Theory of weak localization in a superlattice

W. Szott, C. Jedrzejek,* and W. P. Kirk
Department of Physics, Texas A&M University, College Station, Texas 77843-4242
(Received 30 November 1988)

A theory of weak localization in the parallel and vertical conductivity of a semiconductor superlattice is presented. Small-collisional-broadening calculations were performed as quantum corrections to Boltzmann transport. The most interesting result is that the weak-localization corrections of the conductivity depend on the width of the superlattice miniband. In addition, the derivation has a form similar to the anisotropic theory of Bhatt, Wölfle, and Ramakrishnan for disordered metallic systems.

I. INTRODUCTION

Semiconductor superlattices are multiwell, low-barrier systems, featuring periodic potentials superimposed on the natural lattice by molecular-beam epitaxy.[1] Dispersion along the growth direction causes superlattices to be very anisotropic, three-dimensional systems. This anisotropy, however, is of a different nature than that found in other systems. For example, by changing various parameters in a superlattice (such as the miniband width, a feature that is inaccessible in other systems) it is possible to achieve new insight into quantum transport mechanisms.

From the perspective of device applications the study of superlattices is quite important, especially for high-speed, high-mobility structures, perpendicular transport devices, sequential resonant tunneling structures, ballistic transistors, and superlattice avalanche photodetectors.[2] A wealth of applied work now exists on superlattice applications. On the other hand, with respect to physics several important new results have been obtained recently. The AT&T group (Störmer *et al.*[3]) observed the quantized Hall effect (QHE) and a concomitantly vanishing magnetoresistance in a $GaAs/Al_xGa_{1-x}As$ superlattice. Less clear evidence of the QHE in $CdTe/Hg_{1-y}Cd_yTe$ has been published by Rafol, Woo, and Faurie.[4] The work of Deveaud *et al.*[5] established experimentally the existence of well-defined Bloch states along the superlattice axis. A positive magnetoresistance due to the suppression of antilocalization in a $CdTe/Hg_{1-y}Cd_yTe$ superlattice has been studied experimentally by Moyle, Cheung, and Ong.[6] Recently, we completed new measurements and made extended studies of negative magnetoresistance effects in a $GaAs/Al_xGa_{1-x}As$ superlattice.[7] These experimental results provide a motive for the theoretical work described below.

In this paper we are concerned principally with the weak-localization aspect of superlattice transport properties in the low-collision limit. If one uses a two-dimensional theory of the weak-localization effects, then one is compelled to introduce an *ad hoc* parameter, namely, the number of active layers in order to explain the experimental results. This approach suggests a simplistic picture of a stack of n effective, independent, two-dimensional layers. We do not subscribe to this picture since n is found to be significantly different from the number N of real layers. For example, $n/N=0.3-0.42$ in measurements by Moyle *et al.*[6] and $n/N=0.46-0.6$ in our measurements.[7]

A desirable starting point for low-magnetic-field quantum corrections of transport in superlattices is three-dimensional weak localization in anisotropic disordered electronic systems. Kawabata[8] developed a theory of negative magnetoresistance employing the restrictive assumption of an anisotropic effective mass. A more general theory using an anisotropic diffusion tensor was constructed by Bhatt, Wölfle, and Ramakrishnan.[9] In addition to an anisotropic effective mass tensor, they also allowed the scattering amplitude to depend on direction. However, they did not consider the nonzero magnetic field case ($B\neq 0$). Yang and Das Sarma[10] introduced theoretically a Bloch-type approach (as opposed to approaches that use transmission coefficients) in the calculation of vertical conductivity in superlattices.

The aim of this paper is to provide a theoretical model to interpret experiments that involve weak-localization effects in superlattices, without the restriction of an effective-mass tensor. One of the main results we find is that the conductivity for superlattices is of the form of the anisotropic theory of Bhatt, Wölfle, and Ramakrishnan[9] for three-dimensional disordered metallic systems. However, the diffusion coefficient D_z we derive depends on the width of the miniband. Another interesting result is that, in contrast to the case of vertical conductivity, the parallel conductivity obeys Drude's law at least to within the quasiparticle approximation.

