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Lifetime for resonant tunneling in a transverse magnetic field

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We present a numerical study of the time-dependent resonant tunneling of a Gaussian wave packet through a double-barrier structure in a transverse magnetic field. From the decay rate of the charge trapped in the quantum well, we obtain the field dependence of the lifetime τ_0 of the resonant level. τ_0 is found to exhibit an oscillatory behavior with increasing magnetic field. This effect is explained as due to the field-induced hybridization of the resonant level with the interfacial Landau states corresponding to the semiclassical skipping orbits at the barrier interfaces. An experiment which should allow the observation of such effects in real heterostructures is suggested.

I. INTRODUCTION AND METHOD

The present availability of high-quality semiconductor heterostructures and high magnetic fields has stimulated many experimental studies of magnetotransport in low-dimensional electronic systems. A particular topic of increasing current interest concerns the effect of a transverse magnetic field on the tunnel current through a potential barrier separating, e.g., two semiconductors, where electrons are forced to execute cyclotron motion while tunneling through the barrier in the plane perpendicular to the field.[1-6] In Refs. 1 and 5, for instance, the tunnel current J through single-barrier semiconductor heterostructures in a transverse field B has been studied, and the observed weak oscillatory structures superimposed on an otherwise exponentially decreasing J versus B curve have been interpreted as due to electron tunneling into the "interfacial" Landau levels corresponding to semiclassical skipping orbit.

Similarly, one may ask what would be the effect of a transverse magnetic field in the case of *resonant* tunneling through a double-barrier (DB) structure.[7] For instance, the current-voltage characteristic of DB heterostructures shows marked negative-differential-resistance (NDR) features associated with the presence of the resonant levels in the quantum well (QW)[8] and one interesting question in the present context could concern the effect of an external magnetic field on these NDR characteristics.

As a "zero-order" approach to the complex problem of transport through DB structures in a magnetic field, we wish to consider here the much simpler problem of *wave-packet* tunneling through a DB in a transverse field. As discussed in Sec. III, qualitatively new features, such as oscillations in the resonant-level lifetime due to the presence of the field, are found as a result of our calculations.

Our approach is described in detail in Ref. 9, where the time-dependent tunneling of a wave packet through a thick barrier in a transverse magnetic field was studied. As in Ref. 9, the present calculations are based on the direct solution of the time-dependent Schrödinger equation. The initial wavefunction Ψ_0, described by a localized wave packet, is represented on a discrete spatial grid and the Hamiltonian operation $\hat{H}\Psi_0$ is calculated locally in coordinate space, using the fast-fourier-transform algorithm to evaluate efficiently spatial derivatives. The solution is then propagated in time. For this purpose, we use the Chebychev scheme,[10] which is based upon a suitable polynomial expansion of the time-evolution operator. This scheme has proven to be more efficient and accurate than other methods, in particular when high accuracy and long propagation times are needed at the same time.[9]

Our model Hamiltonian is that of a (spinless) particle, of mass m^* and charge e, subject to a constant and uniform transverse magnetic field $\mathbf{B}=B\hat{\mathbf{z}}$, i.e.,

$$\hat{H}=\frac{1}{2m^*}\left[\mathbf{p}-\frac{e}{c}\mathbf{A}\right]^2+V_{\rm DB}(x)\ , \qquad (1)$$

where $\mathbf{p}\equiv -i\hbar\nabla$, $\mathbf{A}$ is the vector potential, and $V_{\rm DB}(x)$ represents the DB potential. In our case the latter consists of two equal square barriers of width d and height V_0 separated by a distance a, as shown schematically in the inset of Fig. 1. Resonant tunneling through this structure occurs when the energy of the incident particle matches that of an unoccupied discrete state in the well between the two confining barriers. In this case, a strong enhancement of the transmission coefficient takes place, due to the Fabry-Perot-like interfering reflections of the particle wave function in the DB region. This is illustrated in Fig. 1, where the transmission coefficient of the DB shown in the inset is plotted versus the energy of the incident particle. With our choice of DB parameters (given in the caption to Fig. 1) a single broad resonance occurs at $E=0.057$ eV.

