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Effect of surface roughening on chaos in yttrium-iron-garnet spheres

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Roughening the surface of polished spheres of yttrium iron garnet causes the uniform mode of spin-wave precession to scatter into low-wave-vector spin-wave modes, or magnons, not present in the polished sphere. These extra spin-wave modes complicate the previously studied nonlinear behavior of this system, creating intermittency where chaotic transients were previously observed. We measure the threshold power for chaos in these spheres and consider the effects of the different interactions present in the rough spheres on this threshold.

INTRODUCTION

Polished spheres of yttrium iron garnet (YIG), a ferrimagnetic material, have recently been popular as a testing ground for different ideas in nonlinear dynamics.[1-8] Nonlinear effects in the ferromagnetic response of these spheres can give rise to period doubling sequences, chaos, chaotic transients, and other phenomena. We have altered the nonlinear behavior that we previously observed[1-4] in polished YIG spheres by roughening their surfaces, creating new spin waves through a two-magnon scattering process.[9-11] Changing this scattering interaction by changing the roughness of the surface produces intermittency and changes the rf power necessary to drive the system into chaos.

In our experiment[1-4] we place a 20-mil-diam sphere of YIG in a dc magnetic field. The magnetization of the YIG sphere precesses about this dc field, but damping will reduce the precession angle to zero unless we apply an rf magnetic field. We apply this field (at the ferromagnetic resonance frequency) perpendicular to the dc field. The easy axis of magnetization (the [111] axis) of the sphere is parallel to the dc field.

For small rf fields, the sample magnetization precesses uniformly about the dc field. At higher rf powers, nonlinear effects couple the uniform precession mode into spin-wave modes,[12-14] which are spatially periodic variations in the amplitude and/or phase of the precession. If one treats these spin-wave modes as quantum phenomena,[10] they are also called magnons. At a particular rf field, a phenomenon known as the Suhl instability[15] leads to the rapid growth of these spin waves and the breakdown of the ferromagnetic resonance line. Our experimental parameters were such that spin waves were generated via the first-order Suhl instability, for which the rate of transfer of energy from the uniform mode into spin-wave modes goes as first order in the amplitude of the uniform mode,[12] and the first spin waves to be excited have a frequency half that of the driving frequency. From the Landau-Lifshitz equation,[12] one may derive a dispersion relation for spin waves in a sphere

$$\omega_k=[(\omega_0-\omega_m/3+\gamma\beta k^2)\times(\omega_0-\omega_m/3+\beta\gamma k^2+2\omega_m\sin\theta_k)]^{1/2}\ , \quad (1)$$

where ω_k is the frequency of a spin-wave mode with wave vector k, ω_0 is the ferromagnetic resonance frequency γH_0, H_0 is the dc magnetic field, $\omega_m=4\pi\gamma M_s$, where M_s is the saturation magnetization, $\beta=H_{\text{ex}}a^2/M_s$, where a is a lattice constant and H_{ex} is the exchange magnetic field, and θ_k is the angle between a spin wave with wave vector k and the dc magnetic field. As θ_k is changed from 0° to 90°, this dispersion relation sweeps out a band of spin waves known as the spin-wave manifold. We have chosen our experimental parameters so that the first spin-wave mode to be excited lies within this spin-wave manifold for wave vector k near zero.[13] When two or more of these spin waves are excited, their interaction produces a low-frequency (kHz) modulation on the GHz detected signal.[14] This modulation, which may not be periodic, is known as an auto-oscillation. This is what we detect in this experiment.

EXPERIMENTAL PROCEDURE

We held a 20-mil-diam undoped single-crystal YIG sphere inside a quartz tube with Apiezon M grease so that the YIG sphere was inside an excitation coil and near an orthogonal pickup coil. An electromagnet provided a dc field perpendicular to both coils and the easy ([111]) axis of the YIG sphere. Microwave power was provided by an HP 8341 A synthesized sweeper. The signal induced in the pickup coil by the YIG sphere was detected by a crystal detector and amplified. The resulting time series could be viewed on a storage oscilloscope or digitized and transferred to a computer. All measurements were made with the system tuned to the center of the ferromagnetic resonance line located in the range 2.0–3.3 GHz. The YIG sphere was undercoupled to the pickup and excitation coils.

