Effect of virtual screening on the absorption spectrum of a coherently pumped semiconductor

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The dielectric function of a nonresonantly pumped semiconductor is calculated in the generalized random-phase approximation to second order in the pump field. The effect of this "virtual screening" on the exciton ground-state energy E_1 and on its oscillator strength f_1 is then numerically computed and compared with the Stark shift of E_1 and f_1 derived before without screening in the Hartree-Fock approximation. It is found that for a detuning $\Delta=(E_g-\omega_p)/E^0$ of the order of 10 (E_g is the semiconductor band-gap energy, ω_p is the angular frequency of the pump field, and E^0 is the exciton binding energy) the modifications due to screening are small in a three-dimensional (3D) system, but are of the same order of magnitude as the Hartree-Fock corrections in a 2D system and cause the oscillator strength to decrease slightly with increasing pump intensities.

I. INTRODUCTION

In the last two years considerable effort has been devoted to the calculation of the induced nonlinear polarization in a semiconductor under nonresonant short-pulse excitation. In such a system, a blue shift of the exciton absorption line (the so-called Stark shift) as well as a modification of the corresponding oscillator strength are observed experimentally. The behavior of this system can be understood in terms of a many-body theory based on the self-consistent Hartree-Fock (HF) approximation.[1-7] An explicit treatment of the time dependence, even in the context of the Hartree-Fock approximation, is very difficult and has up to now relied on numerical calculation.[3] Interesting analytical results can, however, be obtained in the special case of a constant excitation (or pump) field E_p which is assumed to be adiabatically switched on. This special situation was considered in Refs. 1 and 2. Moreover, this simpler formalism has been shown to be generalizable (to some extent) to the more interesting case of short-pulse excitation.[4,5] In Ref. 2, corrections to the exciton energy E_1 and oscillator strength f_1 due to the electron-photon interaction (including such effects as phase-space filling, Fermion exchange, exciton-exciton interaction, etc.) were derived. These corrections have been numerically evaluated[6,7] in the limit of small-field intensity, i.e., to second order in E_p.

In the work presented here, we want to complete the picture given by the above time-independent Hartree-Fock formalism by considering explicitly the effect of screening. Because of the applied pump field and also because we work in a nonresonant excitation regime, the screened potential is expected to be radically different from the well-known Thomas-Fermi or Debye potentials of the equilibrium theory. A formal theory of the Stark effect including screening has been reported[8] but without any numerical result. We want here to derive explicitly the behavior of the potential and to compare the resulting changes of the exciton energy and oscillator strength with those calculated in Refs. 6 and 7. In this way, we can get an idea of the importance of screening which was, up to now, always neglected in numerical calculations.

This paper is organized in the following way. In Sec. II, we summarize the results of the Hartree-Fock theory for the dynamical Stark effect. In Sec. III, we generalize the equations of Ref. 2 for the optical susceptibility to include screening and find explicit expressions for the screening corrections to the exciton energy and oscillator strength. The dielectric function of the system is worked out in Sec. IV. Section V is devoted to a numerical evaluation, using Monte Carlo integration, of the dielectric function and of the corrections found in Sec. III. These corrections are then compared with those calculated in Refs. 6 and 7. We finally conclude in Sec. IV.

II. REVIEW OF THE HARTREE-FOCK FORMALISM

We consider the simple model of a nondegenerate two-band semiconductor interacting with homogeneous and nonresonant pump field treated in the dipole approximation.[1] We use, for the time dependence of the nonequilibrium Green's functions, the same approximations as those usually found in the literature on this subject. Namely, we consider that the applied field is constant and adiabatically switched on so that the system always remains in its ground state. In this case, the system under consideration is simply described by the usual matrix Green's function $G_{ij}(1,2)$ defined in the electron-hole picture by

$$G(1,2)=\begin{bmatrix} G_e(1,2) & F(1,2) \\ F^\dagger(1,2) & -G_h(2,1) \end{bmatrix} \equiv \begin{bmatrix} -i\langle T\psi_e(1)\psi_e^\dagger(2)\rangle & -i\langle T\psi_e(1)\psi_h(2)\rangle \\ -i\langle T\psi_h^\dagger(1)\psi_e^\dagger(2)\rangle & -i\langle T\psi_h^\dagger(1)\psi_h(2)\rangle \end{bmatrix} . \tag{2.1}$$

There is no need to introduce nonequilibrium Green's-function techniques in a rotating-wave approximation.

The diagonal terms in (2.1) are nonzero because the coherent pump field couples the two bands of the semiconductor. These terms are primarily responsible for the special character of the screening which, in the nonresonant regime that we are considering, is mediated by the excitation of virtual electron-hole pairs. One must have in mind that, in analogy with the case of a superconductor, the ground state of our system consists of a coherent superposition of electron-hole pairs. It is this pairing between electron and hole which is taken into account via the nondiagonal elements of the Green's function.

The "adiabatic" matrix Green's function defined in (2.1) obeys the Dyson equation

$$G_{ij}(1,2)=G^0_{ij}(1,2)+\int d3\,d4\,G^0_{im}(1,3)\Sigma_{mn}(3,4)G_{nj}(4,2)\ , \tag{2.2}$$

where $1,2,3,4=(\mathbf{r}_n,t_n,\sigma_n)$ and σ_n is the spin variable. $G^0(1,2)$ is the diagonal Green's function defined in the absence of Coulomb interaction and pump field. In (2.2) and throughout this paper, summation over repeated indices is implied.

The self-consistent Dyson equation can be solved formally in the Hartree-Fock approximation with static screening [the so-called "screened-exchange approximation" (SEA) but with a static potential]. The formal solution is

$$\begin{aligned}
G_{11}(\mathbf{p},\omega)&=\frac{u_p^2}{\omega+i\delta-\epsilon_{1p}}+\frac{v_p^2}{\omega-i\delta-\epsilon_{2p}}\ ,\\
G_{22}(\mathbf{p},\omega)&=\frac{v_p^2}{\omega+i\delta-\epsilon_{1p}}+\frac{u_p^2}{\omega-i\delta-\epsilon_{2p}}\ ,\\
G_{12}(\mathbf{p},\omega)&=G_{21}(\mathbf{p},\omega)\\
&=u_pv_p\left[\frac{1}{\omega+i\delta-\epsilon_{1p}}-\frac{1}{\omega-i\delta-\epsilon_{2p}}\right]\ ,
\end{aligned} \tag{2.3}$$

where we have used the definitions

$$\begin{aligned}
\eta_{ep}&\equiv E^{\mathrm{CH}}+\frac{E_g}{2}+\frac{p^2}{2m_e}-\frac{\omega_p}{2}+\Sigma_{11}(p)\ ,\\
\eta_{hp}&\equiv-\left[E^{\mathrm{CH}}+\frac{E_g}{2}+\frac{p^2}{2m_h}-\frac{\omega_p}{2}-\Sigma_{22}(p)\right]\ ,\\
\zeta_p&\equiv\frac{\eta_{ep}-\eta_{hp}}{2}\ ,\\
\eta_p&\equiv\frac{\eta_{ep}+\eta_{hp}}{2}\ ,\\
\epsilon_p&\equiv(\eta_p^2+\Sigma_{12}\Sigma_{21})^{1/2}\ ,\\
\epsilon_{1,2p}&\equiv\zeta_p\pm\epsilon_p\ ,\\
u_p^2&\equiv\frac{1}{2}\left[1+\frac{\eta_p}{\epsilon_p}\right],\quad v_p^2=1-u_p^2\ ,\\
u_pv_p&=\frac{\Sigma_{12}(p)}{2\epsilon_p}\ .
\end{aligned} \tag{2.4}$$