We use in (1) the symmetric gauge for the vector potential, i.e., $\mathbf{A}=\mathbf{A}_S\equiv(B/2)(-y,x,0)$. The motion along z (i.e., in the direction of the magnetic field) reduces to free-particle motion, $\sim\exp(ik_z z)$, which can be factorized out. For each k_z, we are thus left with a two-dimensional (2D) Hamiltonian, namely

$$\hat{H}=\frac{1}{2m^*}(p_x^2+p_y^2)+\tfrac{1}{8}m^*\omega_c^2(x^2+y^2)-\tfrac{1}{2}\omega_c L_z+V_{\rm DB}(x)\ . \qquad (2)$$

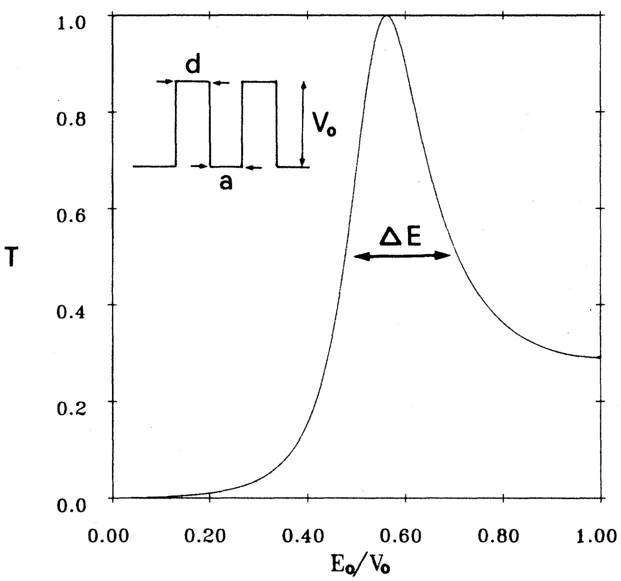

FIG. 1. Transmission coefficient T for the double barrier shown in the inset vs incident energy. The barrier parameters are $V_0=0.1$ eV, $d=35$ Å, and $a=47$ Å. The width $\Delta E \sim \hbar/\tau_0$ of the resonance, as obtained from our numerical results for τ_0, is also shown (horizontal arrow).

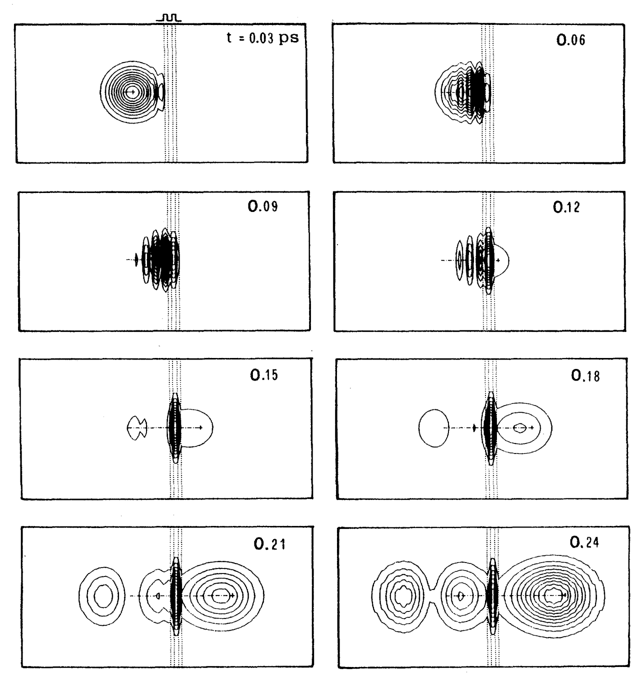

FIG. 2. Contour plots of the charge density $|\Psi(t)|^2$, at different times t, for the scattering of a Gaussian wave packet of width $\sigma=150$ Å by the double-barrier structure shown in the inset of Fig. 1. At $t=0$ the wave packet is moving from the left towards the DB with energy $E_0=0.057$ eV at resonance with the single quasistationary state of the well. The DB edges are shown with vertical dotted lines. The dashed-dotted line shows the wave-packet trajectory in the absence of the DB potential.