The surfaces of the YIG spheres were roughened by rolling polished spheres between a microscope slide and the appropriate grade of abrasive paper. Except where noted, the surface of the sphere was completely covered with pits, as observed through a microscope.

TWO-MAGNON SCATTERING

Pits in the surface of a YIG sphere allow the uniform mode of precession to scatter into low-k spin-wave modes through two-magnon scattering.[9–11] In two-magnon scattering, one uniform precession magnon (quantized spin-wave mode) scatters into one low-k magnon with the same frequency. The wavelength of these low-k spin-wave modes is assumed to be the same as the size of the abrasive particles used to create the pits. This process may be modeled by using a scattering potential based on the interaction of the magnetization $\mathbf{M}$ of the uniform precession with the demagnetization field $\mathbf{H}_D$ of a spherical cavity in an infinite medium. The resulting potential is of the form $\mathbf{H}_D \cdot \mathbf{M}$. This results in a scattering rate from the uniform mode into the low-k spin-wave modes given by[10]

$$\frac{1}{\gamma T_{\mathrm{pit}}} = \frac{\pi^2}{2} M_s \frac{R_{\mathrm{pit}}^3}{R^3 \omega_i} \frac{[(3\cos^2\theta_u - 1)^2 + 1.6]}{\cos\theta_u} \qquad (2)$$

where T_{pit} is the relaxation time of the uniform mode of precession into a low-k spin wave, R_{pit} is the radius of the pits, R is the sample radius, M_s is the saturation magnetization of the sample, γ is the gyromagnetic ratio, θ_u is the angle between the scattered spin-wave mode wave vector and the dc magnetic field, ω is the uniform precession mode resonant frequency, and ω_i is the internal frequency

$$\gamma[H_0 - (4\pi/3)M_s] .$$

RESULTS

The uniform mode which interacts with higher-k spin-wave modes generated by the Suhl instability process now also interacts with low-k spin-wave modes created through two-magnon scattering. The first indication of this extra interaction which is not present in the smooth spheres is a change in the behavior of the low-frequency auto oscillation modulating the microwave signal. Figure 1 shows qualitatively the type of behavior seen as the ferromagnetic resonance line is swept through the spin-wave manifold.

Figure 1(a) is a parameter space plot for a polished sphere[3,4] with pits estimated to be on the order of 0.1 μm. There are three main regions seen in this plot, one where only quasiperiodic auto oscillations are seen, one (shaded) where chaotic transients lead into quasiperiodic auto oscillations, and a region where only chaos is seen. Intermittency does occur in this sample, but only for a very small range of rf power. This plot could be reproduced easily with other polished samples.

Figure 1(b) is, in contrast to Fig. 1(a), a typical parameter space plot for a roughened sphere. This sphere was roughened with 5-μm abrasive. The types of intermittency are distinguished by the fact that in some, the interval between bursts increases as rf power increases, while in others the burst interval decreases. For the first three types of intermittency shown in the legend for Fig. 1(b), the burst interval decreases as rf power increases. For the fourth type, the burst interval increases.

Behavior in the rough spheres was very sensitive to their exact location in the rf-excitation coil, so this plot is only a typical example of the type of behavior seen. The

