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Magnetotransport of an electron-hole plasma in a GaAs quantum well

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Magnetotransport properties of a quasi-two-dimensional electron-hole plasma confined in a GaAs quantum well are examined theoretically. In this, we consider a two-component plasma consisting of minority electrons and majority holes, subjected to crossed electric and magnetic fields. Magnetoconductivity and mobilities of both the minority electrons and majority holes are determined, taking account of the various scattering mechanisms [electron-hole, electron(hole)-phonon, electron(hole)-impurities] which are dynamically screened by the carriers. We also examine the influence of the magnetic field on electron-hole drag and the associated phenomenon of negative minority-electron mobility, obtaining numerical results for a weak magnetic field.

I. INTRODUCTION

Recent observations of negative minority-carrier mobilities in GaAs quantum wells[1] have stimulated much interest in the study of electron-hole plasma transport in these quasi-two-dimensional semiconductor heterostructures.[2,3] Strongly attractive electron-hole scattering, as compared to majority-carrier–lattice scattering, can cause the minority carriers to drift along with majority carriers, resulting in a negative absolute mobility for the minority carriers.[4] Such a "carrier-drag" effect is relatively weak in bulk semiconductors, due to the lower mobility and lower majority-carrier concentration, in contrast to the situation in quasi-two-dimensional semiconductor quantum wells where mobilities as high as 10^6 cm^2/V s, and carrier densities above 10^{12} cm^{-2} are not uncommon. In these ultrapure, high–carrier-concentration samples, scatterings between electrons and holes become as important as those between carriers and the lattice. Furthermore, charged-carrier screening can profoundly affect the transport properties of such high-density two-component plasmas.

In a previous study[3] we examined the carrier-drag problem in the context of an evaluation of the minority-electron mobility in a GaAs quantum well, where we showed that proper account of screening effects in electron-hole scattering, as well as in carrier-lattice scattering, is necessary to obtain reasonable results. In this paper, we generalize our earlier formulation to include an ambient transverse magnetic field perpendicular to the heterojunction surfaces (z direction). Simple physical considerations lead us to expect that the magnetic field will significantly modify the carrier-drag effect: Instead of simply drifting parallel or antiparallel to the electric field direction in the plane, electrons and holes will execute circular motions due to the magnetic Lorentz force (with or without Landau quantization), in addition to the drifting of the orbit centers. In this crossed-field geometry the electron (hole) drift velocity divides into two components, one parallel to the electric field and one perpendicular to it. Only the former is directly related to the negative electron mobility, if any. This parallel component is invariably smaller in magnitude than its counterpart in the field-free ($B=0$) case for a given electric field. Hence, considered alone, this would lead to a reduced electron mobility (positive or negative). This is similar to the argument about the positive change of magnetoresistivity $[\rho(B)>\rho(0)]$ for a two-component bulk system, neglecting interactions between the two components.[5] This geometric tendency toward reduced mobility is opposed by the enhancement of the effectiveness of the electron-hole interaction due to the circularity of magnetic field orbits, increasing the likelihood of electrons and holes being in proximity of each other. Without substantially altering the carrier-lattice scattering, the magnetic field in this respect tends to render the minority carriers more susceptible to being dragged along by the majority carriers. In our analysis of the competition of these opposing influences, we shall examine the role of the magnetic field in carrier screening, and more importantly, its effect on the attractive interaction between electrons and holes.

In the following we discuss the formulation of magnetotransport of an electron-hole plasma confined in a GaAs quantum well in Sec. II, where we derive expressions for the minority-electron mobility and the majority-hole mobility, as well as the magnetoconductivity tensor in the transverse configuration. These expressions are evaluated numerically for weak magnetic field conditions in Sec. III, along with discussions of results obtained. Our calculated results confirm that the negative electron mobility is indeed strongly affected by the applied magnetic field, especially at low temperatures ($T<60$ K). In this, we include low magnetic field corrections to dynamic screening of the electron-hole interaction and phonon scattering interaction, as well as circularity of the carrier orbits in the magnetic field.