Here $L_z \equiv xp_y - yp_x$ is the z component of the angular momentum and $\omega_c \equiv eB/m^*c$ is the cyclotron frequency. Note that when $V_{DB}=0$, (2) reduces to the Hamiltonian of a 2D isotropic harmonic oscillator of frequency $\omega_c/2$.[11]

Because of the finite size of the 2D grid used in our simulations, the initial wave function must be localized in space. We use here a standard two-dimensional Gaussian wave packet:

$$\Psi_0 \equiv \Psi(\mathbf{R};t=0)=(2\pi\sigma^2)^{-1/2}e^{-\mathbf{R}^2/4\sigma^2}e^{i\mathbf{k}_0\cdot\mathbf{R}} , \quad (3)$$

where $\mathbf{R}\equiv(x,y)$, σ is the spatial width of the packet, and $\mathbf{k}_0$ its initial momentum. The use of localized wave packets implies a finite-width distribution of momenta about the average value $\mathbf{k}_0$. Since the Fourier components of energy higher than the top of the barrier propagate through the latter as classical particles, eventually masking the effects due to "pure" tunneling, one must require the portion of the transmitted packet whose plane-wave ponents have energies above the top of the barrier to be much smaller than the under-the-barrier part. It can be shown[9] that a sufficient condition for the above requirement to be satisfied is

$$\sigma^2(k_M-k_0)^2 \gtrsim d\,(k_M^2-k_0^2)^{1/2} , \quad (4)$$

where $k_M=(2m^*V_0/\hbar^2)^{1/2}$. In all our calculations σ is such that the above inequality is satisfied.

II. RESULTS

Consider first the case of zero magnetic field. We show in the sequence of plots of Fig. 2 the time behavior of the calculated charge density $|\Psi(\mathbf{R},t)|^2$ (represented by means of contours of equal height) for a circular wave packet of width $\sigma=150$ Å moving from the left towards the DB (shown with dotted lines in the figure). The wave packet has an initial momentum $\mathbf{k}_0$ such that its energy $E_0 \simeq \hbar^2k_0^2/2m^*=0.057$ eV is equal to that of the quasilocal level inside the well (an effective mass $m^*=0.067m_0$ is used). The energy spread of the packet is $\delta E \simeq \hbar^2k_0/m^*\sigma \sim 0.025$ eV, i.e. comparable with the intrinsic width of the DB resonance, see Fig. 1. The calculations are performed using a 256×256-point square grid of side $L=3000$ Å.

One sees from the sequence of Fig. 2 that a large fraction of the incident wave packet is quickly trapped between the two barriers, giving rise to a high peak in the charge density in this region. In the meantime a double-peaked reflected wave packet is formed, moving to the left in the figure, and a transmitted packet emerges from the right edge of the double barrier, moving to the right. The dash-dotted line represents the classical trajectory that the packet would have traced in the absence of the barrier. The barrier-induced delay of the transmitted packet is apparent. This behavior is typical of resonant tunneling, whereas in the case of wave-packet tunneling through a *single* thick barrier the opposite situation occurs, i.e., the transmitted packet always anticipates with respect to free-packet motion.[12]

Figure 2 shows another general feature of wave-packet tunneling through a DB structure, i.e. the appearance of two reflected Gaussian-like pulses which move with slightly different velocities. This can be understood[13] by noticing that the reflection of the packet is governed by $R \equiv 1 - T$ which has a marked dip at the resonant energy. This roughly divides the momentum distribution of the incident wave packet into two components, each of which gives rise to a separate pulse, as it is indeed observed. In particular, the leftmost peak in the last panel of Fig. 2 is the faster of the two.

The subsequent decay of the peak trapped between the two barriers is quite slow with respect to the time scale required for the separation of the incident packet into well-defined transmitted and reflected parts. The way in which an incoming packet of width σ splits into reflected and transmitted parts depends mainly on σ.[9] However, the decay of the charge Q trapped into the quasistationary state in the well is determined by the DB parameters only, and follows the law[14,15]

$$Q(t) = Q_0 e^{-t/\tau_0} , \tag{5}$$

where

$$\tau_0 \sim \hbar / \Delta E \tag{6}$$

and ΔE is the intrinsic width of the resonance in the DB transmission coefficient. ΔE depends on the "transparency" of the confining barriers through a factor $\simeq \exp(-2Kd)$, where $K = [2m^*(V_0 - E_0)/\hbar^2]^{1/2}$ and E_0 is the resonant energy.[16] We have fitted our numerical values for $Q(t)$, for the sequence of Fig. 2, using the exponential law (5), thus obtaining $\hbar/\tau_0$. Our estimate for this quantity is compared in Fig. 1 with the natural width of the peak in the DB transmission coefficient.