II. PARALLEL CONDUCTANCE

We assume a structure in which the miniband is described by a tight-binding model. The three-dimensional dispersion curve has the following form:

$$\varepsilon(\mathbf{k})=\varepsilon_{\mathbf{k}_\parallel}+\varepsilon_{\mathbf{k}_z}=\frac{\hbar^2k_\parallel^2}{2m_\parallel}+w[1-\cos(k_za)] , \qquad (1)$$

where $2w$ is the bandwidth of the superlattice miniband and a is the superlattice period. The density of states

$g(\varepsilon)$ for this system is

$$g(\varepsilon)=\frac{m_\parallel}{\pi^2\hbar^2 a}\begin{cases}\cos^{-1}\left[\dfrac{-\varepsilon+w}{w}\right], & \varepsilon<2w\\ \pi, & \varepsilon>2w\ .\end{cases} \tag{2}$$

We start with the Kubo formula[11] for conductivity in the following form:

$$\sigma_{ij}(\omega=0)=\hbar\int_{-\infty}^{+\infty}\frac{d\varepsilon}{2\pi}\left[-\frac{\partial f}{\partial\varepsilon}\right]\{P_{ij}^{-+}(\varepsilon)-\mathrm{Re}[P_{ij}^{++}(\varepsilon)]\}\ . \tag{3}$$

Here

$$P_{ij}^{\pm\pm}(\varepsilon)=\frac{2e^2\hbar^2}{m_\parallel}\int\frac{d^3k}{(2\pi)^3}G^{\pm}(\mathbf{k},\varepsilon)G^{\pm}(\mathbf{k},\varepsilon)k_i\Gamma_j(\mathbf{k},\varepsilon)\ , \tag{4}$$

where i,j are x,y; $m_\parallel$ is the effective electron mass, $G^{\pm}(\mathbf{k},\varepsilon)=G_{\mathrm{ret/adv}}(\mathbf{k},\varepsilon)$, f is the Fermi function, and $\Gamma_j^{\pm\pm}$ is the vertex function as defined through the Bethe-Salpeter equation

$$\Gamma_j^{\pm\pm}(\mathbf{k},\varepsilon)=k_j+\int\frac{d^3k'}{(2\pi)^3}W^{\pm\pm}(\mathbf{k},\mathbf{k}';\varepsilon,\varepsilon)G^{\pm}(\mathbf{k}',\varepsilon)\times G^{\pm}(\mathbf{k},\varepsilon)\Gamma_j^{\pm\pm}(\mathbf{k}',\varepsilon)\ . \tag{5}$$

Here $W^{\pm\pm}(\mathbf{k},\mathbf{k}';\varepsilon,\varepsilon)$ is the amplitude of the elastic scattering from impurities in a particle-hole–particle-hole channel.

Neglecting the vertex correction [equivalent to putting $\Gamma_j(\mathbf{k})=k_j$, the approximation used by Yang and Das Sarma for vertical conductivity] the following expression for parallel conductivity at temperature $T=0$ is obtained:

$$\sigma_\parallel=\frac{1}{2\pi}\frac{e^2\hbar^3}{m_\parallel^2}\int_{-\pi/a}^{\pi/a}\frac{dk_z}{2\pi}\times\int\frac{d^2k_\parallel}{(2\pi)^2}k_\parallel^2\times\frac{2\gamma^2}{[(\varepsilon_F-\varepsilon_{\mathbf{k}_\parallel}-\varepsilon_{\mathbf{k}_z}-\delta)^2+\gamma^2]^2}\ , \tag{6}$$

where ε_F is the Fermi energy, and δ and γ are

$$\delta=\mathrm{Re}\Sigma(\varepsilon_F),\quad \gamma=-\mathrm{Im}\Sigma(\varepsilon_F)\ . \tag{7}$$

