Remanent magnetization decay at the spin-glass critical point: A new dynamic critical exponent for nonequilibrium autocorrelations

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A quench of a d-dimensional spin system from a random initial configuration, $\{\mathbf{S}_i(0)\}$, to a critical point is considered. The decay with time t of the autocorrelation with the initial condition is $q_0(t) \equiv \langle \mathbf{S}_i(0) \cdot \mathbf{S}_i(t) \rangle \sim t^{-\lambda_c/z}$, where z is the usual dynamic critical exponent. Naively, $\lambda_c = d$, but I find $\lambda_c < d$ in simulations of pure Ising models in $d=2$, 3 and the $\pm J$ Ising spin glass in $d=3$. This suggests that λ_c is a new critical exponent for *nonequilibrium* dynamics. For a spin glass the decay of $q_0(t)$ is the same as that of the remanent magnetization; the exponent λ_c/z observed in the spin-glass simulation is in good agreement with a recent experimental measurement by Granberg *et al.*

I. INTRODUCTION

For a spin glass, one of the most readily measured nonequilibrium dynamic quantities is the temporal decay of a remanent magnetization. This quantity has often been measured in the spin-glass ordered phase,[1] and has recently been used as a diagnostic for detecting various "aging" effects.[2,3] Here I consider the decay of a remanent magnetization *at* the spin-glass transition temperature, T_c.[4]

The remanent magnetization in a spin glass is a specific example of a much more general nonequilibrium autocorrelation,[3]

$$q_0(t) \equiv \langle \mathbf{S}_i(0) \cdot \mathbf{S}_i(t) \rangle \ , \tag{1}$$

where $\{\mathbf{S}_i(0)\}$ is a *nonequilibrium* disordered initial condition (time zero), t is the time, and the average is over spins i in an infinite system evolving towards equilibrium at a given temperature T. In this paper, I specifically consider critical points $T = T_c$ of both spin glasses and nonrandom ferromagnets. The nonequilibrium initial condition must not have an expectation value of the order parameter that develops below T_c, otherwise the long-time behavior of $q_0(t)$ is dominated by the nonlinear relaxation of that order parameter.[5] It is in this sense that the initial condition is disordered. Furthermore, we require that the correlations of the order parameter in the initial state be only of finite spatial range, ξ_0. These conditions are met rather generally by a quench from the disordered phase $T > T_c$; for the spin glass, a quench *at* T_c produced by turning off a magnetic field also results in a suitably disordered initial condition because the magnetic field does not directly couple to the order parameter. (Note even a small magnetic field suffices; see the discussion in Sec. IV.)

The long-time behavior of $q_0(t)$ at T_c can be obtained analytically for a few models; the following examples are discussed in more detail in Sec. II. In nonrandom Gaussian models, where interactions between different Fourier modes are nonexistent, $q_0(t) \sim t^{-d/z}$, where d is the spatial dimensionality and z is the usual dynamic critical exponent. The exactly solvable one-dimensional Glauber kinetic Ising model[3,6] also reduces to a Gaussian model, so has $q_0(t) \sim t^{-d/z}$ at $T_c = 0$, with $d=1$, $z=2$. The dynamics of the infinite-range (Sherrington-Kirkpatrick) spin glass[4] can be analyzed for $T \geq T_c$: $q_0(t) \sim t^{-6/4}$ at T_c, which can be interpreted as the result appropriate for the upper critical dimension for spin glasses, $d=6$, where $z=4$.

Let us define the nonequilibrium dynamic critical exponent λ_c via

$$q_0(t) \sim t^{-\lambda_c/z} \sim \xi_n^{-\lambda_c} \tag{2}$$

for long times at T_c, where $\xi_n \sim t^{1/z}$ is the growing nonequilibrium correlation length. (Here, and throughout this paper, I assume that the critical point has conventional dynamic scaling.[4,7]) The above-mentioned analytic results suggest that perhaps $\lambda_c = d$ rather generally. (Below T_c, $q_0(t) \sim \xi_n^{-\lambda}$, with a generally different exponent λ, as is discussed in Ref. 3.)