II. FORMULATION

The balance equation analysis presented in Ref. 3 is employed here, with the addition of a constant magnetic field in the z direction. Such an approach has been adopted to study a variety of magnetotransport problems, such as hot-electron magnetotransport in bulk semiconductors,[6,7] in quasi-2D quantum wells,[8] and the linear and nonlinear magnetophonon effects.[9,10] In this paper, we are concerned with steady-state magnetotransport in a two-dimensional electron-hole plasma confined to the GaAs region of a GaAs-$Al_xGa_{1-x}As$ heterostructure, subject to a weak electric field applied parallel to the heterolayer, and a magnetic field perpendicular to the heterolayer. The quantum well is taken to have width a, in which there are N_1 electrons per unit area, with effective mass m_1, and charge $-e$, and N_2 holes per unit area, with effective mass m_2 and charge e. Carrier tunneling out of the well is neglected, and the electrons are assumed to populate only the lowest conduction subband, and holes populate only the topmost valence subband. The confinement of the electron (hole) in the quantum well is described by the envelope function $\xi(z)=(2/a)^{1/2}\cos(\pi z/a)$ for $-a/2<z<a/2$, and it vanishes everywhere else. The wave functions in the plane of the quantum well are the usual Landau states in a magnetic field. The analysis leading to the balance equations closely follows that in Ref. 3 and will not be repeated here. The resulting force-momentum balance equations are also similar in form to the field-free case of Ref. 3 ($B=0$), except for an additional term representing the Lorentz force:

$$-N_1e(\mathbf{E}+\mathbf{v}_1\times\mathbf{B})+\mathbf{f}_1(\mathbf{v}_1)+\mathbf{f}_{12}(\mathbf{v}_1-\mathbf{v}_2)=\mathbf{0}\ , \tag{1}$$

$$N_2e(\mathbf{E}+\mathbf{v}_2\times\mathbf{B})+\mathbf{f}_2(\mathbf{v}_2)-\mathbf{f}_{12}(\mathbf{v}_1-\mathbf{v}_2)=\mathbf{0}\ , \tag{2}$$

where $\mathbf{v}_1$ and $\mathbf{v}_2$ are the electron and hole drift velocities, respectively, and $\mathbf{f}_1(\mathbf{v}_1)$ $[\mathbf{f}_2(\mathbf{v}_2)]$ is the frictional force exerted on the electron (hole) by the lattice (impurities and phonons), and $\mathbf{f}_{12}(\mathbf{v}_1-\mathbf{v}_2)$ is the Coulombic force between the electrons and the holes. In the weak-electric-field limit the force functions may be linearized with respect to the drift velocities as

$$\mathbf{f}_j(\mathbf{v}_j)=\mathbf{v}_jS_j,\quad j=1,2 \tag{3}$$

and

$$\mathbf{f}_{12}(\mathbf{v}_1-\mathbf{v}_2)=(\mathbf{v}_1-\mathbf{v}_2)S\ . \tag{4}$$

Clearly $S_j=f_j'(0)$ and $S=f_{12}'(0)$. Substituting Eqs. (3) and (4) into Eqs. (1) and (2), we obtain the linearized force balance equations

$$-N_1e(\mathbf{E}+\mathbf{v}_1\times\mathbf{B})+\mathbf{v}_1S_1+(\mathbf{v}_1-\mathbf{v}_2)S=\mathbf{0}\ , \tag{5}$$

$$N_2e(\mathbf{E}+\mathbf{v}_2\times\mathbf{B})+\mathbf{v}_2S_2-(\mathbf{v}_1-\mathbf{v}_2)S=\mathbf{0}\ . \tag{6}$$

In the remainder of this paper we take $\mathbf{E}=E\hat{\mathbf{x}}$, $\mathbf{B}=B\hat{\mathbf{z}}$, and $\mathbf{v}_j=v_{jx}\hat{\mathbf{x}}+v_{jy}\hat{\mathbf{y}}$. The vector equations [(5) and (6)] can now be written in component form, which are solved to obtain the drift-velocity components in terms of the external fields

$$v_{1x}=(eE/D)\{N_1(N_2eB)^2S_1+[N_1(S_2+S)-N_2S]\times[S(S_1+S_2)+S_1S_2]\}\ , \tag{7}$$

$$v_{1y}=-(eE/D)(eB)[(N_1N_2eB)^2+N_1^2(S_2+S)^2+N_2^2S(S_1+S)-N_1N_2S(S_2+2S)]\ , \tag{8}$$

and

$$v_{2x}=(eE/D)\{-N_2(N_1eB)^2S_2+[N_1S-N_2(S_1+S)]\times[S(S_1+S_2)+S_1S_2]\}\ , \tag{9}$$

$$v_{2y}=-(eE/D)(eB)[(N_1N_2eB)^2+N_1^2S(S_2+S)+N_2^2(S_1+S)^2-N_1N_2S(S_1+2S)]\ , \tag{10}$$

with

$$D=[N_1N_2(eB)^2+S(S_1+S_2)+S_1S_2]^2+[N_1eB(S_2+S)-N_2eB(S_1+S)]^2\ . \tag{11}$$