The self-energy matrix in the SEA is given by the screened exchange and optical interband self-energies (μ is the interband dipole matrix element):

$$\Sigma(\mathbf{p})=-\sum_{\mathbf{q}}V_s(\mathbf{p}-\mathbf{q})\begin{bmatrix}n_{e\mathbf{q}} & \psi_{\mathbf{q}}\\ \psi^*_{\mathbf{q}} & -n_{h-\mathbf{q}}\end{bmatrix}-\begin{bmatrix}0 & \mu E_p\\ \mu^*E_p^* & 0\end{bmatrix}\ , \tag{2.5}$$

where we have defined a density matrix $\hat{n}_{\mathbf{p}}$ by

$$\hat{n}_{\mathbf{p}}=G(\mathbf{p},t_2=t_1^+)=\begin{bmatrix}n_{e\mathbf{p}} & \psi_{\mathbf{p}}\\ \psi^*_{\mathbf{p}} & 1-n_{h-\mathbf{p}}\end{bmatrix}\ . \tag{2.6}$$

In this equation, n_e,n_h are the electron and hole densities and $2\sum_{\mathbf{p}}\mu^*\psi_{\mathbf{p}}=P_{\mathrm{opt}}$ is the optical polarization. In the SEA, one finds the useful relations

$$\begin{aligned}
\psi_p&=-u_pv_p\ ,\\
n_{ep}&=n_{hp}=v_p^2\ ,\\
(2n_p-1)^2&+4|\psi_p|^2=1\ .
\end{aligned} \tag{2.7}$$

As usual, all calculations are made in the rotating frame[1] with the pump frequency ω_p. In the expression for the electron and hole renormalized energies, we have explicitly included the Coulomb-hole contribution $E^{\mathrm{CH}}=(1/2)\sum_{\mathbf{q}}[V_s(\mathbf{q})-V(\mathbf{q})]$. This contribution[8] is contained in the dynamical Hartree-Fock self-energy and must be retained when approximating the dynamical potential by a static one as we do here. Although the discussion of Ref. 8 applies to an equilibrium system, one can easily convince oneself that it goes through in the case of a pumped system. In particular, there is no such contribution in the nondiagonal elements of the self-energy.

Solving the Dyson equation (2.2) with the self-energy (2.5), one finds for $\psi_{\mathbf{p}}$, using (2.6) and (2.7), the inhomogeneous integral equation

$$\left[E_g+E^{\mathrm{CH}}-\omega_p+\frac{p^2}{2m}-\sum_{\mathbf{q}}V_s(\mathbf{p}-\mathbf{q})[1-(1-4|\psi_{\mathbf{p}}|^2)^{1/2}]\right]\psi_{\mathbf{p}}=(1-4|\psi_{\mathbf{p}}|^2)^{1/2}\left[\mu E_p+\sum_{\mathbf{q}}V_s(\mathbf{p}-\mathbf{q})\psi_{\mathbf{q}}\right]\ , \tag{2.8}$$

where $m=m_em_h/M$ is the reduced mass and $M=m_e+m_h$ the total mass of the electron-hole pair. In three dimensions, the Coulomb potential is given by $V(\mathbf{p})=4\pi e^2/\epsilon_0p^2$, while in two dimensions it is

$V(\mathbf{p})=2\pi e^2/\epsilon_0 p$. Here, ϵ_0 is the dielectric constant of the background material. In this work, the dipole matrix element is assumed to be real. Since the static potential is also real, one sees from (2.8) that, for a nonresonant pump field, ψ_p can also be taken as real.

III. SCREENING CORRECTIONS TO E_1 AND f_1

In principle, one can derive all the correlation functions of this system by functional differentiation of the Dyson's equation (2.2).[9] In this paper, we are interested only in the nonlinear transverse (optical) and longitudinal susceptibilities which are respectively given by $\chi_t=\delta\langle P_{\rm opt}\rangle/\delta E_t$ and $L=\delta\langle\rho\rangle/\delta\phi_t$. E_t is an external transverse test field added to the system and which couples to the nondiagonal matrix element of G_{ij}, and ϕ_t is an external longitudinal test field that couples to the charge density ρ, i.e., to the diagonal element of G_{ij}.

If, in (2.2), the self-energy is taken in the SEA, then functional differentiation of this equation (as usual, however, the screened interaction is not differentiated) with respect to some external field leads to the well-known generalized random-phase approximation (GRPA) for the two-particle Green's functions.[9] The detail of this procedure is given in Refs. 10 and 11 where it was applied to a system of Bose-condensed excitons. With $L_{abmn}(1,2,1',2')\equiv[\delta G_{ab}(1,1')]/[\delta W_{mn}(2',2)]$, where W_{mn} is an external field used to generate the two-particle Green's functions (see Ref. 11), the end result, in Fourier space, is the following set of integral equations for the retarded two-particle Green's functions:

$$\begin{aligned}L_{abmn}(\mathbf{p},\mathbf{k},\mathbf{q};\omega)=&2\int\frac{d\omega'}{2\pi i}G_{am}(p+q)G_{nb}(p)\delta_{\mathbf{p},\mathbf{k}}\\&+2\sum_{\mathbf{p}'}\int\frac{d\omega'}{2\pi i}G_{ai}(p+q)G_{ib}(p)V(\mathbf{q})L_{kkmn}(\mathbf{p}',\mathbf{k},\mathbf{q};\omega)\\&-\sum_{\mathbf{p}'}\int\frac{d\omega'}{2\pi i}G_{ai}(p+q)G_{jb}(p)V_s(\mathbf{p}-\mathbf{p}')L_{ijmn}(\mathbf{p}',\mathbf{k},\mathbf{q};\omega)\ ,\end{aligned}\tag{3.1}$$

where the different two-particle correlation functions L_{abmn} are coupled. In (3.1), $(p+q)\equiv(\mathbf{p}+\mathbf{q},\omega'+\omega)$ and $(p)\equiv(\mathbf{p},\omega')$. The transverse susceptibility corresponds to the function $\chi_t(\mathbf{q};\omega)=-\mu^2\sum_{\mathbf{p},\mathbf{k}}L_{1212}(\mathbf{p},\mathbf{k},\mathbf{q};\omega)$ while the longitudinal susceptibility (the charge-charge correlation function) corresponds to $L(\mathbf{q},\omega)=\sum_{\mathbf{p},\mathbf{k}}L_{iijj}(\mathbf{p},\mathbf{k},\mathbf{q};\omega)$. The proper part $P(\mathbf{q},\omega)$ of this last correlation function enters in the definition of the longitudinal dielectric function

$$\epsilon(\mathbf{q},\omega)=1-V(\mathbf{q})P(\mathbf{q},\omega)\ .\tag{3.2}$$

Notice that the system of Eq. (3.1) which is represented in diagrammatic form in Fig. 1 is equivalent to the variational approach used in Ref. 2 except for the fact that we have replaced the bare Coulomb potential by a screened one [V_s appears in the third term in the right-hand side of (3.1). The bare potential in the second term comes from differentiation of the Hartree part of the self-energy and so is not screened]. Summation over spin variables has been taken care of in (3.1).

