Nonuniversal critical dynamics on the Fibonacci-chain quasicrystal

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An exact real-space renormalization-group calculation of the dynamic exponent z is given for Ising-Glauber dynamics on the Fibonacci-chain quasicrystal. The method is first illustrated for the simple cases of the uniform and alternating Glauber chains, which reproduce the well-known results $z=2$ and $1+|J_A/J_B|$ $(J_A>J_B)$, respectively. For positive exchange interactions the critical Glauber dynamics of the Fibonacci chain is governed by a single, unstable one-cycle (fixed point) of the renormalization-group transformation. However, if one or all the exchange interactions are negative, then the critical dynamics is governed by a three-cycle. In all cases the dynamic exponent z is identical to that obtained for the alternating-bond Glauber chain.

I. INTRODUCTION

The dynamic scaling hypothesis of Halperin and Hohenberg[1,2] relates the time scale τ to the correlation length ξ through the relation $\tau \sim \xi^z$, where z is the dynamic exponent. Furthermore, the exponent z was thought to be universal, depending on the nature of conserved quantities as well as those features (the dimensionality of space, the number of spin components, etc.) which determine the static universality class. It has been seen, however, that in certain systems, such as the alternating-bond Ising chain,[3] the disordered Ising ferromagnetic chain,[4] and $d>1$ Ising systems near the percolation threshold,[5] the dynamic scaling hypothesis appears to break down. More specifically, the breakdown occurs near a zero-temperature fixed point in structures where the dynamics involves thermal activation over barriers. In each case the Ising dynamics is assumed to be of the form originally proposed by Glauber[6] where spins undergo random transitions between the values ± 1 as a result of interactions with neighboring spins and an external heat bath. The time-dependent probability distribution for single-spin-flip Ising-Glauber dynamics system satisfies a master equation of the form

$$\frac{d}{dt}P(\sigma_1,\sigma_2,\ldots,\sigma_N;t)=\sum_n W_n(-\sigma_n)P(\sigma_1,\ldots,-\sigma_n,\ldots,\sigma_N;t)-\left[\sum_n W_n(\sigma_n)\right]P(\sigma_1,\sigma_2,\ldots,\sigma_N;t) , \tag{1.1}$$

where $P(\sigma_1,\ldots,\sigma_N;t)\equiv P(\{\sigma\};t)$ is the probability distribution of the spin configuration, and $W_n(\sigma_n)$ is the probability per unit time that the nth spin flips from the value σ_n to $-\sigma_n$. Furthermore, the expectation value $\bar{q}_n(t)$ of the spins is given by

$$\bar{q}_n(t)=\sum_{\{\sigma\}}\sigma_n P(\{\sigma\};t) , \tag{1.2}$$

where

$$\sum_{\{\sigma\}}\equiv\sum_{\sigma_1=\pm1}\sum_{\sigma_2=\pm1}\cdots\sum_{\sigma_N=\pm1} .$$

Renormalization-group (RG) methods, although initially developed to treat static critical phenomena, have also been extended to dynamic critical phenomena.[1,2,7] Following the development of the static RG, the dynamic RG was essentially perturbative in nature (i.e., ϵ expansion in k space); however, an attractive implementation of the renormalization group as applied to both statics as well as dynamics, is the so-called real-space RG, in such forms as decimation or blocking. Decimation methods have proved to be useful in studying the critical dynamics of both translationally and nontranslationally invariant systems with a discrete scale invariance.[7]

There are various means of implementing the decimation procedure. In the present context of nonlinear Glauber dynamics, decimation has been used in the past at the level of the master equation[8-11] (1.1), where one is forced to make the *ad hoc* assumption that the nonequilibrium probability distribution $P(\{\sigma\};t)$ can be written as

$$P_{\text{eq}}(\{\sigma\};t)\left[1+\sum_j h_j(t)\sigma_j\right] ,$$

where P_{eq} denotes the equilibrium probability distribution. We shall take an alternative approach and work directly with the equations of motion as obtained using (1.2). This has the advantage that for one-dimensional problems with finite-range interactions, this method leads to the exact solution without the need for any simplifying assumptions.