One might expect that for a sufficiently long time, $q_0(t)$ will be essentially the same as the equilibrium relaxation behavior [the analogous equilibrium autocorrelation function has been called $q(t)$ for spin glasses[4]]. However, this is only true for $T > T_c$, where the equilibrium correlation length ξ is finite, and therefore the nonequilibrium correlation length $\xi_n(t)$ can approach ξ at long enough times. For $T \leq T_c$, on the other hand, the equilibrium ξ is infinite, while $\xi_n(t)$ is finite for any finite t, so one never gets truly near equilibrium. Thus, the asymptotic long-time decay of $q_0(t)$ is fundamentally different from that of $q(t)$ for $T \leq T_c$.

In the absence (so far) of an analytical calculation of the effect of mode coupling on λ_c, say in an ϵ expansion,[8] I have performed Monte Carlo simulations to measure $q_0(t)$ at T_c. For $d=3$, the $\pm J$ Ising spin glass has $\lambda_c \simeq 2 - \eta \simeq 2.3 < d$ for effective exponents in the readily numerically accessible regime. However, because of the lack of precise knowledge of T_c, η, and z, and the very slow dynamics (large z), this estimate of λ_c may not be

very reliable. In order to get a nontrivial estimate of λ_c under the most favorable conditions, I have simulated the critical, nonrandom square-lattice Ising ferromagnet, finding $\lambda_c = 1.59 \pm 0.02$ as the effective exponent for times up to 400 Monte Carlo steps per spin (MCS's) and thus correlation lengths up to at least 20 lattice units. The power laws are well established over enough of a range that one can say $\lambda_c < d$ for this $d=2$ Ising model with a high level of confidence. Simulations out to only 40 MCS's suggest $\lambda_c \simeq 2.8$ for the nonrandom $d=3$ Ising ferromagnet.

Thus, it appears that λ_c may indeed be a new exponent, although there is still the possibility of a scaling relation between it and the known independent exponents d, z, η, and ν. To further understand this phenomenon, it would be nice to have more experimental measurements of the long-time decay of the remanent magnetization $m(t)$ *at* T_c in spin glasses. My simulations find $m(t) \sim t^{-\lambda_c/z}$ with $\lambda_c/z \simeq 0.37$ in $d=3$ for the $\pm J$ Ising spin glass. The one experimental measurement of power-law decay of the remanent magnetization near T_c after quenching from a large field that I have found is that of Granberg *et al.*[1] They study the amorphous metallic spin glass $(Fe_{0.15}Ni_{0.85})_{75}P_{16}B_6Al_3$. They find a decay that is faster than a power law for temperatures above a certain temperature, which is probably T_c (although this T_c is $\simeq 1\%$ below the T_c they obtained by other techniques). At this T_c, they find $\lambda_c/z \simeq 0.39$, in surprisingly good agreement with the present simulation result. For $T < T_c$, the decay slows down, as expected.

II. ANALYTIC RESULTS

For simplicity, let us start with a particular disordered initial spin configuration, $\mathbf{S}_i(0)$. Then we may write

$$q_0(t) = \frac{1}{N} \sum_{i=1}^{N} \mathbf{S}_i(0) \cdot \langle \mathbf{S}_i(t) \rangle \tag{3}$$

for a system of N spins, where the average is now over all thermal histories starting from the same initial configuration. [In fact, $q_0(t)$ is self-averaging, so one history for $N \to \infty$ suffices to measure it accurately.] The thermodynamic limit $N \to \infty$ is assumed here and in the following. For a nonrandom ferromagnet it is appropriate to Fourier transform:

$$q_0(t) = \frac{1}{N} \sum_{\mathbf{k}} \mathbf{S}_{\mathbf{k}}(0) \cdot \langle \mathbf{S}_{-\mathbf{k}}(t) \rangle \ . \tag{4}$$

For an initial condition with correlation length ξ_0, $\mathbf{S}_{\mathbf{k}}(0)$ is a random vector with magnitude of order $\xi_0^{(2-\eta)/2}$ for $\xi_0 k \ll 1$. Lengths are here measured in units of the lattice spacing, and ξ_0 is defined such that it is unity in a fully disordered configuration. In a Gaussian model, as well as the one-dimensional Ising-Glauber model, mode coupling does not enter, and we have

$$\langle \mathbf{S}_{\mathbf{k}}(t) \rangle = \mathbf{S}_{\mathbf{k}}(0) e^{-t\Gamma_{\mathbf{k}}} \ . \tag{5}$$

At T_c, we have $\Gamma_{\mathbf{k}} \approx A k^z$ for small k, and thus, $q_0(t) \sim t^{-d/z}$ at long times $t \gg d\xi_0^z / A$. For the infinite-range ferromagnet, all spins are equally coupled and $\Gamma_{\mathbf{k}} = \Gamma$ for all $\mathbf{k} \neq \mathbf{0}$. This results in $q_0(t) \sim e^{-\Gamma t}$; for very high d the initial decay is exponential, followed by a crossover to the power law at long times.

