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Fractional quantum Hall effect with spin reversal

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We study the spin polarization of the ground states and the excited states of the fractional quantum Hall effect, using spherical geometry for finite-size systems. In the absence of the Zeeman energy, the ground states at filling $2/q$, with q odd, are found to be spin unpolarized and nondegenerate for all values of q studied. The energy differences between the spin-polarized and spin-unpolarized states at $\nu=\frac{2}{3}$ and $\frac{2}{5}$ are estimated by extrapolating to the thermodynamic limit. The spin polarization of the lowest-energy excitations are studied and the results agree with recent experiments.

I. INTRODUCTION

Recently, there have been several interesting experiments in the fractional quantum Hall efffect (FQHE) involving electron spins. The discovery[1] of a Hall plateau in the FQHE at the even denomenator filling $\nu=\frac{5}{2}$, and the subsequent tilted magnetic field experiment,[2] strongly suggests the possibility of nonpolarized-electron-spin states. Very recently, magnetic-field-induced phase transitions in the FQHE have been observed,[3,4] which indicates that in the low-field side the ground states could be spin unpolarized at some particular fillings. These findings are widely believed to be evidence for the FQHE involving reserved spins and raise general interest on the subject.

Theoretical studies[5–13] of the spin configurations in the FQHE started a few years ago. It was first pointed out by Halperin,[5] that due to the small Landé g factor in GaAs, some of the fractional quantum Hall states (FQHS) may contain some electrons with reversed spins. In particular, Zhang and Chakraborty[6] showed that the ground states at many filling factors are not spin polarized in the absence of the Zeeman energy. The spin-reversed excitation at filling $\nu=\frac{1}{3}$ (Refs. 7 and 8) was later found to have lower energies than the excitations without spin reversal in the absence of the Zeeman term.

In this paper, we report a systematic numerical investigation of the ground states and the excited states of a two-dimensional electron gas in the FQHE region involving spin degrees of freedom. The numerical calculations of finite systems are carried out using Haldane's spherical geometry. In the absence of the Zeeman energy, we find that the ground states at filling $\nu=2/q$ (with odd q) are all spin unpolarized and nondegenerate, and maintain the rotational and translational invariance. We calculate the energies of these unpolarized ground states at fillings $\nu=\frac{2}{3}$ and $\nu=\frac{2}{5}$, and the results are extrapolated to the thermodynamic limit. The excitation energies at these fillings are also calculated for various spin configurations and the results are consistent with the experimental observations.

II. SPIN-UNPOLARIZED FQHS

Consider a system of electrons confined to the surface of a sphere of radius R, as originally introduced by Haldane.[14] The electrons are confined to the lowest Landau level of a monopole-type magnetic field $B=\hbar N_\phi/2eR^2$. The flux quanta passing through the surface are denoted by N_ϕ and are required to be positive integers. The single electron states may be characterized by the z component of the angular momentum $m_z=-N_\phi/2, -N_\phi/+1, \dots, N_\phi/2$, and spin quantum number of $\pm\frac{1}{2}$. The system containing N_e electrons with Coulomb interaction among them is then solved numerically to study the spin polarization of the ground states and the energy difference between the different spin configurations. The system sizes in this study are varied from four to eight electrons. In the thermodynamic limit, the filling factor is given by $\nu=N_e/N_\phi$. As reported earlier,[6,8] the ground state at $\nu=\frac{1}{3}$ is still found to remain polarized when the spin degrees of freedom are taken into account. This result agrees with the Haldane's generalization of the Laughlin theory[15] of the FQHE involving electron spins.[9] Among the other states studied, we have found a class of states, whose ground states are spin unpolarized (zero total spin), rotational and translational invariant (zero total orbital angular momentum), and nondegenerate. These states are reminiscent of the spin-polarized ground state at $\nu=\frac{1}{3}$. This class of states appears when