Without any pump field and in the simple Elliot's approximation, the optical susceptibility is given by

$$\chi_t(\mathbf{q}=0,\omega)=2\sum_n\frac{f_n^0}{E_n^0-(\omega+i\delta)}=2\sum_n\frac{|\mu\phi_n(r=0)|^2}{E_n^0-(\omega+i\delta)}\ ,\tag{3.3}$$

where $\phi_n(\mathbf{r})$ and E_n^0 are the excitonic wave functions and eigenenergies that are solutions of the Wannier equation (in Fourier space)

$$\left[E_g-\omega_p+\frac{p^2}{2m}\right]\phi_n(\mathbf{p})-\sum_{\mathbf{q}}V(\mathbf{p}-\mathbf{q})\phi_n(\mathbf{q})=E_n^0\phi_n(\mathbf{p})\ .\tag{3.4}$$

In Ref. 2, (3.1) has been solved (for $V_s\to V$) to second order in the pump field, giving a renormalized exciton energy ($E_n^R=E_n^0+\Delta E_n^{\rm HF}$) and oscillator strength ($f_n^R=f_n^0+\Delta f_n^{\rm HF}$) in (3.3). This renormalization occurs

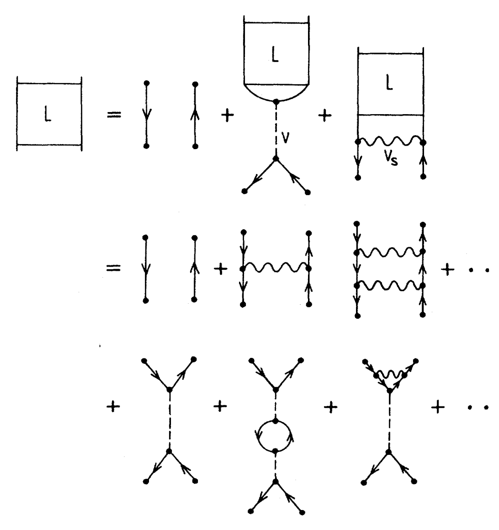

FIG. 1. Diagrammatic expression of Eq. (3.1).

because of additional many-body effects such as exciton-photon interaction, exciton-exciton interaction, phase-space filling, etc., that are neglected in the simple Elliot's picture. For further reference, we refer to them as the Hartree-Fock corrections. In this paper, we consider instead the corrections due to the screened interaction in (3.1), i.e., due to the replacement of V by V_s. These corrections will be denoted by ΔE_n^s and Δf_n^s. As in Ref. 2, we solve the system of Eq. (3.1) to second order in the pump field. This approach presents up to this order no real difficulties although the algebra is a bit cumbersome. We will thus only indicate the main steps of the solution.

Defining a polarization function $P_{abmn}(\mathbf{p},\mathbf{k},\mathbf{q};\omega)$ by

$$P_{abmn}(\mathbf{p},\mathbf{k},\mathbf{q};\omega)=2\int\frac{d\omega'}{2\pi i}G_{am}(p+q)G_{nb}(p)\delta_{\mathbf{p},\mathbf{k}}-\sum_{\mathbf{p}'}\int\frac{d\omega'}{2\pi i}G_{ai}(p+q)G_{jb}(p)V_s(\mathbf{p}-\mathbf{p}')P_{ijmn}(\mathbf{p}',\mathbf{k},\mathbf{q};\omega)\ , \tag{3.5}$$

one can, as usual, rewrite (3.1) as

$$L_{abmn}(\mathbf{p},\mathbf{k},\mathbf{q};\omega)=P_{abmn}(\mathbf{p},\mathbf{k},\mathbf{q};\omega)+\sum_{\mathbf{p}',\mathbf{k}'}P_{abkk}(\mathbf{p},\mathbf{k}',\mathbf{q};\omega)V(\mathbf{q})L_{llmn}(\mathbf{p}',\mathbf{k},\mathbf{q};\omega)\ . \tag{3.6}$$

With $L_{abmn}(\mathbf{q};\omega)\equiv\sum_{\mathbf{p},\mathbf{k}}L_{abmn}(\mathbf{p},\mathbf{k},\mathbf{q};\omega)$ and $P_{abmn}(\mathbf{q};\omega)\equiv\sum_{\mathbf{p},\mathbf{k}}P(\mathbf{p},\mathbf{k},\mathbf{q};\omega)$, Eq. (3.6) can easily be solved in terms of the functions P_{abmn}. One obtains in this way the following exact results:

$$L(\mathbf{q},\omega)=\frac{P(\mathbf{q},\omega)}{\epsilon(\mathbf{q},\omega)}\ , \tag{3.7a}$$

and

$$L_{1212}(\mathbf{q}=0,\omega)=P_{1212}(\mathbf{q}=0,\omega)+\lim_{q\to 0}\frac{[P_{1211}(\mathbf{q},\omega)+P_{1222}(\mathbf{q},\omega)]V(\mathbf{q})[P_{1112}(\mathbf{q},\omega)+P_{2212}(\mathbf{q},\omega)]}{\epsilon(\mathbf{q},\omega)}\ , \tag{3.7b}$$

where $P(\mathbf{q},\omega)\equiv P_{iijj}(\mathbf{q},\omega)$ is effectively the proper part of the retarded function $L(\mathbf{q},\omega)$ as seen from (3.7a) and (3.2). In order to solve for L and χ_t, one needs now only solve the simpler integral equation (3.5).

Using the results of Sec. II for the one-particle Green's functions, the frequency integrals appearing in (3.5) can readily be done and expressed as functions of the coherence factors u_p, v_p. Since $u_p^2\simeq 1+O(E_p^2)$ and $v_p^2\simeq O(E_p^2)$, Eq. (3.5) simplifies considerably to second order in the field. After some algebra, we find ($\alpha\equiv m_h/M$ and $\beta\equiv m_e/M$)

$$\begin{aligned}P(\mathbf{q},\omega)=&\,2\sum_n\sum_{\mathbf{p},\mathbf{k}}\frac{\phi_n(\mathbf{p})\phi_n^*(\mathbf{k})}{(\omega+i\delta)-(E_n^0+q^2/2M)}[(\psi_{\mathbf{p}-\alpha\mathbf{q}}-\psi_{\mathbf{p}+\beta\mathbf{q}})(\psi_{\mathbf{k}-\alpha\mathbf{q}}-\psi_{\mathbf{k}+\beta\mathbf{q}})]\\&-2\sum_n\sum_{\mathbf{p},\mathbf{k}}\frac{\phi_n(\mathbf{p})\phi_n^*(\mathbf{k})}{(\omega+i\delta)+(E_n^0+q^2/2M)}[(\psi_{\mathbf{p}-\beta\mathbf{q}}-\psi_{\mathbf{p}+\alpha\mathbf{q}})(\psi_{\mathbf{k}-\beta\mathbf{q}}-\psi_{\mathbf{k}+\alpha\mathbf{q}})]\ ,\end{aligned} \tag{3.8}$$