Under the present decomposition, we may define the components of the electron and hole absolute mobilities as

$$\mu_{xx}^e=-v_{1x}/E\ , \tag{12a}$$

$$\mu_{yx}^e=-v_{1y}/E\ , \tag{12b}$$

$$\mu_{xx}^h=v_{2x}/E\ , \tag{12c}$$

$$\mu_{yx}^h=v_{2y}/E\ . \tag{12d}$$

Following Ref. 3, we define the various relaxation times through the relations

$$\tau_1=-N_1m_1/S_1\ , \tag{13a}$$

$$\tau_2=-N_2m_2/S_2\ , \tag{13b}$$

$$\tau_{12}=-N_1m_1/S\ , \tag{13c}$$

and

$$\tau_{21}=-N_2m_2/S\ . \tag{13d}$$

In terms of these relaxation times, the components of the mobilities are

$$\mu_{xx}^e=(\mu_0^e/D_1)[(\omega_{c2}\tau_2)^2+(1+N_1m_1\tau_2/N_2m_2\tau_{12}-m_1\tau_2/m_2\tau_{12})(1+\tau_1/\tau_{12}+N_1m_1\tau_2/N_2m_2\tau_{12})]\ , \tag{14}$$

$$\mu_{yx}^e=(\mu_0^e/D_1)(\omega_{c2}\tau_2)[(\omega_{c1}\tau_1)(\omega_{c2}\tau_2)+(m_1/m_2)(\tau_2\tau_{12}+\tau_1\tau_2/\tau_{12}^2)+(m_2\tau_1/m_1\tau_2)(1+N_1m_1\tau_2/N_2m_2\tau_{12})^2-(\tau_1/\tau_{12}+2N_1m_1\tau_1\tau_2/N_2m_2\tau_{12}^2)]\ , \tag{15}$$

$$\mu_{xx}^h=(\mu_0^h/D_2)[(\omega_{c1}\tau_1)^2+(1+N_2m_2\tau_1/N_1m_1\tau_{21}-m_2\tau_1/m_1\tau_{21})(1+\tau_2/\tau_{21}+N_2m_2\tau_1/N_1m_1\tau_{21})]\ , \tag{16}$$

$$\mu_{yx}^h = -(\mu_0^h/D_2)(\omega_{c1}\tau_1)[(\omega_{c1}\tau_1)(\omega_{c2}\tau_2)+(m_2/m_1)(\tau_1\tau_{21}+\tau_1\tau_2/\tau_{21}^2) + (m_1\tau_2/m_2\tau_1)(1+N_2m_2\tau_1/N_1m_1\tau_{21})^2-(\tau_2/\tau_{21}+2N_2m_2\tau_1\tau_2/N_1m_1\tau_{21}^2)] \,, \quad (17)$$

where $\mu_0^e = e\tau_1/m_1$ and $\mu_0^h = e\tau_2/m_2$ are the electron and hole mobilities at $B=0$ and without *e-h* interactions; $\omega_{c1}=eB/m_1$ and $\omega_{c2}=eB/m_2$ are the cyclotron frequencies for the electrons and holes, respectively. Furthermore,

$$D_1 = [(\omega_{c1}\tau_1)(\omega_{c2}\tau_2)+(1+\tau_1/\tau_{12}+N_1m_1\tau_2/N_2m_2\tau_{12})]^2 + [(\omega_{c1}\tau_1)(1+N_1m_1\tau_2/N_2m_2\tau_{12})-(\omega_{c2}\tau_2)(1+\tau_1/\tau_{12})]^2 \,, \quad (18)$$

and

$$D_2 = [(\omega_{c1}\tau_1)(\omega_{c2}\tau_2)+(1+\tau_2/\tau_{21}+N_2m_2\tau_1/N_1m_1\tau_{21})]^2 + [(\omega_{c1}\tau_1)(1+\tau_2/\tau_{21})-(\omega_{c2}\tau_2)(1+N_2m_2\tau_1/N_1m_1\tau_{21})]^2 \,. \quad (19)$$

Equations (14)–(19), along with the magnetoconductivity tensor elements

$$\sigma_{xx} = N_1 e\mu_{xx}^e + N_2 e\mu_{xx}^h \quad (20)$$

and

$$\sigma_{yx} = N_1 e\mu_{yx}^e + N_2 e\mu_{yx}^h \,, \quad (21)$$

describe quite generally magnetotransport in a two-component system under a transverse magnetic field of arbitrary strength, in the presence of interactions between the two types of carriers. Whereas the zero-field limits of the mobilities given by Eqs. (14)–(17) are exactly those presented in Ref. 3, it is also readily shown that upon neglecting interactions between the two components, Eqs. (20) and (21) give the known noninteracting two-component magnetoconductivity.[11]