How should Eq. (5) change when mode coupling is introduced? The simulations presented below support the expectation that, averaging over disordered initial conditions, at T_c one has the scaling form

$$\langle \mathbf{S}_{\mathbf{k}}(0) \cdot \mathbf{S}_{-\mathbf{k}}(t) \rangle \approx t^{(d-\lambda_c)/z} F(tk^z) \ , \tag{6}$$

where $F(x)$ is a scaling function satisfying $F(x) \to$ constant for $x \to 0$. For large x, $F(x)$ presumably falls off faster than any power law. The exponent z is that of the usual equilibrium dynamic scaling. Note that for $\lambda_c < d$, Eq. (6) implies an instability of the low k modes—their averages initially grow before decaying at $t \gtrsim k^{-z}$.

Let us now consider a spin glass with Hamiltonian

$$H = \sum_{i,j} J_{ij} S_i \cdot S_j \ ,$$

where the J_{ij} are quenched random variables. The order that occurs in a spin glass is the establishment of a random local magnetization pattern that is chosen in detail by the specific exchanges J_{ij} and the temperature.[3,4] A uniformly magnetized state, as produced by a large magnetic field, is disordered as far as the spin-glass ordering pattern is concerned. Thus, if our initial state has $\mathbf{S}_i(0) = \mathbf{m}_0$, then

$$q_0(t) = \mathbf{m}_0 \cdot \langle \mathbf{m}(t) \rangle \ ,$$

where $\langle \mathbf{m}(t) \rangle$ is the remanent magnetization. This equivalence of $q_0(t)$ and $m(t)$ is precise for an Ising spin glass with uncorrelated exchanges chosen from symmetric probability distributions: $P(J_{ij}) = P(-J_{ij})$. There the fully polarized initial configuration $S_i(0) = +1$ is statistically equivalent under a gauge transformation ($S_i \to \epsilon_i S_i$, $\epsilon_i = \pm 1$) to any other random initial condition.[3]

Unfortunately, a Gaussian spin-glass model ("soft" spins with *no* $|S|^4$ term) with short-range couplings on a finite-dimensional lattice does not have a proper critical point. Instead, rare, localized modes go unstable at high temperatures. Only for the infinite-range model is there a proper critical point. In the infinite-range relaxational model ["model A" (Ref. 7)], the relaxation rates Γ of the linear response modes (eigenvectors of the matrix J_{ij}) are simply the eigenvalues of a random matrix. In the thermodynamic limit, the distribution of these eigenvalues is known;[4] at T_c one has

$$q_0(t) \sim \int d\Gamma \, p(\Gamma) e^{-\Gamma t} \ ,$$

with $p(\Gamma) \sim \Gamma^{1/2}$ for $\Gamma \to 0$. This results in $q_0(t) \sim t^{-3/2}$. Since the critical exponents of the infinite-range model generally apply at the upper critical dimension, this result appears quite analogous to $\lambda_c = d$: The upper critical dimension for spin glasses is $d=6$, where $z=4$. However, we must note that for the infinite-range model-A *ferromagnet,* we did *not* find that $\lambda_c = 4$, where $d=4$ is the

upper critical dimension; rather $q_0(t)$ decays exponentially even at T_c for this nonrandom system.

III. MONTE CARLO SIMULATIONS

Since this work was motivated by an attempt to understand the remanent magnetization decay in spin glasses, I first simulated the three-dimensional nearest-neighbor $\pm J$ Ising spin glass with spin-flip dynamics (model A) on a simple cubic lattice at its estimated[7,8] critical temperature, $T=1.18J$. The longest runs were on samples of size $10^3=1000$ spins with periodic boundary conditions. For each of 2000 realizations of the exchanges two "replicas" (with identical J_{ij}'s) were started in independent random initial configurations, $[S_i^\alpha(0);\alpha=1,2]$. The quantity

$$\tilde{\chi}_{\rm SG}(t)\equiv\frac{1}{N}\left\langle\left[\sum_{i=1}^{N}S_i^1(t)S_i^2(t)\right]^2\right\rangle \tag{7}$$