It has been realized that for some quite simple systems the dynamic exponent is nonuniversal, generally depending on the system parameters. Droz *et al.*[3] have shown that for the periodic Ising chain with alternating exchange interactions and Glauber dynamics, the dynamic exponent can assume any value greater than 2. In addition, the work of Lage[10] on critical Glauber dynamics has suggested that similar nonuniversal properties are also present in the one-dimensional Potts model. There has, however, been comparatively little work concerned with the breakdown of dynamic scaling on lattices which lack translational invariance. Achiam[12] has studied the critical dynamics of the kinetic Glauber-Ising chain model on various fractal geometries including branching and nonbranching Koch curves and the Sierpinski gasket, his results indicating standard dynamic scaling. More recently Bell and Southern[13] have reported a similar study of the critical dynamics of the ferromagnetic Ising model on the Mandelbrot-Given and Sierpinski gasket fractals. Their results confirm the prediction of a singular dynamic scaling, and cast some doubt as to the validity of the previous work of Achiam.

In this paper we study the critical Glauber dynamics of a one-dimensional Ising chain with a quasiperiodic arrangement of two types of nearest-neighbor exchange constants J_A and J_B. The absence of any translational invariance together with the new feature of quasiperiodicity offers the possibility of still new nonuniversality appearing in the dynamics.

The current interest in quasiperiodic systems or quasicrystals[14] stems from the 1984 discovery by Shechtman and co-workers,[15] of a quasicrystalline phase of AlMn and the subsequent interpretation by Levine and Steinhardt[16] in terms of the aperiodic but space-filling Penrose lattice (of which the Fibonacci chain is the one-dimensional realization). The transfer matrix approach introduced independently by Kohmoto, Kadanoff, and Tang[17] and Ostlund and Pandit,[18] has been used with considerable success in the study of various one-dimensional dynamical models including harmonic excitations (phonons) and tight-binding electrons, for which it is especially suited. This method involves using the Fibonacci inflation rule $A_n \rightarrow A_{n+1}B_{n+1}$, $B_n \rightarrow A_{n+1}$ to construct a third-order difference equation for the trace of the transfer matrix. The electronic or phonon spectrum is then obtained by iterating this equation and examining the asymptotic behavior. One finds that the spectrum is a Cantor set of zero Lebesgue measure corresponding to critical eigenfunctions (i.e., neither extended nor localized). In addition, the existence of a quantity which was invariant under the action of the trace map was demonstrated and used to determine exponents describing the two- and six-cycle scaling near special points in the spectrum.

In a recent series of papers[19,20] an exact real-space renormalization group approach has been given which provides an *a priori* exact determination of the average density or integrated density of states, as well as the full wave-vector and frequency-dependent response function. The decimation procedure does not rely on any translational invariance and exploits in a very direct way the hierarchical properties of the quasicrystal.

The present paper uses the decimation technique to provide an exact description of the nonuniversal critical Glauber dynamics of the Fibonacci chain quasicrystal. We arrive at the treatment of the quasicrystal via discussions of two simpler systems which help to illustrate the development of the method. The paper is organized as follows: In Sec. II we briefly outline how, using the decimation approach directly at the level of the equations of motion, the dynamic exponent $z=2$ for the uniform one-dimensional Glauber chain, can be obtained with a minimum of effort. In Sec. III we perform a similar analysis for the alternating-bond Ising chain, where it is shown that detailed knowledge of the renormalization-group trajectory is required in order to extract the nonuniversal exponent z. Finally in Sec. IV, we present the real-space renormalization-group treatment of the Fibonacci-Glauber chain.

II. THE UNIFORM ONE-DIMENSIONAL GLAUBER CHAIN

In this section we consider a simple model which was originally proposed by Glauber to describe the dynamics of a system of Ising spins whose Hamiltonian is given by

$$H=-\sum_{\langle ij\rangle} J_{ij}\sigma_i\sigma_j \ .$$

Following Glauber we take our rates to be of the form

$$W_n(\sigma_n)=\tfrac{1}{2}\alpha[1-\tfrac{1}{2}\gamma\sigma_n(\sigma_{n-1}+\sigma_{n+1})] \ , \qquad (2.1)$$

where α is the basic spin-flip rate, taken to be unity for convenience, and $\gamma=\tanh(2K)$ with $K=J/k_BT$. The equation of motion for the magnetization $\bar{q}_n(t)$ of the nth site is obtained by differentiating (1.2) with respect to t and using (1.1) and (2.1). It is straightforward to show that

$$\frac{d}{dt}\bar{q}_n(t)=-\bar{q}_n(t)+\tfrac{1}{2}\gamma[\bar{q}_{n-1}(t)+\bar{q}_{n+1}(t)] \ . \qquad (2.2)$$