The derivatives of the frictional forces are, as in the zero-magnetic-field case,[3] given by $(j=1,2)$

$$S_j = \left[\frac{\partial}{\partial v_j} f_j(v_j)\right]_{v_j=0} = \sum_{\mathbf{q}} q_x^2 (2\pi e^2/\kappa q)^2 \tilde{N}(q) \left[\frac{\partial}{\partial\omega}\hat{\Pi}_2^{(j)}(\mathbf{q},\omega)\right]_{\omega=0} + (2/T)\sum_{\mathbf{Q}\lambda} |M_j(\mathbf{Q},\lambda)|^2 |I(iq_z)|^2 q_x^2[-n'(\Omega_{\mathbf{Q}\lambda}/T)]\hat{\Pi}_2^{(j)}(\mathbf{q},\Omega_{\mathbf{Q}\lambda}) \,, \quad (22)$$

and

$$S = \left[\frac{\partial}{\partial v} f_{12}(v)\right]_{v=0} = (1/T)\sum_{\mathbf{q}} q_x^2 [V(q)]^2 \int_{-\infty}^{\infty} (d\omega/\pi)[-n'(\omega/T)]\hat{\Pi}_2^{(1)}(\mathbf{q},\omega)\hat{\Pi}_2^{(2)}(-\mathbf{q},-\omega) \,. \quad (23)$$

However, unlike the zero-field case, these quantities now depend on the magnetic field through the imaginary part of the density-density correlation functions $\hat{\Pi}_2^{(j)}(\mathbf{q},\omega)$.

In Eqs. (22) and (23) the 3D wave vector $\mathbf{Q}$ is decomposed as $\mathbf{Q}=(\mathbf{q},q_z)$, with $q=|\mathbf{q}|$. The first term in Eq. (22) represents the contribution from carrier-impurity scattering, with $\tilde{N}(q)$ being an effective impurity density. We consider the case where there are ionized dopants within a narrow space-charge layer with area density N_i at a distance s from the interface, in the barrier of the $Al_xGa_{1-x}As$ side. Hence

$$\tilde{N}(q) = N_i Z^2 \exp(-2qs)[I(q)]^2 \,, \quad (24)$$

with $I(q)=\int dz\, \exp(-qz)|\xi(z)|^2$ being a form factor.

The imaginary parts of the density-density correlation functions of the quantum-well system with only electrons $(j=1)$ or holes $(j=2)$ present are, in the random-phase approximation (RPA),

$$\hat{\Pi}^{(j)}(\mathbf{q},\omega) = \Pi^{(j)}(\mathbf{q},\omega)/[1-V(q)\Pi^{(j)}(\mathbf{q},\omega)] \,, \quad (25)$$

where $\Pi^{(j)}(\mathbf{q},\omega)$ are the 2D density-density correlation functions for noninteracting electrons (holes) in the presence of a magnetic field. $V(q)$ is the electron-electron (hole-hole) matrix element of the Coulomb potential

$$V(q) = (2\pi e^2/\kappa q)\int dz \int dz' \exp(-q|z-z'|) \times |\xi(z)|^2 |\xi(z')|^2 \,, \quad (26)$$

where κ is the background dielectric constant. We have neglected the difference between the dielectric constants of the GaAs region and the $Al_xGa_{1-x}As$ region (which is small) and the accompanying image potential. The second term in Eq. (22) comes from phonon scattering where a 3D plane-wave representation is adopted to describe equilibrium phonons at temperature T (the hot-phonon effect is not considered here, since it does not significantly change the results, see Ref. 2) with wave vector $\mathbf{Q}$, frequency $\Omega_{\mathbf{Q}\lambda}$, in branch λ, which couples to the carriers through the matrix element $M_j(\mathbf{Q},\lambda)$. $n(x)=[\exp(x)-1]^{-1}$, and $n'(x)=dn(x)/dx$. (Units in which $\hbar=k_B=1$ are used in all the expressions.)