was measured to monitor the growing spin-glass (SG) correlations.[3] At T_c, this is expected by scaling to grow as $\tilde{\chi}_{\rm SG}(t)\sim t^{(2-\eta)/z}$. The results for $q_0(t)$ and $\tilde{\chi}_{\rm SG}(t)$ versus t are shown in a log-log plot in Fig. 1. The results shown in Fig. 1 do not appear to be affected by the finite sample size, as determined by seeing when and how finite-size effects appear in smaller samples. The slopes for the time range 100–2500 MCS's are $(2-\eta)/z\simeq 0.39$ and $\lambda_c/z\simeq 0.37$. This suggests that λ_c is less than or roughly equal to $(2-\eta)$, and thus quite a bit less than $d=3$. The current estimates of the critical exponents are[9] $z=6\pm1$ and[10] $2-\eta=2.3\pm0.2$. However, for this system there is a large uncertainty in T_c; according to Bhatt and Young,[8] T_c could perhaps be as high as $1.3J$. Furthermore, the data in Fig. 1 are only for times that are quite short by spin-glass standards. Since this estimate of $\lambda_c\simeq 2.2$ for the spin glass probably has large uncertainties, let us examine a much more accessible critical point.

The nonrandom nearest-neighbor square-lattice ($d=2$) Ising ferromagnet has[11] $T_c=2J/\ln(1+\sqrt{2})$. The static critical exponents are known exactly,[11] $\eta=\frac{1}{4}$, $\nu=1$, and the usual dynamic critical exponent for spin-flip (model A) dynamics is[12] $z=2.15\pm0.05$. For this system, I have simulated lattice sizes of up to 57^2 spins with periodic boundary conditions. Data for smaller sizes show that the results presented here for $N=57^2$ are not finite-size affected. Averages were taken over 50 000 independent random initial conditions run for 398 MCS's using a single spin-flip heat bath algorithm ($398\simeq 10^{2.6}$). The results for $q_0(t)$ and $\langle M^2(t)\rangle/N$ are presented in Fig. 2, where $M(t)$ is the total magnetization of the sample. Well established power-law fits are seen for times 10–398 MCS's. The scaling expectation is that $\langle M^2(t)\rangle\sim t^{(2-\eta)/z}$; the slope in Fig. 2 agrees with this within the accuracy of our knowledge of z. The fit to $q_0(t)$ gives $\lambda_c/z=0.74\pm0.01$; here and in the following the error bars are statistical only; there is also the possibility of some systematic error due to measuring at too early times.

To check the scaling proposed in Eq. (6), I have also measured $\langle M(0)M(t)\rangle/N$; this corresponds to $\mathbf{k}=\mathbf{0}$ in Eq. (6). To directly get an estimate of λ_c that does not depend on knowing z, let us define the nonequilibrium correlation length

$$\xi_n\equiv\left[\frac{\langle M^2(t)\rangle}{N}\right]^{1/(2-\eta)}\sim t^{1/z}. \tag{8}$$

Given this definition and Eq. (6), we expect

$$\langle M(0)M(t)\rangle\sim\xi_n^{d-\lambda_c}. \tag{9}$$

The log-log plot suggested by Eq. (9) is Fig. 3. The slope there indicates $\lambda_c=1.59\pm0.02$. This is completely consistent with the above-mentioned estimates of λ_c/z and z, and thus the scaling proposal in Eq. (6).

To see the dimensionality dependence of λ_c, I have also examined the $d=3$ simple cubic nonrandom Ising fer-

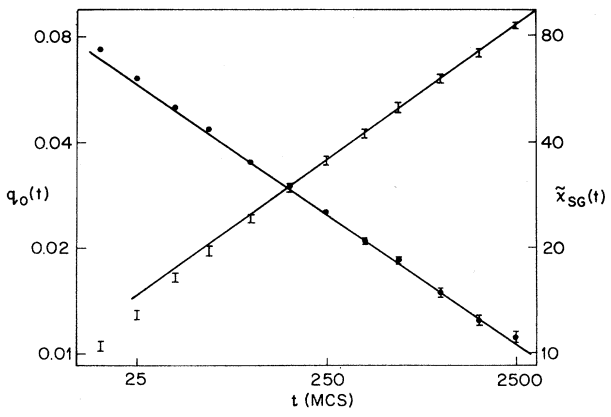

FIG. 1. Results for the $\pm J$ Ising spin glass at $T=1.18J\simeq T_c$ in $d=3$ dimensions. The solid circles show $q_0(t)$, which is identical to the remanent magnetization after starting in a fully saturated configuration. The unadorned error bars show $\tilde{\chi}_{\rm SG}(t)$.