$$N_\phi=m(N_e-1)+\alpha N_e/2\,, \tag{1}$$

with m =odd and $\alpha = \pm 1$. They correspond to the filling factors $\nu = 2/(2m+\alpha)$ in the thermodynamic limit. We wish to point out that Haldane's hierarchical formula appropriate[9] for the spin-polarized systems will replace N_e by $(N_e - 1)$ for the second term in Eq. (1). In Table I we list all the states of Eq. (1) which we have examined. A few years ago, Haldane[9] used the truncated-pseudopotential method to construct the principal incompressible Laughlin-type FQHS with a total spin of zero. The states in his theory correspond to the case $\alpha = -1$ in Eq. (1). In this case, such as at $\nu = \frac{2}{5}$, there are explicit Laughlin-type wave functions to describe the unpolarized FQHS. The case $\alpha = +1$, however, does not fall into Haldane's category, and is not yet well understood from the wave-function point of view. Examples are the unpolarized states at $\nu = \frac{2}{3}$ and $\frac{2}{7}$. Our small system studies strongly suggest that the states in Eq. (1) belong to the same type of spin-unpolarized FQHS. We note that the same results for the spin polarization of the ground states at fillings $2/q$ (q =odd) were obtained in the rectangular geometry,[6,13] which shows that this result is not a geometric effect. But the spherical geometry retains the full symmetry of the infinite system, hence it is the best one to study the symmetry problem, as shown in our study of the nondegeneracy of the unpolarized ground states.

The Coulomb interaction we studied does not depend on electron spins explicitly. The spin dependence of the eigenstates is purely a quantum phenomenon, and is due to the symmetry requirement on the wave function. A possible explanation for the unpolarized ground states at these fillings is that the electrons with opposite spins may form bound pairs, but the explicit Laughlin-type wave functions[5] for the states with $\alpha = +1$ need to be refined to satisfy the Fock cyclic condition.[9]

We now turn to the discussion of the ground-state energies. In order to get rid of the system size dependence due to the nonzero curvature on a sphere, we adopt a method suggested by Morf, d'Ambrumenil, and Halperin.[16] This method uses a size- and filling-dependent magnetic length unit l_0' to measure the energy. l_0' is related to the magnetic length $l_0 = (\hbar c/eB)^{1/2}$ by

$$l_0' = (\nu N_\phi / N_e)^{1/2} l_0 . \qquad (2)$$

l_0' equals the magnetic length l_0 in the thermodynamic limit. This method has been well tested and extensively used[17] to extract the properties of a bulk system in the spin-polarization problem of the FQHE. In this study, the flux N_ϕ for the unpolarized state is obtained from Eq. (1) while N_ϕ is obtained from Haldane's hierarchical formula[9] for the polarized state, i.e., replacing N_e by $(N_e - 1)$ in the second term on the right-hand side of Eq. (1). In Fig. 1, we plot the unpolarized-state energies at $\nu = \frac{2}{3}$ for systems with up to eight electrons. Also plotted are the lowest energies of the polarized states at the same filling. These data are obtained from the known results of $\nu = \frac{1}{3}$ and $\nu = 1$ polarized states[17] by using the electron-hole symmetry. For the polarized case, the energy of an N-electron system on a sphere with N_s degeneracy in the lowest Landau level (denoted as ν state) is related to the energy of its conjugate system, with $N_s + 1 - N$ electrons on the same sphere (filling $1 - \nu$) by[18]

$$(N_s + 1 - N)\varepsilon_{1-\nu} = N\varepsilon_\nu + (N_s + 1 - 2N)\varepsilon_1 , \qquad (3)$$