Finally, Laplace transforming both sides of (2.2) yields

$$(\Omega+1)q_n=\frac{\gamma}{2}(q_{n-1}+q_{n+1}) \ , \qquad (2.3)$$

where Ω is the variable conjugate to t, and $q_n(\Omega)$ is the Laplace transform of $\bar{q}_n(t)$. As is well known, Eq. (2.3) is a generalized diffusion equation, and has the same form as those describing the linear dynamics of harmonic excitations (phonons), Heisenberg spin waves at zero temperature or tight-binding electrons.

Real-space renormalization-group (RSRG) methods have used in the past[7,21] on equations of type (2.3) with considerable success, yielding for phonon, magnon, and electron systems, information on both critical exponents as well as spectral properties (e.g., density or integrated density of states). One method of implementing the RSRG (Ref. 22) would be to decimate half the sites followed by spatial rescaling by a factor $b=2$, in such a way that (2.3) retains its original form. The advantage of this is that it isolates the static scaling $[\gamma'=(\gamma^2/2-\gamma^2)]$ from the dynamic scaling

$$\left[\Omega'=\frac{2\Omega(2+\Omega)}{2-\gamma^2}\right] .$$

An alternative would be to rewrite (2.3) as

$$q_n=\Gamma(q_{n-1}+q_{n+1}) \ , \qquad (2.4)$$

where $\Gamma=\gamma/2(1+\Omega)$. The decimation by $b=2$ then leads to the single recursion relation

$$\Gamma'=\frac{\Gamma^2}{1-2\Gamma^2} \tag{2.5}$$

which contains within it information on both the static and dynamic scaling. The advantage of using the equation of motion in the form (2.4) is that it reduces the dimension of the renormalization-group parameter space by a factor of 2, and as we shall see later this will lead to enormous simplifications.

The recursion relation (2.5) has three fixed points $\Gamma^*=-1,0,\frac{1}{2}$. On physical grounds we know that the critical point for this model must correspond to $\gamma=1$ and $\Omega=0$, which obviously corresponds to the fixed point $\Gamma^*=\frac{1}{2}$. Working to leading order in $\delta\Gamma=\frac{1}{2}-\Gamma$ leads to the linear relation

$$\delta\Gamma'=4\delta\Gamma \ . \tag{2.6}$$

Now, for $\Omega \ll 1$ and $T \ll 1$, $\delta\Gamma$ has the form

$$\delta\Gamma \approx \Omega+2e^{-4K} \tag{2.7}$$

and so using (2.6), together with the known temperature scaling in the form $\gamma'=\gamma^2/(2-\gamma^2)$, it follows immediately that

$$\Omega' \sim 4\Omega \ , \tag{2.8}$$

implying $z=2$. It is worth mentioning that the particular situation of the fixed point and the critical point coinciding encountered in this example is the exception rather than the rule, and in the next two sections we shall meet situations where this is not the case and a procedure based on projecting back from the fixed point must be employed.

III. THE ALTERNATING-BOND GLAUBER CHAIN

Perhaps the best-known model exhibiting nonuniversal critical dynamics is the alternating-bond Ising chain with Glauber dynamics, a model in which the exchange interaction J_{ij} alternates between two values J_A and J_B. Using a domain-wall argument of Cordery,[23] Droz and co-workers[3] have shown the dynamic exponent to be nonuniversal and given by $z=1+J_A/J_B$ for Glauber dynamics, and $z=3+2J_A/J_B$ $(J_A>J_B>0)$ for Kawazaki dynamics. More recently Luscombe[24] has given a detailed analysis of both relaxational (Glauber) dynamics as well as diffusive (Kawazaki) dynamics for the alternating-bond Ising chain, generalizing the work of Droz *et al.* to couplings of arbitrary sign. It is of interest to note that the value of z $(z=4+|J_A/J_B|)$, obtained by Luscombe for the case of Kawazaki dynamics, differs from that found originally by Droz *et al.*, suggesting that the faster mechanisms for domain diffusion differ in the two treatments.