and

$$\chi_t(\mathbf{q}=0)=2\sum_n\frac{f_n^R}{E_n^R-(\omega+i\delta)}\ , \tag{3.9}$$

where now $f_n^R\equiv f_n^0+\Delta f_n^{\mathrm{HF}}+\Delta f_n^s$ and $E_n^R\equiv E_n^0+\Delta E_n^{\mathrm{HF}}+\Delta E_n^s$. The screening corrections are given by

$$\begin{aligned}\Delta E_n^s&=\Delta E_n^{(1)}+\Delta E_n^{(2)}\\&=\sum_{\mathbf{p}}\Delta V(\mathbf{p})-\sum_{\mathbf{p},\mathbf{k}}\phi_n(\mathbf{p})\Delta V(\mathbf{p}-\mathbf{k})\phi_n^*(\mathbf{k})\ ,\end{aligned} \tag{3.10}$$

and for the oscillator strength

$$\Delta f_n^s=-\mu^2\sum_{n'\neq 1}\sum_{\mathbf{p},\mathbf{k}}\sum_{\mathbf{p}',\mathbf{k}'}\left[\frac{\phi_n(\mathbf{p})\Delta V(\mathbf{p}-\mathbf{k})\phi_{n'}^*(\mathbf{k})\phi_n^*(\mathbf{p}')\phi_{n'}(\mathbf{k}')+n\rightleftarrows n'}{E_{n'}-E_n}\right]\ . \tag{3.11}$$

In (3.10) and (3.11)

$$\begin{aligned}\Delta V(\mathbf{p})&=V_s(\mathbf{p})-V(\mathbf{p})\\&=V(\mathbf{p})\left[\frac{1}{1+\Delta\epsilon(\mathbf{p})}-1\right]\\&\simeq -V(\mathbf{p})\Delta\epsilon(\mathbf{p})\ ,\end{aligned} \tag{3.12}$$

since, as we will see, $\Delta\epsilon(\mathbf{p})\ll 1$ for large enough detuning $[\Delta\equiv(E_g-\omega_p)/E^0]$, where $E^0=me^4/2\epsilon_0^2$ is the 3D excitonic Rydberg. Note that the first correction to the exciton energy in (3.10) is the Coulomb-hole contribution already discussed in Sec. II. For the special case $n=1$ (the only one of interest here), we can express the screening corrections as a function $\Delta\epsilon_p$, only. We find with

$y \equiv qa_0$, where $a_0 = \epsilon_0/me^2$ is the 3D excitonic Bohr radius as follows.

In 3D,

$$\frac{\Delta E_1^{(1)}}{E^0} = -\frac{4}{\pi} \int_0^\infty dy\, \Delta\epsilon(y) , \quad (3.13)$$

$$\frac{\Delta E_1^{(2)}}{E^0} = \frac{64}{\pi} \int_0^\infty dy \frac{\Delta\epsilon(y)}{(4+y^2)^2} . \quad (3.14)$$

In 2D,

$$\frac{\Delta E_1^{(1)}}{E^0} = -2 \int_0^\infty dy\, \Delta\epsilon(y) , \quad (3.15)$$

$$\frac{\Delta E_1^{(2)}}{E^0} = 128 \int_0^\infty dy \frac{\Delta\epsilon(y)}{(16+y^2)^{3/2}} . \quad (3.16)$$

To evaluate the corrections to f_1, we need to introduce a few more definitions. Firstly, we need the Green's function of the Wannier equation defined by

$$[\omega + i\delta - (E_g - \omega_p + p^2/2m + q^2/2M)] G_{ex}(\mathbf{p},\mathbf{k},\mathbf{q};\omega) + \sum_{\mathbf{p}'} V(\mathbf{p}-\mathbf{p}') G_{ex}(\mathbf{p}',\mathbf{k},\mathbf{q};\omega) = -\delta_{\mathbf{p},\mathbf{k}} , \quad (3.17)$$

which has the explicit solution

$$G_{ex}(\mathbf{p},\mathbf{k},\mathbf{q};\omega) = -\sum_n \frac{\phi_n(\mathbf{p})\phi_n^*(\mathbf{k})}{\omega + i\delta - (E_n^0 + q^2/2M)} . \quad (3.18)$$

Secondly, we need the reduced exciton Green's function $G_{ex}^{red}(\mathbf{p},\mathbf{k},\mathbf{q};\omega)$ defined as in (3.18), but with the restriction $n \neq 1$ in the summation. The screening corrections to Δf_n^s can now be simplified (with $f_1^0 = |\mu\phi_1(r=0)|^2$) to

$$\frac{\Delta f_1^s}{f_1^0} = -\frac{2}{\phi_1(r=0)} \sum_{\mathbf{p}} \Delta\epsilon(\mathbf{p}) V(\mathbf{p}) F(\mathbf{p}) , \quad (3.19)$$

where

$$F(\mathbf{p}) \equiv \int d\mathbf{r}' e^{i\mathbf{p}\cdot\mathbf{r}'} \phi_1(r') G_{ex}^{red}(r=0,r',q=0;\omega=0) . \quad (3.20)$$

The Fourier transform of the Green's function is given by

$$G_{ex}^{red}(\mathbf{r},\mathbf{r}',\mathbf{q};\omega) = \int d\mathbf{p}\, d\mathbf{k}\, e^{i\mathbf{p}\cdot\mathbf{r} - i\mathbf{k}\cdot\mathbf{r}'} G_{ex}^{red}(\mathbf{p},\mathbf{k},\mathbf{q};\omega) . \quad (3.21)$$

The reduced exciton Green's function $G_{ex}^{red}(r=0,r',q=0;\omega=0)$ is given[12,13] by ($z \equiv r'/a_0$ and $\gamma = 0.577\,216$ is the Euler's constant)

$$G_{ex}^{red}(r=0,r',q=0;\omega=0) = \frac{e^{-z}}{2\pi a_0^3 E^0} \left[\frac{1}{2z} + 2.5 - \gamma - \ln(2z) - z \right] \text{ in 3D} ,$$
$$= \frac{e^{-2z}}{2\pi a_0^2 E^0} [3 - \gamma - \ln(4z) - 4z] \text{ in 2D} . \quad (3.22)$$

Equation (3.19) can finally be written as follows.