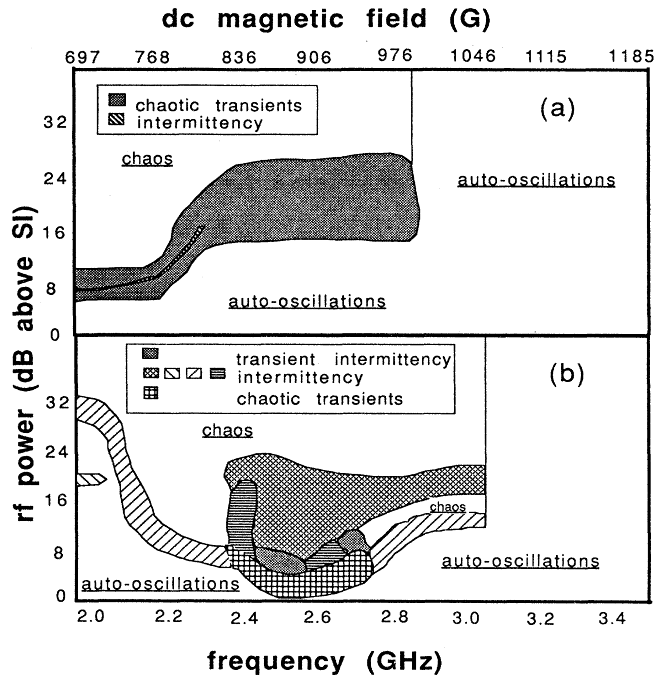

FIG. 1. Parameter space plots of the behavior of auto oscillations in YIG spheres driven at the ferromagnetic resonance frequency. (a) is for polished spheres, while (b) is for spheres roughened with 5-μm abrasive.

method that we use to mount the spheres, i.e., floating them in grease, minimizes vibration noise but makes exact positioning of the spheres impossible. The grease does allow the sphere to rotate slightly, so that the [111] axis of the sphere will tend to be closely aligned with the dc magnetic field. The process of roughening the spheres does not produce a perfectly uniform surface, however. Very small variations in the orientation the sphere or its position relative to the excitation coil will change the relative locations of different surface regions. This will affect the observed dynamics. The position of the sphere relative to the excitation coil could vary by as much as $\frac{1}{2}$ mm each time a different sphere was used. There are also several [111] axes in the sphere, but we do not know exactly which one is aligned with the dc magnetic field. The general outlines of the different behavior regions did stay the same when the sample was moved. The important contrast with Fig. 1(a) is the much greater number of behavior regions seen, including much intermittency. Rougher spheres showed similar behavior, with some differences to be noted below.

Spheres that were polished showed very little sensitivity to their location relative to the excitation coil. The rf magnetic field at the sample changes with the location of the sample relative to the excitation coil. The narrow width of the ferromagnetic resonance line for the polished spheres (on the order of 0.2 G) indicates that the variation in field across a 20-mil YIG sphere is very small.

Figure 2 is a time series showing the intermittency seen

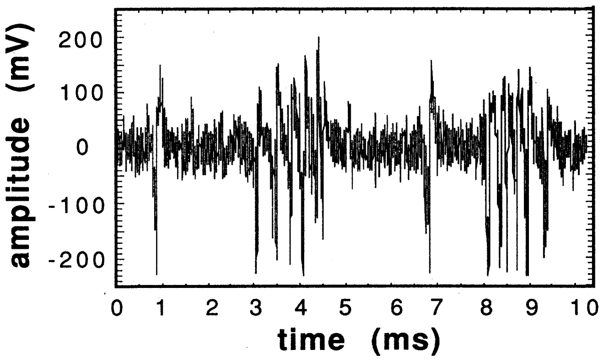

FIG. 2. Experimental time series showing bursting or intermittency in a rough YIG sphere. Measured burst interval is the time from the start of one large burst to the start of the next.

in a sphere roughened with 5-μm abrasive. The small amplitude component of this intermittency appears to be similar to chaotic waveforms seen in the polished spheres. The dimension of the small component is about 3.3, which is the same as the dimension of low-power chaos in the polished sphere.[1–4] The large bursts were never present for a long enough time to allow a dimension calculation. Figure 3(a) is a Fourier amplitude spectrum for chaos in a polished sphere, while Fig. 3(b) is a Fourier amplitude spectrum for intermittency similar to that shown in Fig. 2. The spectra are similar in the high-frequency region, but the Fourier spectrum of the intermittency contains an extra broad peak at low frequencies caused by the switching of the waveform between large and small amplitude states.