Here Σ is the retarded self-energy due to the point impurity scattering in the renormalized Born approximation. In the limit of the quasiparticle approximation ($\gamma\to 0$) $\sigma_\parallel$ becomes

$$\sigma_\parallel=\frac{e^2}{\pi\hbar^2}\frac{\tau}{a}\begin{cases}\dfrac{1}{\pi}\left[(\varepsilon_F'-w)\cos^{-1}\left[\dfrac{w-\varepsilon_F'}{2}\right]+[\varepsilon_F'(2w-\varepsilon_F')]^{1/2}\right], & 0<\varepsilon_F'<2w\\ \varepsilon_F'-w, & \varepsilon_F'>2w\end{cases} \tag{8}$$

where $\varepsilon_F'=\varepsilon_F-\delta$ and the relaxation time $\tau=\hbar/2\gamma$. If we make a further assumption that the density of states is approximated by the free-particle formula so that the density of carriers n becomes

$$n=\int_0^{\varepsilon_F-\delta}g(\varepsilon)d\varepsilon\ , \tag{9}$$

then we recover the parallel conductivity (in the semiclassical approximation) as the simple Drude form, viz.,

$$\sigma_\parallel=\frac{ne^2\tau}{m_\parallel}\ . \tag{10}$$

It should be emphasized that even when the mean free path is smaller than the superlattice period, in which the Bloch picture fails, the proper two-dimensional limit is obtained from Eq. (10) when $w\to 0$.

The result as expressed by Eq. (10) should be contrasted with the vertical conductivity expression of Yang and Das Sarma[10] in which they used analogous approximations plus the relatively naive assumption that the effective mass m_z in the z direction is taken at the bottom of the miniband. They found

$$\sigma_z=\frac{n^*e^2\tau}{m_z}\ . \tag{11}$$

However, the effective carrier density n^* is not explicitly connected with the real carrier density. The superlattice structure is absolutely essential for vertical conductivity as shown by Yang and Das Sarma[10] and for a weak-localization correction of the conductivity as we demonstrate in Sec. III.

III. WEAK-LOCALIZATION CORRECTION TO CONDUCTIVITY

We use the standard Kubo formalism of conductivity in a spatially uniform field to calculate a quantum correction of the dynamic conductivity,

$$\sigma_{ij}(\omega)=2e^2\int\frac{d\varepsilon}{2\pi}\frac{f(\varepsilon-\hbar\omega)-f(\varepsilon)}{\omega}\times\int\frac{d^3k}{(2\pi)^3}\int\frac{d^3k'}{(2\pi)^3}\Pi_{ij}^{+-}(\mathbf{k},\mathbf{k}';\varepsilon,\varepsilon-\hbar\omega)\ . \tag{12}$$

At the outset we consider the case of small γ; hence mutually canceling divergent parts are omitted in Eq. (12), in contrast to the previously used expression (3) for the static conductivity. The current-current correlation function Π is defined as follows (also see Fig. 1):

$$\Pi_{ij}^{+-}(\mathbf{k},\mathbf{k}';\varepsilon,\varepsilon-\hbar\omega) = v_i(\mathbf{k})v_j(\mathbf{k}')G^+(\mathbf{k},\varepsilon)G^-(\mathbf{k},\varepsilon-\hbar\omega)G^+(\mathbf{k}';\varepsilon) \times G^-(\mathbf{k}',\varepsilon-\hbar\omega)W^{+-}(\mathbf{k},\mathbf{k}';\varepsilon,\varepsilon-\hbar\omega)\ , \quad (13)$$

where $v_i(\mathbf{k})$ is the ith component of carrier velocity $\mathbf{v}(\mathbf{k})$. We now specialize to simple cases and use the following standard approximations: (i) $T=0$, (ii) $\omega\rightarrow 0$, (iii) isotropy in x-y plane, i.e., $\sigma_{xx}=\sigma_{yy}$, (iv) only one miniband is important for transport, in which $\varepsilon_F>2w$, (v) isotropic δ-function scattering from impurities in r space, and (vi) small γ. From (iii) it follows that $\sigma_\parallel$ is proportional to $\mathbf{v}_\parallel(\mathbf{k})\cdot\mathbf{v}_\parallel(\mathbf{k}')/2$. Also, since the product G^+G^- has a strong maximum at the Fermi energy and backscattering is responsible for weak localization, then

$$\mathbf{v}_\parallel(\mathbf{k})\cdot\mathbf{v}_\parallel(\mathbf{k}')\rightarrow -\mathbf{v}_\parallel^2(\varepsilon_F,\mathbf{k}_z)\ . \quad (14a)$$