The additional bond periodicity present in the alternating-bond Ising chain necessitates introducing two transition rates which generalize naturally from (2.1) to

$$W_{2n}(\sigma_{2n})=\tfrac{1}{2}\alpha[1-\tfrac{1}{2}\sigma_{2n}(\gamma^+\sigma_{2n-1}+\gamma^-\sigma_{2n+1})] \tag{3.1a}$$

$$W_{2n+1}(\sigma_{2n+1})=\tfrac{1}{2}\alpha[1-\tfrac{1}{2}\sigma_{2n+1}(\gamma^-\sigma_{2n}+\gamma^+\sigma_{2n+2})] \ , \tag{3.1b}$$

where

$$\gamma^{\pm}=\tanh(K_A+K_B)\pm\tanh(K_A-K_B) \ .$$

Then in complete analogy with the steps that led to (2.4) it is straightforward to show that the equations of motion are given by

$$q_{2n}=\Gamma_1 q_{2n-1}+\Gamma_2 q_{2n+1} \ , \tag{3.2a}$$

$$q_{2n+1}=\Gamma_2 q_{2n}+\Gamma_1 q_{2n+2} \ , \tag{3.2b}$$

where $\Gamma_1=\gamma^+/2(\Omega+1)$ and $\Gamma_2=\gamma^-/2(\Omega+1)$. For the alternating-bond Glauber chain the natural decimation scale factor is $b=3$ since then we can map the chain onto an exact replica of itself with a third as many sites and renormalized parameters Γ_1' and Γ_2' given by

$$\Gamma_1'=\frac{\Gamma_1^2\Gamma_2(1-\Gamma_1^2)}{1-2(\Gamma_1^2+\Gamma_2^2)+\Gamma_1^2\Gamma_2^2+\Gamma_1^4+\Gamma_2^4} \ , \tag{3.3a}$$

$$\Gamma_2'=\frac{\Gamma_1\Gamma_2^2(1-\Gamma_2^2)}{1-2(\Gamma_1^2+\Gamma_2^2)+\Gamma_1^2\Gamma_2^2+\Gamma_1^4+\Gamma_2^4} \ . \tag{3.3b}$$

The recursion relations (3.3) have two fixed points $(\Gamma_1^*,\Gamma_2^*)=(\frac{1}{2},\frac{1}{2})$ and $(-\frac{1}{2},-\frac{1}{2})$. However, for positive couplings $J_A \geq J_B>0$ we expect, on physical grounds, that the critical points should correspond to $(\Gamma_1^c,\Gamma_2^c)=(\frac{1}{2},\frac{1}{2})$ and (1,0). Whereas the isotropic critical point $(\frac{1}{2},\frac{1}{2})$ coincides with the fixed point, the more interesting anisotropic critical point (1,0) clearly does not. Linearizing about the fixed point $(\frac{1}{2},\frac{1}{2})$ leads to a largest eigenvalue $\lambda=9$ and the scaling field formed from the eigenvector associated with this largest eigenvalue only describes the deviation from criticality for the isotropic situation, yielding $\psi\approx\Omega+2r^{-2K_A}$ for $T\ll 1$ and $\Omega \ll 1$ and hence $z=2$. In contrast, the deviation from criticality for the anisotropic case is measured by the quantity $r=(1-\Gamma_1)/\Gamma_2$, criticality corresponding to being on the line $\Gamma_2=1-\Gamma_1$. Indeed, it is easily checked that starting anywhere along this critical line, the system scales, according to (3.3), into the fixed point $(\frac{1}{2},\frac{1}{2})$. Assuming we deviate only slightly from this critical line then after many scalings we arrive at a neighborhood of the fixed point, where the system experiences the repulsive effect of the eigenvalue $\lambda=9$, and so it is the quantity $\psi\equiv r-r_c=r-1$, which must be taken to scale with the eigenvalue $\lambda=9$. It is straightforward to show that this scaling field has the low-temperature form $\psi=\Omega\epsilon_B/\epsilon_A$, where $\epsilon_\alpha=e^{-2K_\alpha}$. Then using the known scaling of ψ $(\psi'\sim 9\psi)$ together with the scaling of r (which can be deduced from the known scaling of the correlation length ξ) it is easy to show that $z=1+J_A/J_B$. The remaining three critical points $(\Gamma_1^c,\Gamma_2^c)=(0,1)$, $(-1,0)$, $(0,-1)$ can be treated in a similar manner and lead in all cases to the results $z=1+|J_A/J_B|$ $(J_A>J_B)$ and $z=1+|J_B/J_A|$ $(J_B>J_A)$. In Fig. 1 we show the parameter space spanned by (Γ_1,Γ_2) together with the fixed points the