Mobility calculations in the present case are similar to those carried out previously for the $B=0$ case.[3] The only complication comes from the magnetic field dependence of the density-density correlation function, whose noninteracting limit is well known,[12,13] and is commonly given in terms of a series of associated Laguerre polynomials. However, it is also a well-known fact that the δ-function nature of the 2D density of states in a quantizing magnetic field gives rise to serious divergences in the density-density correlation function. To avoid this difficulty, a Landau level linewidth has to be introduced, which accounts for the level broadening due to lattice scatterings.[14] On the other hand, our main concern in the present study is the influence of the magnetic field on the electron-hole scattering through the change in the motion of the carriers from straight-line orbits to circular orbits. Such circular motion occurs even for relatively low magnetic fields where Landau quantization is unimportant. We may take advantage of this to simplify our considerations by expanding the full magnetic-field-dependent density-density correlation function, and keep only terms of lowest order in magnetic field strength ($\sim B^2$). Such an expansion is valid, provided that the magnetic field is weak in accordance with[15]

$$\omega_c/\omega \ll 1, \quad (m\omega_c^2/q^2)(1/T \text{ or } 1/\mu) \ll 1 , \quad \omega_c(1/T \text{ or } 1/\mu) \ll 1 , \tag{27}$$

($1/T$ or $1/\mu$ for nondegenerate or degenerate cases, respectively). Here $\omega_c = eB/m$ is the cyclotron frequency for a particle with mass m and charge e; μ is the chemical potential. Under the conditions of Eq. (27), it is readily shown, following the procedure outlined in Ref. 15 for the 3D case, that the 2D noninteracting density-density correlation function in a weak magnetic field is given as

$$\Pi(q,\omega) = [1 + h_1 T\, d/d\mu + (h_2/4)T^2 d^2/d\mu^2 + (h_3/24)T^3 d^3/d\mu^3] P(q,\omega) , \tag{28}$$

where

$$h_1 = m\omega_c^2/8q^2T , \tag{29}$$

$$h_2 = (\omega_c^2/T^2)(\tfrac{5}{12} - m^2\omega_c^2/q^4) , \tag{30}$$

$$h_3 = (\omega_c^2/T^2)(q^2/16mT + m^3\omega^4/Tq^6 - m\omega^2/2Tq^2) , \tag{31}$$

and $P(q,\omega)$ is the field-free ($B=0$) density-density correlation function for noninteracting particles,[16] which for arbitrary temperature and particle concentration is given by ($P = P_1 + iP_2$)

$$P_1(q,\omega) = -(m/2\pi q)\int_0^1 dx\,(1-x)^{-1/2}[(q/2 - m\omega/q) f_0(x(q/2 - m\omega/q)^2/2m) + (\omega \to -\omega)] , \tag{32}$$

and

$$P_2(q,\omega) = -(m/2\pi q)\int_0^\infty dx\, x^{-1/2}[f_0(x/2m + (q/2 - m\omega/q)^2/2m) + (\omega \to -\omega)] , \tag{33}$$

with $f_0(\varepsilon) = \{\exp[(\varepsilon-\mu)/T] + 1\}^{-1}$. A short derivation of Eq. (28) is outlined in the Appendix.

III. NUMERICAL RESULTS AND DISCUSSION

The formulation presented above is used to calculate numerically the minority-electron mobility and majority-hole mobility as functions of the lattice temperature, in the presence of a weak magnetic field. The lattice scattering mechanisms considered here are polar-optic-phonon Fröhlich coupling and acoustic deformation potential coupling with electrons. For holes, in addition to the aforementioned couplings we also include the nonpolar-optic-phonon coupling. The same material parameters of Ref. 3 are used here. These are electron density $N_1 = 3\times 10^{10}$ cm^{-2}, hole density $N_2 = 1.5\times 10^{11}$ cm^{-2}, quantum-well width 112 Å, electron effective mass $m_1 = 0.07m_0$ (m_0 is the free-electron mass), hole effective mass $m_2 = 0.4m_0$, GaAs static dielectric constant 12.9, optical dielectric constant 10.8, longitudinal optical-phonon frequency 35.4 meV, lattice mass density 5.31 g cm^{-3}, longitudinal sound velocity 5.29×10^5 cm s^{-1}, transverse sound velocity 2.48×10^5 cm s^{-1}, conduction-band deformation potential 8.5 eV, and valence-band deformation potential 9.0 eV.

Screening of the electron-hole interaction and carrier-phonon interaction is included making use of the RPA density-density correlation function of Eq. (25), which is calculated with the aid of Eqs. (28)–(33). To simplify the numerical calculation we have used the zero-temperature density-density correlation function in place of Eqs. (32) and (33), and have taken the chemical potentials to be independent of the magnetic field.