At a fixed rf power, the distribution of times from the start of one large burst to the start of the next for all types of intermittency seen fits the distribution

$$\mathcal{P}(t)=\exp(-t/\langle t\rangle)\ .$$

As rf power is increased, the average separation of the intermittent bursts varies. Figure 4 shows typical plots of average time between bursts as a function of rf power. Figure 4(a) (rf frequency is 2.5 GHz) shows an average time between bursts that decreases with rf power, while in 4(b) (rf frequency is 2.4 GHz) both an increase and decrease are seen. Because the nonlinear behavior in a roughened sphere is very sensitive to its exact location in the excitation coil, these cases are meant only as typical examples of the types of behavior seen in a roughened sphere. The general regions of behavior that we see do not vary greatly when the sphere is moved slightly, but the exact curves of intermittent separation versus power do vary.

The type of intermittency that we see in these rough spheres fits the type described by Grebogi and coworkers.[16–18] In their theory, bursts between two different forms of chaos that show an exponential distribution in time are caused by the merging of two different chaotic attractors. As rf power varies, the average time interval between bursts increases or decreases according to a scaling law. For intermittency with bursts that become closer together in time as rf power increases, this scaling law is of the form

$$\langle t\rangle=K/(P-P_c)^{\gamma}\ .$$

In this relation, P stands for driving power and P_c is the critical power at which the time between bursts goes to infinity. This corresponds to the two attractors just touching each other. The collision of two attractors is

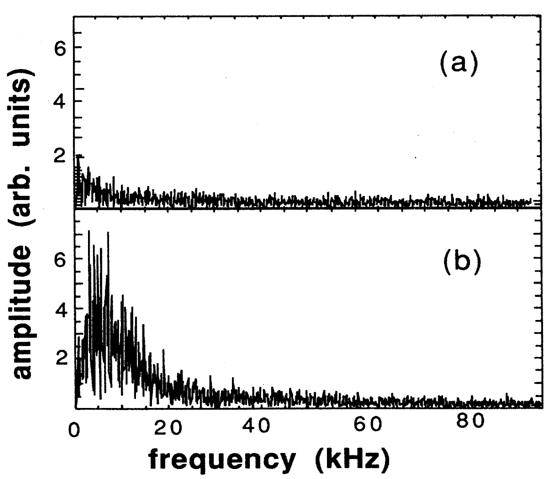

FIG. 3. (a) Fourier amplitude spectrum for chaos in a polished YIG sphere. (b) Fourier amplitude spectrum for intermittency such as that shown in Fig. 2.

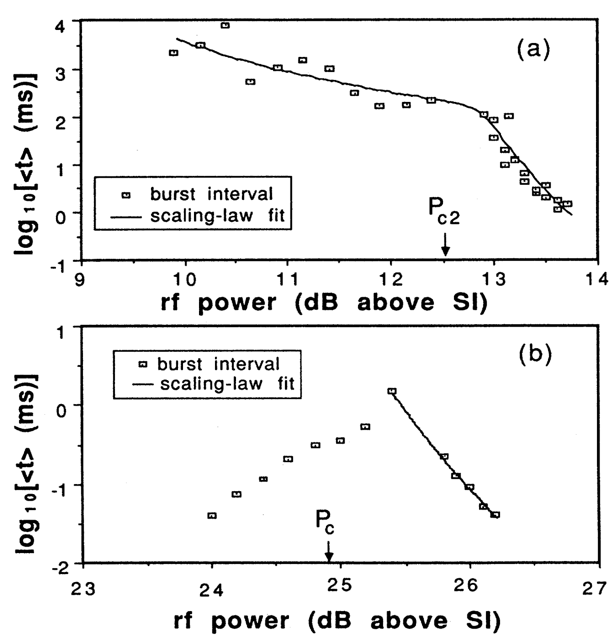

FIG. 4. (a) Average time between bursts and combined scaling law fit for intermittency at 2.5 GHz. (b) is the average time between bursts and a scaling law fit for intermittency at 2.4 GHz.

called a crisis.[16–18]

For the example of intermittency shown in Fig. 4(a), it is obvious that one scaling law will not fit the observed burst separation as function of rf power. Previously,[1,4] we have used a combined scaling law to fit a similar plot of chaotic transient length versus power when the chaotic attractor overlapped the boundaries of the basins of attraction of two nonchaotic attractors. If we combine two power laws in the same way here, adding average burst intervals as reciprocals, we have a net scaling law

$$\langle t \rangle = \frac{K_1 K_2}{[K_2(P-P_{c1})^{\gamma_1} + K_1(P-P_{c2})^{\gamma_2}]} . \quad (3)$$

The term $(P-P_{c2})$ is set to zero for $P < P_{c2}$. For the plot in Fig. 4(a), γ_1 is 0.77, γ_2 is 5.2, P_{c1} is 2.8 dB, and P_{c2} is 12.5 dB.