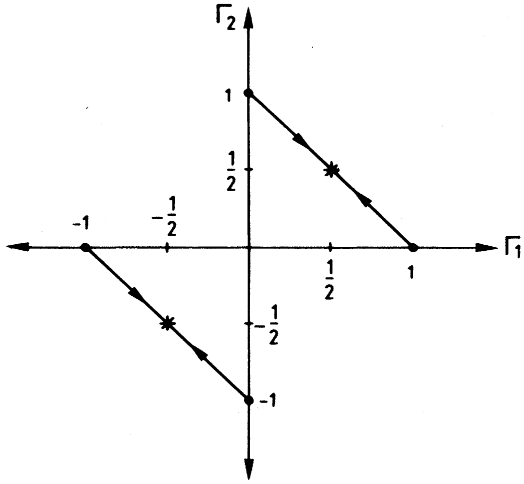

FIG. 1. The renormalization-group parameter space for the alternating-bond Glauber chain.

critical points, the critical trajectories, and the RG flow along these trajectories.

IV. THE FIBONACCI-GLAUBER CHAIN

We now consider the Fibonacci-chain quasicrystal whose dynamics is assumed to be of the Glauber type. The statics of this model have been studied in the past[25,26] and are completely determined by the Ising Hamiltonian

$$H=-\sum_{\langle ij\rangle} J_{ij}\sigma_i\sigma_j ,$$

where it is assumed that the exchange interactions J_{ij} are arranged according to the Fibonacci sequence. In what follows it is convenient to imagine the Fibonacci chain as a lattice $\mathcal{L}$ of sites which is the union of three distinct sublattices $\mathcal{L}_\alpha$, $\mathcal{L}_\beta$, and $\mathcal{L}_\gamma$, such that $\mathcal{L}=\mathcal{L}_\alpha\cup\mathcal{L}_\beta\cup\mathcal{L}_\gamma$, where the distinction between sites in these sublattices is made entirely on the basis of their local environments.[19] The dynamics of the model is introduced through a set of transition rates $W_j(\sigma_j)$, which may assume one of three values $W_\alpha(\sigma_n)$, $W_\beta(\sigma_n)$, $W_\gamma(\sigma_n)$, depending on the nearest neighbors of site n. To be specific we write

$$W_\alpha(\sigma_n)=\tfrac{1}{2}\alpha[1-\tfrac{1}{2}\gamma\sigma_n(\sigma_{n-1}+\sigma_{n+1})] \quad \text{if } n\in\mathcal{L}_\alpha , \tag{4.1a}$$

$$W_\beta(\sigma_n)=\tfrac{1}{2}\alpha[1-\tfrac{1}{2}\sigma_n(\gamma^+\sigma_{n-1}+\gamma^-\sigma_{n+1})] \quad \text{if } n\in\mathcal{L}_\beta , \tag{4.1b}$$

$$W_\gamma(\sigma_n)=\tfrac{1}{2}\alpha[1-\tfrac{1}{2}\sigma_n(\gamma^-\sigma_{n-1}+\gamma^+\sigma_{n+1})] \quad \text{if } n\in\mathcal{L}_\gamma , \tag{4.1c}$$

where $\gamma=\tanh(2K_A)$ and

$$\gamma^\pm=\tanh(K_A+K_B)\pm\tanh(K_A-K_B) .$$

Then using (1.2) it is straightforward to show that the equations of motion are given by

$$q_n=\Gamma_1(q_{n-1}+q_{n+1}), \quad \text{if } n\in\mathcal{L}_\alpha , \tag{4.2a}$$

$$q_n=\Gamma_2 q_{n-1}+\Gamma_3 q_{n+1}, \quad \text{if } n\in\mathcal{L}_\beta , \tag{4.2b}$$

$$q_n=\Gamma_4 q_{n-1}+\Gamma_5 q_{n+1}, \quad \text{if } n\in\mathcal{L}_\gamma , \tag{4.2c}$$

where $\Gamma_1=\gamma/2(1+\Omega)$, $\Gamma_2=\Gamma_5=\gamma^+/2(1+\Omega)$, and $\Gamma_3=\Gamma_4=\gamma^-/2(1+\Omega)$. Note that although $\Gamma_2=\Gamma_5$ and $\Gamma_3=\Gamma_4$ initially, an extension of parameter space occurs, and as a result they differ even after the first stage of decimation. It is thus necessary to start with the most general situation where they are different.