Results of our numerical computation are presented in the figures. Minority-electron mobility element μ_{xx}^e is plotted as a function of temperature in Fig. 1 for several values of the magnetic field strength. A similar set of calculation for the majority-hole mobility element μ_{xx}^h is shown in Fig. 2. The corresponding tensor elements μ_{yx}^e and μ_{yx}^h are presented in Fig. 3 and Fig. 4, respectively.

The conditions for the occurrence of negative minority-electron mobility (corresponding to μ_{xx}^e here) have been discussed extensively in the literature.[1–4] The most crucial requirement is that the electron-hole scattering dominate over hole-phonon scattering, which in the case of $B=0$ is given in terms of the electron-hole scattering time τ_{12} and the hole-phonon scattering time τ_2 as $\tau_{12}/m_1 < \tau_2/m_2$. In the present case this condition is modified by the introduction of the magnetic field. The modified condition is easily obtained from Eq. (14), which gives the criterion for negative minority-electron mobili-

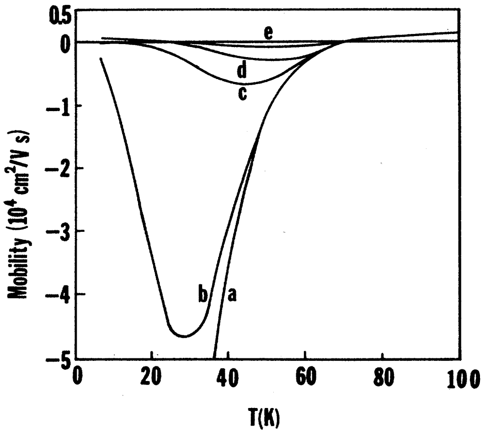

FIG. 1. μ^e_{xx} vs temperature for various magnetic field strengths. (a) $B=0.0$, (b) $B=0.1$, (c) $B=0.5$, (d) $B=1.0$, (e) $B=5.0$ T.

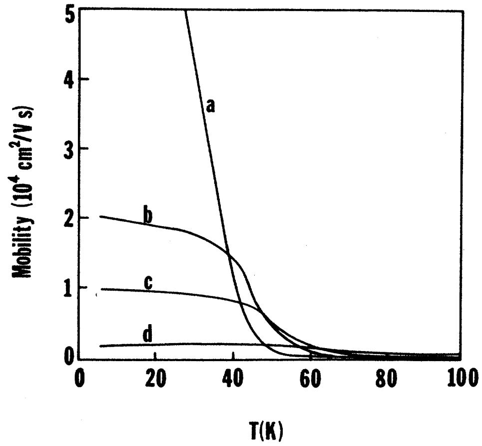

FIG. 3. μ^e_{yx} vs temperature for various magnetic field strengths. (a) $B=0.1$, (b) $B=0.5$, (c) $B=1.0$, (d) $B=5.0$ T.

ty, in the usual situation ($N_1 m_1 \tau_2 / N_2 m_2 \tau_{12} \ll 1$, $\tau_1/\tau_{12} \ll 1$), as

$$(\tau_{12}/m_1)[1+(\omega_{c2}\tau_2)^2] < \tau_2/m_2 \ . \tag{34}$$

It is easy to identify the two competing effects of the magnetic field in the above inequality if we ignore for the moment the change of τ_2 due to the field. The left-hand side of Eq. (34) contains two factors, τ_{12} and $1+(\omega_{c2}\tau_2)^2$. The former, τ_{12}, decreases with increasing magnetic field because of the enhancement of electron-hole scattering associated with their greater proximity due to the bending of the carrier orbits by the magnetic field, while the latter obviously increases in connection with the general geometrical increase of magnetoresistance. The combined result is such that the negative minority-electron mobility is reduced by the magnetic field as shown in Fig. 1, which indicates that the general geometrical increase of magnetoresistance dominates the determination of the electron mobility. Figure 1 clearly shows that the magnitude of the electron mobility is reduced as the magnetic field is increased. The majority-hole mobility is similarly affected by the magnetic field, and Fig. 2 shows its reduction by the magnetic field.