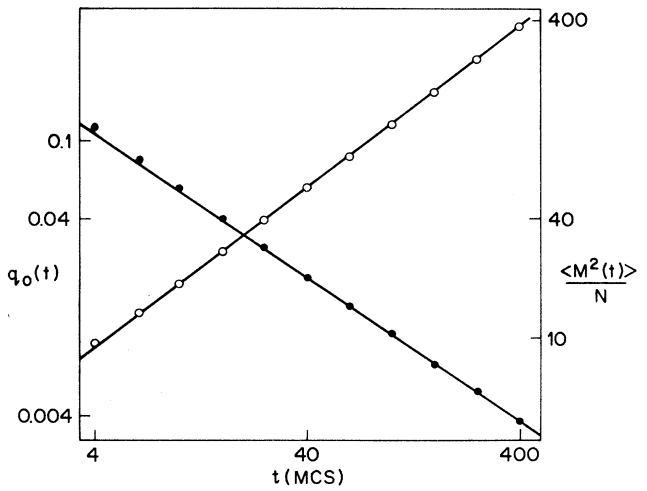

FIG. 2. Data for the $d=2$ nonrandom Ising ferromagnet at T_c. The solid circles show $q_0(t)$, while open circles show the growth of $\langle M^2(t)\rangle$ with exponent $(2-\eta)/z=0.81\pm0.01$.

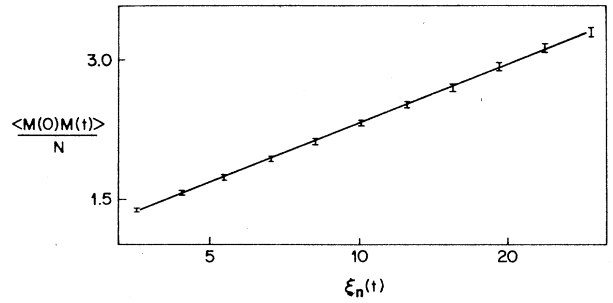

FIG. 3. Growth of $\langle M(0)M(t)\rangle$ with $\xi_n(t)$. These data represent the time range, 4–398 MCS's, shown in Fig. 2.

romagnet at $T=4.512J\simeq T_c$. Here accuracy comparable to that obtained in $d=2$ for 400 MCS's can, with the same computer time, only be obtained up to 40 MCS's. At these very early times $\lambda_c\simeq 2.8$. Thus, λ_c appears to be closer to d here than in $d=2$, as might be expected if $\lambda_c=d$ for $d\geq 4$, where a Gaussian fixed point governs the critical dynamics.[8]

IV. DISCUSSION

In the above simulations, the quench is effectively from infinite initial magnetic field. Let us now consider, for the spin glass, the dependence on the initial magnetic field h of the remanent magnetization decay $m(t)$ after an isothermal field quench at T_c. Experimentally, one may not have available a saturating (effectively infinite) magnetic field, and it is likely to be most convenient to use the smallest possible field.

The equilibrium spin-glass correlation length at T_c with an applied field scales for small h as[4,13] $\xi_h\sim h^{-4/(d+2-\eta)}$, and the relaxation time diverges for $h\to 0$ as $\tau_h\sim\xi_h^z$. Let us first assume that equilibrium in the presence of the field is attained by letting the system sit at T_c with the field on for a waiting time $t_w>\tau_h$. If the system is then quenched instantaneously and isothermally to $h=0$, the initial relaxation for times $t\ll\tau_h$ will be on length scales where the correlations are little affected by the applied field. The resulting initial decay will therefore be that of linear response at equilibrium,[9,13] namely $m(t)\sim ht^{-(d-2+\eta)/2z}$. For long times $t\gg\tau_h$, we should enter the long time decay, $m(t)\sim t^{-\lambda_c/z}$, and the two regimes should be connected by a crossover scaling form:

$$m(t)\sim ht^{-(d-2+\eta)/2z}R(t/\tau_H)\ . \tag{10}$$

In order to have the correct short- and the long-time behavior, the scaling function must behave as $R(x)\to$const for $x\to 0$, and $R(x)\sim x^{(d-2+\eta-2\lambda_c)/2z}$ for $x\to\infty$. The resulting long-time behavior for small h is

$$m(t)\sim t^{-\lambda_c/z}h^{(3d-2+\eta-4\lambda_c)/(d+2-\eta)}\ , \tag{11}$$

which, given the above-mentioned estimates of η and λ_c, has the surprising feature that the remanent magnetization at fixed time $t\gg\tau_h$ increases with decreasing h.[14] This suggests that experimentally one should actually make the measurement of the long-time decay of $m(t)$ with a rather small preparing field h in order to find the maximum signal.