where ε_ν, $\varepsilon_{1-\nu}$, and ε_1 are the energies per electron for systems corresponding to N, $N_s + 1 - N$, and $N_s + 1$ electrons, respectively. In particular, we have for an infinite system, $2\varepsilon_{2/3} = \varepsilon_{1/3} + \varepsilon_1$. This equation is used to obtain the energies of the polarized states in Fig. 1. Shown in Fig. 2 are the unpolarized-state energies at $\nu = \frac{2}{5}$ for systems with up to six electrons. The polarized-state energies are quoted from Ref. 17 for comparison. Since a good linear dependence of energy on $1/N_e$ is obtained in the spin-polarized case, we linearly extrapolate spin-unpolarized data to the bulk system. We estimate $\varepsilon_{2/3} = -0.527$ and $\varepsilon_{2/5} = -0.439$ in units of $e^2/\varepsilon l_0$. In the same unit, the energy gains per electron in the unpolarized state against polarized ones are 0.009 and 0.006 at $\nu = \frac{2}{3}$ and $\frac{2}{5}$, respectively.

TABLE I. A list of systems given by Eq. (1), where the ground states are found to be spin unpolarized in the absence of the Zeeman term.

ν	m	α	N_e	N_ϕ
$\frac{2}{3}$	1	1	4, 6, 8	5, 8, 11
$\frac{2}{5}$	3	−1	4, 6	7, 12
$\frac{2}{7}$	3	1	4	11
$\frac{2}{9}$	5	−1	4	13

III. EXCITATIONS AT $\nu = \frac{2}{3}$ AND $\frac{2}{5}$

We study charged excitations in this section. The fractional charges of the excitations in FQHE are independent

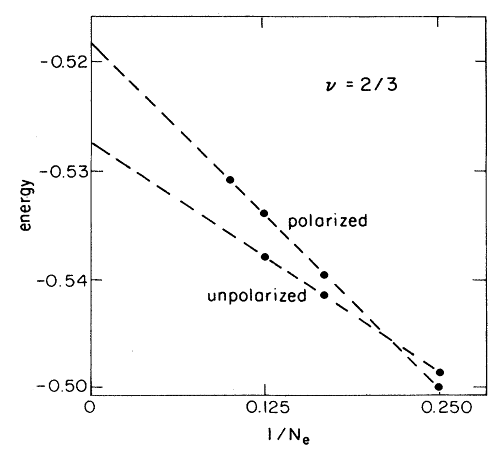

FIG. 1. The ground-state energies per electron in spin-polarized and unpolarized states at $\nu = \frac{2}{3}$, as functions of $1/N_e$, in units of $e^2/\varepsilon l_0'$. The dashed lines are the linear fittings. The Zeeman term is not included.

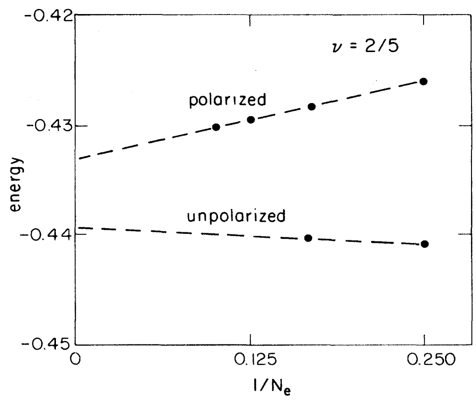

FIG. 2. The ground-state energies per electron in spin-unpolarized states at $\nu=\frac{2}{5}$, in units of $e^2/\varepsilon l_0'$. The energies in spin-polarized states are quoted from Ref. 17 for comparison. The dashed lines are the linear fittings. The Zeeman term is not included.

of the spin polarization, and are $e^*=\pm e/3$ and $\pm e/5$ for $\nu=\frac{2}{3}$ and $\frac{2}{5}$, respectively. The spin of the excitations (defined as the total spin change in the excited state) depends on the spin polarization of the ground state. For the spin-unpolarized ground states, the lowest quasiparticle and quasihole states have spin $\frac{1}{2}$. However, the excitations of spin-polarized ground states may or may not involve spin reversal, with spin being an integer, 1 or 0. Interger spin of the excitations is a consequence of the ferromagnetic ground state. The excitation gap 2Δ (in units of $e^2/\varepsilon l_0'$) of a finite system, defined as the energy to create a well-separated pair of quasiparticle and quasihole, is given by