It is possible that this combined scaling law describes a situation in which three attractors merge, rather than just two. In this scenario, at P_{c1}, a large amplitude attractor would just touch a smaller amplitude attractor. The system would spend most of the time on the small part of the new attractor, occasionally finding its way onto the large part, causing a large amplitude burst. As the rf power increases, the overlap between the two formerly separate attractors would increase, causing the large bursts to occur more often. At a driving power equal to P_{c2}, another large attractor would just touch the small attractor. The rate at which this second large attractor merged with the small attractor as rf driving power increased would be larger than the rate at which the first larger attractor merged, leading to a larger critical exponent above P_{c2} than below.

In Fig. 4(b), a single scaling law fit to the region where burst separation is decreasing gives an exponent of 3.9 and a critical power of 24.8 dB. The region in Fig. 4(b) where burst spacing is increasing with rf power appears to follow a combination scaling law of a form similar to that used in Fig. 4(a), but we do not have enough data points here to give a reliable fit to this type of scaling law.

While roughening the surface of the YIG spheres adds the extra interactions with the uniform mode that lead to intermittency, it also causes greater damping of this mode, evidenced by a greater full width at half maximum (FWHM) for the ferromagnetic resonance line (Fig. 5) and a greater rf power needed to reach the Suhl instability (Fig. 6).

Unlike the Suhl instability, the rf power needed to drive this system into chaos (Fig. 6) does not increase as abrasive size increases. We believe that the extra low-k spin-waves generated by surface pit scattering are at least partly responsible for offsetting the extra damping of the uniform mode, which would tend to increase the chaos threshold. For the coarsest abrasive, 20 μ, the chaos threshold actually decreases.

We study the onset of chaos because it is the most distinct benchmark of the nonlinear behavior of this system that we could find. The onset power for quasiperiodic auto oscillations was very sensitive to experimental parameters, while the onset power for chaos was not as sensitive to rf frequency, dc magnetic field, or sample posi-

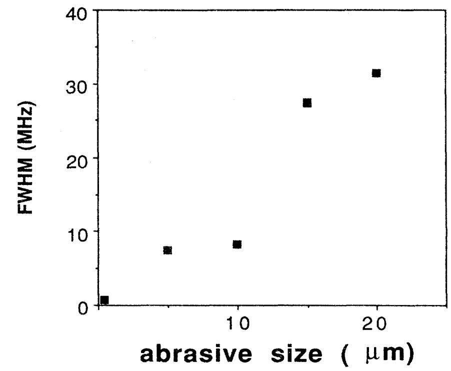

FIG. 5. Full width at half maximum of the ferromagnetic resonance line in roughened YIG spheres.

tion. In the polished (~0.1-μm pit size) sphere, we took the onset of chaos to be the point where chaotic transients give way to stable chaos at an rf frequency of 2.5 GHz. In the rougher spheres, we use the point at which intermittency gives way to stable chaos (see Fig. 1). There may be errors as large as ± 1 dB in our estimation for the onset of chaos in the rough spheres.