If the system is prepared in too small a field, then equilibrium will not be attained and the results will be waiting-time dependent for $t_w<\tau_h$. In this regime, we should have $m(t)\approx ht^{-(d-2+\eta)/2z}\tilde{R}(t/t_w)$, with $\tilde{R}(x)$ being presumably a quantitatively different function from $R(x)$ in Eq. (10), but having the same behavior for $x\to 0$ and $x\to\infty$.

The above discussion of the field dependence of $m(t)$ assumes that we know T_c; in reality, there is always quite a bit of uncertainty in T_c, so we should also examine the temperature dependence of $m(t)$. For T near T_c, there is a critical relaxation time[4,9,13] that diverges at T_c as $\tau_c\sim|T-T_c|^{-z\nu}$. For times $t\ll\tau_c$, the system behaves as if it is critical; deviations from criticality in the correlation functions become significant for $t\gtrsim\tau_c$. For $t\gg\tau_h$, we thus should expect the scaling form

$$m(t)\sim t^{-\lambda_c/z}G[(T-T_c)t^{1/z\nu}]\ , \tag{12}$$

with $G(0)=$constant, $G(x)\sim|x|^{\lambda_c\nu}(\ln|x|)^{-\lambda/\psi}$ for $x\to-\infty$,[13] and $G(x)$ decaying faster than any power law, presumably as some sort of "stretched" exponential, for $x\to+\infty$. Thus the long-time curvature of $m(t)$ versus t on a log-log plot changes sign at T_c. This might actually serve as yet another way of estimating T_c.

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[1]P. Granberg *et al.*, Phys. Rev. B **35**, 2075 (1987).

[2]See, for example, Ph. Refrigier *et al.*, J. Phys. (Paris) **48**, 1533 (1987); P. Granberg *et al.*, Phys. Rev. B **38**, 7097 (1988); G. J. M. Koper and H. J. Hilhorst, J. Phys. (Paris) **49**, 429 (1988).

[3]D. S. Fisher and D. A. Huse, Phys. Rev. B **38**, 373 (1988).

[4]For a review of spin glasses, see K. Binder and A. P. Young, Rev. Mod. Phys. **58**, 801 (1986).

[5]R. Bausch, E. Eisenriegler, and H. K. Janssen, Z. Phys. B **36**, 179 (1979).

[6]See, for example, K. Kawasaki, in *Phase Transitions and Critical Phenomena,* edited by C. Domb and M. S. Green (Academic, London, 1972), Vol. 2, p. 465.

[7]P. C. Hohenberg and B. I. Halperin, Rev. Mod. Phys. **49**, 435 (1977).

[8]After this work was completed I received a letter from A. J. Bray pointing out a paper by H. K. Janssen, B. Schaub, and B. Schmittmann [Z. Phys. B **73**, 539 (1989)] where an exponent equivalent to λ_c is calculated in an ϵ expansion

($\epsilon=4-d$). I have not been able to decipher the ϵ^2 term in that paper, but to lowest order they obtain $\lambda_c=d-\epsilon/6$ for the Ising case, in reasonable agreement with the present simulation results.

[9]A. T. Ogielski, Phys. Rev. B **32**, 7384 (1985).

[10]R. N. Bhatt and A. P. Young, Phys. Rev. B **37**, 5606 (1988).

[11]See, for example, R. J. Baxter, *Exactly Solved Models in Statistical Mechanics* (Academic, London, 1982).

[12]S. Tang and D. P. Landau, Phys. Rev. B **36**, 567 (1987).

[13]D. S. Fisher and D. A. Huse, Phys. Rev. B **38**, 386 (1988).

[14]Such an increase in $m(t)$ with decreasing h has been observed for $T<T_c$ over a fairly wide field range in Cu:Mn by P. Nordblad, L. Lundgren, and L. Sandlund, Europhys. Lett. **3**, 235 (1987).