$$2\Delta=N_e(\varepsilon_+ + \varepsilon_- - 2\varepsilon_\nu)+2e^{*2}\left(\frac{\nu}{2N_e}\right)^{1/2}, \qquad (4)$$

where ε_+ and ε_- are the energies per electron in systems with one quasiparticle and one quasihole, respectively. The calculated excitation gaps for various spin configurations are listed in Table II. In particular, we find that for the $\nu=\frac{2}{5}$ spin-polarized state, a quasiparticle excitation with spin reversal is energetically very favorable. However, the energy gain by reversing one spin in the quasihole state is much less. This is similar to the spin-reversal excitations[7,8] at $\nu=\frac{1}{3}$, and it is relevant to the recent experimental findings as we will discuss later.

The spin of a quasihole in the unpolarized state at $\nu=\frac{2}{5}$ has been discussed by Haldane[9] and studied numerically by Rezayi.[8] We have found similar behavior for the spin of the quasiparticle at $\nu=\frac{2}{3}$ unpolarized state. The spin distribution of the excitation is well localized, and strongly tied to the charge profile as shown in Fig. 3. Therefore, the quasiparticle excitation at $\nu=\frac{2}{3}$ is a true spin-$\frac{1}{2}$ object. For the quasiparticle at $\nu=\frac{2}{5}$ and quasihole at $\nu=\frac{2}{3}$, the spin distributions have some indications of being localized, but the systems we have studied seem too small to give a clear demonstration of the localized nature.

IV. COMPARISON WITH EXPERIMENTS

To compare the above results to experiment, we must include the Zeeman energy. The Zeeman energy favors the spin antiparallel to the external magnetic field **B**. The Zeeman energy per electron in the spin-polarized state is $E_Z=-\frac{1}{2}\mu_B gB$ and is zero in the unpolarized state. The unpolarized state is stable if the exchange energy gain overcomes the loss in the Zeeman energy. In the tilted field experiments,[2-4] the field component perpendicular to the two-dimensional plane is kept fixed while the total field B changes. The Zeeman energy is proportional to the total field. We express the Zeeman energy in units of $e^2/\varepsilon l_0$ in order to compare the Coulomb energies obtained from Sec. III. Consider the case with perpendicular component field $B_\perp=6$ T, which is about the value in the reported experiments. Using the values $g=0.4$ and $\varepsilon=13$, we obtain $E_Z=-0.001B$, where B is in units of tesla and $B\geq B_\perp$. We find that gains in exchange energy in the unpolarized states at $\nu=\frac{2}{5}$ (0.006) and $\frac{2}{3}$ (0.009) estimated for a bulk system are barely enough to compensate the loss in the Zeeman energy, even at $B\simeq B_\perp$. In our calculations, the layer thickness effect has not been included. This effect usually results largely in a reduction of the energy scale in the Coulomb term, hence a reduction of the exchange energy gain in the spin-unpolarized state.[19] Therefore, one expects that the Zeeman energy dominates and the ground states at $\nu=\frac{2}{3}$ and $\frac{2}{5}$ are spin polarized. The experimental findings[3,4] with spin-unpolarized ground states are at fillings $\nu=\frac{4}{3}$ and $\frac{8}{5}$. These are the conjugate states of fillings $\nu=\frac{2}{3}$ and $\frac{2}{5}$, respectively, if we assume the absence of the higher Landau-level mixing.

TABLE II. Excitation gaps at $\nu=\frac{2}{3}$ and $\frac{2}{5}$.