In 3D,

$$\frac{\Delta f_1^s}{f_1^0} = -\frac{16}{\pi} \int_0^\infty dy\, \Delta\epsilon(y) \left[\frac{y^4 + 24y^2 + 16}{2(4+y^2)^3} + \frac{y - 4/y}{(4+y^2)^2} \arctan\left[\frac{y}{2}\right] + \frac{2}{(4+y^2)^2} \ln\left[\frac{4+y^2}{4}\right] \right] . \quad (3.23)$$

In 2D,

$$\frac{\Delta f_1^s}{f_1^0} = -\frac{4}{\pi} \int_0^\infty dy\, \Delta\epsilon(y) \int_0^{\pi/2} d\theta \left[\frac{24y^2}{(16+y^2)^{5/2}} + \frac{16y\cos\theta}{(16+y^2\cos^2\theta)^2} \arctan\left[\frac{y\cos\theta}{4}\right] + \frac{16 - y^2\cos^2\theta}{(16+y^2\cos^2\theta)^2} \ln\left[\frac{16+y^2\cos^2\theta}{16}\right] \right] . \quad (3.24)$$

If $\Delta\epsilon$ was a constant, then we could write $\chi_t(q=0,\omega) = 2\sum_n (|\mu\phi_n^s(r=0)|^2)/[E_n^s - (\omega+i\delta)]$, where ϕ_n^s, E_n^s are solutions of the Wannier equation (3.4) with $V(q) \to V_s(q) = V(q)/(1+\Delta\epsilon)$. In this case, we would have obviously the simple relations $\Delta E_1^2/E^0 = 2\Delta\epsilon$ (3D) or $8\Delta\epsilon$ (2D) and $\Delta f_1^s/f_1^0 = -3\Delta\epsilon$ (3D) or $-2\Delta\epsilon$ (2D). As we will see in the next section, these simple relations are acceptable approximations in 3D but do not apply in the 2D case.

IV. DIELECTRIC FUNCTION IN THE GRPA

To evaluate the corrections of Sec. III, we need a numerically computable expression for the change in the dielectric function $\Delta\epsilon(\mathbf{q}) = -V(\mathbf{q})P(\mathbf{q})$ with $P(\mathbf{q})$ given by (3.8). First of all, we need an expression for $\psi_{\mathbf{p}}$ to order E_p^2. From (2.8), we easily see that, to this order,

$$\psi_{\mathbf{p}} = \mu E_p \sum_n \frac{\phi_n(\mathbf{p})\phi_n^*(r=0)}{E_n^0 - \omega_p} = \mu E_p \int d\mathbf{r}\, e^{i\mathbf{p}\cdot\mathbf{r}} G_{ex}(r,r'=0,q=0;\omega=0) . \quad (4.1)$$

So, using (4.1), (3.18), and (3.21), we get from (3.8)

$$P(q)=-4(\mu E_p)^2\int d\mathbf{r}\,d\mathbf{r}'G_{\rm ex}(\mathbf{r},-\mathbf{r}',\mathbf{q};\omega=0)G_{\rm ex}(r,r'=0,q=0;\omega=0)G_{\rm ex}(r=0,r',q=0;\omega=0)$$
$$\times[\cos(\alpha\mathbf{q}\cdot\mathbf{R})+\cos(\beta\mathbf{q}\cdot\mathbf{R})-\cos(\mathbf{q}\cdot\mathbf{R}_0)-\cos(\mathbf{q}\cdot\mathbf{R}_{00})]\ , \tag{4.2}$$

where $R\equiv|\mathbf{r}+\mathbf{r}'|$, $R_0\equiv|\alpha\mathbf{r}-\beta\mathbf{r}'|$, $R_{00}\equiv|\beta\mathbf{r}-\alpha\mathbf{r}'|$.

We use for the function $G_{\rm ex}(\mathbf{r},\mathbf{r}',\mathbf{q};\omega=0)$ in 3D the integral representation[14]

$$G_{\rm ex}(\mathbf{r},\mathbf{r}',\mathbf{q};\omega=0)=\frac{\bar\delta\bar a}{2\pi a_0^3E^0B(\bar a)}\int_0^1 dz\frac{\exp(-\bar\delta\bar f a_2/2)}{(1-z^{\bar a})^2}\int_0^1(1-t^{\bar a})\exp(\bar\delta\bar g a_1/2)\left[1+\frac{\bar\delta a_1a_2(\bar g+\bar f)}{2(a_2-a_1)}\right]\ , \tag{4.3}$$

where we have defined

$$\delta\equiv[(E_g-\omega_p)/E^0]^{1/2},$$
$$\bar\delta\equiv[(E_g-\omega_p+q^2/2M)/E^0]^{1/2}\ , \tag{4.4}$$

$$a\equiv\frac{\delta}{\delta-1},\quad \bar a\equiv\frac{\bar\delta}{\bar\delta-1}\ , \tag{4.5}$$

$$\bar f\equiv\frac{1+z^{\bar a}}{1-z^{\bar a}},\quad \bar g\equiv(2t^{\bar a}-1)\ , \tag{4.6}$$

$$a_0a_1\equiv r+r'-|\mathbf{r}-\mathbf{r}'|,\quad a_0a_2\equiv r+r'+|\mathbf{r}-\mathbf{r}'|\ , \tag{4.7}$$

$$B(\bar a)\equiv\int_0^1 dt(1-t^{\bar a})^{1-1/\bar a}\ . \tag{4.8}$$

Setting $\bar a=1$ in (4.3) gives the noninteracting Green's function $G_{\rm ex}^0(\mathbf{r},\mathbf{r}',\mathbf{q};\omega=0)$ defined by (3.17) with $V=0$. The other Green's function appearing in (4.2), i.e., $G_{\rm ex}(r,r'=0,q=0;\omega=0)$ can be obtained from (4.3) by the substitution $\bar a\to a$ and $r'=0$. $G_{\rm ex}^0(r,r'=0,q=0;\omega=0)$ is then obtained from this last function by setting $a=1$.

Equation (4.2) defines the generalized random-phase approximation (GRPA) to $P(q)$. Its diagrammatic expression is depicted in Fig. 2(a). The random-phase approximation (RPA) depicted in Fig. 2(b) is easily obtained from (4.2) by replacing the first Green's function in this equation by its noninteracting counterpart (i.e., setting $\bar a=1$ in the 3D integral representation as discussed above). A third approximation to the polarization is obtained by replacing all Green's functions in (4.2) by their noninteracting counterpart (i.e., setting $\bar a=1$ and $a=1$). This last approximation is depicted in Fig. 2(c) where g is defined by (2.2) but with $V=0$. We will refer to this approximation as the noninteracting approximation (NIA). These different approximations will be discussed later.

The analysis is essentially the same in 2D. Due to some computational problems, we will, however, consider only the RPA in this case. For this case, we use for the integral representation of the Green's function $G_{\rm ex}(r,r'=0,q=0;\omega=0)$

$$G_{\rm ex}(r,r'=0,q=0;\omega=0)=\frac{a}{2\pi a_0^2E^0}\int_0^1 dz\frac{\exp(-\delta rf/a_0)}{1-z^a}\ , \tag{4.9}$$

where (4.4) is unchanged, but

$$a\equiv\frac{2\delta}{\delta-2}\ , \tag{4.10}$$

$$f\equiv\frac{1+z^a}{1-z^a}\ . \tag{4.11}$$

The noninteracting Green's function $G_{\rm ex}^0(\mathbf{r},\mathbf{r}',\mathbf{q};\omega=0)$ is found from (4.9) by the substitution $a=2$, $\delta\to\bar\delta$ and $r\to|\mathbf{r}-\mathbf{r}'|$.