The effect of a magnetic field $B = 2.5$ T on the scattering process of Fig. 2 is shown in the sequence of snapshots of Fig. 3. As at $B = 0$, a rather large fraction of the tunneling packet builds up quickly between the two barriers and subsequently decays outside. The reflected and transmitted packets, both largely distorted from their zero-field counterparts, evolve with opposite curvatures, as required by their oppositely directed velocity.[9] In particular, far from the barrier their average trajectory is circular, as expected from semiclassical reasoning, the radii of curvature depending on the average momenta of the two packets.[9] Note again the appearance of a double-peaked reflected packet, similarly to the zero-field case of Fig. 2. The spatial oscillations in the charge density associated with the reflected wave packet in the last panel of Fig. 3 are due to the interference of the wave function with its own periodically repeated tail, and are thus unphysical.

We are interested here in the effect of the magnetic field on the lifetime τ_0 of the resonant level in the quantum well. We found that the decay of the trapped peak in Fig. 3 follows strictly the exponential law (5) also in the presence of the magnetic field, from which the lifetime of the resonant level as a function of B, $\tau_0(B)$, can be extracted. The values of $\tau_0(B)$ as calculated from our

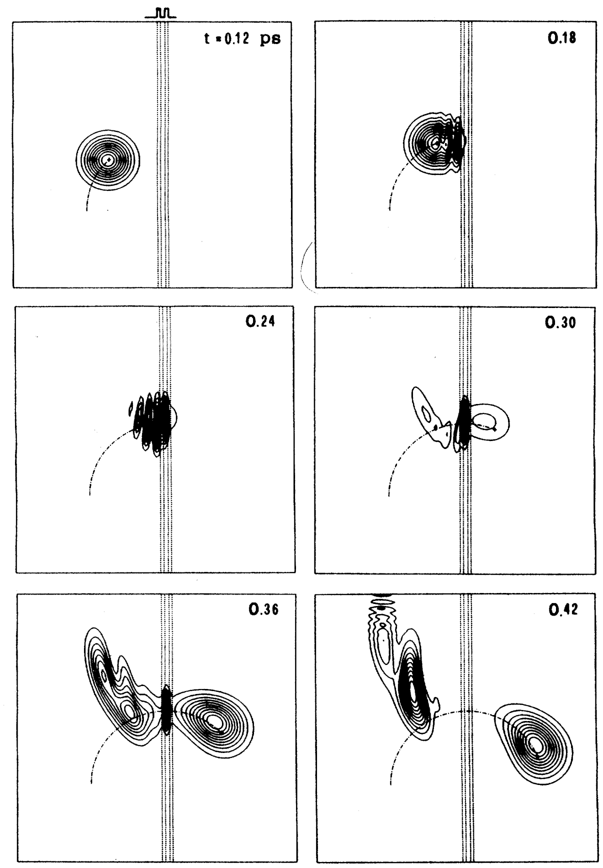

FIG. 3. Contour plots of the charge density $|\Psi(t)|^2$ (same barrier and packet parameters as in Fig. 2), in a magnetic field $B = 2.5$ T perpendicular to the plane of the figure. The dashed-dotted line shows the wave-packet trajectory in the absence of the DB potential: the center of the wave-packet orbit coincides with the center of the barrier.

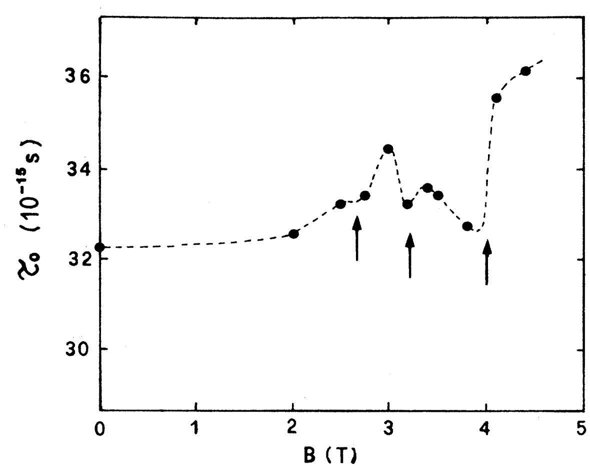

FIG. 4. Lifetime τ_0 of the resonance as a function of the magnetic field strength. The dots are the results from our simulations, while the dashed line is only to guide the eye. The arrows are drawn at field values where a crossing between interfacial Landau levels ("skipping orbit" levels) and the quasilocal level inside the well occurs (see Fig. 6).

time-dependent simulations are shown in Fig. 4 for several values of B, up to 5 T. A nonmonotonic behavior with the field B is apparent.