Similarly,

$$\mathbf{v}_z(\mathbf{k})\cdot\mathbf{v}_z(\mathbf{k}')\rightarrow -\mathbf{v}_z^2(\varepsilon_F,\mathbf{k}_z)\ . \quad (14b)$$

Furthermore from (iv)

$$|\mathbf{v}_\parallel(\varepsilon_F,\mathbf{k}_z)| = \frac{1}{\hbar}\left|\frac{\partial\varepsilon}{\partial\mathbf{k}_\parallel}\right|_{(\varepsilon=\varepsilon_F)} = \left[\frac{2\{\varepsilon_F-w[1-\cos(k_z a)]\}}{m_\parallel}\right]^{1/2} \approx \left[\frac{2\varepsilon_F}{m_\parallel}\right]^{1/2}, \quad (15a)$$

and

$$|\mathbf{v}_z(\varepsilon_F,\mathbf{k}_z)| = \frac{wa}{\hbar}|\sin(k_z a)|\ . \quad (15b)$$

From the maximally crossed diagrams that are responsible for weak localization, we note that W^{+-} is a function of $\mathbf{k}+\mathbf{k}'=\mathbf{q}$. This allows us to separate Π^{+-} into two parts, in which one part does not involve W^{+-}. We evaluate Eq. (13) with the help of (ii)

$$\int\frac{d^2k_\parallel}{(2\pi)^2}\int\frac{dk_z}{2\pi}\frac{\mathbf{v}_\parallel^2(\varepsilon_F)}{2}G^+G^-G^+G^- = \frac{4\pi g_\parallel(\varepsilon_F)}{a}\left[\frac{\tau}{\hbar}\right]^2\frac{D_\parallel}{\hbar}\ . \quad (16a)$$

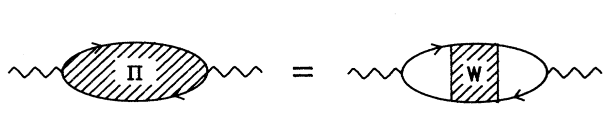

FIG. 1. The diagrammatic expression for correlation function Π vs scattering amplitude W.

In an analogous way

$$\int\frac{d^2k_\parallel}{(2\pi)^2}\int\frac{dk_z}{2\pi}\mathbf{v}_z^2(\varepsilon_F,k_z)G^+G^-G^+G^- = \frac{4\pi g_\parallel(\varepsilon_F)}{a}\left[\frac{\tau}{\hbar}\right]^2\frac{D_z}{\hbar}\ . \quad (16b)$$

Here $g_\parallel(\varepsilon_F)$ is the projected density of states, which in general reads

$$g_\parallel(\varepsilon,k_z)=2\int\frac{d^2k_\parallel}{(2\pi)^2}\delta[\varepsilon-\varepsilon(\mathbf{k}_z)-\varepsilon(\mathbf{k}_\parallel)] = \frac{m_\parallel}{\pi\hbar^2}\theta(\varepsilon-\varepsilon(\mathbf{k}_z))\ . \quad (17)$$

For $\varepsilon>2w$, $g_\parallel(\varepsilon,k_z)=g_\parallel(\varepsilon)$. While the effective diffusion constant $D_\parallel$ has the standard form

$$D_\parallel=\frac{\mathbf{v}_\parallel^2(\varepsilon_F)\tau}{2}\ , \quad (18a)$$

we obtain the important new result that the effective diffusion constant in the z direction is given by

$$D_z=\left[\frac{wa}{\hbar}\right]^2\frac{\tau}{2}\ . \quad (18b)$$

Next we find W^{+-} by solving the Bethe-Salpeter equation as diagrammed in Fig. 2 using the assumption of isotropic δ-function scattering at temperature $T=0$. The result is

$$W^{+-}(\mathbf{q};\varepsilon_F,\varepsilon_F-\hbar\omega) = \frac{N_i|V|^2}{1-N_i|V|^2\int\frac{d^3p}{(2\pi)^3}G^+(\mathbf{p},\varepsilon_F)G^-(\mathbf{q}-\mathbf{p},\varepsilon_F-\hbar\omega)}\ , \quad (19)$$

where N_i is the density of scatterers and V is the constant scattering potential in k space. Use of the Born approximation yields

$$N_i|V|^2=\frac{\hbar a}{2\pi g_\parallel(\varepsilon_F)\tau}\ . \quad (20)$$

Consequently, after some manipulations, for small $\mathbf{q}$ (backscattering)