Combining the results of this section, we finally get for the change in the dielectric function as follows.

In 3D,

$$\Delta\epsilon_{\rm GRPA}(y)=32\left[\frac{\mu E_p}{E^0}\right]^2\frac{\delta^2\bar\delta a^2\bar a}{y^2B(\bar a)}\int_0^1 dR\int_0^1 dR'\int_0^1 dz\int_0^1 dz'\int_0^1 dz''\int_{-1}^1 ds\int_0^1 dt$$
$$\times(1-t^{\bar a})^{1/\bar\delta}\left[1+\frac{\bar\delta a_1a_2(\bar g+\bar f)}{2(a_2-a_1)}\right]\left[\frac{R^2R'^2\exp\left[-\dfrac{\bar\delta\bar f_2}{2}-\delta f'x-\delta f''x'+\dfrac{\bar\delta\bar g_1}{2}\right]}{(1-R)^4(1-R')^4(1-z^{\bar a})^2(1-z'^a)^2(1-z''^a)^2}\right]$$
$$\times\left[\frac{\sin(\alpha y|\mathbf{x}+\mathbf{x}'|)}{\alpha y|\mathbf{x}+\mathbf{x}'|}+\frac{\sin(\beta y|\mathbf{x}+\mathbf{x}'|)}{\beta y|\mathbf{x}+\mathbf{x}'|}-\frac{\sin(y|\alpha\mathbf{x}-\beta\mathbf{x}'|)}{y|\alpha\mathbf{x}-\beta\mathbf{x}'|}-\frac{\sin(y|\beta\mathbf{x}-\alpha\mathbf{x}'|)}{y|\beta\mathbf{x}-\alpha\mathbf{x}'|}\right]\ , \tag{4.12}$$

where, in this formula,

$$a_1\equiv x+x'-|\mathbf{x}+\mathbf{x}'|,\quad a_2\equiv x+x'+|\mathbf{x}+\mathbf{x}'|\ , \tag{4.13}$$

$$\mathbf{x}\equiv\frac{\mathbf{R}}{1-R},\quad \mathbf{x}'\equiv\frac{\mathbf{R}'}{1-R'}\ , \tag{4.14}$$

$$f \equiv \left[\frac{1+z'^a}{1-z'^a} \right], \quad f' \equiv \left[\frac{1+z''^a}{1-z''^a} \right] . \tag{4.15}$$

In 2D,

$$\Delta\epsilon_{\mathrm{RPA}}(y) = 2 \left[\frac{\mu E_p}{E^0} \right]^2 \frac{2a^2}{\pi^2 y} \int_0^1 dR \int_0^1 dR' \int_0^1 dz \int_0^1 dz' \int_0^1 dz'' \int_0^{2\pi} d\theta \int_0^{2\pi} d\theta'$$
$$\times \left[\frac{RR' \exp(-\delta \bar{f} |\mathbf{x}+\mathbf{x}'| - \delta f' x - \delta f'' x')}{(1-R)^3 (1-R')^3 (1-z^2)(1-z'^a)(1-z''^a)} \right]$$
$$\times [\cos(\alpha y x \cos\theta + \alpha y x' \cos\theta') + \cos(\beta y x \cos\theta + \beta y x' \cos\theta')$$
$$- \cos(\alpha y x \cos\theta - \beta y x' \cos\theta') - \cos(\beta y x \cos\theta - \alpha y x' \cos\theta')] , \tag{4.16}$$

where x, x', f, f' are defined as in (4.14) and (4.15) and

$$s \equiv \cos(\mathbf{R}, \mathbf{R}') ,$$
$$\cos\theta \equiv \cos(\mathbf{R}, \mathbf{q}) , \tag{4.17}$$
$$\cos\theta' \equiv \cos(\mathbf{R}', \mathbf{q}) ,$$
$$\bar{f} \equiv \left[\frac{1+a^2}{1-a^2} \right] . \tag{4.18}$$

In this paper, E^0 and a_0 are always taken as the 3D excitonic Rydberg and Bohr radius, respectively.

Although the final expressions for $\Delta\epsilon$ look rather complicated, they are easily evaluated by using a Monte Carlo integration routine. In this case, the number of dimensions is not a problem if the integrand is a smooth enough function. This is the case for the integrals in (4.12) and (4.16). These integrands include sin and cos functions that oscillate, but these oscillations are cut off by the rapidly decreasing exponential function. We have verified that the Monte Carlo routine effectively gives a rapidly convergent result for a root-mean-square relative dispersion σ of the order of 10%. The screening corrections to the energy and oscillator strength are also evaluated by Monte Carlo integration. For them, we need only one more integration over the variable y (two for the special case of f_1 in 2D).

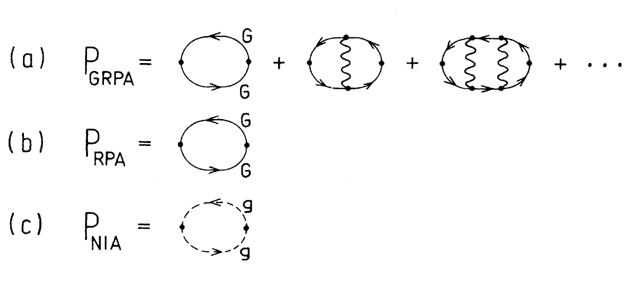

FIG. 2. Diagrammatic expansion of $P(q)$ in the (a) GRPA, (b) RPA, and (c) NIA. G and g are defined in the text.

In concluding this section, we wish to discuss the result (3.8) for $P(q)$. The diagrammatic expression of this function was given in Fig. 2(a). To order E_p^2, as we have seen, all terms in Fig. 2(a) contribute to the sum. Of course, this approximation for $P(q)$ does not contain all terms of this order since we have only summed up here a definite set of diagrams known as the GRPA. The GRPA has, however, the advantage to be a well-known approximation giving reasonable results in exciton systems (see Ref. 12 and references given therein). In the language of Baym and Kadanoff,[10] it is a "conserving approximation," meaning that the usual sum rules are satisfied by $P(q)$. The GRPA has already been considered in a formal theory for the Stark effect in Ref. 8 (in this paper, however, no result for the screening corrections were derived).

As can be seen from the denominators of $P(q)$ in (3.8), this function includes only single-pair processes involving the creation or destruction of virtual excitons in state $n, \mathbf{q}$ (with the energy measured with respect to ω_p in the rotating frame). This $P(q)$ should not be confused with excitonic screening which implies the more general scattering processes $(n, \mathbf{q}) \to (n', \mathbf{q})$ involving two pairs. To get excitonic screening, one needs to sum a more important set of diagrams than the one represented in Fig. 2(a). As we already mentioned in Sec. II, the ground state of the system considered here consists of a coherent superposition of electron-hole pairs. This is reflected by the fact that the poles of the frequency sums $\int d\omega' G_{im}(\omega') G_{nj}(\omega+\omega')$ appearing in (3.1) contain only the pair processes $\pm(\epsilon_1 - \epsilon_2)$, i.e., there are no processes involving the scattering of single particles. In the pumped system considered in this paper, $P(q)$ is thus completely different from the usual screening found in an equilibrium plasma system and which is mostly due to the motion of free carriers. Here, the screening is of insulating character and disappears when $E_p = 0$, since then there is no virtual polarization in the system.