III. DISCUSSION

According to the simplified approach described in Ref. 17, where the magnetic field was assumed to be confined within the DB region, a monotonic increase of $\tau_0(B)$ is expected, due to the narrowing of the resonance with increasing magnetic field (see Fig. 1 in Ref. 17). The above approximation might be expected to hold in the case of real DB heterostructures, if electrons are injected and collected from a heavily doped semiconductor emitter and collector, respectively, where the Landau levels are expected to be smeared out by disorder. However, if the carrier mean free path outside the barrier could be made comparable or longer than a few Larmor radii, then the full magnetic level structure outside the DB region become important, and the results of Ref. 17 should then be revised.

In order to explain the oscillations of $\tau_0(B)$ in Fig. 4 (i.e., the appearance of minima in the lifetime for certain values of the magnetic fields) we consider first the stationary-state solutions associated with the Hamiltonian (1). These states are more conveniently calculated by using the Landau gauge for the vector potential, $\mathbf{A}=\mathbf{A}_L\equiv(0,Bx,0)$. In this case the motion in the y direction is free-particle-like, $\sim\exp(ik_y y)$, and the wave function $\Phi(x)$ describing the motion along the x direction satisfies a one-dimensional Schrödinger equation with a potential $V_{\rm eff}=\frac{1}{2}m^*\omega_c^2(x-X_0)^2+V_{\rm DB}(x)$. Here $X_0\equiv\hbar k_y c/eB$ is interpreted, as usual, as the center of the cyclotron orbit associated with the nth Landau level. When $V_{\rm DB}=$const, one is left with the familiar harmonic-oscillator Hamiltonian whose solutions are equidistant Landau levels having the same energy $\hbar\omega_c(n+\frac{1}{2})$ for all values of k_y: in a semiclassical picture, these states correspond to closed circular orbits in the x-y plane. In the presence of the barrier potential $V_{\rm DB}(x)$, however, the translational invariance along x is broken