In contrast to the pure and alternating Glauber chains where, because of the periodicity any rescaling factor b can be used, the discrete scale invariance of the Fibonacci-chain quasicrystal under the irrational length scale factor $b=\tau$, dictates that only special sites are ever eliminated.[19] In Fig. 2 we illustrate the decimation for a Fibonacci chain containing nine sites. The recursion relations one obtains for such a decimation are given by

$$\Gamma_1'=\frac{\Gamma_2\Gamma_3\Gamma_4}{\Gamma_3-\Gamma_4(\Gamma_2^2+\Gamma_3^2)} , \tag{4.3a}$$

$$\Gamma_2'=\frac{\Gamma_2\Gamma_4}{1-\Gamma_3\Gamma_4} , \tag{4.3b}$$

$$\Gamma_3'=\frac{\Gamma_2\Gamma_4}{\Gamma_3(1-\Gamma_2\Gamma_4)} , \tag{4.3c}$$

$$\Gamma_4'=\frac{\Gamma_1}{1-\Gamma_1\Gamma_2} , \tag{4.3d}$$

where we have used the fact that $\Gamma_2^{(n)}\Gamma_4^{(n)}=\Gamma_3^{(n)}\Gamma_5^{(n)}$ for all n to reduce the dimensionality of our RG parameter space by one.

For positive-exchange interactions J_A and J_B, the criticality of the Fibonacci-Glauber chain is governed by the fixed point $(\frac{1}{2},\tau^{-2},\tau^{-1},\tau^{-1})$ of the recursion relations (4.3), where $\tau=(1+\sqrt{5})/2$ is the Golden mean. However, on physical grounds we expect the transition to occur at $T=0$ and $\Omega=0$ and so clearly the critical points must be $(\Gamma_1^c,\Gamma_2^c,\Gamma_3^c,\Gamma_4^c)=(\frac{1}{2},1,0,0)$, $(\frac{1}{2},0,1,1)$, and $(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2})$ corresponding to $J_A>J_B>0$, $J_B>J_A>0$, and $J_A=J_B=0$, respectively. Linearizing about the fixed point leads to the largest eigenvalue of τ^2, however, as encountered

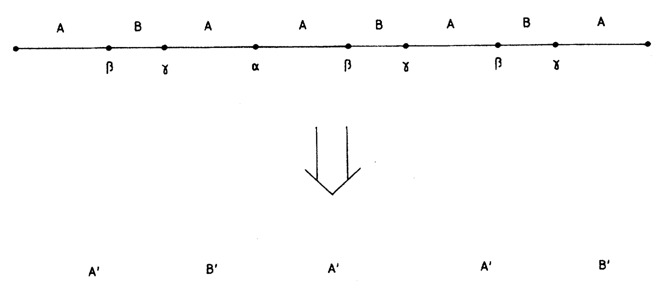

FIG. 2. The decimation procedure for a Fibonacci chain with nine sites.

in the previous section, the fixed point and the critical points do not coincide, and so the scaling field constructed from the eigenvectors corresponding to the eigenvalue $\lambda=\tau^2$ will not describe any deviation from criticality for any physically accessible situation.

We have been able to identify the critical RG trajectory connecting all the critical points. For the case of positive-exchange couplings the critical trajectory is a line in the four-dimensional parameter space, defined by the equations

$$\Gamma_1=\tfrac{1}{2}, \quad \Gamma_2=\Gamma_3=1, \quad \Gamma_3=\Gamma_4 . \tag{4.4}$$