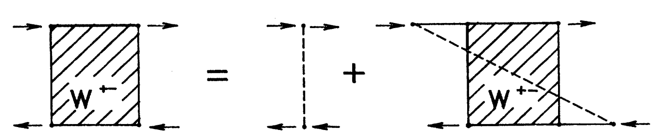

FIG. 2. The Bethe-Salpeter equation for particle-hole scattering amplitude W^{+-} in the maximally crossed diagram approximation.

$$W^{+-}(\mathbf{q};\varepsilon_F,\varepsilon_F-\hbar\omega)=\frac{\hbar a}{2\pi g_{\parallel}(\varepsilon_F)\tau}\frac{1}{D_{\parallel}\tau\mathbf{q}_{\parallel}^2+D_z\tau\mathbf{q}_z^2-i\omega\tau} . \tag{21}$$

Combining Eqs. (3), (13), (14), and (21) we obtain the weak-localization (WL) conductivity

$$\sigma_{\alpha,\mathrm{WL}}=-\frac{2e^2}{\pi\hbar}D_\alpha\tau \times\int\frac{d^2q_{\parallel}}{(2\pi)^2}\int\frac{dq_z}{2\pi}\frac{1}{D_{\parallel}\tau\mathbf{q}_{\parallel}^2+D_z\tau\mathbf{q}_z^2-i\omega\tau} , \tag{22}$$

where $\alpha=\parallel$ or z. Thus, the correction to the conductivity of the superlattice has a form analogous to the correction for an anisotropic three-dimensional disordered metallic system first derived by Bhatt, Wölfle, and Ramakrishran.[9] However, the effective diffusion coefficient along the z direction D_z has a character that is different from the diffusion coefficients in the other directions. That is

$$D_z=\bar{v}_z^2\tau , \tag{23}$$

where

$$\bar{v}_z=\frac{v_{z,\max}}{\sqrt{2}}=\frac{1}{\sqrt{2}}\frac{wa}{\hbar} \tag{24}$$

is the average Fermi velocity in the z direction, independent of the carrier density, and provided that $\varepsilon_F>2w$.

D_z may be easily controlled over a large range of values by changing various superlattice parameters such as barrier width and height and well width. D_z can also be measured directly since the scale-dependent part of the conductivity takes the form

$$\sigma_{\alpha,\mathrm{WL}}=\frac{e^2}{\pi^3\hbar}\frac{D_\alpha}{(D_{\parallel}^2D_z\tau_{ph})^{1/2}} , \tag{25}$$

where τ_{ph} is a dephasing scattering time.

For the case we have considered (i.e., $\varepsilon_F>2w$), it is worth mentioning that our results for $\sigma_{\alpha,\mathrm{WL}}$ (Eq. 22) and $\sigma_{\parallel}$ (Eq. 8) together with σ_z obtained by Yang and Das Sarma[10] fulfill a useful scaling relation

$$\frac{\sigma_{\parallel,\mathrm{WL}}}{\sigma_{\parallel}}=\frac{\sigma_{z,\mathrm{WL}}}{\sigma_z} . \tag{26}$$

It should be noted that Eq. (26) is the same as the $B=0$ case of the general relation derived by Bhatt, Wölfle, and Ramakrishnan[9] although this theory is not concerned with superlattices.