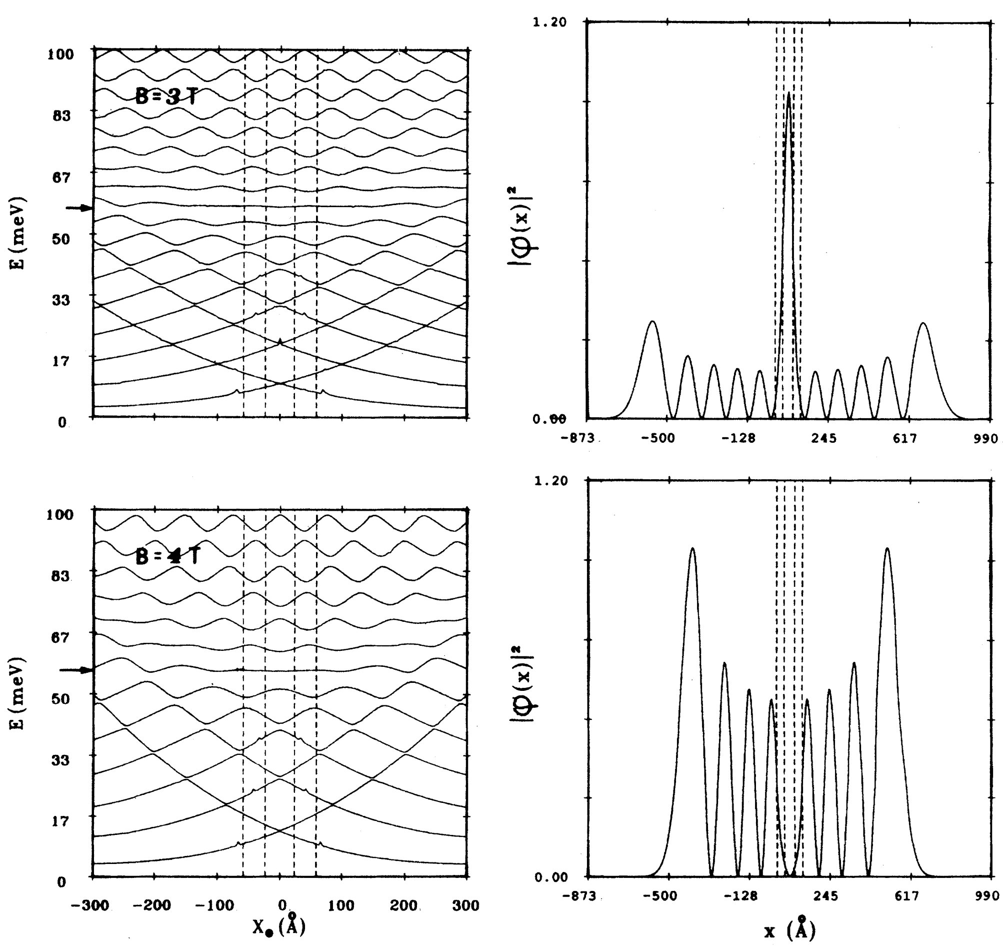

FIG. 5. Left: Eigenvalue spectra of the double-barrier of Fig. 1 in a magnetic field, as a function of the cyclotron orbit center X_0, for $B=3$ and 4 T. The arrows indicate where the resonant level is at $B=0$. Right: probability density $|\Phi(x)|^2$ for the wave function of the flat level at about 60 meV, at $X_0=0$ (for the same two values of the magnetic field). The DB position is indicated with dashed lines.

and the eigenvalues of (1) depend on X_0. In this case, the stationary states are just the quantized version of the semiclassical "skipping orbits" at a potential step,[3,18] where electrons with energy less than the step height are repeatedly reflected at the interface, thus moving parallel to the barrier edge.

In the two left panels of Fig. 5 we show the calculated stationary states in a magnetic field of 3 and 4 T respectively, as a function of the cyclotron orbit center X_0. The value $B \sim 4$ T is one of the special values of the magnetic field at which a minimum in the lifetime τ_0 is observed (see Fig. 4). The resonant level is clearly visible in the two left panels of Fig. 5 as the flat level at about 60 meV (the arrows show where the resonant level lies at $B=0$). In the right part of Fig. 5 we show the square moduli of the wave functions $\Phi(x)$ for this particular level, for $X_0=0$ (i.e., the cyclotron orbit center is right in the middle of the well). From Fig. 5 one sees that, although at $B=3$ T a large fraction of the charge is localized in the well region, at $B=4$ T the electron is essentially localized *outside* the DB region.

Roughly speaking, one can think of the level structures of Fig. 5 as obtained by coupling the set of Landau levels, modified into "skipping orbit" levels by the presence of the two confining barriers, with the localized state inside the well. We show in the upper part of Fig. 6 how the effective potential $V_{\text{eff}}=\frac{1}{2}m^*\omega_c^2x^2+V_{\text{DB}}(x)$, which generates the eigenvalues at $X_0=0$ in Fig. 5, can be approximately decomposed into two parts, one giving rise to the interfacial Landau levels associated with skipping orbits outside the barriers (solid lines), and the other giving rise to the quantized level in the well (dashed line). The dispersion with the field of these levels is shown in Fig. 6. Note that the position of the resonant level is almost field independent. When the resonant level does not match any of the "skipping orbit" levels outside the barrier, its associated wave function is largely localized within the barriers (as it happens, e.g., at $B=3$ T in Fig. 5). On the other hand, for those particular values of the field for which a crossing occurs (as, e.g., at $B\sim 4$ T), the hybridization between the two levels gives rise to a state characterized by the electron charge being localized mostly outside the barrier region. This corresponds to fast escape of the particle from the quasibound level by resonant tunneling into the skipping orbits. According to the above picture, at each crossing a reduction in the lifetime of the resonant level is to be expected. In Fig. 6, three values of B for which a crossing takes place are visible. As shown in Fig. 4, these values do indeed roughly coincide with the observed dips in $\tau_0(B)$.