Starting at the critical point $(\frac{1}{2},1,0,1)$ we flow, in one step, to the critical point $(\frac{1}{2},0,1,1)$; a further iteration takes us to the isotropic critical point $(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2})$ from which we flow, after infinitely many iterations, to the fixed point $(\frac{1}{2},\tau^{-2},\tau^{-1},\tau^{-1})$. The identification of the scaling field ψ associated with each of these critical points, namely that combination of the variables (Γ_i, $i=1,\ldots,4$) which will scale according to the eigenvalue τ^2, is made in a similar way as in Sec. III. In the case of the isotropic critical point one finds that $\psi_I=\Gamma_2+\Gamma_4-1$ which, for $T\ll 1$ and $\Omega\ll 1$ has the form given in (2.7) yielding $z=2$ as expected. For the anisotropic critical point $(\frac{1}{2},1,0,0)$ we find that the deviation from criticality is proportional to $r=\Gamma_3/(1-\Gamma_2)$ leading to a scaling field which, for $T\ll 1$ and $\Omega\ll 1$, is given by $\psi_A\approx\Omega\epsilon_B/\epsilon_A$, yielding $z=1+J_A/J_B$ as found previously in Sec. III, for the alternating Glauber chain. Similarly, we find $z=1+J_B/J_A$ for the other anisotropic critical point $(\frac{1}{2},0,1,1)$, again identical to the result obtained for the alternating Glauber chain.

We have also investigated the case where some (or all) of the exchange interactions are negative, and find that the critical dynamics is no longer governed by a simple one-cycle (fixed point) as was the case just described, but instead by the following three-cycle:

$$(\tfrac{1}{2},\tau^{-2},-\tau^{-1},-\tau^{-1})\to(-\tfrac{1}{2},-\tau^{-2},\tau^{-1},\tau^{-1})\to(-\tfrac{1}{2},-\tau^{-2},-\tau^{-1},\tau^{-1}) . \tag{4.5}$$

Linearizing about this three-cycle yields the largest eigenvalue of $\lambda=\tau^6$ which, as expected, is the cube of the one-cycle eigenvalue. For the remaining five critical points we find the dynamic exponent to be given by $z=1+|J_A/J_B|$ $(J_A>J_B)$ and $z=1+|J_B/J_A|$ $(J_B>J_A)$, excluding the remaining isotropic critical point corresponding to $J_A=J_B<0$ for which $z=2$. In Fig. 3 we indicate all eight critical points and their relations to the one- and three-cycles [note that the points $(\frac{1}{2},\frac{1}{2},-\frac{1}{2},-\frac{1}{2})$ and $(-\frac{1}{2},-\frac{1}{2},\frac{1}{2},\frac{1}{2})$ are not critical points].

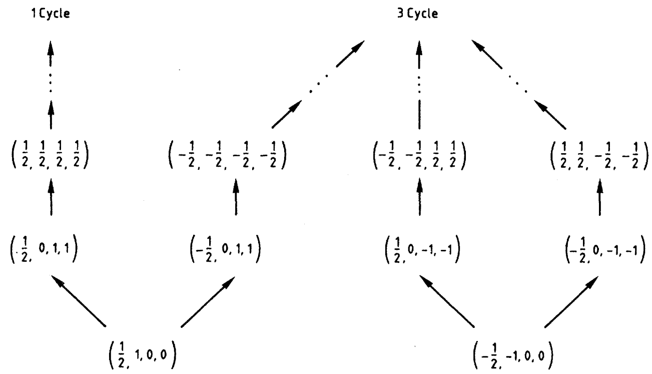

FIG. 3. The critical points for the Fibonacci-Glauber chain and their relation to the one-cycle (fixed point) and three-cycle.

V. CONCLUSIONS

In conclusion, we have presented an exact real-space renormalization group (decimation) calculation of the dynamic exponent z for the Fibonacci-chain quasicrystal with Ising-Glauber dynamics. The method is illustrated using the well-known uniform and alternating-bond Glauber chains, where we reproduce the standard results. For the aperiodic Fibonacci-Glauber chain we found the dynamic exponent to be nonuniversal and identical to that obtained for the alternating-bond Glauber chain. It can be verified that domain-wall arguments, along the lines given by Droz *et al.*,[3] also give the same result. However, in light of the work of Luscombe,[24] methods based on the diffusion of domain walls should be used with caution.

The method presented here is particularly well suited to linear systems with interactions of finite range, where in most cases, it enables one to obtain exact results without the need for *ad hoc* assumptions. Furthermore, a straightforward extension of the decimation method also allows one to extract the correlation functions, and this is currently being investigated.

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