In the RPA introduced earlier, $P(q)$ is approximated by retaining only the first bubble in the ladder diagrams of Fig. 2(a). In our pumped system, we expect this approximation to give a smaller $\Delta\epsilon$ since by doing this we take into account only the virtually excited free states in the summation over n [the denominator for the free states is bigger than for the bound states and so the resulting $P(q)$ is smaller]. In the RPA, $P(q)$ includes only processes involving the creation or destruction of excitons in some free state. The effect of the ladder diagrams in Fig. 2(a) is to bring in the bound states in (3.8), giving to the gas a bigger polarizability than in the RPA. As we mentioned above, with the integral representations used here, $\Delta\epsilon_{\mathrm{GRPA}}(\bar{a}\rightarrow 1,a)\rightarrow\Delta\epsilon_{\mathrm{RPA}}(a)$ in 3D.

The NIA introduced above is a lowest-order approximation which takes into account no bound states. It is calculated from the RPA with the help of the relation $\Delta\epsilon_{\mathrm{RPA}}\,[a\rightarrow 1$ (3D) or 2 (2D)$]\rightarrow\Delta\epsilon_{\mathrm{NIA}}$. This last approximation is useful because in this simple case, the dielectric functions can be reduced to the following much simpler expressions.

In 3D,

$$\Delta\epsilon(y)=\frac{64}{\pi\delta^5}\left[\frac{\mu E_p}{E^0}\right]^2\int_0^\infty dz\frac{z^4}{(1+z^2+y^2/4)^5}\int_0^1 dx\frac{x^2}{(u^2x^2-1)^2(1-g^2u^2x^2)}\ . \tag{4.19}$$

In 2D,

$$\Delta\epsilon(y)=\frac{64y}{\pi\delta^5}\left[\frac{\mu E_p}{E^0}\right]^2\int_0^\infty dz\frac{z^3}{(1+z^2+y^2/4)^5}\int_0^{\pi/2} d\theta\frac{\cos^2\theta}{(u^2\cos^2\theta-1)^2(1-g^2u^2\cos^2\theta)}\ , \tag{4.20}$$

where $g\equiv(m_h-m_e)/M$, $u\equiv yz(1+z^2+y^2/4)$, and $y\equiv qa_0/\delta$. To evaluate these expressions, one need only use a simple Riemann integration routine. We will thus be able to compare the result of the Monte Carlo routine for this NIA with that of the Riemann integration. This gives us a way to judge the validity of our Monte Carlo approach for the RPA and GRPA approximations.

V. NUMERICAL RESULTS

We now give the results of the numerical integration for the dielectric function and the corrections to E_1 and f_1. We first show, in Fig. 3, the result of the comparison between the Riemann and the Monte Carlo integrations for the NIA, as explained in Sec. IV. We see that at least in this case, the Monte Carlo approach gives a very good result. We expect that this is also true for the other two approximations (RPA and GRPA) since they are obtained from the same integrand by modifying only one or two parameters.

In Fig. 4, $\Delta\epsilon(y)$ in the GRPA and in 3D is evaluated for different values of the mass ratio, showing that this factor has indeed little influence. The same trend persists for the other two approximations and in 2D. Note that when $\mathbf{q}\rightarrow 0$, $P(q)$ depends only on the reduced mass m. The influence of the detuning parameter is also shown in the range $\Delta>10$. As we see, the screening is very sensitive to this parameter. When the detuning is increased by a factor of 2, $\Delta\epsilon$ is decreased by at least a factor 10 (and even more in 2D). In this range, however, the condition $\Delta\epsilon\ll 1$ is satisfied if we take $F\equiv(\mu E_p)/E^0=1$. In 3D or 2D GaAs and assuming the same (3D) effective mass for both structures for simplicity, this value of the field corresponds[6] to an intensity of approximately 30 $\mathrm{MW/cm^2}$. According to Ref. 6, this value of the intensity also corresponds to the region where the linear theory is expected to be valid. For $\Delta<10$ (not shown), $\Delta\epsilon$ increases even faster when the detuning is decreased. In this last range,

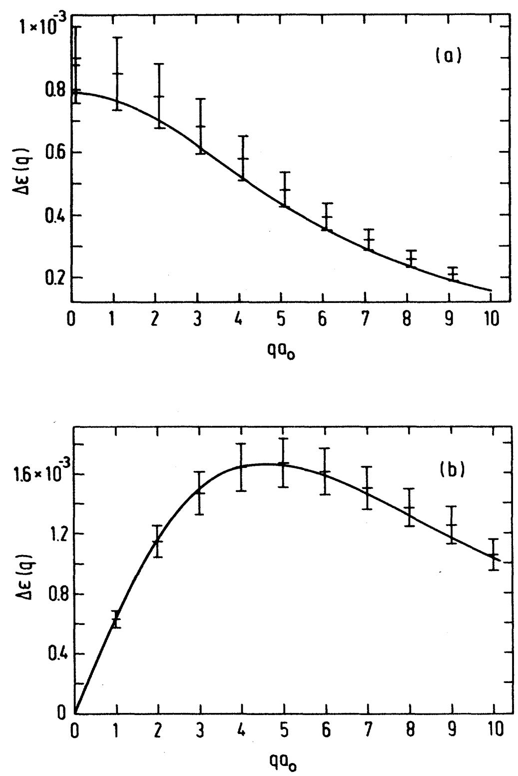

FIG. 3. (a) Comparison between Riemann (solid line) and Monte Carlo integrations (+) for $\Delta\epsilon(q)$ in the NIA for 3D. Parameters: $\Delta=10$, $\beta=0.1$, $\alpha=0.9$, and $F=1$. The error bars represent a relative root-mean-square deviation $\sigma\simeq 10\%$ for the Monte Carlo integration. (b) Comparison between Riemann (solid line) and Monte Carlo integrations (+) for $\Delta\epsilon(q)$ in the NIA for 2D. Parameters: $\Delta=10$, $\beta=0.1$, $\alpha=0.9$, and $F=1$. The error bars represent a $\sigma\simeq 10\%$ for the Monte Carlo integration.

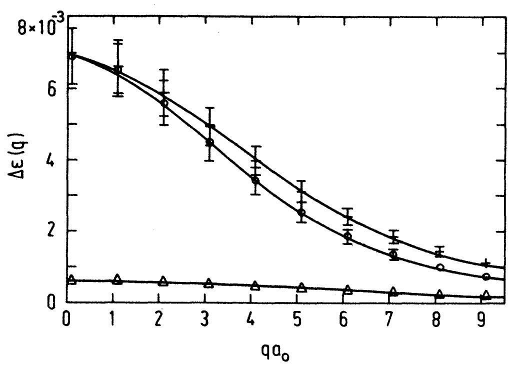

FIG. 4. $\Delta\epsilon(q)$ in the GRPA for different values of the mass ratios and detuning for 3D ($\sigma \simeq 10\%$) and with $F=1$. Parameters: $+$, $\Delta=10$, $\beta=0.1$, $\alpha=0.9$; $\circ$, $\Delta=10$, $\beta=0.45$, $\alpha=0.55$; $\triangle$, $\Delta=20$, $\beta=0.1$, $\alpha=0.9$.