While we cannot say how the extra low-k spin-wave modes interact with the uniform mode to change the threshold of chaos in roughened spheres, we can estimate how roughening the spheres affects the balance of energy between the uniform mode and the low-k modes. We use the same energy balance equations used by Fletcher, Spencer, and LeCraw.[19] In these equations, they use the approximation that the only interactions are between the uniform mode and the crystal lattice, the uniform mode and the low-k spin-wave mode that it scatters into, and the low-k spin-wave mode and the lattice:

$$\frac{dn_t}{dt} = P - \frac{1}{T_{ul}} n_u - \frac{1}{T_{dl}} (n_t - n_u) , \quad (4)$$

$$\frac{dn_u}{dt} = P - \frac{1}{T_{ul}} n_u - \frac{1}{T_{ud}} (n_u - n_k) , \quad (5)$$

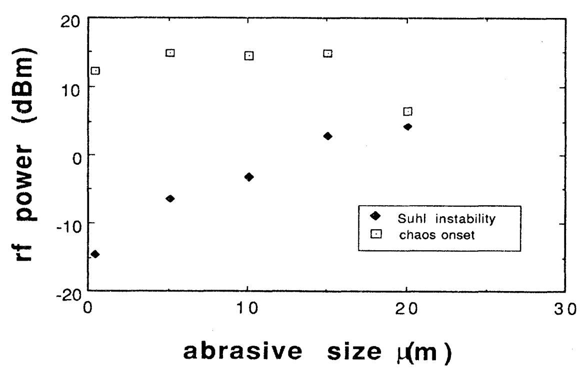

FIG. 6. rf power necessary for Suhl instability and onset of chaos in roughened YIG spheres.

where n_t is the total number of magnons (quantized spin waves) excited, n_u is the number of uniform precession magnons, n_k is the number of low-k magnons created by surface pit scattering, P is the driving power, T_{ul} is the decay time from the uniform precession mode to the lattice, T_{dl} is the time for the new low-k magnons to decay to the lattice, and T_{ud} is the time for uniform mode magnons to scatter into low-k magnons via surface pit scattering. The approximation of the uniform mode interacting only with the low-k spin-wave mode and the lattice is definitely not true above the Suhl instability. We use this approximation, however, because the behavior of the system below the Suhl instability may be related to the behavior above the Suhl instability.

We consider what happens in equilibrium, when the time derivatives on the left-hand side of Eqs. (4) and (5) are zero. From our data in Fig. 5, the width of the ferromagnetic resonance line in the roughened spheres is between 10 and 60 times as large as in the polished sphere. This means that the width of the resonance line in the polished spheres, $1/T_{ul}$, is much smaller than $1/T_{ud}$. From measurements of this system,[19] $1/T_{ud}$ and $1/T_{dl}$ are both much larger in the roughened spheres than $1/T_{ul}$, so we drop all terms with $1/T_{ul}$. With this approximation, the ratio of the occupation number of the low-k spin-waves to the occupation number of the uniform mode is

$$\frac{n_k}{n_u} \approx \frac{T_{dl}}{T_{ud}+T_{dl}} , \tag{6}$$

where T_{ud} is, in this approximation, equal to the reciprocal of the ferromagnetic resonance linewidth.

Fletcher, Spencer, and LeCraw[19] show that T_{dl} increases with magnon wavelength. Therefore, as abrasive size increases, T_{ud} decreases, T_{dl} increases, and the ratio of n_k to n_u increases. We cannot find any simple relation between this ratio and the power at which chaos begins, but this number does increase to oppose damping. The actual occupation numbers of each mode decrease with damping. This suggests that the ratio of occupation numbers in each mode or some similar quantity may be important in determining the onset of chaos.

CONCLUSION

We were able to use low-k spin-wave modes produced by two-magnon surface irregularity scattering to alter the nonlinear behavior seen in single-crystal YIG spheres. By changing the size of the surface pits that we introduced in the YIG spheres, we could change the rate at which these low-k modes were produced and the rate at which they decayed to the lattice. Changing pit size also altered the damping of the uniform mode. We were able to observe many different types of intermittency in these rough spheres, although roughening these samples also made them very sensitive to their exact position in the rf excitation coil.

We also measured the change in the onset of chaos as different abrasive sizes were used. We found that the threshold power for chaos was affected by more than just the damping of the uniform mode. Using a very crude approximation, we speculated on some possible causes for the behavior of the chaos threshold. A more satisfactory study of the effect of the extra low-k spin-waves on the threshold must await more sophisticated numerical work.

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