (42) are the following.

(a) The *temperature dependence* of $(M_0 - M_T)/M_0$ is *quasilinear* in that it is directly proportional to the product of T and a slowly varying function of T.

(b) The *position dependence* of $(M_0 - M_T)/M_0$ is an *exponential* decay from the crystal surface $\eta = 0$ toward the crystal's interior ($\eta > 0$).

(c) The newly introduced[4] lowest-order surface anisotropy constant K_{ss} is indispensible for our mechanism, in contrast to Néel's[5] constant K_s. Specifically, K_{ss} determines not only the penetration depth $A/(2K_{ss})$ but even the entire existence of the quasilinear $(M_0 - M_T)/M_0$.

III. DISCUSSION

A. Extension to weakened surface exchange

As mentioned in Sec. I, the possibility of a quasilinear temperature dependence of $(M_0 - M_T)/M_0$ has already been suggested in previous theoretical work[15] for temperatures near the Curie temperature and for situations in

which the exchange interactions at the surface of a ferromagnet differ from those in its interior. It is shown in the following that an explicit prediction of a quasilinear T dependence of $(M_0-M_T)/M_0$ for the case of a weakened surface exchange can be obtained by suitably adapting Eq. (42). To do this, we replace the exchange stiffness coefficient A by its surface value A_s in each of the three places in Eq. (42) where A arises from the boundary condition at $\eta=0$ rather than from the equation of motion. Specifically, we replace K_{ss}/A^2 by $K_{ss}/(A_s A)$, replace K_{ss}/A by K_{ss}/A_s, and replace K_{ss}^2/A by $K_{ss}^2 A/A_s^2$. Stated equivalently, we replace throughout Eq. (42) the quantity K_{ss} by its effective value

$$K_{ss}^e = K_{ss} A/A_s \ . \tag{43}$$

It follows that if the positive quantities A_s and A satisfy the relation $A_s/A<1$, i.e., if the surface exchange is weakened, then the use of K_{ss}^e in place of K_{ss} in Eq. (42) is equivalent to the use of a spatially uniform A and an increased K_{ss}. Our surface-anisotropy-based mechanism is seen, therefore, to contain a weakened-surface-exchange model as a special case.

B. Digression on volume spin waves

The thermally excited modes in a ferromagnet include volume spin waves as well as surface spin waves of the kind treated in this paper. It is instructive, therefore, to refer briefly to the effects of volume spin waves on the temperature and position dependence of the spontaneous magnetization. Some of these effects were predicted by us in an early calculation.[10] already mentioned in Sec. I, which includes exchange interactions but neither volume anisotropy nor demagnetizing fields. In that calculation we omitted surface anisotropy and used free ("unpinned") boundary conditions. This was the first use of nonperiodic boundary conditions in a thermal magnetization problem. As discussed in Ref. 12, the free boundary condition remains valid even in the presence of surface anisotropy provided the latter is negligibly small compared to Ak, where the magnitude of k is predominately the $1/\Lambda$ of Eq. (36).

C. Roles of surface anisotropy

Returning to the calculation of $(M_0-M_T)/M_0$ given in Sec. II, we now summarize some of the crucial roles of the surface anisotropy constant K_{ss}. We recall, for this purpose, the prediction (c) at the end of Sec. II, the important approximation (19), and the inequality (21). In addition, we note that K_{ss} has a role which has not yet been mentioned, namely its tendency to destabilize the calculated surface magnetization. To see this, note that if the value of K_{ss} and hence α were sufficiently small, then the integral J [see Eqs. (40) and (41)] would be kept from diverging even if β included solely H_K rather than $H_K+2\pi M_0$. But if K_{ss} is sufficiently large, then a β arising from H_K alone would not prevent the integral J from diverging. It is, in fact, this role of K_{ss} which led us to introduce a demagnetizing field for helping to stabilize J. Our use of H_K and the demagnetizing field implicitly assumes, of course, that the sample is a single ferromagnetic domain. This can be accomplished either by choosing the value of the thickness l to be sufficiently small [but still large compared to the penetration depth $A/(2K_{ss})$] or by applying an external magnetic field parallel to H_K.

Particularly interesting is the manner in which K_{ss} causes the T dependence of the surface magnetization to be quasilinear. The essential point is that the integration (originally summation) over modes contained in Eq. (33) involves only two dimensions (namely ξ and z) rather than three. This, in turn, arises from the fact that the component of the wave vector along η, which we call p, is forced by the boundary condition at $\eta=0$ to have only the single value of $p=K_{ss}/A$ rather than several values. The semi-infinite ferromagnet we are considering [i.e., one described by the approximation (19), etc.] behaves, therefore, similarly to the two-dimensional ferromagnet treated, for example, in Keffer's[19] review of spin waves.

D. Interpretation of experimental results

As noted in Sec. I, a quasilinear T dependence of $(M_0-M_T)/M_0$ was first observed experimentally[2,3] in MnF_2-covered Fe(110). Additional experimental results[7–9] on this T dependence and on the position dependence of $(M_0-M_T)/M_0$ were published more recently. The newest work, by Tang *et al.*,[9] provides considerable experimental support for the theory of the present paper. Reference 9 clearly shows a quasilinear T dependence of $(M_0-M_T)/M_0$ at the Fe-MnF_2 interface ($\eta=0$) and also at a distance of 5 atomic layers ($\eta=1.014\times10^{-7}$ cm) below this interface. At a sufficiently large distance, namely 10 atomic layers, the observed[8] T dependence is no longer quasilinear but proportional to $T^{3/2}$. Using Eq. (42) of the present paper, which Tang *et al.*[9] quote in a regrettably incorrect manner, they infer the value $K_{ss}=5.9$ ergs/cm^2 from the ratio of the observed slopes of $(M_0-M_T)/M_0$ versus T at $\eta=0$ and $\eta=1.014\times10^{-7}$ cm. In reference to the discussion given in Sec. III C, we find that the value $K_{ss}=5.9$ ergs/cm^2 is sufficiently large to require the use of the $2\pi M_0$ term in β [see Eq. (38)] to keep the integral J [see Eqs. (40) and (41)] from divering. If we had to work with $K_{ss}\lesssim1$ erg/cm^2 (but not, say, $K_{ss}\approx1.5$ ergs/cm^2), then the $2\pi M_0$ term in β would not have to be used. We also note that even if the "semi-infinite" sample thickness l is as small as 50 Å, the value $K_{ss}=5.9$ ergs/cm^2 means (with the use of $A=2.0\times10^{-6}$ erg/cm) that $\exp(-2K_{ss}l/A)$ is as small as 0.053, i.e., barely small enough to satisfy one of the assumption underlying Eq. (31).

The physical origin of a K_{ss} value as large as 5.9 ergs/cm^2 is presently unknown. It is worth recalling, therefore, that our surface-anisotropy-based mechanism contains a weakened-surface-exchange model as a special case. This means, according to Eq. (43), that if the value of A/A_s for MnF_2-covered Fe(110) were roughly 10, then the above-mentioned experimental data[9] could be explained by some more reasonable value (e.g., $K_{ss}<1$) of the true surface anisotropy. In any case, it appears that Eqs. (42) and (43) of the present paper constitute a possible interpretation of experimental results on the T dependence and position dependence of $(M_0-M_T)/M_0$ in MnF_2-covered Fe(110).

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