As in the zero-field case, electron-hole drag depends strongly on the lattice temperature. Since the electron-hole scattering is more effective at low temperatures, while the phonon scattering is weak at such temperatures, one expects to observe negative minority-electron mobility at low temperatures. Our calculated μ^e_{xx} (Fig. 1) for all magnetic field strengths becomes positive at temperatures 60–70 K. The lower temperature (60 K) corresponds to $B=5$ T, while the higher temperature is for $B=0$. Beyond 70 K, optical-phonon scattering becomes so strong that the magnetic field has very little effect on the carrier mobilities. This is seen in all the figures presented, and is expected from simple physical reasoning. For the nonquantizing fields considered here, the most important of the magnetic field effects can be measured by the parameter $\omega_c\tau$, where τ is a typical scattering time. If τ is small (which is the case for optic-phonon scattering above $\sim$50 K), such that $\omega_c\tau \ll 1$, the carrier travels through a very small part of the circular orbit before being scattered again. These small arcs deviate only

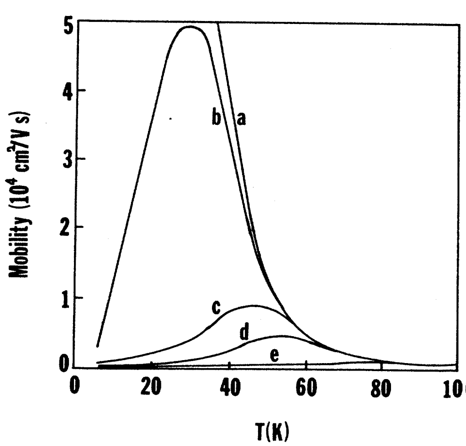

FIG. 2. μ^h_{xx} vs temperature for various magnetic field strengths. (a) $B=0.0$, (b) $B=0.1$, (c) $B=0.5$, (d) $B=1.0$, (e) $B=5.0$ T.

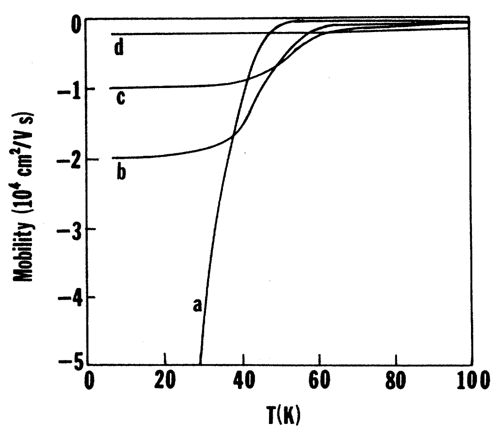

FIG. 4. μ^h_{yx} vs temperature for various magnetic field strengths. (a) $B=0.1$, (b) $B=0.5$, (c) $B=1.0$, (d) $B=5.0$ T.

slightly from straight-line paths, rendering the magnetic field ineffective in changing the carrier transport.

At extremely low temperatures ($T \lesssim 10$ K), carrier-lattice scatterings become negligibly weak (we do not consider impurity scattering here), and this creates a qualitatively different physical situation. In this case the carriers circle the magnetic field direction $\mathbf{B}=B\hat{\mathbf{z}}$ indefinitely, and a static electric field $\mathbf{E}=E\hat{\mathbf{x}}$ results in the same drift velocities $-(E/B)\hat{\mathbf{y}}$ for both electrons and holes [this is also obtained upon setting $S_1=S_2=0$ in Eqs. (7)–(10)]. Consequently, the frictional Coulomb force between electrons and holes $\mathbf{f}_{12}(\mathbf{v}_1-\mathbf{v}_2)=(\mathbf{v}_1-\mathbf{v}_2)S$ also vanishes because of the zero relative velocity, for any nonzero applied magnetic field, just as Dreicer found for a gas plasma.[17] In terms of the mobility elements, this leads to the results $\mu_{xx}^e=\mu_{xx}^h=0$, and $\mu_{yx}^e=-\mu_{yx}^h=1/B$, for any nonvanishing magnetic field. These results are clearly demonstrated in our calculations: In Figs. 1 and 2 the diagonal elements of the mobilities for $B\neq 0$ tend to zero as the temperature approaches zero, whereas in Figs. 3 and 4 the off-diagonal elements of the mobilities tend to $1/B$ or $-1/B$ in the same limit.

To summarize, we have presented a general formulation for the problem of magnetotransport in a quasi-2D system with two types of carriers for arbitrary magnetic field strength, taking into account carrier-lattice interactions, as well as interactions between different types of carriers, both dynamically screened by interactions among like carriers. Our formulation has been applied to calculate electron and hole mobilities in a weak magnetic field. The calculated results presented here clarify the roles played by the magnetic field, the lattice temperature, and the scattering mechanisms, in determining the magnetotransport properties of such a two-component system. Special attention has been focused on the influence of the magnetic field on electron-hole drag, and the associated phenomenon of negative minority-carrier mobility. We have found that the general geometrical magnetoresistance increase dominates over magnetic enhancement of electron-hole scattering except at very low temperature, where phonon scattering becomes ineffective. In the latter case, we find that even a small magnetic field can reduce the velocity of electrons relative to holes to zero, and thus drastically reduce the effectiveness of electron-hole scattering. As such magnetic fields are readily available, it will be of interest to examine this breakdown of carrier drag in favor of Hall drift experimentally.