Filling and spin polarization	Spin of excitation Quasiparticle	Quasihole	Excitation gap (units of $e^2/\varepsilon l_0'$) $N_e=3$	$N_e=5$	$N_e=7$
$\frac{2}{3}$ ↑↓	$\frac{1}{2}$	$\frac{1}{2}$	0.132	0.120	0.111
$\frac{2}{5}$ ↑↓	$\frac{1}{2}$	$\frac{1}{2}$	0.067	0.056	
$\frac{2}{5}$ ↑↑	0	0	0.104	0.084	
$\frac{2}{5}$ ↑↑	1	0	0.045	0.029	
$\frac{2}{5}$ ↑↑	0	1	0.084	0.063	

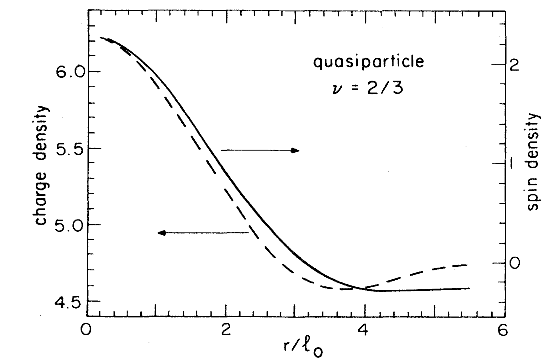

FIG. 3. Charge and spin profiles, in units of $4\pi\rho R^2$ with ρ the charge and spin density, respectively, for a quasiparticle of the spin-unpolarized $\nu=\frac{2}{3}$ state. $N_e=5$, $N_\phi=6$, and the total spin is $\frac{1}{2}$.

Since the difference between the Zeeman energy and the exchange energy is quite small, some other effect, such as Landau-level mixing and disorder, may be crucial to determine the spin polarization of the ground state at these fillings.

In the work of Eisenstein *et al.*,[4] the activation energy as a function of the total magnetic field has been reported for $\nu=\frac{8}{5}$. In the low-field case, the activation energy decreases as the field increases, while it increases in the higher-field case. This behavior has been discussed as being quantitatively consistent with a transition from the spin-unpolarized to polarized FQHS, with the assignment of spin of quasiparticle and quasihole to be $\frac{1}{2}$ in the former state, and one spin reversal in the quasiparticle-quasihole pair in the polarized ground state.

Our calculations at $\nu=\frac{2}{5}$ show that for the polarized ground state the excitation energy is about $0.04e^2/\varepsilon l_0$ lower for the spin-1 quasiparticle than that of the spin-0 quasiparticle. This means that the lower quasiparticle state is with one spin reversed. The value $0.04e^2/\varepsilon l_0$ is large enough to compensate the Zeeman energy loss $0.02e^2/\varepsilon l_0$ at a field of 10 T. On the other hand, the gain of the quasihole energy by reversing a spin is small, and is not large enough to overcome the loss in Zeeman energy, especially if we consider the reduction of the energy gain due to the finite layer thickness.[7,19] Therefore, our results may identify the excitations in the high-field region in the experiment[4] of $\nu=\frac{8}{5}$ to be the quasihole with one spin reversal and quasiparticle without spin reversal. Notice that the quasihole at $\nu=\frac{8}{5}$ corresponds to the quasiparticle at $\frac{2}{5}$ and vice versa.

V. CONCLUSION

We have studied the spin configurations of the ground and excited states in the FQHE, using a numerical diagonalization method for small systems on a spherical surface. We found the ground states at filling $\nu=2/q$, with q odd, are spin unpolarized in the absence of the Zeeman energy. This finding is consistent with the experimental results by Clark *et al.*[3] These states are all rotational and translational invariant and belong to the incompressible FQHS. We estimated the energy differences of a bulk system between spin-polarized and unpolarized states at fillings $\nu=\frac{2}{5}$ and $\frac{2}{3}$. Our results on the spin polarization of the excitations at $\nu=\frac{2}{5}$ agree with the experimental results reported by Eisenstein *et al.*[4]

Finally, we wish to point out that with the evidences for spin-unpolarized FQHE at $\nu=\frac{2}{3}$ from the small system studies, as well as from the related experiments, it is important to propose an appropriate Laughlin-type wave function at this filling.

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