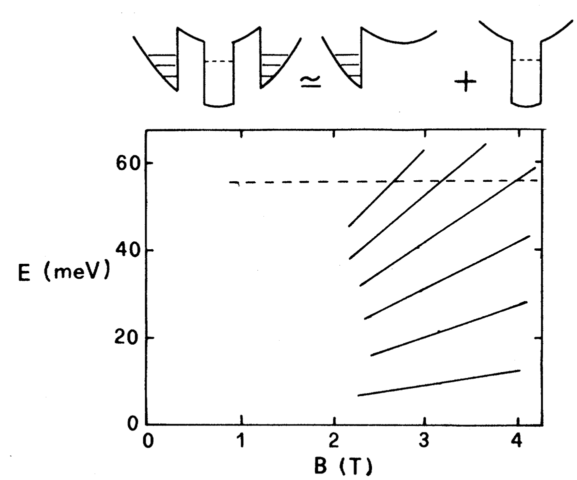

FIG. 6. B dependence of the interfacial "skipping orbits" Landau levels (solid lines) and of the resonant level in the quantum well (dashed line). The two sets of levels, which are assumed to be uncoupled, arise from the effective potential profiles sketched in the upper part of the figure, as described in the text. The three values of B where a crossing between solid lines and the dashed line occurs are reported in Fig. 4 with arrows. At these values of B, the skipping orbit frequency and the resonance frequency resonate, and enhanced tunneling between them is to be expected.

To substantiate our results, we propose an experiment, which could be easily implemented within the existing GaAs/AlAs heterojunction technology, and which should be capable of allowing a direct observation of the above effect. The experiment, based on time-resolved photoluminescence spectroscopy, would be simply a reedition of that presented in Ref. 19, where the lifetime of a resonant state inside a GaAs/$Ga_{1-x}Al_xAs$ DB structure was measured. In these experiments subpicosecond light pulses are used to selectively excite electron–heavy-hole pairs localized in the QW, by tuning of the energy of the light source. The subsequent temporal decay of the photoluminescence intensity from the quantum well is monitored, allowing to measure directly the resonance lifetime. By turning on a transverse magnetic field, the resonance lifetime should exhibit very similar oscillations to those shown in Fig. 4, with minima directly corresponding to tunneling out of the resonant level into the skipping orbits outside the well.[20]

Of course a sufficiently long mean-free-path for electrons $\lambda > l_B \equiv (\hbar c/|e|B)^{1/2}$ must be attained outside the barrier, allowing for the electron to execute several cyclotron or skipping orbits before being scattered: this implies the use of high-quality samples with smooth interfaces. Moreover, the coherence of the electron wave function must be preserved during tunneling, i.e., one must have $\tau_0 \ll \tau_{\text{sc}}$, where τ_{sc} is the lifetime for scattering in the direction parallel to the QW layer. The latter condition implies relatively broad resonances, i.e., the transparency $\sim \exp(-2Kd)$ of the confining barriers should not be too small. For instance, using the same DB parameters as in our calculations, $\tau_0 \sim 0.04$ ps (see Fig. 4), while for state-of-the-art GaAs/AlAs heterostructures $\tau_{\text{sc}} \lesssim 1$ ps.[19]

In summary, we have calculated the lifetime τ_0 of a quasilocal level in an ideal double-barrier structure placed in a transverse magnetic field B, using a direct numerical solution of the 2D time-dependent Schrödinger equation for wave-packet tunneling. We show that the observed oscillatory behavior of τ_0 versus B can be explained in terms of resonant couplings of the quasistationary state in the well with the interfacial Landau levels corresponding to the semiclassical skipping orbits at the barrier interfaces. We also propose a possible experimental setup which should allow a direct observation of this effect in real heterostructure systems.

We finally stress again that the problem of transport of electrons through a DB in a magnetic field under realistic conditions requires a much more complex treatment than the simple, single wave-packet approach described here. Among others, two major effects should be taken into account. First, for a tunnel current to be established, a bias must be applied across the DB structure. This makes the structure asymmetric, and may in part destroy the simple matching conditions described in this paper (see Fig. 6). The second effect is related to the buildup of space charge both in the emitter and/or collector electrodes and inside the well region when an appreciable steady-state current flows through the DB device.[21] In particular, under steady-state conditions the effect of the space-charge buildup inside the well is roughly to make both barriers more transparent.[14] The effect on $\tau_0(B)$ should in principle be obtained through a self-consistent calculation.[22] Qualitatively, we may expect that the overall result will be a slight reduction of the τ_0 versus B structures of Fig. 4.

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