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APPENDIX

In the following we outline how one may expand the magnetic-field-dependent 2D density-density correlation function when ω_c is small [in the sense of Eq. (27)]. Such a procedure is easier to carry out starting from the integral representation of Horing and Yildiz,[12] than from the conventional infinite-series representation.[13,14] From Ref. 12 one can write ($\mathcal{P}$ denotes principal value)

$$\Pi_1(q,\omega)=\mathcal{P}\int\frac{d\omega'}{2\pi}\int\frac{d\omega''}{2\pi}\frac{\omega''}{\omega^2-\omega''^2}f_0(\omega')R(\omega',\omega'',q)\ , \tag{A1}$$

and

$$\Pi_2(q,\omega)=-\tfrac{1}{2}\int\frac{d\omega'}{2\pi}f_0(\omega')R(\omega',\omega,q)\ , \tag{A2}$$

where

$$R(\omega,\omega',q)=m\omega_c\int_{-\infty}^{\infty}dy\,e^{-i\omega' y/2}\int_{-i\infty+\delta}^{i\infty+\delta}\frac{ds}{2\pi i}e^{\omega s}\frac{\sinh(\omega' s/2)}{\tanh(\omega_c s/2)}\exp\left[\frac{q^2}{2m\omega_c}\frac{\cos(\omega_c y/2)-\cosh(\omega_c s/2)}{\sinh(\omega_c s/2)}\right]\ . \tag{A3}$$

For small ω_c, $R(\omega,\omega',q)$ can be expanded in a power series of ω_c, which to order ω_c^2 has the form

$$R(\omega,\omega',q)=2m\int_{-\infty}^{\infty}dy\,e^{-i\omega' y/2}\int_{-i\infty+\delta}^{i\infty+\delta}\frac{ds}{2\pi i}e^{(\omega-q^2/8m)s}\sinh(\omega' s/2)\frac{1}{s}\left[1+\frac{\omega_c^2 s^2}{12}+\frac{q^2\omega_c^2}{384ms}(y^2+s^2)^2\right]e^{-q^2y^2/8ms}\ . \tag{A4}$$

The y integration can be evaluated as the Fourier transform of a Gaussian along with derivatives, resulting in

$$R(\omega,\omega',q)=2m\left[\frac{8\pi m}{q^2}\right]^{1/2}\int_{-i\infty+\delta}^{i\infty+\delta}\frac{ds}{2\pi i}s^{-1/2}\sinh(\omega' s/2)(1+H_1 s+H_2 s^2+H_3 s^3)e^{[\omega-q^2/8m-m(\omega')^2/2q^2]}\ , \tag{A5}$$

with

$$H_1=\frac{m\omega_c^2}{8q^2},\quad H_2=\frac{5\omega_c^2}{48}-\frac{m^2\omega_c^2(\omega')^2}{4q^4},\quad H_3=\frac{q^2\omega_c^2}{384m}+\frac{m^3\omega_c^2(\omega')^4}{24q^6}-\frac{m\omega_c^2(\omega')^2}{48q^2}\ .$$

Using the identity $s\exp(\omega s)=(d/d\omega)\exp(\omega s)$, the powers of s can all be expressed in terms of ω derivatives, and the s integral is now of standard form[18]

$$R(\omega,\omega',q)=m\left[\frac{8\pi m}{q^2}\right]^{1/2}\left[1+H_1\frac{d}{d\omega}+H_2\frac{d^2}{d\omega^2}+H_3\frac{d^3}{d\omega^3}\right]\sum_{\pm}(\pm)\frac{\Theta(\omega-q^2/8m-m\omega'^2/2q^2\pm\omega'/2)}{(\omega-q^2/8m-m\omega'^2/2q^2\pm\omega'/2)^{1/2}}\,, \tag{A6}$$

where $\Theta(x)$ is the Heaviside unit-step function. Upon substituting this into (A1) and (A2), one recognizes that the ω derivatives can be turned into μ derivatives through integration by parts, and that the ω'' integral in (A1) is a tabulated Hilbert transform.[19] After redefining the constants H_1, H_2, and H_3, one obtains the expressions given by Eqs. (28)–(33).

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