Our model gives the proper two-dimensional limit for $\sigma_{\parallel,\mathrm{WL}}$ when the width of the miniband tends to zero (or $D_z\to0$). Denoting c_x, c_y, and c_z as the dimensions of the sample and N as the number of periods of the superlattice, then the three-dimensional resistivity is given by

$$\rho_{xx}^{(3)}=R_{xx}\frac{c_yc_z}{c_x} . \tag{27}$$

The two-dimensional resistivity per period of the superlattice is then

$$\rho_{xx}^{(2)}=R_{xx}\frac{c_yN}{c_x} , \tag{28}$$

where R_{xx} is the measured resistance. Consequently,

$$\sigma_{xx}^{(2)}=\sigma_{xx}^{(3)}a , \tag{29}$$

where $a=c_z/N$ is the spatial separation of a superlattice period. Substituting into Eq. (22) $D_z=0$ and using $(-\pi/a,\pi/a)$ as the range of the q_z integration, then Eq. (29) is recovered. In this limit there is no need to keep the cutoff of q_z equal $\sim\pi/(D_z\tau)^{1/2}$ because there is no diffusion in the z direction.

Our main result can be generalized to provide a quantum correction of the magnetoresistance when the magnetic field is parallel to the superlattice growth direction. Taking into account quantization of the electron orbits in the plane perpendicular to the magnetic field $\mathbf{B}$, one gets

$$\sigma_{\alpha,\mathrm{WL}}(B)=-\frac{2e^2D_\alpha}{\pi\hbar}\frac{eB}{\pi\hbar}\int\frac{dq_z}{2\pi}\sum_{n=0}^{n_{\max}}\frac{1}{D_zq_z^2+D_{\parallel}\frac{4eB}{\hbar}(n+\frac{1}{2})+\tau_{ph}^{-1}} , \tag{30}$$

where $n_{\max}$ is the cutoff. After integrating over q_z

$$\sigma_{\alpha,\mathrm{WL}}(B)=\frac{e^2}{\pi^3\hbar}\frac{1}{l}C_\alpha\sum_{n=0}^{n_{\max}}\frac{1}{\left[n+\frac{1}{2}+\frac{B_0}{B}\right]^{1/2}}\tan^{-1}\left[\frac{q_{z,\max}\left[\frac{D_z}{D_{\parallel}}\right]^{1/2}\frac{l}{2}}{\left[n+\frac{1}{2}+\frac{B_0}{B}\right]^{1/2}}\right] , \tag{31}$$

where

$$C_{\parallel}=\left[\frac{D_{\parallel}}{D_z}\right]^{1/2},\quad C_z=\left[\frac{D_z}{D_{\parallel}}\right]^{1/2} , \tag{32}$$

$l=(\hbar/eB)^{1/2}$ is the magnetic length, and the characteristic dephasing field B_0 due to scattering is given by

$$B_0=\frac{\hbar}{4eD_{\parallel}\tau_{ph}} . \tag{33}$$

Finally $q_{z,\max}$ is the cutoff for diffusive motion in the z direction; $q_{z,\max} \sim \pi/(D_z\tau)^{1/2}$.

If we now apply the assumptions that were used by Kawabata,[8] ($\omega_c\tau, \hbar/\tau\varepsilon_F \ll 1$, $\omega_c \equiv eB/m_\parallel$) we obtain the following expression for the weak-localization contribution to the magnetoconductivity:

$$\Delta\sigma_\alpha(B) = \sigma_{\alpha,\mathrm{WL}}(B) - \sigma_{\alpha,\mathrm{WL}}(0) = \frac{e^2}{2\pi^2\hbar l} C_\alpha F(\delta) , \quad (34)$$

where

$$F(\delta) = \sum_{n=0}^{\infty} \left[2[(n+1+\delta)^{1/2} - (n+\delta)^{1/2}] - \frac{1}{(n+\frac{1}{2}+\delta)^{1/2}} \right] , \quad (35)$$

and

$$\delta = \frac{l^2}{4D_\parallel \tau_{ph}} . \quad (36)$$

Application of magnetic field, in general, also affects impurity scattering via the relaxation time [see Eq. (20)], which is inversely proportional to the density of states $g(\varepsilon_F)$ evaluated at the Fermi surface. This density of states oscillates as a function of magnetic field. The effect could become important at higher fields,[12] causing (in the case of small level broadening) "jumps" of the Fermi energy from one Landau level to the next as the magnetic field increases. However, when the magnetic field is weak ($\omega_c\tau \le 1$) the oscillations of the density of states are negligible due to the large broadening of Landau levels compared to the level spacing. As was shown by Ando,[13] the scattering rate in the limit of small magnetic fields is given by