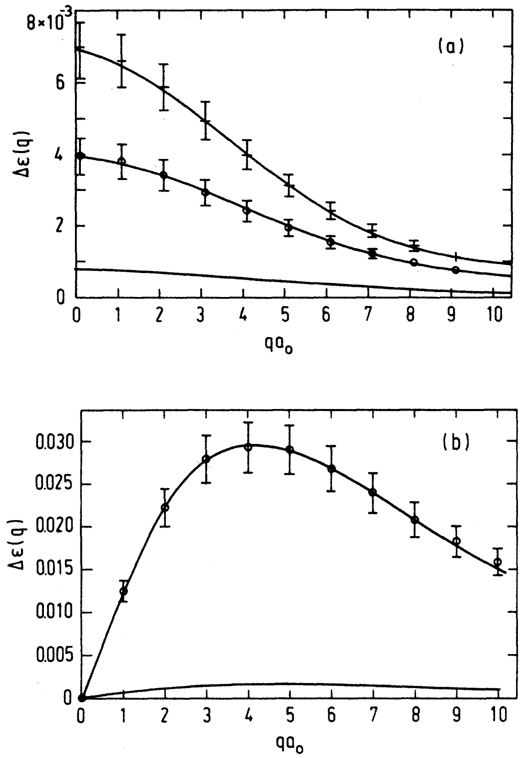

FIG. 5. Comparison of the three approximations (see text) to $\Delta\epsilon(q)$ in 3D ($\sigma \simeq 10\%$) with $F=1$. $\Delta=10$; $+$, GRPA; $\circ$, RPA; (——), NIA. (b) Comparison of the two approximations (see text) to $\Delta\epsilon(q)$ in 2D ($\sigma \simeq 10\%$), $F=1$, $\Delta=10$; $\circ$, RPA; (——), NIA.

$\Delta\epsilon$ can become very big for a given field strength. But in order to fulfill still the condition $\Delta\epsilon \ll 1$ the field strengths for which the results are valid become smaller and smaller for decreasing detuning. Comparing the results for GRPA with those for NIA (see Fig. 5), we see that $\Delta\epsilon(\text{GRPA}) \to \Delta\epsilon(\text{NIA})$ for large detunings as it should be since for large values of the detuning parameter, the excitonic Green's function tends toward the noninteracting function.

In Fig. 5, the three different approximations to $\Delta\epsilon$ discussed earlier are compared. From these curves, we see that the strength of the screening depends much on the approximation considered. Inclusion of more and more correlations leads to an increase in the dielectric function as expected. Notice that the behavior of $\Delta\epsilon$ is quite different in 3D and 2D. This is normal since (for $q \to 0$) $P(q) \propto q^2$ in both dimensions while $V(q) \propto q^{-2}$ (3D) and $V(q) \propto q^{-1}$ (2D).

Table I lists the screening corrections to the exciton energy and oscillator strength. These are compared with the HF corrections to Ref. 6 calculated for 3D and 2D GaAs structures (note that in Ref. 6, the detuning parameter is defined with respect to the exciton level E_1). We have taken the values $\beta=0.1, \alpha=0.9$ appropriate for 3D GaAs in both 3D and 2D results. From this comparison, we see that the screening corrections in 3D and in the GRPA are small compared with the HF corrections for $\Delta > 10$. One can thus neglect them in this range of detuning. For $\Delta \simeq 5$, however, they are already important, in-

TABLE I. Screening and HF corrections to E_1 and f_1. The HF corrections are in parentheses. All energy corrections are to be multiplied by $(\mu E_p/E^0)^2$. Parameters are given in the text.

Δ	$\Delta E_1^{(1)}/E^0$	$\Delta E_1^{(2)}/E^0$	$\Delta E_1^s/E^0$	$\Delta f_1^s/f_1^0$
(2D)				
8	-1.90 ± 0.09	0.470 ± 0.020	-1.43 ± 0.11	-0.110 ± 0.010
			(2.00)	(-0.16)
14	-0.20 ± 0.01	0.033 ± 0.002	-0.17 ± 0.01	-0.010 ± 0.001
			(0.66)	(0.002)
24	-0.037 ± 0.005	0.0038 ± 0.0002	-0.033 ± 0.005	-0.0013 ± 0.0001
			(0.25)	(0.002)
(3D)				
5	-0.51 ± 0.03	0.18 ± 0.02	-0.33 ± 0.05	-0.16 ± 0.01
			(1.28)	(0.27)
11	-0.036 ± 0.002	0.0082 ± 0.0005	-0.028 ± 0.003	-0.0090 ± 0.0005
			(0.47)	(0.062)
21	-0.0056 ± 0.0006	$0.000\,94\pm0.000\,08$	-0.0047 ± 0.0007	-0.0011 ± 0.0001
			(0.185)	(0.015)

creasing much faster than the HF corrections with decreasing Δ. In 2D the screening corrections are of the same order as the HF corrections already in the RPA and, for $\Delta \simeq 10$, one can expect somewhat larger corrections in the GRPA. For the oscillator strength, the correction is even bigger than the HF one, leading to a change in the sign of this quantity. The resulting 2D oscillator strength thus decreases always slightly with increasing pump intensity. For large detunings ($\Delta \simeq 21$) this correction gets smaller than in the HF case and the correction to the energy is then negligible. Notice, however, that the HF corrections to the oscillator strength[6] are quite singular in 2D since they change sign around $\Delta = 14$. For $\mu E_p / E^0 = 1$ (which corresponds approximately to a field intensity of 30 MW/cm^2 as we mentioned earlier), the corrections (HF and screening) to the oscillator strength are, however, very small (of the order of 2%). At the moment, it is not clear how these corrections can be compared with experimental values. The aim of this paper was indeed to compare the effect of screening on previous corrections, not with experimental values.

Finally, we note that, in 3D, the potential can reasonably be approximated by a constant in the calculation of ΔE_1^2 and Δf_1^s. With $\Delta\epsilon(q=0)=0.004$ for $\Delta = 11$, we see that the simple expressions $\Delta E_1^2 / E^0 \simeq 2\Delta\epsilon$ and $\Delta f_1^s / f_1^0 \simeq -3\Delta\epsilon$ given in Sec. III are not too wrong. Because of the behavior of the potential in 2D, this type of approximation is, of course, not possible.

VI. CONCLUSION

We have evaluated the screened potential in the GRPA and RPA approximations for the case of a semiconductor submitted to an applied pump field. In the limit of low intensity (second order in the pump field) and large detuning $\Delta > 10$, the resulting screening corrections to the exciton energy and oscillator strength in 3D are small. In 2D, screening effects are more pronounced and result in a slowly decreasing oscillator strength with increasing pump intensity. Only for very large detuning do the 2D screening corrections also become negligible with respect to previous corrections.[6]

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