$$\frac{1}{\tau} = \frac{1}{\tau_0} \left[1 + 2\cos\left[\frac{\varepsilon_F}{\hbar\omega_c}\right] \exp\left[-\frac{\pi}{\omega_c\tau_0}\right] + \cdots \right] , \quad (37)$$

where τ_0 is the relaxation time when $B=0$. The exponential factor ensures a weak dependence of the relaxation time on magnetic field. For example, if we consider fields $B \le 0.1$ T and the relaxation time $\tau = 0.1$ ps as estimated from our recent measurements,[7] then $\omega_c\tau < 0.005$; consequently, $\tau = \tau_0$ to extremely high accuracy.

In summary, we have derived formulas for parallel conductivity in the semiclassical approximation and for weak-localization corrections of the parallel and vertical conductivity in superlattices. We find that, subject to all approximations mentioned in Sec. II, (semiclassical) parallel conductivity is not sensitive to the superlattice structure. Instead, only the density of states is modified.

On the other hand the weak-localization correction is affected much more profoundly by the superlattice structure. We learn that any theory assuming a stack of independent, two-dimensional heterostructures is clearly inadequate because our final result cannot be reduced to a two-dimensional formula for a finite miniband width. Also, the weak-localization theory for anisotropic three-dimensional systems, as presented by Kawabata, is not applicable to superlattices because the assumption of a simple anisotropic mass tensor concept is too restrictive to properly account for anisotropy in superlattices. Instead, a theory using an effective diffusion tensor has to be used.[9] A magnetoconductivity tensor relationship for superlattices is derived in this work.

ACKNOWLEDGMENTS

This material is based in part on work supported by the Texas Advanced Technology Program (Grant No. 3611), the Texas A&M University Board of Regents Available University Fund—sponsored Materials Science and Engineering Program (AUF-24424-IV), and by the Department of Commerce—National Bureau of Standards (60NANB7D0740). One of the authors (C.J.) acknowledges partial travel support from the Polish Ministry of Science, Higher Education, and Technology Project No. CPBP-01-03.

*Permanent address: Department of Physics, Jagellonian University, Cracow, Poland.

[1]L. Esaki and R. Tsu, IBM Research Note RC-2418 (1969); IBM J. Res. Dev. **14**, 61 (1970).

[2]V. Narayanamurti, in *Proceedings of the 18th International Conference on the Physics of Semiconductors, Stockholm, 1986,* edited by O. Engström (World Scientific, Singapore, 1987), p. 3.

[3]H. L. Störmer, J. P. Eisenstein, A. C. Gossard, W. Wiegmann, and K. Baldwin, Phys. Rev. Lett. **56**, 85 (1986).

[4]S. Rafol, K. C. Woo, and J. P. Faurie, in Ref. 2, p. 773.

[5]B. Deveaud, J. Shah, T. C. Damen, B. Lambert, and A. Regreny, Phys. Rev. Lett. **58**, 2582 (1987).

[6]J. K. Moyle, J. T. Chueng, and N. P. Ong, Phys. Rev. B **35**, 5639 (1987).

[7]W. Szott, C. Jedrzejek, and W. P. Kirk (unpublished).

[8]A. Kawabata, J. Phys. Soc. Jpn. **49**, 628 (1981).

[9]R. N. Bhatt, P. Wölfle, and T. V. Ramakrishnan, Phys. Rev. B **32**, 569 (1985).

[10]S.-R. Eric Yang and S. Das Sarma, Phys. Rev. B **37**, 10090 (1988).

[11]G. D. Mahan, *Many-Particle Physics* (Plenum, New York, 1981).

[12]J. P. Eisenstein, H. L. Störmer, V. Narayanamurti, A. Y. Cho, A. C. Gossard, and C. T. Wu, Phys. Rev. Lett. **55**, 875 (1985).

[13]T. Ando, J. Phys. Soc. Jpn. **37**, 1233 (1974).